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The Sheaf of the Groups Formed by Topological Generalized Group over Topological Spaces

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Abstract

In the present paper, we show how to construct an algebraic sheaf by means of the toplogical generalized group defined by Molaei in [1] by considering both homotopy and sheaf theory.

1. INTRODUCTION AND PRELIMINARIES

Generalized groups were deduced from a geometrical structure and introduce by Molaei [1]. It is an fascinating extension. Because each element has an identity element for itself in Molaei's generalized groups. So generalized groups is not admitting the unique identity element for each element. If this feature is considered, every group is a generalized group. Properties of generalized groups as a new structure from algebraic, and topological viewpoints are studied. [1–7].

As an algebraic structure generalized group has important physical reasons for its definition in the unified guage theory. The unified theory has directly important connection with the geometry of space. It describes particles and their interactions in a quantum mechanical manner and the geometry of the space-time through which they are moving. Currently, the most promising is super-string theory in which the so called elementary particles are described as vibration of tiny (planck-length) closed loops of strings. In this theory the classical law of physics, such as electromagnetism and general relativity, are modified at time distances comparable to the length of the string. This notion of 'quantum space-time' is the goal of unified theory of physical forces.

Therefore the unified theory offers a new insight into the structure, order and measures of the quantum world of the entire universe. It is known that unified theories are based on the geometry of a space and the metric can determine the geometry [8]. Because of this physical forces mathematicians and physicists have been working on constructing an convenient unified theory kind of twistor and isotopies theories. Now generalized groups are well known as a structure that used for constructing of unified geometric and electroweak theories which is builded on Minkowskian axioms and gravitational theories which is builded on Riemannian axioms.

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Furthermore this kind of structure appears in genetic codes. Generalized groups have been applied to DNA analysis by transforming the set of DNA sequences to generalized group in [9].

Another important concept in this present paper are sheaves which were originally introduced by Leray [10] in 1946. The modified definition of sheaves now used was given by Lazard, and appeared first in the Cartan Sem. [11] 1950-51. Sheaf theory provides a language for the discussion of geometric objects of many different kinds. Nowadays it is applied in topology and (more primarily) in modern algebraic geometry, where it has been used successfully as a tool for the solution of several problems which are existed for a long time.

Yildiz constructed an algebraic sheaf by means of the topological group in [12]. This is our motivation for costructing a sheaf by the means of the topological generalized group in this paper. We replace topological group with topological generalized group construct an algebraic sheaf by means of the topological generalized group introduce in [1].

In this section we gave fundamental definitions and notions in connection with the generalized groups, topological generalized groups and sheaves. We can start by giving some basic notions of generalized group that was first introduce in 1999 [1].

Definition 1. [1] Let G be generalized group and a non-empty set. An operation is called multiplication subject, if

(i) $(ab)c = a(bc)$, for all $a, b, c \in G$ (associative law);
(ii) There exists a unique $e(a) \in G$ for each $a \in G$, such that $ae(a) = e(a)a = a$
(iii) There exists $a^{-1} \in G$ for each $a \in G$, such that $a\, a^{-1} = a^{-1}\, a = e(a)$ are valid on G.

Example 1. [7] The set $G = \left\{ \begin{bmatrix} a & b \\ c & d \end{bmatrix} : a, b, c \text{ and } d \text{ are real numbers} \right\}$ with the operation

$$\left(\begin{bmatrix} a & b \\ c & d \end{bmatrix}, \begin{bmatrix} e & f \\ g & h \end{bmatrix} \right) \to \begin{bmatrix} a & f \\ g & d \end{bmatrix},$$

is a generalized group. If

$$e(A) = \begin{bmatrix} a & f \\ g & d \end{bmatrix} \text{ and } A^{-1} = \begin{bmatrix} a & f \\ g & d \end{bmatrix},$$

for all $A \in G$, where $e(A)$ and A^{-1} are the identity and the inverse of matrix all $A \in G$respectively.

