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An Adapted Approach for Self-Exciting Threshold Autoregressive Disturbances in Multiple Linear Regression

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Abstract

Ordinary least squares method is usually used for parameter estimation in multiple linear regression models when all regression assumptions are satisfied. One of the problems in multiple linear regression analysis is the presence of serially correlated disturbances. Serial correlation can be formed by autoregressive or moving average models. There are many studies in the literature including parameter estimation in regression models especially with autoregressive disturbances. The motivation of this study is that whether serially correlated disturbances are defined by a different type of nonlinear process and how this process is analyzed in multiple linear regression. For this purpose, a nonlinear time series process known as self-exciting threshold autoregressive model is used to generate disturbances in multiple linear regression models. Two-stage least squares method used in the presence of autoregressive disturbances is adapted for dealing with this new situation and comprehensive experiments are performed in order to compare efficiencies of the proposed method with the others. According to numerical results, the proposed method can outperform under the type of self-exciting threshold autoregressive autocorrelation problem when compared to ordinary least squares and two-stage least squares.

1. INTRODUCTION

A multiple linear regression model including two or more independent variables can be defined in matrix notation as

$$\boldsymbol{y} = \boldsymbol{X\beta} + \boldsymbol{\varepsilon}, \tag{1}$$

where $\boldsymbol{y}$ is an $n \times 1$ vector of responses, $\boldsymbol{X}$ is an $n \times k$ matrix of observations on $k - 1$ independent variables, $\boldsymbol{\beta}$ is a $k \times 1$ vector of unknown parameters, $\boldsymbol{\varepsilon}$ is an $n \times 1$ vector of random disturbances. The random disturbances should have zero mean and constant variance. Also, there should be no near linear relationships among independent variables for obtaining efficient parameter estimates by using ordinary least squares (OLS) method

$$\hat{\boldsymbol{\beta}} = \left(\boldsymbol{X}'\boldsymbol{X}\right)^{-1}\boldsymbol{X}'\boldsymbol{y}. \tag{2}$$

There is a problem in multiple linear regression analysis when serially correlated disturbances exist. Generally, the autocorrelation problem among disturbances is defined by using an autoregressive (AR) model. There are many studies relating to overcoming the problem in different types of models. Cochrane and Orcutt [1] introduced a method by evaluating the autocorrelation structure in linear regression. Also, the full maximum likelihood approach was given by Beach and MacKinnon [2] as an alternative to

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Cochrane-Orcutt method. Gallant and Goebel [3] defined two-stage least squares (TSLS) method for obtaining efficient parameter estimates in the presence of AR disturbances in nonlinear regression. Glasbey [4-6] studied on real different data sets having autocorrelated disturbances and considered asymptotically efficient estimators. Huang and Huang [7] proposed a parameter elimination method in the presence of many parameters and autocorrelated disturbances. In recent years, Asikgil and Erar [8] considered a modification for TSLS in nonlinear regression. Moreover, Asikgil [9] examined some alternative methods on seemingly unrelated regressions with high-order AR disturbances. All the methods given in the literature were usually considered for AR disturbances. In this paper, TSLS is adapted for self-exciting threshold autoregressive (SETAR) disturbances in multiple linear regression models. The rest of the paper is organized as follows. Sec. 2 presents the theoretical overview of SETAR models and TSLS is extended to the new situation given as the presence of SETAR disturbances. In Sec. 3, a comprehensive Monte Carlo simulation study is carried out under different conditions in order to examine the relative efficiencies. Conclusions are given in the final section.

2. MATERIALS AND METHODS

2.1. Self-Exciting Threshold Autoregressive Model

Nonlinear time series modeling and forecasting have become popular in recent years especially for financial data. Many nonlinear time series models have been considered in the literature. Some of them are the bilinear model, the threshold autoregressive model and the Markov switching model. One of the useful classes of nonlinear models is threshold autoregressive (TAR) given by Tong [10, 11]. This model deals with several nonlinear characteristics such as asymmetry in declining and rising patterns of a series. It is based on piecewise linear models which means that the threshold process divides the space into *r* regimes (*r* > 1) with an AR model in each regime [12, 13].

A time series z_t is an *r*-regime self-exciting TAR (SETAR) if it takes the form

$$z_t = \phi_0^{(j)} + \sum_{\ell=1}^{q_j} \phi_\ell^{(j)} z_{t-\ell} + \upsilon_t^{(j)}, \qquad \tau_{j-1} \le z_{t-d} < \tau_j, \tag{3}$$

where *r*, *d* and $(q_1, q_2, \ldots, q_r)$ are positive integers and $j = 1, 2, \ldots, r$. The thresholds are $-\infty = \tau_0 < \tau_1 < \cdots < \tau_r = \infty$ and (*j*) is used to denote the regime. $\upsilon_t^{(j)}$ denotes identical independently distributed sequences with mean zero and variance σ_j^2 and they are mutually independent for different *j*. *d* is called the delay parameter. Tong [11] suggested denoting (3) as $\text{SETAR}(d; q_1, q_2, \ldots, q_r)$. The class of SETAR models has been widely used in different areas in the literature such as financial studies [14], medical studies [15], actuarial studies [16].

In SETAR models, the threshold values are usually unknown and need to be estimated with the other parameters. Chan and Tong [17] studied on estimating thresholds in SETAR models and Chan [18] showed that the parameter estimates obtained by using OLS is strongly consistent for a stationary ergodic SETAR model. Moreover, Chan and Cheung [19] examined the performance of robust generalized-M (GM) estimates in SETAR models in the presence of additive outliers. On the other hand, Baragona, Battaglia and Cucina [20] used genetic algorithms for parameter estimation in different SETAR models. In this paper, grid search approach given in the literature is used to estimate the threshold parameter for two-regime models. This approach points out that the threshold value is an element of the series. Therefore, the model given in (3) is estimated to each value of z_t by using OLS. Only 70-80% middle of the series is examined in order to provide the necessary amount of observations in each regime. In each fitted model sum of squared residuals is obtained for each of the potential threshold. The threshold value corresponding to the model with the least sum of squared residuals can be preferred [21].

2.2. Adapted Two-Stage Least Squares Method

TSLS method considered by Gallant and Goebel [3] can give efficient parameter estimates by using

$$\tilde{\boldsymbol{\beta}} = \left(\boldsymbol{X}'\hat{\boldsymbol{P}}'\hat{\boldsymbol{P}}\boldsymbol{X}\right)^{-1}\left(\boldsymbol{X}'\hat{\boldsymbol{P}}'\hat{\boldsymbol{P}}\boldsymbol{y}\right), \tag{4}$$

when the model in (1) includes AR disturbances. In (4), $\hat{\boldsymbol{P}}$ denotes an $n \times n$ transformation matrix defined by

$$\hat{\boldsymbol{P}} = \begin{bmatrix} & \sqrt{\hat{\sigma}^2}\,\hat{\boldsymbol{P}}_q & & & \mathbf{0} & & \\ \hat{a}_q & \hat{a}_{q\text{-}1} & \cdots & \hat{a}_1 & 1 & & \\ & \hat{a}_q & \hat{a}_{q\text{-}1} & \cdots & \hat{a}_1 & 1 & \\ & \ddots & \ddots & & \ddots & \ddots & \ddots \\ & & \hat{a}_q & \hat{a}_{q\text{-}1} & \cdots & \hat{a}_1 & 1 \end{bmatrix}, \tag{5}$$

