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THE ANALYSIS OF THE TWO PARAMETER EXPONENTIAL DISTRIBUTION BASED ON PROGRESSIVE TYPE II CENSORED DATA

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Abstract: In this study, the estimation of the parameters of the two parameters exponential distribution based on progressive Type II censored data is considered. Assuming both location and scale parameters to be unknown, the maximum likelihood(ML), penalized maximum likelihood (PML) and Bayes estimators are obtained. The results indicate that the PMLE is the same as the uniformly minimum variance unbiased estimator (UMVUE). Further, the mean square errors of proposed estimators both analytical and Monte Carlo simulation study for different types of censoring schemes are computed. The results reveal that the PMLE and Bayes estimators outperform the MLE ones. This paper comes to an end by presenting an illustrative example.

Key words: Bayes Estimators; Exponential Distribution; Maximum Likelihood Estimators; Penalized Maximum Likelihood Estimators; Progressive Type II Censored Data.

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1. Introduction

The two parameters exponential distribution has many real world applications. It can be used to model the data such as service time of agents in a system (Queuning Theory), the time it takes before your next telephone call, the time until a radioactive particle decays, the distance between mutations on a DNA strand, and the extreme values of annual snowfall or rainfall. The probability density function (p.d.f) of the two parameters exponential distribution is given by:

$$f_{\eta,\theta}(x) = \frac{1}{\theta}e^{-\frac{(x-\eta)}{\theta}}, \quad x > \eta, \theta > 0, \eta \in \mathbf{R}. \tag{1.1}$$

Due to its importance, the estimation of the parameters of this distribution have been considered by many authors. [8] and [6] considered the earlier studied upon the estimation of exponential parameters. More recent works can also be found in [7].

In reliability and survival analysis studies, the experimenter may not always obtain complete information on failure times for all experimental units. Therefore, the censored data obtained from such experiments and censoring occurs commonly. Recently, progressive censoring scheme introduced has gained special attention in theoretical and applied statistics due to the flexibility in removing the pre-specified number of item during the test. Suppose n items are put in the life test. At the time of the first failure, R_1 of the surviving $n-1$ items are removed randomly from the test. Then after the second failure R_2 out of $n-R_1-2$ remaining survived items are removed. This test continued until the m^{th} failure occur. At this stage, all the remaining $R_m = n-R_1-R_2-...-R_{m-1}-m$ survived units withdrawn from the test. Which m and $\mathbf{R}=(R_1,R_2,...,R_m)$ are prefixed. Many inferences and studies have been done based on progressive censored data. One

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may refer to [4], [1], [3] and [5] for more details.
In section 2 the maximum likelihood, Penalized maximum likelihood and Bayes estimators for both of the unknown parameters were considered. Section 3 contains the simulation study for assessing the performance of the proposed estimators. At the end of paper a summary and conclusion are presented.

2. Proposed Estimators

In this section we obtain the Maximum likelihood (ML) and penalized maximum likelihood PML estimators of the parameters based on Progressive Type II censored data. Let $x_1 = x_{1:m:n}, x_2 = x_{2:m:n}, ..., x_m = x_{m:m:n}$ are the progressive Type II censored data from a continuous population X with the p.d.f $f(.)$ and c.d.f $F(.)$, then the likelihood of the observations is given as:

$$L(\theta,\eta) = A\prod_{i=1}^{m} f(x_i,m,n)[1-F(x_i,m,n)]^{r_i}, \tag{2.1}$$

where

$$A = n(n-r_1-1)\cdots(n-\sum_{i=1}^{m-1}(1+r_i)).$$

2.1. ML Estimators

For obtaining the MLE,

$$L(\theta,\eta) = A\frac{1}{\theta^m}e^{-\frac{1}{\theta}\sum_{i=1}^{m}(1+r_i)(x_i-\eta)} \qquad ; x_i \geq \eta. \tag{2.2}$$

And logarithm equation (2.2) is:

$$lnL(\theta,\eta) = \ln A - mln\theta - \frac{1}{\theta}\sum_{i=1}^{m}(1+r_i)(x_i-\eta); \qquad x_i \geq \eta \tag{2.3}$$

The equation (2.3) is maximized with respect to η by taking $\eta = x_{(1)}$. To get the MLE for θ, the following equation should be solved.

