Academic Platform Journal of Engineering and Science 7-1, 52-55, 2019

Academic Platform Journal of Engineering and Science

journal homepage: http://apjes.com/

Total Irregularity of Indu-Bala Product of Graphs

*Zeynep Nihan Berberler
[*]Faculty of Science, Department of Computer Science, Dokuz Eylul University,
35160, Izmir/TURKEY, zeynep.berberler@deu.edu.tr,

Research Paper Received Date: 04.07.2017 Accepted Date: 07.08.2018

Abstract

The total irregularity of a simple undirected graph G is defined as $irr_t(G)=\frac{1}{2}\sum_{u,v\in V(G)}|d_G(u)-d_G(v)|$, where $d_G(u)$ denotes the degree of a vertex $u\in V(G)$. The Indu-Bala product of G_1 and G_2 is denoted by $G_1\nabla G_2$ and is obtained from two disjoint copies of the join $G_1\vee G_2$ of G_1 and G_2 by joining the corresponding vertices in the two copies of G_2. In this paper, the total irregularity of $G_1\nabla G_2$ is obtained in terms of the total irregularities of G_1 and G_2.

Keywords: Irregularity of a graph; Total irregularity of a graph

1. INTRODUCTION

In this paper, finite, simple and undirected graphs $G=(V,E)$ with vertex set V, edge set E are considered.

In [1] the total irregularity of a graph is defined as

$$irr_t(G)=\frac{1}{2}\sum_{u,v\in V(G)}|d_G(u)-d_G(v)|,$$

where $d_G(u)$ denotes the degree of a vertex $u\in V(G)$.

This parameter has attracted much attention. For recent related work on total irregularity, we refer the reader to [2-3,4-7,8-13] and the references therein.

Recently, in [14], a new graph operation, so-called Indu-Bala product of graphs, is defined by Indulal and Balakrishnan. The join of two disjoint graphs G_1 and G_2 with disjoint vertex sets $V(G_1)$ and $V(G_2)$ and edge sets $E(G_1)$ and $E(G_2)$ is the graph $G=G_1+G_2$ with vertex set $V(G)=V(G_1)\cup V(G_2)$ and edge set $E(G)=E(G_1)\cup E(G_2)\cup\{(u,v):u\in V(G_1),v\in V(G_2)\}$. Let $V(G_1)=\{u_1,u_2,\dots,u_{n_1}\}$ and $V(G_2)=\{v_1,v_2,\dots,v_{n_2}\}$. The Indu-Bala product $G_1\nabla G_2$ of G_1 and G_2 is obtained by taking a disjoint copy $G_1'\vee G_2'$ of $G_1\vee G_2$ with vertex sets $V(G_1')=\{u_1',u_2',\dots,u_{n_1}'\}$ and $V(G_2')=\{v_1',v_2',\dots,v_{n_2}'\}$ and then making v_i adjacent with v_i' for each $i=1,2,\dots,n_2$.

By the definition of Indu-Bala product, for every vertex u_i, v_j, u_i', and v_j' ($1\le i\le n_1$, $1\le j\le n_2$), it holds that

$d_{G_1\nabla G_2}(u_i)=d_{G_1\nabla G_2}(u_i')=d_{G_1}(u_i)+n_2$, for $1\le i\le n_1$;

$d_{G_1\nabla G_2}(v_j)=d_{G_1\nabla G_2}(v_j')=d_{G_2}(v_j)+n_1+1$, for $1\le j\le n_2$.

In this paper, the total irregularity of Indu-Bala product of graphs are computed and exact formula in terms of the total irregularities of the underlying graphs is derived.