where $\hat{\boldsymbol{P}}_q$ is a $q \times q$ matrix calculated by using the Cholesky decomposition $\hat{\boldsymbol{\Gamma}}_q^{-1} = \hat{\boldsymbol{P}}_q'\hat{\boldsymbol{P}}_q$, $\hat{\sigma}^2 = \hat{\gamma}(0) + \hat{\boldsymbol{a}}'\hat{\boldsymbol{\gamma}}_q$ and $\left(\hat{a}_1, \ldots, \hat{a}_{q\text{-}1}, \hat{a}_q\right)$ are parameter estimates of the AR(q) model formed by disturbances. $\hat{\boldsymbol{\Gamma}}_q$ is a matrix presented as

$$\hat{\boldsymbol{\Gamma}}_q = \begin{bmatrix} \hat{\gamma}(0) & \hat{\gamma}(1) & \cdots & \hat{\gamma}(q-1) \\ \hat{\gamma}(1) & \hat{\gamma}(0) & \cdots & \hat{\gamma}(q-2) \\ \vdots & \vdots & \cdots & \vdots \\ \hat{\gamma}(q-1) & \hat{\gamma}(q-2) & \cdots & \hat{\gamma}(0) \end{bmatrix}, \tag{6}$$

where its elements are the estimated variance and covariances obtained by

$$\hat{\gamma}(h) = \frac{1}{n}\sum_{i=1}^{n\text{-}h}\hat{\varepsilon}_i\hat{\varepsilon}_{i+h}, \quad h = 0,\ 1,\ \ldots,\ q. \tag{7}$$

In this paper, an adapted TSLS (ATSLS) method is proposed for parameter estimation since SETAR disturbances are considered in (1). Firstly, the residuals are obtained from (1) by using OLS. Then, the threshold parameters are estimated by using grid search and the regimes are determined. The observations are classified for each regime in view of residuals. The model can be designed by using partitions

$$\begin{bmatrix} \boldsymbol{y}_1 \\ \cdots\cdots \\ \boldsymbol{y}_2 \\ \cdots\cdots \\ \vdots \\ \cdots\cdots \\ \boldsymbol{y}_r \end{bmatrix} = \begin{bmatrix} \boldsymbol{X}_1 \\ \cdots\cdots \\ \boldsymbol{X}_2 \\ \cdots\cdots \\ \vdots \\ \cdots\cdots \\ \boldsymbol{X}_r \end{bmatrix}\boldsymbol{\beta} + \begin{bmatrix} \boldsymbol{\varepsilon}_1 \\ \cdots\cdots \\ \boldsymbol{\varepsilon}_2 \\ \cdots\cdots \\ \vdots \\ \cdots\cdots \\ \boldsymbol{\varepsilon}_r \end{bmatrix}, \tag{8}$$

where $\boldsymbol{y}_j$ and $\boldsymbol{X}_j$ denote an $n_j \times 1$ vector of responses and an $n_j \times k$ matrix of observations for jth regime, respectively. The sample sizes can be different for each regime and

$$\sum_{j=1}^{r} n_j = n. \tag{9}$$

ATSLS estimators can be obtained by using transformed system

$$\tilde{\tilde{\boldsymbol{\beta}}} = \left(\boldsymbol{W}'\boldsymbol{W}\right)^{-1}\left(\boldsymbol{W}'\boldsymbol{U}\right), \tag{10}$$

where

$$\boldsymbol{W} = \begin{bmatrix} \hat{\boldsymbol{P}}_1\boldsymbol{X}_1 \\ \cdots\cdots \\ \hat{\boldsymbol{P}}_2\boldsymbol{X}_2 \\ \cdots\cdots \\ \vdots \\ \cdots\cdots \\ \hat{\boldsymbol{P}}_r\boldsymbol{X}_r \end{bmatrix}, \quad \boldsymbol{U} = \begin{bmatrix} \hat{\boldsymbol{P}}_1\boldsymbol{y}_1 \\ \cdots\cdots \\ \hat{\boldsymbol{P}}_2\boldsymbol{y}_2 \\ \cdots\cdots \\ \vdots \\ \cdots\cdots \\ \hat{\boldsymbol{P}}_r\boldsymbol{y}_r \end{bmatrix}. \tag{11}$$

The point to be noted in (11) is that each regime can consist of different types of models. Therefore, it is clear that the transformation matrix can be separated for each regime.

3. NUMERICAL RESULTS

Some comprehensive Monte Carlo experiments have been conducted under special conditions to evaluate the performances of OLS, TSLS and ATSLS. The vector of parameters given in (1) is considered as $\boldsymbol{\beta} = \begin{bmatrix} 10 & 2 & -5 \end{bmatrix}'$, the independent variables $\left(\boldsymbol{X}_1, \boldsymbol{X}_2\right)$ are distributed from independently identically N(15, 25) and SETAR disturbances used in these experiments are derived in view of the studies of Tong and Lim [22], Chan and Cheung [19], Gibson and Nur [23]. The results of the Monte Carlo experiments are implemented by MATLAB R2010a.

In these experiments different scenarios are considered. Five different stationary two-regime SETAR models are used to generate disturbances. Because of the validity in real life problems only two-regime models are assessed in this study. Also, there is computational burden when dealing with higher regimes models. In every model, 500 samples are generated for each different sample size (n = 50, 100, 250, 500) and each different standard deviation (σ = 0.5, 1, 3) of υ_t. Therefore, there are twelve different situations for each parameter examined by using five different SETAR models. For these SETAR models, the plots of each series of ε_t with n = 500 and σ = 1 are illustrated in Figures 1-5. One important characteristic of these models used for the generation of disturbances in this study is expressed that $E\left(\varepsilon_t\right)$ may not be zero although they have no constant terms. Therefore, it is strictly recommended to use a constant term $\left(\beta_0\right)$ in multiple linear regression models.

The first SETAR model for disturbances is defined by

$$\varepsilon_t = \begin{cases} 0.5\varepsilon_{t\text{-}1} + \upsilon_t & \text{if} \quad \varepsilon_{t\text{-}1} \le 0 \\ 0.8\varepsilon_{t\text{-}1} + \upsilon_t & \text{if} \quad \varepsilon_{t\text{-}1} > 0 \end{cases}, \tag{12}$$

where $\upsilon_t \sim \text{NID}\left(0, \sigma^2\right)$, the delay is 1 and the threshold is 0. This model is denoted as SETAR(1; 1, 1). Figure 1 shows a series from (12) and the threshold is determined by a horizontal line. The series ε_t is geometrically ergodic $\left(\text{i.e. } \phi_1^{(1)} < 1,\ \phi_1^{(2)} < 1 \text{ and } \phi_1^{(1)}\phi_1^{(2)} < 1\right)$ [24] and stationary.

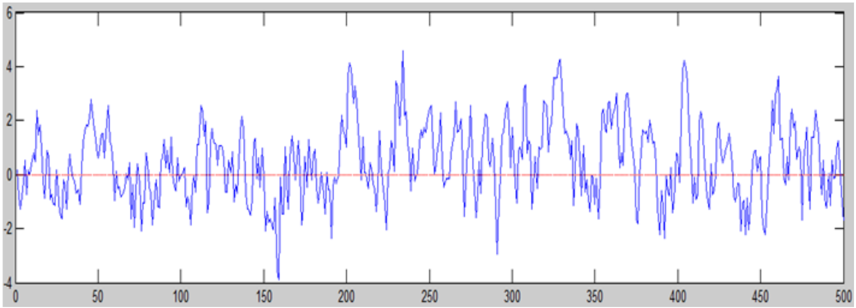

Figure 1. *Plot of a simulated SETAR(1; 1, 1) series*

The results in the presence of disturbances from (12) are given in Table 1. The means, standard deviations, mean squared errors (MSE) and efficiencies (Eff = MSE_{OLS} / $\text{MSE}_{\text{Method}}$) of β_1 and β_2 are obtained by OLS, TSLS for AR(1) and ATSLS in different conditions. It can be said that all the methods give unbiased estimations. The standard deviations and MSE for TSLS and ATSLS methods approach when the sample size increases. However, ATSLS is more efficient than the other methods in view of Eff.