$$\frac{\partial}{\partial\theta}lnL(\theta,\eta) = -\frac{m}{\theta^2} - \frac{1}{\theta^2}\sum_{i=1}^{m}(1+r_i)(x_i-\hat{\eta}) = 0 \tag{2.4}$$

Therefore (2.5) and (2.6) are obtained.

$$\hat{\eta} = x_{(1)} \tag{2.5}$$

$$\hat{\theta} = \frac{1}{m}\sum_{i=1}^{m}(1+r_i)(x_i-\hat{\eta}). \tag{2.6}$$

THEOREM 1. *Let $x_{1:m:n}, \cdots, x_{m:m:n}$ be order statistics form $EXP(\theta,\eta)$. Then*

$$Z_1 = n(x_{1:m:n}-\eta), ..., Z_m = (n-\sum_{i=1}^{m-1} r_i - m)(x_{m:m:n} - x_{m-1:m:n}), \tag{2.7}$$

$1 \leq m \leq n-1$ *are independent and identically distributed with common distribution $EXP(\theta)$.*

PROOF. The proof can be found in [1].
Now, by making use of 1,the expectation, bias and the variance of $\hat{\theta}$ and $\hat{\eta}$ are as follows

$$E(\hat{\theta}) = \theta - \frac{\theta}{m}, \tag{2.8}$$

$$Var(\hat{\theta}) = \frac{m-1}{m^2}\theta^2, MSE(\hat{\theta}) = \frac{\theta^2}{m}. \tag{2.9}$$

2.2. Penalized MLE

The most commonly used method of estimation is Maximum Likelihood Estimation. Although, under some regularity conditions, the MLE method has proper properties such as consistency and efficiency. But it is too conservative because it always chooses the minimum of the sample to estimate the location parameter. The penalized maximum likelihood estimators for both parameters will be Uniformly Minimum Variance Unbiased Estimators (UMVUE).

$$L^*(\theta,\eta) = A(x_1-\eta)\frac{1}{\theta^m}e^{-\frac{1}{\theta}\sum_{i=1}^m(1+r_i)(x_i-\eta)} \qquad ;x_i \geq \eta \tag{2.10}$$

Logarithm of the likelihood function is

$$lnL^*(\theta,\eta) = \ln A + \ln(x_1-\eta) - m\ln\theta - \frac{1}{\theta}\sum_{i=1}^m(1+r_i)(x_i-\eta); x_i \geq \eta \tag{2.11}$$

By solving the following equations

$$\begin{cases} \frac{\partial}{\partial\theta}lnL^*(\theta,\eta) &= -\frac{m}{\theta}+\frac{1}{\theta^2}\sum_{i=1}^m(1+r_i)(x_i-\eta)=0 \\ \frac{\partial}{\partial\eta}lnL^*(\theta,\eta) &= -\frac{1}{x_{(1)}-\eta}+\frac{1}{\theta}\sum_{i=1}^m(1+r_i)=0, \end{cases} \tag{2.12}$$

It is easy to verify that the PMLEs are

$$\Rightarrow \begin{cases} \theta^* &= \frac{1}{m-1}\sum_{i=1}^m(1+r_i)(x_{(i)}-x_{(1)}) \\ \eta^* &= x_{(1)} - \frac{\theta^*}{n} \end{cases} \tag{2.13}$$

Considering the PMLEs are exactly the UMVUE! Now, by using theorem 1, the expectation of θ^* is:

$$E(\theta^*) = \frac{1}{m-1}E(\sum_{i=1}^m(1+r_i)(x_{(i)}-x_{(1)})) = \frac{1}{m-1}(m-1)\theta = \theta.$$

Thus θ^* is unbiased estimator of θ. Also, The variance of θ^* is:

$$Var(\theta^*) = \frac{1}{(m-1)^2}(m-1)\theta^2 = \frac{\theta^2}{m-1}.$$

Consider that $x_{(1)}$ and θ^* are independent, and the expectation of η^* is given by

$$E(\eta^*) = E(x_{(1)}) - \frac{E(\theta^*)}{n} = \frac{\theta}{n}+\eta-\frac{\theta}{n} = \eta.$$

So η^* is an unbiased estimator of η. The η^* variance is:

$$Var(\eta^*) = \frac{m\theta^2}{n^2(m-1)}.$$

2.3. Bayes Estimators

In this part the estimation in the Bayes context will be considered. In this regard, the following prior distribution to the parameters will be applied as follows:

$$\pi_1(\eta|\theta) \propto \frac{1}{\theta}$$

and

$$\pi_2(\theta) \propto \theta^{-(\alpha+1)}e^{-\frac{\beta}{\theta}}, \qquad \theta>0, \alpha>0, \beta>0$$