2. MAIN RESULTS

Theorem 2.1. *Let G_1 and G_2 be graphs with n_1 and n_2 vertices, respectively. Then*

$$irr_t(G_1\nabla G_2)=4\left(irr_t(G_1)+irr_t(G_2)+\sum_{i=1}^{n_1}\sum_{j=1}^{n_2}\left|d_{G_1}(u_i)+n_2-\left(d_{G_2}(v_j)+n_1+1\right)\right|\right).$$

Proof. The vertex set of $G_1\nabla G_2$ can be partitioned into four subsets as

Corresponding Author: Faculty of Science, Department of Computer Science, Dokuz Eylul University,
35160, Izmir/TURKEY, zeynep.berberler@deu.edu.tr

Doi: 10.21541/apjes.376397

$$V_1=\{u_i\in V(G_1\nabla G_2):u_i\in V(G_1)\}\ \ (1\le i\le n_1),$$
$$V_2=\{v_i\in V(G_1\nabla G_2):v_i\in V(G_2)\}\ \ (1\le i\le n_2),$$
$$V_3=\{v_i'\in V(G_1\nabla G_2):v_i'\in V(G_2')\}\ \ (1\le i\le n_2),$$
$$V_4=\{u_i'\in V(G_1\nabla G_2):u_i'\in V(G_1')\}\ \ (1\le i\le n_1).$$

From the definition of graph total irregularity, it follows that

$$irr_t(G_1\nabla G_2)=\frac{1}{2}\sum_{\substack{u\in V_i,v\in V_j\\(1\le i,j\le 4)}}\left|d_{G_1\nabla G_2}(u)-d_{G_1\nabla G_2}(v)\right|.$$

The contribution of the vertices in V_1 to the total irregularity of $G_1\nabla G_2$ is given by

$$irr_{t_1}(G_1\nabla G_2)=\frac{1}{2}\sum_{\substack{u\in V_1,v\in V_i\\(1\le i\le 4)}}\left|d_{G_1\nabla G_2}(u)-d_{G_1\nabla G_2}(v)\right|.$$

We start to compute with

$$\frac{1}{2}\sum_{u,v\in V_1}\left|d_{G_1\nabla G_2}(u)-d_{G_1\nabla G_2}(v)\right|=\frac{1}{2}\sum_{i=1}^{n_1}\sum_{\substack{j=1\\j\ne i}}^{n_1}\left|d_{G_1\nabla G_2}(u_i)-d_{G_1\nabla G_2}(v_j)\right|.$$

By substituting the values of parameters in terms of the degrees of the vertices of G_1, we compute

$$=\frac{1}{2}\sum_{i=1}^{n_1}\sum_{\substack{j=1\\j\ne i}}^{n_1}\left|\left(d_{G_1}(u_i)+n_2\right)-\left(d_{G_1}(v_j)+n_2\right)\right|=\frac{1}{2}\sum_{i=1}^{n_1}\sum_{\substack{j=1\\j\ne i}}^{n_1}\left|d_{G_1}(u_i)-d_{G_1}(v_j)\right|$$
$$=\sum_{u,v\in V(G_1)}\left|d_{G_1}(u)-d_{G_1}(v)\right|=irr_t(G_1)\ . \tag{1}$$
$$\frac{1}{2}\sum_{u\in V_1,v\in V_2}\left|d_{G_1\nabla G_2}(u)-d_{G_1\nabla G_2}(v)\right|=\frac{1}{2}\sum_{i=1}^{n_1}\sum_{j=1}^{n_2}\left|d_{G_1\nabla G_2}(u_i)-d_{G_1\nabla G_2}(v_j)\right|.$$

By substituting the values of parameters in terms of the degrees of the vertices of G_1 and G_2, we receive

$$=\frac{1}{2}\sum_{i=1}^{n_1}\sum_{j=1}^{n_2}\left|\left(d_{G_1}(u_i)+n_2\right)-\left(d_{G_2}(v_j)+n_1+1\right)\right|. \tag{2}$$
$$\frac{1}{2}\sum_{u\in V_1,v\in V_3}\left|d_{G_1\nabla G_2}(u)-d_{G_1\nabla G_2}(v)\right|=\frac{1}{2}\sum_{i=1}^{n_1}\sum_{j=1}^{n_2}\left|d_{G_1\nabla G_2}(u_i)-d_{G_1\nabla G_2}(v_j')\right|.$$

Since $d_{G_1\nabla G_2}(v_i)=d_{G_1\nabla G_2}(v_i')$ for $\forall v_i\in V(G_2)$ and $\forall v_i'\in V(G_2')$ $(1\le i\le n_2)$, we get the same equality in (2).