The second SETAR disturbances are given by

$$\varepsilon_t = \begin{cases} 0.34\varepsilon_{t\text{-}1} + 0.13\varepsilon_{t\text{-}2} + \upsilon_t & \text{if} \quad \varepsilon_{t\text{-}1} \le 1 \\ 0.55\varepsilon_{t\text{-}1} + 0.18\varepsilon_{t\text{-}2} + \upsilon_t & \text{if} \quad \varepsilon_{t\text{-}1} > 1 \end{cases}, \tag{13}$$

where $\upsilon_t \sim \text{NID}\left(0, \sigma^2\right)$, the delay is 1 and the threshold is 1. A series from SETAR(1; 2, 2) defined by (13) is presented in Figure 2. Although (13) has similar piecewise models, the first regime contains more observations than the second regime.

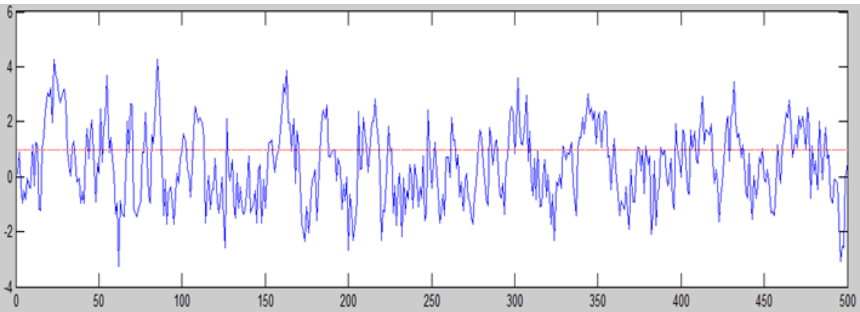

Figure 2. *Plot of a simulated SETAR(1; 2, 2) series*

The results in the presence of these type disturbances are given in Table 2 pointing out the performances of OLS, TSLS for AR(2) and ATSLS in different conditions. It can be seen that ATSLS has better performance than the others for all conditions by evaluating especially MSE and Eff.

Table 1. *Simulation results (approximate values) in the presence of SETAR(1; 1, 1) disturbances*

Parameter	n	σ	OLS				TSLS				ATSLS			
			Mean	Std. Dev.	MSE	Eff	Mean	Std. Dev.	MSE	Eff	Mean	Std. Dev.	MSE	Eff
β_1	50	0.5	2.0139	0.0144	$4.00*10^{-4}$	1	2.0015	0.0123	$1.52*10^{-4}$	2.63	2.0006	0.0114	$1.31*10^{-4}$	3.05
		1	1.9954	0.0368	$13.7*10^{-4}$	1	2.0022	0.0224	$5.05*10^{-4}$	2.72	2.0010	0.0220	$4.85*10^{-4}$	2.83
		3	1.9413	0.0880	$112*10^{-4}$	1	1.9924	0.0605	$37.2*10^{-4}$	3.01	1.9945	0.0588	$34.8*10^{-4}$	3.21
	100	0.5	1.9983	0.0123	$1.54*10^{-4}$	1	1.9999	0.0076	$0.58*10^{-4}$	2.64	1.9998	0.0075	$0.56*10^{-4}$	2.74
		1	2.0080	0.0237	$6.27*10^{-4}$	1	1.9992	0.0152	$2.32*10^{-4}$	2.70	2.0000	0.0144	$2.09*10^{-4}$	3.01
		3	1.9694	0.0764	$67.8*10^{-4}$	1	1.9980	0.0424	$18.1*10^{-4}$	3.75	1.9990	0.0409	$16.7*10^{-4}$	4.05
	250	0.5	1.9979	0.0062	$0.43*10^{-4}$	1	2.0002	0.0045	$0.20*10^{-4}$	2.15	2.0002	0.0044	$0.19*10^{-4}$	2.26
		1	2.0058	0.0139	$2.26*10^{-4}$	1	2.0006	0.0099	$0.98*10^{-4}$	2.30	2.0003	0.0097	$0.94*10^{-4}$	2.40
		3	1.9931	0.0437	$19.6*10^{-4}$	1	1.9995	0.0289	$8.37*10^{-4}$	2.34	1.9991	0.0282	$7.95*10^{-4}$	2.46
	500	0.5	2.0024	0.0044	$0.25*10^{-4}$	1	2.0002	0.0033	$0.11*10^{-4}$	2.33	2.0001	0.0032	$0.10*10^{-4}$	2.52
		1	1.9957	0.0106	$1.32*10^{-4}$	1	2.0002	0.0067	$0.45*10^{-4}$	2.92	2.0004	0.0066	$0.44*10^{-4}$	3.01
		3	1.9949	0.0372	$14.1*10^{-4}$	1	1.9992	0.0200	$3.99*10^{-4}$	3.53	1.9992	0.0197	$3.87*10^{-4}$	3.64
β_2	50	0.5	-5.0108	0.0185	$4.59*10^{-4}$	1	-5.0019	0.0125	$1.61*10^{-4}$	2.86	-5.0010	0.0116	$1.36*10^{-4}$	3.38
		1	-5.0171	0.0432	$21.6*10^{-4}$	1	-5.0009	0.0250	$6.25*10^{-4}$	3.45	-5.0006	0.0242	$5.84*10^{-4}$	3.69
		3	-5.0494	0.0665	$68.7*10^{-4}$	1	-5.0014	0.0492	$24.3*10^{-4}$	2.83	-5.0017	0.0478	$22.9*10^{-4}$	3.00
	100	0.5	-5.0108	0.0109	$2.35*10^{-4}$	1	-4.9995	0.0078	$0.61*10^{-4}$	3.85	-4.9992	0.0076	$0.58*10^{-4}$	4.06
		1	-4.9969	0.0280	$7.93*10^{-4}$	1	-4.9993	0.0139	$1.94*10^{-4}$	4.09	-4.9995	0.0135	$1.83*10^{-4}$	4.35
		3	-5.0375	0.0592	$49.2*10^{-4}$	1	-5.0043	0.0399	$16.1*10^{-4}$	3.05	-5.0042	0.0386	$15.1*10^{-4}$	3.26
	250	0.5	-4.9950	0.0064	$0.66*10^{-4}$	1	-4.9997	0.0050	$0.26*10^{-4}$	2.60	-4.9997	0.0049	$0.24*10^{-4}$	2.74
		1	-4.9888	0.0143	$3.32*10^{-4}$	1	-4.9994	0.0098	$0.96*10^{-4}$	3.44	-4.9999	0.0097	$0.93*10^{-4}$	3.56
		3	-4.9827	0.0392	$18.3*10^{-4}$	1	-4.9988	0.0274	$7.55*10^{-4}$	2.43	-4.9993	0.0267	$7.11*10^{-4}$	2.58
	500	0.5	-5.0008	0.0053	$0.29*10^{-4}$	1	-5.0001	0.0033	$0.11*10^{-4}$	2.73	-5.0001	0.0032	$0.10*10^{-4}$	2.89
		1	-4.9963	0.0104	$1.22*10^{-4}$	1	-4.9999	0.0065	$0.43*10^{-4}$	2.85	-5.0001	0.0064	$0.41*10^{-4}$	3.00
		3	-5.0141	0.0289	$10.3*10^{-4}$	1	-5.0008	0.0187	$3.50*10^{-4}$	2.94	-5.0001	0.0182	$3.31*10^{-4}$	3.11