The posterior distribution can easily be found as:

$$\pi(\theta,\eta|x) \propto \frac{1}{\theta^{m+\alpha+2}} e^{-\frac{1}{\theta}[\sum_{i=1}^{m}(1+r_i)(x_i-\eta)+\beta]}, \qquad \theta > 0, \eta \leq x_{(1)}.$$

By taking the marginal integration, the marginal posterior distribution can be computed as:

$$\pi(\theta|x) = \int \pi(\theta,\eta|x)d\eta$$

Thus,

$$\pi(\theta|x) \propto \frac{1}{\theta^{m+\alpha+1}} e^{-\frac{1}{\theta}[\sum_{i=1}^{m}(1+r_i)(x_i-x_{(1)})+\beta]}.$$

It is easy to verify the marginal posterior distribution of $\theta|x$ is an Inverse gamma. That is:

$$\theta|x \sim IG(m+\alpha, \sum_{i=1}^{m}(1+r_i)(x_i-x_{(1)})+\beta).$$

Thus, under the squared error loss (SEL), the Bayes estimators of the scale parameter can be found as

$$\delta_\theta^{SEL} = E(\theta|x) = \frac{\sum_{i=1}^{m}(1+r_i)(x_i-x_{(1)})+\beta}{m+\alpha-1},$$

With the similar discussion, the posterior distribution of η given x can be computed as:

$$\pi(\eta|x) = \frac{n(m+\alpha)\left(\sum_{i=1}^{m}(1+r_i)(x_i-x_{(1)})+\beta\right)^{m+\alpha}}{\left(\sum_{i=1}^{m}(1+r_i)(x_i-\eta)+\beta\right)^{m+\alpha+1}},$$

Therefore under SEL the Bayes estimator of the location parameter can be obtained as:

$$E(\eta|x) = \int \eta\pi(\eta|x)d\eta = \int_{-\infty}^{x_{(1)}} \eta \frac{n(m+\alpha)\left(\sum_{i=1}^{m}(1+r_i)(x_i-x_{(1)})+\beta\right)^{m+\alpha}}{\left(\sum_{i=1}^{m}(1+r_i)(x_i-\eta)+\beta\right)^{m+\alpha+1}} d\eta =$$

$$\underbrace{(m+\alpha)\left(\sum_{i=1}^{m}(1+r_i)(x_i-x_{(1)})+\beta\right)^{m+\alpha}}_{A} \int_{-\infty}^{x_{(1)}} \frac{n\eta}{\underbrace{\left(\sum_{i=1}^{m}(1+r_i)(x_i-\eta)+\beta\right)}_{u}^{m+\alpha+1}} d\eta$$

$$\begin{cases} du = -n d\eta \\ \eta = \frac{\sum_{i=1}^{m}(1+r_i)x_i+\beta-u}{n} \end{cases}$$

$$= A\int -\frac{\sum_{i=1}^{m}(1+r_i)x_i+\beta-u}{nu^{m+\alpha+1}} du = A\left(\frac{\sum_{i=1}^{m}(1+r_i)x_i+\beta}{n(m+\alpha)u^{m+\alpha}} - \frac{1}{n(m+\alpha-1)u^{m+\alpha-1}}\right)$$

$$\delta_\eta^{SEL} = E(\eta|x) = x_{(1)} - \frac{\sum_{i=1}^{m}(1+r_i)(x_i - x_{(1)}) + \beta}{n(m+\alpha-1)}.$$

It is obvious that the risk of the proposed Bayes estimators can not be achieved analytically. Consequently, a simulation study will be conducted in next section.

3. Simulation Study

In order to assess the performance of proposed estimators a Monte Carlo simulation study for different values of n, m and R will be conducted. It is obvious that the MSEs of the MLE and PMLE are free from $\mathbf{R}$ which is ambiguous in practical situation to choose appropriate progressive censoring scheme. The Bayes estimators of the parameters can not be computed analytically, so the risk of the estimators is computed by simulation through following steps:

Algorithm:

1. Based on the Algorithm in [2], the progressive Type II censored sample for the given values of R, n, m, η and θ can be generated.
2. Compute The values of the proposed estimators based on the sample in step 1.
3. Repeat the previous steps for 1000 times and then obtain the average and variance of the estimators from 1000 repeatations.