$$\frac{1}{2}\sum_{u\in V_1,v\in V_4}\left|d_{G_1\nabla G_2}(u)-d_{G_1\nabla G_2}(v)\right|=\frac{1}{2}\sum_{i=1}^{n_1}\sum_{j=1}^{n_1}\left|d_{G_1\nabla G_2}(u_i)-d_{G_1\nabla G_2}(u_j')\right|.$$

Since $d_{G_1\nabla G_2}(u_i)=d_{G_1\nabla G_2}(u_i')$ for $\forall u_i\in V(G_1)$ and $\forall u_i'\in V(G_1')$, we get the same equality in (1).

$$irr_{t_1}(G_1\nabla G_2)=2irr_t(G_1)+\sum_{i=1}^{n_1}\sum_{j=1}^{n_2}\left|d_{G_1}(u_i)+n_2-\left(d_{G_2}(v_j)+n_1+1\right)\right|.$$

The contribution of the vertices in V_2 to the total irregularity of $G_1\nabla G_2$ is given by

$$irr_{t_2}(G_1\nabla G_2)=\frac{1}{2}\sum_{\substack{u\in V_2,v\in V_i\\(1\le i\le 4)}}\left|d_{G_1\nabla G_2}(u)-d_{G_1\nabla G_2}(v)\right|.$$

We start to compute with

$$\frac{1}{2}\sum_{u\in V_2,v\in V_1}\left|d_{G_1\nabla G_2}(u)-d_{G_1\nabla G_2}(v)\right|=\frac{1}{2}\sum_{i=1}^{n_2}\sum_{j=1}^{n_1}\left|d_{G_1\nabla G_2}(u_i)-d_{G_1\nabla G_2}(v_j)\right|$$
$$=\frac{1}{2}\sum_{i=1}^{n_2}\sum_{j=1}^{n_1}\left|\left(d_{G_2}(u_i)+n_1+1\right)-\left(d_{G_1}(v_j)+n_2\right)\right|. \tag{3}$$
$$\frac{1}{2}\sum_{u,v\in V_2}\left|d_{G_1\nabla G_2}(u)-d_{G_1\nabla G_2}(v)\right|=\frac{1}{2}\sum_{i=1}^{n_2}\sum_{\substack{j=1\\j\ne i}}^{n_2}\left|d_{G_1\nabla G_2}(u_i)-d_{G_1\nabla G_2}(v_j)\right|.$$

By substituting the values of parameters in terms of the degrees of the vertices of G_2, we receive

$$=\frac{1}{2}\sum_{i=1}^{n_2}\sum_{\substack{j=1\\j\ne i}}^{n_2}\left|\left(d_{G_2}(u_i)+n_1+1\right)-\left(d_{G_2}(v_j)+n_1+1\right)\right|=\frac{1}{2}\sum_{i=1}^{n_2}\sum_{\substack{j=1\\j\ne i}}^{n_2}\left|d_{G_2}(u_i)-d_{G_2}(v_j)\right|$$
$$=\frac{1}{2}\sum_{u,v\in V(G_2)}\left|d_{G_2}(u)-d_{G_2}(v)\right|=irr_t(G_2). \tag{4}$$
$$\frac{1}{2}\sum_{u\in V_2,v\in V_3}\left|d_{G_1\nabla G_2}(u)-d_{G_1\nabla G_2}(v)\right|=\frac{1}{2}\sum_{i=1}^{n_2}\sum_{j=1}^{n_2}\left|d_{G_1\nabla G_2}(u_i)-d_{G_1\nabla G_2}(u_j')\right|$$
$$=\frac{1}{2}\sum_{i=1}^{n_2}\sum_{j=1}^{n_2}\left|\left(d_{G_2}(u_i)+n_1+1\right)-\left(d_{G_2}(u_j)+n_1+1\right)\right|=\frac{1}{2}\sum_{i=1}^{n_2}\sum_{j=1}^{n_2}\left|d_{G_2}(u_i)-d_{G_2}(u_j)\right|$$
$$=\frac{1}{2}\sum_{u,v\in V(G_2)}\left|\left(d_{G_2}(u)-d_{G_2}(v)\right)\right|=irr_t(G_2). \tag{5}$$
$$\frac{1}{2}\sum_{u\in V_2,v\in V_4}\left|d_{G_1\nabla G_2}(u)-d_{G_1\nabla G_2}(v)\right|=\frac{1}{2}\sum_{i=1}^{n_2}\sum_{j=1}^{n_1}\left|d_{G_1\nabla G_2}(u_i)-d_{G_1\nabla G_2}(v_j')\right|$$
$$=\frac{1}{2}\sum_{i=1}^{n_2}\sum_{j=1}^{n_1}\left|\left(d_{G_2}(u_i)+n_1+1\right)-\left(d_{G_1}(v_j)+n_2\right)\right|. \tag{6}$$