Table 2. *Simulation results (approximate values) in the presence of SETAR(1; 2, 2) disturbances*

Parameter	n	σ	OLS				TSLS				ATSLS			
			Mean	Std. Dev.	MSE	Eff	Mean	Std. Dev.	MSE	Eff	Mean	Std. Dev.	MSE	Eff
β_1	50	0.5	2.0031	0.0161	$2.68*10^{-4}$	1	2.0003	0.0115	$1.32*10^{-4}$	2.03	2.0003	0.0106	$1.12*10^{-4}$	2.40
		1	1.9856	0.0284	$10.1*10^{-4}$	1	2.0021	0.0209	$4.42*10^{-4}$	2.29	2.0011	0.0202	$4.08*10^{-4}$	2.48
		3	2.0506	0.0946	$115*10^{-4}$	1	2.0091	0.0605	$37.4*10^{-4}$	3.08	2.0087	0.0594	$36.0*10^{-4}$	3.20
	100	0.5	2.0026	0.0136	$1.92*10^{-4}$	1	2.0001	0.0086	$0.74*10^{-4}$	2.58	1.9997	0.0083	$0.68*10^{-4}$	2.81
		1	2.0150	0.0174	$5.27*10^{-4}$	1	2.0016	0.0143	$2.07*10^{-4}$	2.54	2.0010	0.0140	$1.98*10^{-4}$	2.66
		3	1.9490	0.0640	$67.0*10^{-4}$	1	1.9979	0.0486	$23.7*10^{-4}$	2.83	1.9977	0.0479	$23.0*10^{-4}$	2.91
	250	0.5	2.0033	0.0067	$0.56*10^{-4}$	1	2.0001	0.0049	$0.24*10^{-4}$	2.35	2.0001	0.0048	$0.23*10^{-4}$	2.44
		1	2.0083	0.0113	$1.95*10^{-4}$	1	2.0002	0.0086	$0.74*10^{-4}$	2.65	2.0002	0.0084	$0.70*10^{-4}$	2.78
		3	2.0078	0.0500	$25.6*10^{-4}$	1	2.0008	0.0286	$8.20*10^{-4}$	3.12	2.0005	0.0280	$7.86*10^{-4}$	3.25
	500	0.5	1.9968	0.0041	$0.27*10^{-4}$	1	1.9999	0.0033	$0.11*10^{-4}$	2.51	2.0000	0.0032	$0.10*10^{-4}$	2.68
		1	2.0045	0.0085	$0.92*10^{-4}$	1	2.0004	0.0067	$0.44*10^{-4}$	2.08	2.0003	0.0066	$0.43*10^{-4}$	2.14
		3	2.0182	0.0248	$9.48*10^{-4}$	1	2.0004	0.0194	$3.75*10^{-4}$	2.53	2.0001	0.0191	$3.64*10^{-4}$	2.61
β_2	50	0.5	-4.9872	0.0173	$4.63*10^{-4}$	1	-4.9990	0.0140	$1.97*10^{-4}$	2.35	-4.9984	0.0135	$1.84*10^{-4}$	2.51
		1	-5.0287	0.0246	$14.3*10^{-4}$	1	-5.0052	0.0217	$4.99*10^{-4}$	2.87	-5.0054	0.0209	$4.66*10^{-4}$	3.07
		3	-5.0913	0.0818	$150*10^{-4}$	1	-5.0195	0.0647	$45.7*10^{-4}$	3.29	-5.0178	0.0617	$41.3*10^{-4}$	3.64
	100	0.5	-4.9920	0.0115	$1.96*10^{-4}$	1	-4.9996	0.0077	$0.60*10^{-4}$	3.29	-4.9997	0.0075	$0.56*10^{-4}$	3.53
		1	-4.9817	0.0199	$7.29*10^{-4}$	1	-4.9983	0.0142	$2.06*10^{-4}$	3.54	-4.9988	0.0140	$1.96*10^{-4}$	3.71
		3	-5.0528	0.0548	$57.9*10^{-4}$	1	-5.0058	0.0427	$18.5*10^{-4}$	3.12	-5.0050	0.0417	$17.6*10^{-4}$	3.28
	250	0.5	-4.9956	0.0060	$0.55*10^{-4}$	1	-4.9998	0.0046	$0.21*10^{-4}$	2.59	-4.9999	0.0045	$0.20*10^{-4}$	2.72
		1	-5.0003	0.0139	$1.94*10^{-4}$	1	-4.9998	0.0089	$0.79*10^{-4}$	2.44	-4.9996	0.0087	$0.75*10^{-4}$	2.57
		3	-4.9666	0.0339	$22.7*10^{-4}$	1	-4.9992	0.0271	$7.35*10^{-4}$	3.08	-5.0002	0.0266	$7.09*10^{-4}$	3.20
	500	0.5	-4.9978	0.0045	$0.25*10^{-4}$	1	-5.0000	0.0033	$0.11*10^{-4}$	2.38	-5.0001	0.0032	$0.11*10^{-4}$	2.43
		1	-4.9930	0.0088	$1.27*10^{-4}$	1	-4.9995	0.0068	$0.47*10^{-4}$	2.70	-4.9997	0.0066	$0.44*10^{-4}$	2.88
		3	-5.0329	0.0258	$17.5*10^{-4}$	1	-5.0029	0.0206	$4.35*10^{-4}$	4.03	-5.0026	0.0202	$4.14*10^{-4}$	4.22

The third SETAR disturbances are derived by increasing the delay and the orders of piecewise models

$$\varepsilon_t = \begin{cases} 0.61\varepsilon_{t\text{-}1} - 0.31\varepsilon_{t\text{-}2} - 0.06\varepsilon_{t\text{-}3} + \upsilon_t & \text{if} \quad \varepsilon_{t\text{-}2} \le 0 \\ 0.93\varepsilon_{t\text{-}1} - 0.45\varepsilon_{t\text{-}2} + 0.19\varepsilon_{t\text{-}3} + \upsilon_t & \text{if} \quad \varepsilon_{t\text{-}2} > 0 \end{cases}, \tag{14}$$

where υ_t is the same as defined before, the delay is 2 and the threshold is 0. A visual representation of SETAR(2; 3, 3) defined by (14) is demonstrated in Figure 3. Since the piecewise models have high orders, the sample size should be large enough to implement this analysis. Therefore, n < 50 can be useless and dramatic for obtaining the results.