Example 2. [6] Let $G = \mathbb{R} \times \{\mathbb{R} \setminus \{0\}\}$. Then with the multiplication $(a, b)(c, d) = (bc, bd)$ is a generalized group. Then, $e(a, b) = (a/b\, , 1)$ and $(a, b)^{-1} = (a/b^2\, , 1\,/\,b)$ for all $(a, b) \in G$.

Example 3. [13] If the set of G with the multiplication m is generalized group, therefore we can say that the set of $G \times G$ is a generalized group. with the multiplication

$$m_1\big((a, b), (c, d)\big) = \big(m(a, c), m(b, d)\big).$$

For this generalized group the identity element and inverse element is defined by $e_1(a, b) = \big(e(a), e(b)\big)$ $(a, b)^{-1} = (a^{-1}, b^{-1})$ for each $(a, b) \in G \times G$ respectively.

Theorem 1. [7] Let G be a a generalized group. Then each $a \in G$ has a unique inverse.

Example 4. [7,14] Let $S = \{1,2\}$. Then S with the binary operation: $2.2 = 2, 2.1 = 1.2 = 2, 1.1 = 1$ is a semigroup. Since the identity of 2 is not unique in S, S is a semigroup, but it is not a generalized group, One can easily deduce that every group is a generalized group from Definition 1. But the next lemma shows that the converse of the the definition of the generalized group may not be true.

Lemma 1. [4] If G is a generalized group and $ab = ba$ for all $a, b \in G$, then G is an abelian group.

Definition 2. [6] Let H be a non-empty subset of a generalized group G. Then H is a generalized subgroup of G if and only if for all $a, b \in H, ab^{-1} \in H$.

Theorem 2. [7] If G be a generalized group and

$$G_a = e^{-1}\{e(a)\}=\{x \in G: e(x) = e(a)\}$$

for $a \in G$, then G_a is a generalized subgroup of G. Furthermore, G_a is a group.

Let us enumarate some elementary features related to the structure of generalized groups with following lemma.

Lemma 2. [2] If G is a generalized group, then

(i) $e(a) = e(a^{-1})$ and $e(e(a)) = e(a)$ where $a \in G$.
(ii) $(a^{-1})^{-1} = a$ where $a \in G$.
(iii) The set $\{G_a = e^{-1}\{e(a)\}: a \in G\}$ is a partitation of groups for G.

We here state definition of a topological generalized group which was defined by Molaei [1] and give simpliest features of generalized groups from topological view was introduced in [1, 7] .

Definition 3. [7] Let G is is a set which satisfies the following conditions:

(i) G is generalized group;
(ii) G is a Hausdorff topological space;
(iii) The mappings

$$m_1: G \times G \to G, (a, b) \to ab$$

and

$$m_2: G \to G, a \to a^{-1}$$

are continuous mappings.

Then G is called a topological generalized group.

Let $a \in G$ and define the product of G topological group on $G_a = e^{-1}(\{e(a)\})$, then G_a is a topological group, and G is disjoint union of these topological groups so it can be written in the form $G = \vee_{a \in G} G_a$.

Example 5. [7] Every non-empty Hausdorff topological space G with the operation:

$$m: G \times G \to G$$
$$(a, b) \mapsto a$$

is a topological generalized group.

Example 6. [1] Let $G = IR \times (IR \setminus \{0\})$ is a set and define a topology induced by a Euclidean metric and the multiplication $(a, b).(c, d) = (bc, bd)$ on G. Then G is a topological generalized group.

Definition 4. [15] Let X, S both topological spaces, and $\pi: S \to X$ be a locally topological map. Then the pair $S = (S, \pi)$ or shortly S is called a sheaf over X.

In the definition of a sheaf, X is not assumed to satisfy any separation axioms (see in [16]). S is called the sheaf space, π the projection map, and X the base space. Let x be an any point in X and V be an open neighborhood of x. A section over V is a continuous map $s: V \to S$ such that $\pi \circ s = id_V$.