By the equations (3), (4), (5) and (6), we compute

$$irr_{t_2}(G_1\nabla G_2)=2irr_t(G_2)+\sum_{i=1}^{n_2}\sum_{j=1}^{n_1}\left|d_{G_2}(u_i)+n_1+1-\left(d_{G_1}(v_j)+n_2\right)\right|.$$

Also, the contribution of the vertices in V_3 to the total irregularity of $G_1\nabla G_2$ is given by

$$irr_{t_3}(G_1\nabla G_2)=\frac{1}{2}\sum_{\substack{u\in V_3,v\in V_i\\(1\le i\le 4)}}\left|d_{G_1\nabla G_2}(u)-d_{G_1\nabla G_2}(v)\right|.$$

We start to compute with

$$\frac{1}{2}\sum_{u\in V_3,v\in V_1}\left|d_{G_1\nabla G_2}(u)-d_{G_1\nabla G_2}(v)\right|=\frac{1}{2}\sum_{i=1}^{n_2}\sum_{j=1}^{n_1}\left|d_{G_1\nabla G_2}(u_i')-d_{G_1\nabla G_2}(v_j)\right|$$
$$=\frac{1}{2}\sum_{i=1}^{n_2}\sum_{j=1}^{n_1}\left|\left(d_{G_2}(u_i)+n_1+1\right)-\left(d_{G_1}(v_j)+n_2\right)\right|. \tag{7}$$
$$\frac{1}{2}\sum_{u\in V_3,v\in V_2}\left|d_{G_1\nabla G_2}(u)-d_{G_1\nabla G_2}(v)\right|=\frac{1}{2}\sum_{i=1}^{n_2}\sum_{j=1}^{n_2}\left|d_{G_1\nabla G_2}(u_i')-d_{G_1\nabla G_2}(u_j)\right|$$
$$=\frac{1}{2}\sum_{i=1}^{n_2}\sum_{j=1}^{n_2}\left|\left(d_{G_2}(u_i)+n_1+1\right)-\left(d_{G_2}(u_j)+n_1+1\right)\right|=\frac{1}{2}\sum_{i=1}^{n_2}\sum_{j=1}^{n_2}\left|d_{G_2}(u_i)-d_{G_2}(u_j)\right|$$
$$=\frac{1}{2}\sum_{u,v\in V(G_2)}\left|d_{G_2}(u)-d_{G_2}(v)\right|=irr_t(G_2). \tag{8}$$

$$\frac{1}{2}\sum\nolimits_{u,v\in V_3}\left|d_{G_1\nabla G_2}(u)-d_{G_1\nabla G_2}(v)\right|=\frac{1}{2}\sum_{i=1}^{n_2}\sum_{\substack{j=1\\ j\neq i}}^{n_2}\left|d_{G_1\nabla G_2}(u_i')-d_{G_1\nabla G_2}(u_j')\right|$$

$$=\frac{1}{2}\sum_{i=1}^{n_2}\sum_{\substack{j=1\\ j\neq i}}^{n_2}\left|\left(d_{G_2}(u_i)+n_1+1\right)-\left(d_{G_2}(u_j)+n_1+1\right)\right|=\frac{1}{2}\sum_{i=1}^{n_2}\sum_{\substack{j=1\\ j\neq i}}^{n_2}\left|d_{G_2}(u_i)-d_{G_2}(u_j)\right|$$

$$=\frac{1}{2}\sum\nolimits_{u,v\in V(G_2)}\left|d_{G_2}(u)-d_{G_2}(v)\right|=irr_t(G_2). \tag{9}$$

$$\frac{1}{2}\sum\nolimits_{u\in V_3,v\in V_4}\left|d_{G_1\nabla G_2}(u)-d_{G_1\nabla G_2}(v)\right|=\frac{1}{2}\sum_{i=1}^{n_2}\sum_{j=1}^{n_1}\left|d_{G_1\nabla G_2}(u_i')-d_{G_1\nabla G_2}(v_j')\right|$$

$$=\frac{1}{2}\sum_{i=1}^{n_2}\sum_{j=1}^{n_1}\left|\left(d_{G_2}(u_i)+n_1+1\right)-\left(d_{G_1}(v_j)+n_2\right)\right|. \tag{10}$$