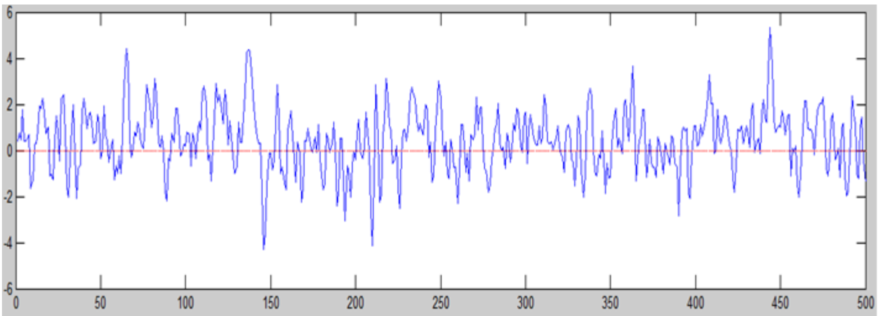

Figure 3. *Plot of a simulated SETAR(2; 3, 3) series*

The results in the presence of these type disturbances are given in Table 3. OLS, TSLS for AR(3) and ATSLS give unbiased estimations in all cases by assessing the means. Although ATSLS is more efficient than the other methods, the performances of TSLS and ATSLS approach in some cases. The standard deviations, MSE and Eff of TSLS and ATSLS become closer for some small standard deviations of υ_t and some large sample sizes. However, the results of this experiment are similar as before and lead to the same conclusions.

The fourth SETAR model for disturbances is designed by using different model structures for the first and second regime

$$\varepsilon_t = \begin{cases} 0.85\varepsilon_{t\text{-}1} + \upsilon_t & \text{if} \quad \varepsilon_{t\text{-}1} \le 0 \\ 0.52\varepsilon_{t\text{-}1} + 0.10\varepsilon_{t\text{-}2} + \upsilon_t & \text{if} \quad \varepsilon_{t\text{-}1} > 0 \end{cases}, \tag{15}$$

where υ_t is the same as defined before, the delay is 1 and the threshold is 0. This model is represented by SETAR(1; 1, 2) and a series from (15) is displayed in Figure 4. According to Figure 4, it can be decided in view of the threshold value that the first regime has more observations than the second regime.

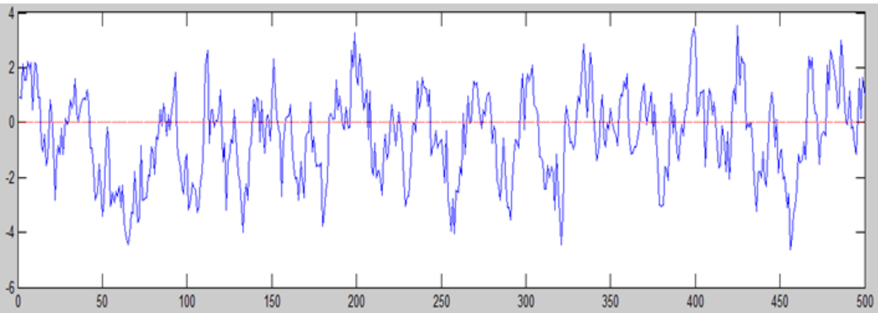

Figure 4. *Plot of a simulated SETAR(1; 1, 2) series*

Table 3. *Simulation results (approximate values) in the presence of SETAR(2; 3, 3) disturbances*

Parameter	n	σ	OLS				TSLS				ATSLS			
			Mean	Std. Dev.	MSE	Eff	Mean	Std. Dev.	MSE	Eff	Mean	Std. Dev.	MSE	Eff
β_1	50	0.5	2.0071	0.0123	$2.01*10^{-4}$	1	2.0018	0.0094	$0.91*10^{-4}$	2.20	2.0017	0.0088	$0.80*10^{-4}$	2.50
		1	2.0133	0.0300	$10.8*10^{-4}$	1	2.0035	0.0205	$4.33*10^{-4}$	2.49	2.0026	0.0199	$4.03*10^{-4}$	2.67
		3	1.9554	0.1346	$201*10^{-4}$	1	1.9836	0.0914	$86.2*10^{-4}$	2.33	1.9921	0.0880	$78.0*10^{-4}$	2.58
	100	0.5	1.9928	0.0114	$1.82*10^{-4}$	1	1.9991	0.0087	$0.76*10^{-4}$	2.38	1.9993	0.0085	$0.73*10^{-4}$	2.50
		1	1.9929	0.0199	$4.49*10^{-4}$	1	1.9988	0.0145	$2.12*10^{-4}$	2.12	1.9988	0.0142	$2.03*10^{-4}$	2.21
		3	2.0442	0.0595	$54.9*10^{-4}$	1	2.0042	0.0443	$19.8*10^{-4}$	2.77	2.0046	0.0427	$18.5*10^{-4}$	2.97
	250	0.5	1.9952	0.0065	$0.66*10^{-4}$	1	1.9997	0.0047	$0.22*10^{-4}$	2.93	1.9998	0.0047	$0.22*10^{-4}$	3.00
		1	2.0065	0.0151	$2.70*10^{-4}$	1	2.0000	0.0104	$1.09*10^{-4}$	2.48	1.9999	0.0103	$1.06*10^{-4}$	2.53
		3	2.0160	0.0447	$22.5*10^{-4}$	1	2.0002	0.0292	$8.54*10^{-4}$	2.64	1.9993	0.0283	$8.00*10^{-4}$	2.82
	500	0.5	1.9978	0.0054	$0.34*10^{-4}$	1	1.9999	0.0033	$0.11*10^{-4}$	3.18	1.9999	0.0032	$0.10*10^{-4}$	3.33
		1	2.0056	0.0099	$1.29*10^{-4}$	1	2.0002	0.0064	$0.41*10^{-4}$	3.14	2.0001	0.0063	$0.40*10^{-4}$	3.22
		3	1.9694	0.0273	$16.8*10^{-4}$	1	1.9989	0.0197	$3.89*10^{-4}$	4.33	1.9994	0.0195	$3.81*10^{-4}$	4.41
β_2	50	0.5	-5.0025	0.0167	$2.83*10^{-4}$	1	-5.0012	0.0112	$1.28*10^{-4}$	2.22	-5.0008	0.0108	$1.17*10^{-4}$	2.42
		1	-5.0212	0.0407	$21.1*10^{-4}$	1	-5.0060	0.0253	$6.74*10^{-4}$	3.13	-5.0055	0.0243	$6.20*10^{-4}$	3.40
		3	-4.9094	0.0857	$155*10^{-4}$	1	-4.9843	0.0642	$43.7*10^{-4}$	3.56	-4.9894	0.0600	$37.1*10^{-4}$	4.19
	100	0.5	-5.0072	0.0107	$1.67*10^{-4}$	1	-5.0008	0.0078	$0.61*10^{-4}$	2.72	-5.0006	0.0075	$0.57*10^{-4}$	2.91
		1	-5.0203	0.0224	$9.14*10^{-4}$	1	-5.0014	0.0159	$2.56*10^{-4}$	3.57	-5.0017	0.0155	$2.43*10^{-4}$	3.76
		3	-5.0711	0.0611	$87.8*10^{-4}$	1	-5.0071	0.0444	$20.2*10^{-4}$	4.34	-5.0086	0.0434	$19.6*10^{-4}$	4.49
	250	0.5	-5.0046	0.0094	$1.10*10^{-4}$	1	-5.0000	0.0055	$0.30*10^{-4}$	3.67	-5.0001	0.0053	$0.28*10^{-4}$	3.94
		1	-5.0136	0.0143	$3.89*10^{-4}$	1	-5.0007	0.0092	$0.86*10^{-4}$	4.54	-5.0006	0.0091	$0.84*10^{-4}$	4.64
		3	-4.9723	0.0387	$22.6*10^{-4}$	1	-4.9956	0.0279	$7.95*10^{-4}$	2.85	-4.9965	0.0274	$7.66*10^{-4}$	2.96
	500	0.5	-5.0007	0.0066	$0.45*10^{-4}$	1	-5.0004	0.0034	$0.12*10^{-4}$	3.93	-5.0003	0.0033	$0.11*10^{-4}$	4.05
		1	-5.0059	0.0090	$1.16*10^{-4}$	1	-4.9994	0.0070	$0.49*10^{-4}$	2.38	-4.9993	0.0069	$0.48*10^{-4}$	2.42
		3	-5.0241	0.0350	$18.0*10^{-4}$	1	-5.0011	0.0210	$4.42*10^{-4}$	4.08	-5.0010	0.0208	$4.32*10^{-4}$	4.17