Let us denote the collection of all sections of S, by $\Gamma(V,S)$ and recall the Whitney sum.

Definition 5. [17,18] Let $(S_1,\pi_1),(S_2,\pi_2),\dots,(S_k,\pi_k)$ be sheaves on X. Construct product $M_W = \Gamma(W,S_1)\times\Gamma(W,S_2)\times\dots\times\Gamma(W,S_k)$ for $V,W \subset X$ open sets. Let Γ_V^W: $M_W \to M_V$ defined by Γ_V^W(s)=$(s_1|_V, s_2|_V,\dots,s_k|_V)$ for $(s_1,s_2,\dots,s_k) \in M_W$ and $V \subset W$. Then $\{M_W,\Gamma_V^W\}$ is a presheaf. The Whitney sum of $S_1,S_2\dots,S_k$ sheaves is a sheaf defined by this presheaf and denoted by $S^* = S_1 \oplus S_2 \oplus \dots \oplus S_k$.

Now we can say that the Whitney sum of sheaves $(S_1,\pi_1),(S_2,\pi_2),\dots,(S_k,\pi_k)$:

$$\begin{aligned} S^* = S_1 \oplus \dots \oplus S_k &:= \{\sigma = (\sigma_1,\dots,\sigma_k) \in S_1 \times \dots \times S_k : \pi_1(\sigma) = \cdots = \pi_k(\sigma)\} \\ &= \bigvee_{x\in X}((S_1)_x \times \dots \times (S_k)_x) \end{aligned}$$

is a set over X topological spaces. Then the map π: $S^* = S_1 \oplus S_2 \oplus \dots \oplus S_k \to X$, $\pi(\sigma) = (\pi_i \circ P_i)(\sigma)$ is a local homeomorphism, hence $S^* = S_1 \oplus S_2 \oplus \dots \oplus S_k$ is a sheaf over X.

Theorem 3. [19] Let (S_i,π_i), $i = 1,\dots,k$ be sheaves and $S^* = S_1 \oplus S_2 \oplus \dots \oplus S_k$ be Whitney sum of $S_1,S_2\dots,S_k$. Then there is bijection $\pi: S_x^* \to (S_1)_x \times (S_2)_x \times\dots\times (S_k)_x$ defined by $(W,(s_1,s_2,\dots,s_k))_x \to (s_1(x),s_2(x),\dots,s_k(x))$.

Theorem 4. [19] Let $(S_i,\pi_i), i = 1,\dots,k$ be sheaves on X. Then the canonic projection

$$P_i: S_1 \oplus S_2 \oplus \dots \oplus S_k \longrightarrow S_i,\ P_i(\sigma_1,\sigma_2,\dots,\sigma_k) = \sigma_i$$

is a sheaf morphism.

Let $s_i \in \Gamma(W_i,S_i)$ for $i = 1,\dots,k$. Define $s_1 \oplus \dots \oplus s_k: W \longrightarrow S^* = S_1 \oplus S_2 \oplus \dots \oplus S_k$, such that$(s_1 \oplus \dots \oplus s_k)(x) = \big(s_1(x),s_2(x),\dots,s_k(x)\big)$. Clearly $(s_1,s_2,\dots,s_k) \in M_W$ and $r(s_1,s_2,\dots,s_k) = \big(W,(s_1,s_2,\dots,s_k)\big)_x = \big(s_1(x),s_2(x),\dots,s_k(x)\big) = (s_1 \oplus \dots \oplus s_k)(x)$.
Therefore since $s_1 \oplus \dots \oplus s_k = r(s_1,s_2,\dots,s_k) \in \Gamma(W,S_1 \oplus S_2 \oplus \dots \oplus S_k)$ we have

$$\Gamma(W,S_1 \oplus S_2 \oplus \dots \oplus S_k) = \Gamma(W,S_1)\times\Gamma(W,S_2)\times\dots\times\Gamma(W,S_k).$$

Furthermore since $W \subset X$ is open set and π is a local homeomorphism $s(W)$ is open set in S, and S is union of these type of open sets. Also if $s_1,s_2 \in \Gamma(W,S)$ and $s_1(x) = s_2(x)$ for $x \in X$ then $s_1 = s_2$ in W. So we can say that every element of S can be seen as a substance of sections in S.