3.1. Simulation results

The simulation results are given in table 2-5 based on the schemes from table 1 while assuming $\alpha = \beta = 1$. The followings can be concluded for the $\hat{\eta}$ and $\hat{\theta}$ respectively,

• Generally, both the MSE and biases of Bayes estimators are less than MSE's and biases of MLEs. While the PMLEs has the least MSE and biases. Also the MSE's of various schemes are approximately the same. Further the Bayes estimators have always positive biases. Finally, as m/n increases, the values of MSEs decreases.

• The Bayes estimators have less biases than the MLEs but slightly less than the PMLEs. Moreover for a fixed values of n, as m increases, the biases and MSEs become smaller. Finally, for a large sample size of n all proposed estimators have similar behaviours.

TABLE 1. progressive censor with n=20

$\theta=0,\eta=1$			MLE		$PMLE$		$Bayes$	
m	$r\ vector$		$\hat{\eta}$	$\hat{\theta}$	$\hat{\eta}$	$\hat{\theta}$	$\hat{\eta}$	$\hat{\theta}$
6	$(14,0^{*5})$	$Bias$	0.0499	-0.1556	$-7.5e-05$	0.0133	$-.0006$	0.011
		MSE	0.005	0.1623	0.0025	0.1991	0.0027	0.1382
	$(0^{*5},14)$	$Bias$	0.0515	-0.1429	0.0015	0.0286	0.0003	0.0238
		MSE	0.0051	0.1567	0.0015	0.1971	0.0028	0.1368
	$(3,2^{*4},3)$	$Bias$	0.0504	-0.165	0.0004	0.0019	0.0003	0.0016
		MSE	0.005	0.1651	0.0024	0.1985	0.0028	0.1378
8	$(12,0^{*7})$	$Bias$	0.052	-0.1264	0.002	-0.0016	0.002	-0.0014
		MSE	0.0053	0.1196	0.0026	0.1353	0.0029	0.1036
	$(0^{*7},12)$	$Bias$	0.0515	-0.1131	0.0015	0.0136	0.0009	0.0119
		MSE	0.0051	0.1149	0.0025	0.1335	0.0027	0.1022
	$(3,1^{*6},3)$	$Bias$	0.0491	-0.1116	-0.0008	0.0153	-0.0015	0.0134
		MSE	0.0048	0.1215	0.0024	0.1427	0.0027	0.1093

TABLE 2. progressive censor with n=30

$\theta=0,\eta=1$			MLE		PMLE		Bayes	
m	r vector		$\hat{\eta}$	$\hat{\theta}$	$\hat{\eta}$	$\hat{\theta}$	$\hat{\eta}$	$\hat{\theta}$
6	$(24,0^{*^5})$	Bias	0.0322	−0.1308	−0.0011	0.043	−0.0023	0.0358
		MSE	0.0019	0.1686	0.0009	0.22	0.001	0.1528
	$(0^{*^5},24)$	Bias	0.0321	−0.1683	−0.0012	−0.002	−0.0012	−0.0016
		MSE	0.0019	0.1674	0.0009	0.2002	0.001	0.139
	(4^{*^6})	Bias	0.034	−0.1666	0.0006	$9.6e-05$	0.0006	$7.99e-05$
		MSE	0.0023	0.1665	0.0012	0.1998	0.0013	0.1387
8	$(22,0^{*^7})$	Bias	0.0334	−0.1243	0.0001	0.0007	$9.5e-05$	0.0006
		MSE	0.0022	0.122	0.0011	0.1392	0.0012	0.1066
	$(0^{*^7},22)$	Bias	0.032	−0.1294	−0.0012	−0.005	−0.0011	−0.0044
		MSE	0.0019	0.1341	0.0009	0.1533	0.001	0.1173
	$(2,3^{*^6},2)$	Bias	0.0337	−0.1199	0.0004	0.0058	0.0002	0.0051
		MSE	0.0023	0.1277	0.0011	0.148	0.0012	0.1133
10	$(20,0^{*^9})$	Bias	0.035	−0.1131	0.0017	−0.0146	0.0021	−0.0131
		MSE	0.0025	0.0956	0.0012	0.1024	0.0014	0.0829
	$(0^{*^9},20)$	Bias	0.0358	−0.0811	0.0025	0.0209	0.0018	0.0188
		MSE	0.0025	0.0971	0.0013	0.1122	0.0014	0.0908
	$(2^{*^{10}})$	Bias	0.0326	−0.0986	−0.0007	0.0015	−0.0007	0.0014
		MSE	0.002	0.1024	0.001	0.1144	0.0011	0.0926