Hence, we receive

$$irr_{t_3}(G_1\nabla G_2)=2irr_t(G_2)+\frac{1}{2}\sum_{i=1}^{n_2}\sum_{j=1}^{n_1}\left|d_{G_2}(u_i)+n_1+1-\left(d_{G_1}(v_j)+n_2\right)\right|.$$

The contribution of the vertices in V_4 to the total irregularity of $G_1\nabla G_2$ is given by

$$irr_{t_4}(G_1\nabla G_2)=\frac{1}{2}\sum\nolimits_{\substack{u\in V_4,v\in V_i\\ (1\le i\le 4)}}\left|d_{G_1\nabla G_2}(u)-d_{G_1\nabla G_2}(v)\right|.$$

We start to compute with

$$\frac{1}{2}\sum\nolimits_{u\in V_4,v\in V_1}\left|d_{G_1\nabla G_2}(u)-d_{G_1\nabla G_2}(v)\right|=\frac{1}{2}\sum_{i=1}^{n_1}\sum_{j=1}^{n_1}\left|d_{G_1\nabla G_2}(u_i')-d_{G_1\nabla G_2}(u_j)\right|$$

$$=\frac{1}{2}\sum_{i=1}^{n_1}\sum_{j=1}^{n_1}\left|\left(d_{G_1}(u_i)+n_2\right)-\left(d_{G_1}(u_j)+n_2\right)\right|=\frac{1}{2}\sum_{i=1}^{n_1}\sum_{j=1}^{n_1}\left|d_{G_1}(u_i)-d_{G_1}(u_j)\right|$$

$$=\frac{1}{2}\sum\nolimits_{u,v\in V(G_1)}\left|d_{G_1}(u)-d_{G_1}(v)\right|=irr_t(G_1). \tag{11}$$

$$\frac{1}{2}\sum\nolimits_{u\in V_4,v\in V_2}\left|d_{G_1\nabla G_2}(u)-d_{G_1\nabla G_2}(v)\right|=\frac{1}{2}\sum_{i=1}^{n_1}\sum_{j=1}^{n_2}\left|d_{G_1\nabla G_2}(u_i')-d_{G_1\nabla G_2}(v_j)\right|$$

$$=\frac{1}{2}\sum_{i=1}^{n_1}\sum_{j=1}^{n_2}\left|\left(d_{G_1}(u_i)+n_2\right)-\left(d_{G_2}(v_j)+n_1+1\right)\right|. \tag{12}$$

$$\frac{1}{2}\sum\nolimits_{u\in V_4,v\in V_3}\left|d_{G_1\nabla G_2}(u)-d_{G_1\nabla G_2}(v)\right|=\frac{1}{2}\sum_{i=1}^{n_1}\sum_{j=1}^{n_2}\left|d_{G_1\nabla G_2}(u_i')-d_{G_1\nabla G_2}(v_j')\right|.$$

Since $d_{G_1\nabla G_2}(v_j)=d_{G_1\nabla G_2}(v_j')$ for $\forall v_j\in V(G_2)$ and $\forall v_j'\in V(G_2')$ $(1\le j\le n_2)$, we get the same equality in (12).

$$\frac{1}{2}\sum\nolimits_{u,v\in V_4}\left|d_{G_1\nabla G_2}(u)-d_{G_1\nabla G_2}(v)\right|=\frac{1}{2}\sum_{i=1}^{n_1}\sum_{j=1}^{n_1}\left|d_{G_1\nabla G_2}(u_i')-d_{G_1\nabla G_2}(u_j')\right|$$

$$=\frac{1}{2}\sum_{i=1}^{n_1}\sum_{j=1}^{n_1}\left|\left(d_{G_1}(u_i)+n_2\right)-\left(d_{G_1}(u_j)+n_2\right)\right|=\frac{1}{2}\sum_{i=1}^{n_1}\sum_{j=1}^{n_1}\left|d_{G_1}(u_i)-d_{G_1}(u_j)\right|$$

$$=\frac{1}{2}\sum\nolimits_{u,v\in V(G_1)}\left|d_{G_1}(u)-d_{G_1}(v)\right|=irr_t(G_1). \tag{13}$$