Table 4. *Simulation results (approximate values) in the presence of SETAR(1; 1, 2) disturbances*

Parameter	n	σ	OLS				TSLS-1				TSLS-2				ATSLS			
			Mean	Std. Dev.	MSE	Eff	Mean	Std. Dev.	MSE	Eff	Mean	Std. Dev.	MSE	Eff	Mean	Std. Dev.	MSE	Eff
β_1	50	0.5	1.9980	0.0141	$2.04*10^{-4}$	1	1.9999	0.0098	$0.96*10^{-4}$	2.14	2.0001	0.0099	$0.98*10^{-4}$	2.08	2.0002	0.0091	$0.83*10^{-4}$	2.45
		1	2.0125	0.0326	$12.2*10^{-4}$	1	2.0024	0.0209	$4.42*10^{-4}$	2.76	2.0029	0.0211	$4.54*10^{-4}$	2.68	2.0023	0.0198	$3.99*10^{-4}$	3.06
		3	1.9661	0.1307	$182*10^{-4}$	1	1.9931	0.0803	$65.0*10^{-4}$	2.81	1.9927	0.0803	$65.0*10^{-4}$	2.81	1.9991	0.0751	$56.4*10^{-4}$	3.23
	100	0.5	2.0053	0.0116	$1.63*10^{-4}$	1	2.0001	0.0073	$0.53*10^{-4}$	3.08	2.0003	0.0072	$0.52*10^{-4}$	3.13	2.0002	0.0069	$0.48*10^{-4}$	3.38
		1	1.9908	0.0197	$4.73*10^{-4}$	1	1.9992	0.0130	$1.71*10^{-4}$	2.77	1.9990	0.0128	$1.64*10^{-4}$	2.89	1.9992	0.0121	$1.48*10^{-4}$	3.20
		3	1.9770	0.0794	$68.3*10^{-4}$	1	1.9956	0.0449	$20.3*10^{-4}$	3.36	1.9939	0.0452	$20.8*10^{-4}$	3.28	1.9945	0.0432	$19.0*10^{-4}$	3.60
	250	0.5	2.0057	0.0068	$0.79*10^{-4}$	1	2.0000	0.0048	$0.23*10^{-4}$	3.41	2.0000	0.0048	$0.23*10^{-4}$	3.43	1.9999	0.0046	$0.21*10^{-4}$	3.72
		1	2.0107	0.0141	$3.13*10^{-4}$	1	2.0008	0.0087	$0.76*10^{-4}$	4.11	2.0008	0.0086	$0.74*10^{-4}$	4.22	2.0007	0.0082	$0.68*10^{-4}$	4.60
		3	2.0137	0.0561	$33.4*10^{-4}$	1	1.9999	0.0290	$8.43*10^{-4}$	3.96	1.9994	0.0289	$8.38*10^{-4}$	3.98	1.9994	0.0282	$7.95*10^{-4}$	4.20
	500	0.5	2.0022	0.0057	$0.37*10^{-4}$	1	2.0001	0.0031	$0.10*10^{-4}$	3.76	2.0001	0.0031	$0.10*10^{-4}$	3.74	2.0001	0.0030	$0.09*10^{-4}$	4.00
		1	1.9936	0.0111	$1.63*10^{-4}$	1	1.9997	0.0071	$0.51*10^{-4}$	3.22	1.9997	0.0070	$0.49*10^{-4}$	3.35	1.9996	0.0068	$0.47*10^{-4}$	3.50
		3	1.9830	0.0315	$12.8*10^{-4}$	1	1.9989	0.0197	$3.88*10^{-4}$	3.30	1.9991	0.0196	$3.85*10^{-4}$	3.33	1.9994	0.0192	$3.68*10^{-4}$	3.48
β_2	50	0.5	-5.0057	0.0168	$3.16*10^{-4}$	1	-5.0006	0.0114	$1.30*10^{-4}$	2.44	-5.0008	0.0113	$1.28*10^{-4}$	2.47	-5.0002	0.0107	$1.14*10^{-4}$	2.77
		1	-5.0217	0.0258	$11.4*10^{-4}$	1	-5.0031	0.0207	$4.37*10^{-4}$	2.60	-5.0034	0.0205	$4.31*10^{-4}$	2.64	-5.0024	0.0189	$3.63*10^{-4}$	3.13
		3	-5.0164	0.1117	$127*10^{-4}$	1	-4.9966	0.0687	$47.4*10^{-4}$	2.69	-4.9967	0.0676	$45.8*10^{-4}$	2.78	-4.9978	0.0655	$43.0*10^{-4}$	2.96
	100	0.5	-4.9935	0.0107	$1.58*10^{-4}$	1	-4.9998	0.0077	$0.59*10^{-4}$	2.67	-4.9996	0.0078	$0.61*10^{-4}$	2.57	-4.9996	0.0075	$0.56*10^{-4}$	2.81
		1	-4.9849	0.0205	$6.49*10^{-4}$	1	-4.9999	0.0134	$1.79*10^{-4}$	3.62	-5.0002	0.0134	$1.79*10^{-4}$	3.63	-5.0004	0.0128	$1.63*10^{-4}$	3.98
		3	-5.0295	0.0825	$76.7*10^{-4}$	1	-5.0016	0.0444	$19.7*10^{-4}$	3.89	-5.0021	0.0446	$19.9*10^{-4}$	3.85	-5.0012	0.0427	$18.2*10^{-4}$	4.21
	250	0.5	-4.9970	0.0085	$0.82*10^{-4}$	1	-4.9999	0.0045	$0.21*10^{-4}$	3.96	-4.9999	0.0046	$0.21*10^{-4}$	3.94	-5.0000	0.0044	$0.19*10^{-4}$	4.26
		1	-5.0047	0.0152	$2.52*10^{-4}$	1	-5.0002	0.0092	$0.84*10^{-4}$	2.99	-5.0003	0.0091	$0.82*10^{-4}$	3.07	-5.0001	0.0088	$0.78*10^{-4}$	3.23
		3	-4.9989	0.0561	$31.5*10^{-4}$	1	-4.9992	0.0285	$8.14*10^{-4}$	3.87	-4.9992	0.0288	$8.28*10^{-4}$	3.80	-4.9994	0.0281	$7.89*10^{-4}$	3.99
	500	0.5	-4.9996	0.0062	$0.38*10^{-4}$	1	-4.9999	0.0035	$0.12*10^{-4}$	3.17	-4.9999	0.0035	$0.12*10^{-4}$	3.21	-4.9999	0.0034	$0.11*10^{-4}$	3.34
		1	-5.0039	0.0111	$1.37*10^{-4}$	1	-5.0001	0.0066	$0.43*10^{-4}$	3.18	-5.0002	0.0064	$0.42*10^{-4}$	3.31	-5.0001	0.0063	$0.40*10^{-4}$	3.42
		3	-4.9993	0.0302	$9.12*10^{-4}$	1	-4.9985	0.0176	$3.12*10^{-4}$	2.94	-4.9988	0.0176	$3.12*10^{-4}$	2.93	-4.9989	0.0174	$3.03*10^{-4}$	3.01

The results in the presence of SETAR(1; 1, 2) disturbances are given in Table 4. The means, standard deviations, MSE and Eff of β_1 and β_2 are obtained by OLS, TSLS for AR(1) denoted as TSLS-1, TSLS for AR(2) denoted as TSLS-2 and ATSLS in different conditions. According to Table 4, it is seen that similar results can be obtained by using TSLS-1 and TSLS-2 which are more efficient than OLS. However, ATSLS is the best one for obtaining efficient parameter estimates when comparing MSE and Eff of these four approaches.