2. MAIN RESULTS

2.1. The Sheaf of The Groups Formed by Topological Generalized Group Over Topological Spaces

Let $\mathcal{C}$ be the category of the topological spaces X satisfying the property that all pointed spaces (X,x) with $x \in X$ have same homotopy type. This category includes all topological vector spaces.

Let us take $X \in \mathcal{C}$ as a base set if (P,p_0) pointed topological space is any topological group with identity element p_0 as base point. Then the set of homotopy class of homotop maps preserving the base point from (X,x) to (P,p_0)obtained for each $x \in X$, (X,x) pointed topological spaces i.e. $S(X) = \bigvee_{x\in X}[(X,x),(P,p_0)]$. Thus $S(X)$ is a set over X.

If (P,p_0) pointed topological space is any topological group with the identity element of the group is p_0, we can construct a sheaf over X by using following theorem which is given by Yildiz [12].

Theorem 5. [12] Let (P,p_0) be any pointed topological group with the identity element $p_0$and $X \in \mathcal{C}$. If $\pi: S(X) \to X$ such that $\pi(\sigma) = \pi([f]_x) = x$ for $\sigma = [f]_x \in S(X)$, $x \in X$ then there is the natural topology over $S(X)$ such that π is locally topological with respect to this topology. Thus the pair (S,π) is a sheaf over X.

In Theorem 5, Yildiz by defining $S(X) = \bigvee_{x\in X}[(X,x),(P,p_0)]$ and $\pi: S(X) \to X$ such that $\pi(\sigma) = x$, $x \in X$ and a mapping $s: V \to S(X)$ as follows:

If $x_0 \in X$, then there exists a group $[(X,x_0),(P,p_0)]$ in $S(X)$. If y is any point in V, Then $(X,x_0)$and (X,y) are having same homotopy type where $V = V(x_0)$ open neighborhood of x_0 in X. Therefore, there is a homotopy equivalence map $\Phi: (X,x_0) \to (X,y)$.

$$\begin{tikzcd}
(P,p_0) & (X,x_0) \arrow[l, "f"'] \\
 & (X,y) \arrow[u, "\Phi"'] \arrow[ul, "h = f\circ\Phi"]
\end{tikzcd}$$

Figure 1. *Diagram*

Hence from the diagram in Figure 1, the map $h = f\circ\Phi: (X,y) \to (P,p_0)$ is continuous and base-point preserving. $[h]_y \in [(X,y),(P,p_0)]$ is a homotopy class of map $f\circ\Phi = h$.

Therefore, we define $s(y) = [h]_y$. In this way s is well defined and $(\pi\circ s)(y) = \pi(s(y)) = y$ for each $y \in V$. Therefore $\pi\circ s = I_V$. Thus s is called a section of $S(X)$ over V.

Let us denote the collection of all sections of $S(X)$, by $\Gamma(V,S)$. A topology-base is constructed on $S(X)$ by using s(V)=$\bigvee_{y\in V}[h]_y$,

$$\beta = \{s(V) : V = V(x) \subset X, x \in X, s \in \Gamma(V,S)\}.$$

Thus gives a natural topology on $S(X)$. Therefore $S(X)$ is a topological space.

Therefore the sheaf $(S(X),\pi)$ given by Theorem 5 is a sheaf of the homotopic groups formed by topological group P over (X,x) pointed topological spaces [12]. The stalk of the sheaf $(S(X),\pi)$ over X is the group $[(X,x),(P,p_0)] = \pi^{-1}$ (x) denoted by $S(X)_x$ for every $x \in X$.