Hence,

$$irr_{t_4}(G_1\nabla G_2)=2irr_t(G_1)+\sum_{i=1}^{n_1}\sum_{j=1}^{n_2}\left|\left(d_{G_1}(u_i)+n_2\right)-\left(d_{G_2}(v_j)+n_1+1\right)\right|.$$

Summing the contributions of the vertex sets V_1, V_2, V_3 and V_4, we finally obtain the desired result of $irr_t(G_1\nabla G_2)=\sum_{i=1}^{4}irr_{t_i}(G_1\nabla G_2)$. Thus, the proof holds.

3. CONCLUDING REMARKS

Graph products play a significant role in pure and applied mathematics, and computer science and many of the problems can be easily handled if the related underlying graphs are regular or close to regular [4]. Therefore in many applications and problems, it is of great importance to know how irregular a given graph is.

We focus our investigation to the study of how the total irregularity of a graph changes with operations based on graph products. Indu-Bala product of graphs is a novel graph operation. In this paper, we consider the total irregularity of simple undirected graphs under Indu-Bala product. Exact formula is given to compute the total irregularity of Indu-Bala product of graphs in terms of the total irregularities and vertex degrees of underlying graphs.

REFERENCES

[1].H. Abdo, S. Brandt and D. Dimitrov, "The total irregularity of a graph", Discrete Math. Theor. Comput. Sci., vol. 16, no. 1, pp. 201-206, 2014.

[2].B. Zhou, "On irregularity of graphs", Ars Combin., vol. 88, pp. 55-64, 2008.

[3].D. Dimitrov and R. Škrekovski, "Comparing the irregularity and the total irregularity of graphs", Ars Math. Contemp., vol. 9, pp. 45–50, 2015.

[4].H. Abdo and D. Dimitrov, "The irregularity of graphs under graph operations", Discuss. Math. Graph Theo., vol. 34, no. 2, pp. 263–278, 2014.

[5].H. Abdo and D. Dimitrov, "The total irregularity of graphs under graph operations", Miskolc Math. Notes, vol. 15, pp. 3–17, 2014.

[6].H. Abdo and D. Dimitrov. "The Total Irregularity of Some Composite Graphs", International Journal of Computer Applications, vol. 122, no. 21, pp. 1-9, 2015.

[7].H. Abdo, N. Cohen and D. Dimitrov, "Graphs with maximal irregularity", Filomat, vol. 28, no. 7, pp. 1315-1322, 2014.

[8].M.A. Henning and D. Rautenbach, "On the irregularity of bipartite graphs", Discrete Math., vol. 307, pp. 1467-1472, 2007.

[9].M.O. Albertson, "The irregularity of a graph", Ars Combin, vol. 46, pp. 219–225, 1997.

[10]. M. Tavakoli, F. Rahbarnia and A.R. Ashrafi, "Some new results on irregularity of graphs", J. Appl. Math. Inform., vol. 32, pp. 675-685, 2014.

[11]. W. Luo and B. Zhou, "On the irregularity of trees and unicyclic graphs with given matching number", Util. Math., vol. 83, pp. 141-147, 2010.

[12]. L.H. You, J.S. Yang and Z.F. You, "The maximal total irregularity of unicyclic graphs", Ars Comb., vol. 114, pp. 153–160, 2014.

[13]. L.H. You, J.S. Yang, Y.X. Zhu and Z.F. You, "The maximal total irregularity of bicyclic graphs", Journal of Applied Mathematics 2014, Article ID 785084, http://dx.doi.org/10.1155/2014/785084.

[14]. G. Indulal and R. Balakrishnan, “Distance spectrum of Indu-Bala product of graphs”, AKCE International Journal of Graphs and Combinatorics, vol. 13, pp. 230-234, 2016.