The last SETAR disturbances are generated by using the model

$$\varepsilon_t = \begin{cases} 0.5\varepsilon_{t\text{-}1} + 2\upsilon_t & \text{if} \quad \varepsilon_{t\text{-}1} \le 1 \\ \\ 0.8\varepsilon_{t\text{-}1} + \upsilon_t & \text{if} \quad \varepsilon_{t\text{-}1} > 1 \end{cases}, \tag{16}$$

where υ_t is the same as defined before, the delay is 1 and the threshold is 1. This SETAR(1; 1, 1) model is designed in a specific form through each regime. ε_t is stationary and Figure 5 shows a series from (16). The characteristics of nonlinearity can be seen more clearly in Figure 5. Asymmetry is evident in the downward-upward jumps and the movement of the series describes the notion of a limit cycle.

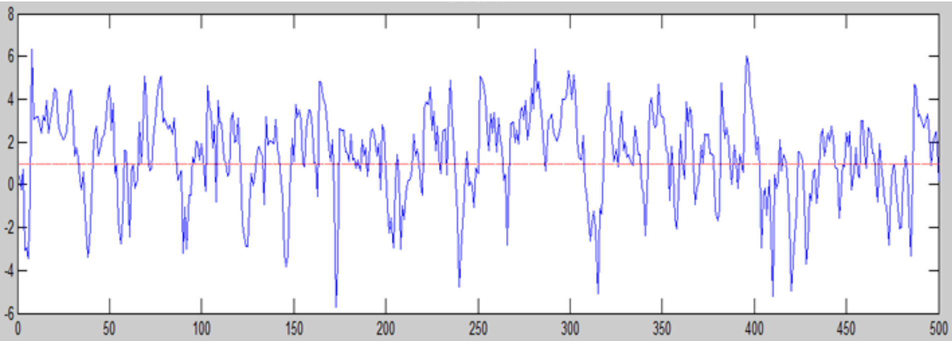

Figure 5. *Plot of a simulated SETAR(1; 1, 1) series with a specific form*

The results in the presence of these type disturbances are given in Table 5. It can be determined in view of MSE and Eff that ATSLS has the best efficiency for each different condition. However, the values of MSE and Eff for TSLS and ATSLS can be closer for large sample sizes.

In order to investigate how ATSLS works on small sample cases, the simulation study is performed for $n = 25$ and the results are given in Table 6. TSLS used for (15) represents the most efficient one of TSLS-1 and TSLS-2. According to Table 6, it can be seen that ATSLS has better performance than the others also for the small sample size.

Table 5. *Simulation results (approximate values) in the presence of SETAR(1; 1, 1) disturbances with a specific form*

Parameter	n	σ	OLS				TSLS				ATSLS			
			Mean	Std. Dev.	MSE	Eff	Mean	Std. Dev.	MSE	Eff	Mean	Std. Dev.	MSE	Eff
β_1	50	0.5	2.0084	0.0200	$4.71*10^{-4}$	1	2.0008	0.0138	$1.91*10^{-4}$	2.47	2.0005	0.0131	$1.71*10^{-4}$	2.76
		1	1.9584	0.0350	$29.6*10^{-4}$	1	1.9933	0.0307	$9.88*10^{-4}$	2.99	1.9961	0.0299	$9.06*10^{-4}$	3.26
		3	1.9284	0.1711	$344*10^{-4}$	1	1.9896	0.1038	$109*10^{-4}$	3.16	1.9910	0.0995	$99.8*10^{-4}$	3.45
	100	0.5	2.0083	0.0186	$4.15*10^{-4}$	1	2.0009	0.0112	$1.26*10^{-4}$	3.31	2.0010	0.0110	$1.22*10^{-4}$	3.41
		1	1.9748	0.0268	$13.5*10^{-4}$	1	1.9987	0.0193	$3.76*10^{-4}$	3.59	1.9992	0.0189	$3.58*10^{-4}$	3.77
		3	2.0639	0.0996	$140*10^{-4}$	1	2.0102	0.0664	$45.1*10^{-4}$	3.10	2.0061	0.0642	$41.6*10^{-4}$	3.37
	250	0.5	2.0057	0.0097	$1.27*10^{-4}$	1	2.0000	0.0064	$0.41*10^{-4}$	3.07	2.0000	0.0063	$0.39*10^{-4}$	3.25
		1	1.9883	0.0182	$4.67*10^{-4}$	1	1.9994	0.0126	$1.60*10^{-4}$	2.92	1.9996	0.0124	$1.55*10^{-4}$	3.02
		3	2.0477	0.0586	$57.0*10^{-4}$	1	2.0027	0.0408	$16.7*10^{-4}$	3.42	2.0022	0.0398	$15.9*10^{-4}$	3.60
	500	0.5	2.0048	0.0074	$0.78*10^{-4}$	1	2.0002	0.0046	$0.21*10^{-4}$	3.75	2.0001	0.0045	$0.20*10^{-4}$	3.90
		1	2.0035	0.0171	$3.05*10^{-4}$	1	1.9999	0.0091	$0.83*10^{-4}$	3.66	1.9998	0.0090	$0.81*10^{-4}$	3.76
		3	2.0278	0.0402	$23.9*10^{-4}$	1	2.0018	0.0265	$7.07*10^{-4}$	3.38	2.0013	0.0262	$6.88*10^{-4}$	3.47
β_2	50	0.5	-5.0138	0.0189	$5.49*10^{-4}$	1	-5.0018	0.0132	$1.78*10^{-4}$	3.08	-5.0017	0.0128	$1.66*10^{-4}$	3.30
		1	-5.0630	0.0529	$67.6*10^{-4}$	1	-5.0098	0.0417	$18.4*10^{-4}$	3.68	-5.0098	0.0400	$17.0*10^{-4}$	3.99
		3	-5.0950	0.1746	$395*10^{-4}$	1	-5.0188	0.1055	$115*10^{-4}$	3.44	-5.0076	0.1001	$101*10^{-4}$	3.92
	100	0.5	-5.0103	0.0172	$4.03*10^{-4}$	1	-5.0010	0.0112	$1.27*10^{-4}$	3.17	-5.0009	0.0108	$1.16*10^{-4}$	3.46
		1	-4.9761	0.0278	$13.4*10^{-4}$	1	-4.9972	0.0206	$4.34*10^{-4}$	3.09	-4.9981	0.0199	$3.99*10^{-4}$	3.37
		3	-5.0622	0.0978	$134*10^{-4}$	1	-5.0009	0.0652	$42.6*10^{-4}$	3.15	-5.0004	0.0628	$39.5*10^{-4}$	3.40
	250	0.5	-5.0050	0.0106	$1.36*10^{-4}$	1	-5.0002	0.0070	$0.48*10^{-4}$	2.82	-5.0002	0.0068	$0.46*10^{-4}$	2.93
		1	-4.9923	0.0191	$4.26*10^{-4}$	1	-4.9993	0.0120	$1.45*10^{-4}$	2.93	-4.9993	0.0117	$1.38*10^{-4}$	3.08
		3	-5.0435	0.0519	$45.9*10^{-4}$	1	-5.0025	0.0347	$12.1*10^{-4}$	3.79	-5.0013	0.0342	$11.7*10^{-4}$	3.91
	500	0.5	-5.0000	0.0067	$0.45*10^{-4}$	1	-5.0000	0.0043	$0.18*10^{-4}$	2.47	-5.0000	0.0042	$0.18*10^{-4}$	2.53
		1	-4.9907	0.0126	$2.45*10^{-4}$	1	-5.0000	0.0093	$0.86*10^{-4}$	2.83	-5.0003	0.0091	$0.83*10^{-4}$	2.93
		3	-4.9839	0.0404	$18.9*10^{-4}$	1	-4.9994	0.0281	$7.90*10^{-4}$	2.39	-4.9998	0.0279	$7.77*10^{-4}$	2.43