$\Gamma(V,S)$ is a group with pointwise multiplication defined by

$$(s_1s_2)(y) = s_1(y)s_2(y),\ s_1,s_2 \in \Gamma(V,S) \text{ and } y \in V.$$

And in this group the identity element is $I: V \to S$ which is obtained by means of the identity element of $[(X,x),(P,p_0)]$ and the inverse element of $s \in \Gamma(V,S)$ is $s^{-1} \in \Gamma(V,S)$ which is obtained by the inverse element of $[(X,x),(P,p_0)]$. Therefore $(S(X),\pi)$ is an algebraic sheaf with the operation $(.): S(X) \otimes S(X) \to S(X)$ (that is, $(\sigma_1,\sigma_2) \to \sigma_1.\sigma_2$ for every $\sigma_1,\sigma_2 \in S(X)$ is continuous [12].

Now let begin to construct the sheaf over X by the finite pointed topological generalized group P. We begin with constructing the Whitney sum of sheaves $S_1(X), \dots, S_k(X)$ i. e. $S^*(X) = S_1(X) \oplus \dots \oplus S_k(X)$.

Let us now define the map $\pi: S^*(X) \to X$, $\pi(\sigma) = (\ \pi_i \circ P_i)(\sigma)$ for where P_i is a canonic projection for $i = 1, \dots, k$.
If $x_0 \in X$, then there exists groups $[(X, x_0), (P, p_i)]$ in $S_i(X)$ for $i = 1, \dots, k$. Let $\sigma = (\sigma_1, \sigma_2, \dots, \sigma_k) = \left([h_1]_{x_0}\ , [h_2]_{x_0}\ , \dots, [h_k]_{x_0}\right)$ be a homotopy class in the group $\prod_{i=1,\dots,k} [(X, x_0), (P, p_i)]$. If y is any point in V, then (X, x_0) and (X, y) are having the same homotopy type. Therefore, there is a homotopy equivalence map Φ: $(X, y) \longrightarrow (X, x_0)$. Hence from the diagram in Figure 2,

$$\begin{tikzcd}
(P, p_i) & (X, x_0) \arrow[l, "f_i"'] \\
 & (X, y) \arrow[u, "\Phi"'] \arrow[ul, "h_i = \mathrm{f}_i \circ \Phi"]
\end{tikzcd}$$

Figure 2. *Diagram*

the map $h_i = f_i \circ \Phi$: $(X, y) \longrightarrow (P, p_i)$ is continuous and base point preserving for $i = 1, \dots, k$. $[h_i]_y \in [(X, y), (P, p_i)]$ for $i = 1, \dots, k$ is a homotopy class of map $f_i \circ \Phi = h_i$.

If $x_0 \in X$ is an arbitrarily fixed point, then let us denote $V = V(x_0)$ open neighborhood of x_0 in X. Now, we can define a mapping $s = (s_1, \dots, s_k)$: $V \to S^*(X)$, as follows:

If y is any point in V, then we define $s(y) = (s_1, s_2, \dots, s_k)(y) = (s_1(y), s_2(y), \dots, s_k(y))$ for $s_i(y) = [h_i]_y$, $i = 1, \dots, k$. In this way s is well defined and

1. $(\pi \circ s)(y) = (\pi_i\ \circ P_i \circ s)(y) = \pi_i\left(P_i\big(s_1(y), s_2(y), \dots, s_k(y)\big) = \pi_i(s_i(y)\right) = y$ for each $y \in V$. Therefore $\pi \circ s = I_V$.

2. If, x_0 is an arbitrary fixed point in V,

$$\begin{aligned} s(x_0) &= (s_1, \dots, s_k)(x_0) = \big(s_1(x_0), \dots, s_k(x_0)\big) \\ &= \left(\ [f_1\ \circ I_x]_{x_0}\ \ , \dots, [f_k\ \circ I_x]_{x_0}\ \right) = \left([f_1]_{x_0}, \dots, [f_k]_{x_0}\right) \end{aligned}$$

for $V = V(x_0)$. Hence it can be written as s(V)=$\prod_{i=1,\dots,k} s_i(V) = \prod_{i=1,\dots,k}\left(\bigvee_{y \in V}\ [h_i]_y\right)$.