Table 6. *Simulation results (approximate values) for the small sample size of different SETAR-type autocorrelated disturbances*

Equation	Parameter	n	σ	OLS				TSLS				ATSLS			
				Mean	Std. Dev.	MSE	Eff	Mean	Std. Dev.	MSE	Eff	Mean	Std. Dev.	MSE	Eff
(12)	β_1	25	0.5	2.0015	0.0217	$4.73*10^{-4}$	1	1.9975	0.0149	$2.28*10^{-4}$	2.07	1.9987	0.0140	$1.99*10^{-4}$	2.38
			1	2.0209	0.0423	$22.3*10^{-4}$	1	2.0059	0.0288	$8.63*10^{-4}$	2.58	2.0023	0.0273	$7.52*10^{-4}$	2.96
			3	1.8833	0.1231	$288*10^{-4}$	1	1.9589	0.0940	$105*10^{-4}$	2.73	1.9678	0.0901	$91.6*10^{-4}$	3.14
(13)	β_1	25	0.5	2.0153	0.0198	$6.27*10^{-4}$	1	2.0053	0.0162	$2.92*10^{-4}$	2.15	2.0040	0.0152	$2.48*10^{-4}$	2.53
			1	2.0293	0.0536	$37.3*10^{-4}$	1	2.0119	0.0381	$16.0*10^{-4}$	2.34	2.0123	0.0372	$15.3*10^{-4}$	2.43
			3	1.8655	0.1251	$337*10^{-4}$	1	1.9359	0.1037	$149*10^{-4}$	2.27	1.9484	0.1008	$128*10^{-4}$	2.63
(15)	β_1	25	0.5	1.9959	0.0387	$15.1*10^{-4}$	1	1.9955	0.0255	$6.69*10^{-4}$	2.26	1.9970	0.0231	$5.43*10^{-4}$	2.79
			1	2.0100	0.0605	$37.6*10^{-4}$	1	2.0030	0.0396	$15.8*10^{-4}$	2.38	2.0026	0.0373	$14.0*10^{-4}$	2.69
			3	2.0589	0.1280	$199*10^{-4}$	1	2.0134	0.0877	$78.6*10^{-4}$	2.53	2.0120	0.0830	$70.4*10^{-4}$	2.82
(16)	β_1	25	0.5	2.0032	0.0385	$14.9*10^{-4}$	1	2.0040	0.0246	$6.24*10^{-4}$	2.39	2.0025	0.0231	$5.39*10^{-4}$	2.77
			1	1.9624	0.0501	$39.2*10^{-4}$	1	1.9906	0.0392	$16.3*10^{-4}$	2.41	1.9941	0.0363	$13.5*10^{-4}$	2.90
			3	2.1211	0.1280	$310*10^{-4}$	1	2.0234	0.1054	$117*10^{-4}$	2.66	2.0171	0.1016	$106*10^{-4}$	2.93
(12)	β_2	25	0.5	-4.9876	0.0187	$5.01*10^{-4}$	1	-4.9981	0.0148	$2.23*10^{-4}$	2.25	-4.9987	0.0138	$1.93*10^{-4}$	2.60
			1	-4.9573	0.0434	$37.1*10^{-4}$	1	-4.9868	0.0310	$11.4*10^{-4}$	3.26	-4.9875	0.0295	$10.3*10^{-4}$	3.61
			3	-4.9290	0.0875	$127*10^{-4}$	1	-4.9868	0.0611	$39.1*10^{-4}$	3.25	-4.9869	0.0602	$37.9*10^{-4}$	3.35
(13)	β_2	25	0.5	-4.9861	0.0246	$8.01*10^{-4}$	1	-4.9948	0.0180	$3.50*10^{-4}$	2.29	-4.9962	0.0166	$2.89*10^{-4}$	2.77
			1	-5.0726	0.0563	$84.3*10^{-4}$	1	-5.0398	0.0472	$38.2*10^{-4}$	2.21	-5.0324	0.0442	$30.0*10^{-4}$	2.81
			3	-4.8593	0.1401	$394*10^{-4}$	1	-4.9375	0.1131	$167*10^{-4}$	2.36	-4.9459	0.1111	$153*10^{-4}$	2.58
(15)	β_2	25	0.5	-4.9960	0.0236	$5.73*10^{-4}$	1	-4.9999	0.0169	$2.84*10^{-4}$	2.01	-4.9995	0.0153	$2.35*10^{-4}$	2.44
			1	-4.9962	0.0513	$26.4*10^{-4}$	1	-4.9986	0.0320	$10.3*10^{-4}$	2.57	-4.9996	0.0298	$8.87*10^{-4}$	2.98
			3	-5.0889	0.1261	$238*10^{-4}$	1	-5.0181	0.0882	$81.1*10^{-4}$	2.93	-5.0157	0.0843	$73.5*10^{-4}$	3.24
(16)	β_2	25	0.5	-5.0041	0.0303	$9.36*10^{-4}$	1	-4.9998	0.0214	$4.57*10^{-4}$	2.05	-5.0008	0.0196	$3.86*10^{-4}$	2.42
			1	-4.9458	0.0702	$78.6*10^{-4}$	1	-4.9843	0.0542	$31.8*10^{-4}$	2.47	-4.9859	0.0515	$28.5*10^{-4}$	2.75
			3	-5.1463	0.1482	$434*10^{-4}$	1	-5.0477	0.1261	$182*10^{-4}$	2.39	-5.0332	0.1207	$157*10^{-4}$	2.77

4. CONCLUSION AND DISCUSSION

This paper is concerned with the multiple linear regression model in which the disturbances are coming from a class of nonlinear models known as SETAR-type autocorrelation. Five different SETAR models are used to generate the autocorrelated disturbances in this study. Moreover, the parameter estimations are examined under different conditions in the presence of SETAR disturbances. An adapted TSLS method has been proposed for estimating parameters under this new situation.

The main conclusion achieved from the numerical results is emphasized that ATSLS can give more efficient parameter estimates than the others for multiple linear regression in the presence of SETAR disturbances. Also, the efficiencies of TSLS and ATSLS can approach to each other in some cases. However, the results can vary to the experiments performed. Therefore, the efficiencies for TSLS and ATSLS may or may not generalize to different SETAR models and conditions like some other studies in the literature. In future studies, the problem of autocorrelated disturbances can be evaluated in view of multiple-threshold SETAR models with high-dimensional structures on large data sets.

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CONFLICT OF INTEREST

No conflict of interest was declared by the author.

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