If we can define $s(V)$ as an open set, then it can be easily shown that the family

$$\beta = \left\{ s(V) \coloneqq \prod_{i=1,\dots,k} s_i(V) :\ V = V(x) \subset X, x \in X,\ s_i\ \in \Gamma(V, S_i) \right\}$$

is a topology-base on $S^*(X)$. Thus $S^*(X)$ is a topological space.

Now we can show that $\pi: S^*(X) \to$X is local topological.

If $\sigma = [h]_y \in S^*(X)$ and $y \in X$, then $\pi(\sigma) = \pi([h]_y) = y$. Therefore, there is a map $s: V \to S^*(X)$ such that $s(y) = \sigma,\ \ y \in V$. Now, let us assume that $U(\sigma) = s(V)$ and $\pi|_U = \pi^*$.

1. The map $\pi^* = \pi|_U: U \to V$ is injective. Because for any $\sigma_1, \sigma_2 \in s(V)$ by using homotopy properties (see in [20-23]), one can see that there are points y_1, y_2 respectively in V such that

$$\sigma_1 = s(y_1) = (s_1, s_2, \dots, s_k)(y_1) = \big(s_1(y_1), s_2(y_1), \dots, s_k(y_1)\big)$$
$$= \big([f_1 \circ \Phi]_{y_1}, [f_2 \circ \Phi]_{y_1}, \dots, [\Phi \circ f_k]_{y_1}\big),$$

$$\sigma_2 = s(y_2) = (s_1, s_2, \dots, s_k)(y_2) = \big(s_1(y_2), s_2(y_2), \dots, s_k(y_2)\big)$$
$$= \big([\, f_1 \circ \Phi']_{y_2}, [\, f_2 \circ \Phi']_{y_2}, \dots, [\, f_k \circ \Phi']_{y_2}\big).$$

That is, we have the following diagrams in Figure 3, for $i = 1, \dots, k$.

$$\begin{tikzcd} (P,p_i) & (X,x_0) \arrow[l, "f_i"'] \\ & (X,y_1) \arrow[u, "\Phi"'] \arrow[ul, "h_i = f_i \circ \Phi"] \end{tikzcd} \qquad \begin{tikzcd} (P,p_i) & (X,x_0) \arrow[l, "f_i"'] \\ & (X,y_2) \arrow[u, "\Phi'"'] \arrow[ul, "h'_i = f_i \circ \Phi'"] \end{tikzcd}$$

Figure 3. *Diagrams*

If $\pi^*(\sigma_1) = \pi^*(\sigma_2)$, then

$$\pi^*\big(s(y_1)\big) = \pi^*\big(s(y_2)\big) \Longrightarrow \pi^*\Big(\big([f_1 \circ \Phi]_{y_1}, [f_2 \circ \Phi]_{y_1}, \dots, [f_k \circ \Phi]_{y_1}\big)\Big)$$

$$= \ \pi^*([f_1 \circ \Phi']_{y_2}, [f_2 \circ \Phi']_{y_2}, \dots, [f_k \circ \Phi']_{y_2}) \Longrightarrow y_1 = y_2.$$

Since for each $i = 1, \dots, k$,

$$\Phi \sim \Phi' \Longrightarrow f_i \circ \Phi \sim f_i \circ \Phi' \Longrightarrow [f_i \circ \Phi]_{y_1} = [f_i \circ \Phi']_{y_2} \Longrightarrow \sigma_1 = \sigma_2.$$

2. The map $\pi^* = \pi|_U: U \to V$ is continuous. In fact, if $\sigma \in U = s(V) \ \Longrightarrow \pi^*(\sigma) = y \in V$ and $W = W_y \subset V$ is neighbourhood of y, then $s(W) \subset U = s(V)$ is neighborhood of σ and $\pi^*(s(W)) = W \subset V$. So π^* is continuous.

3. $\pi^{*^{-1}} = (\pi|_U)^{-1} = s: V \to U = s(V)$ is continuous. In fact, if y is any point in V, $\ s(y) = \sigma \in U$ and $U' = U'(\sigma) \subset U$ is a neighborhood of y in V and $s(\pi|_U)(U') \subset U$. So $\pi^{*^{-1}}$ is continuous.
Therefore π is locally topological map. Now we can give the following theorem.

Theorem 6. Let $(P, p_i)_{i=1,\dots,k}$ be any pointed finite topological generalized group with the identity elements $p_1, p_2, \dots, p_k$ and $X \in \mathcal{C}$. If

$$S^*(X) = S_1(X) \oplus S_2(X) \oplus \dots \oplus S_k(X) \ \text{ and } \pi\text{: } S^*(X) \to X$$

such that

$$\pi\ (\sigma) = (\pi_i \circ P_i)([h_1]_x, [h_2]_x, \dots,\ [h_k]_x) = x,\ \ i = 1, \dots, k,$$

for $\sigma \in S^*(X)$ and $x \in X$, then there is the natural topology based on $S^*(X)$, such that π is locally topological with respect to this natural topology. Thus the pair $(S^*(X), \pi)$ is a sheaf over X.

Definition 6. The sheaf $(S^*(X), \pi)$ given by Theorem 6 is called sheaf of the groups formed by the finite pointed topological generalized groups over (X, x), $x \in X$ pointed topological spaces.

Definition 7. The group $\prod_{i=1,\ldots,k} [(X, x), (P, p_i)] = \pi^{-1}(x)$ is called the stalk of the sheaf $(S^*(X), \pi)$ over X and denoted by $S^*(X)_x$ for every $x \in X$.

Now, if $x \in X$ is an arbitrarily fixed point and V is open neighborhood of x in X, the mapping $s: V \to S^*(X)$ as defined in the construction of topology of $S^*(X)$, is called section of $S^*(X)$, over V. Let us denote the collection of all sections of $S^*(X)$, by $\Gamma(V, S^*(X))$.

Theorem 7. $\Gamma(V, S^*(X))$ is a group with the operation

$$(s_1 s_2)(y) = s_1(y) s_2(y), s_1, s_2 \in \Gamma(V, S^*(X))$$

where $y \in V$.

Proof. If we consider pointwise multiplication

$$\left(s_1^i, s_2^i\right)(y) = s_1^i(y) s_2^i(y), s_1^i, s_2^i \in \Gamma(V_i, S_i(X))$$

and $y \in V_i$ which is defined on$\in \Gamma(V_i, S_i(X))$ for $i = 1, \ldots, k$. Proof follows from that the operation of production is well-defined and closed. Clearly, the operation of production is associative and the mapping $I: V \to S^*(X)$ is identity element which is obtained by means of the identity element of $\prod_{i \in I} [(X, x), (P, p_i)]$. On the other hand, the any inverse element of $s \in \Gamma(V, S^*(X))$, namely, $s^{-1} \in \Gamma(V, S^*(X))$ which is obtained by means of the homotopy inverses of pointed groups (P, p_i) for $i = 1, \ldots, k$. Hence $\Gamma(V, S^*(X))$ is a group.

From the Theorem 6, $(S^*(X), \pi)$ is an algebraic sheaf with the continuous operation

$$(.): S^*(X) \otimes S^*(X) \to S^*(X),$$
$$(\sigma_1, \sigma_2) \to \sigma_1 . \sigma_2$$

where $\sigma_1, \sigma_2 \in S^*(X)$.

CONFLICTS OF INTEREST

No conflict of interest was declared by the authors.

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