TABLE 3. progressive censor with n=50

$\theta=0,\eta=1$			MLE		PMLE		Bayes	
m	r vector		$\hat{\eta}$	$\hat{\theta}$	$\hat{\eta}$	$\hat{\theta}$	$\hat{\eta}$	$\hat{\theta}$
8	$(42,0^{*^7})$	Bias	0.021	−0.1259	0.001	−0.0011	0.0009	−0.0009
		MSE	0.0009	0.128	0.0004	0.1465	0.0004	0.1121
	$(0^{*^7},42)$	Bias	0.0195	−0.1224	−0.0005	−0.0029	−0.0005	0.0025
		MSE	0.008	0.1184	0.0004	0.135	0.0004	0.1033
	$(6,5^{*^6},6)$	Bias	0.0202	−0.1382	0.0002	0.0151	0.0004	−0.0132
		MSE	0.0008	0.1147	0.0004	0.1251	0.0004	0.0958
10	$(40,0^{*^9})$	Bias	0.0206	−0.0929	0.0006	0.0079	0.0005	0.0071
		MSE	0.0008	0.1038	0.0004	0.1175	0.0004	0.0952
	$(0^{*^9},40)$	Bias	0.0195	−0.1016	−0.0005	−0.0017	−0.0004	−0.0015
		MSE	0.0008	0.0972	0.0004	0.1072	0.0004	0.0868
	$(4^{*^{10}})$	Bias	0.0202	−0.1057	0.0002	0.0063	0.0003	−0.0057
		MSE	0.0008	0.0952	0.0004	0.1037	0.0004	0.084
15	$(35,0^{*^{14}})$	Bias	0.0209	−0.0828	0.0009	−0.0173	0.0012	−0.0161
		MSE	0.0009	0.0656	0.0004	0.0677	0.0004	0.059
	$(0^{*^{14}},35)$	Bias	0.0194	−0.0739	0.0005	−0.0078	−0.0004	−0.0072
		MSE	0.0007	0.0654	0.0003	0.0689	0.0003	0.06
	$(5,2^{*^{13}},4)$	Bias	0.0207	−0.06666	0.0007	$5.13e-06$	0.0006	$4.7e-06$
		MSE	0.0008	0.0708	0.0004	0.0762	0.0004	0.0664

TABLE 4. progressive censor with n=100

$\theta=0,\eta=1$			*MLE*		*PMLE*		*Bayes*	
m	$r\,vector$		$\hat{\eta}$	$\hat{\theta}$	$\hat{\eta}$	$\hat{\theta}$	$\hat{\eta}$	$\hat{\theta}$
15	$(85,0^{*^{14}})$	*Bias*	0.0099	−0.0618	−0.0001	0.0052	−0.0001	0.0048
		MSE	0.0002	0.0737	0.0001	0.0803	0.0001	0.07
	$(0^{*^{14}},85)$	*Bias*	0.0106	−0.0776	0.0006	−0.0117	0.0007	−0.0109
		MSE	0.0002	0.0681	0.0001	0.0714	0.0001	0.0622
	$(10,5^{*^{13}},10)$	*Bias*	0.01	−0.0708	$6.34e-05$	−0.0044	0.0001	−0.0041
		MSE	0.00019	0.0729	$9.6e-05$	0.078	0.0001	0.0679
20	$(80,0^{*^{19}})$	*Bias*	0.0098	−0.0524	−0.0002	−0.0026	−0.0002	−0.0024
		MSE	0.00018	0.0502	$8.9e-05$	0.0526	$9.3e-05$	0.0474
	$(0^{*^{19}},80)$	*Bias*	0.0106	−0.0661	−0.0006	−0.0169	0.0007	−0.0161
		MSE	0.0002	0.0499	0.0001	0.0508	0.0001	0.0458
	$(4^{*^{20}})$	*Bias*	0.0097	−0.0548	−0.0002	−0.0051	−0.0002	−0.0048
		MSE	0.00018	0.0504	$8.9e-05$	0.0525	$9.3e-05$	0.0474
25	$(75,0^{*^{24}})$	*Bias*	0.0105	−0.0388	0.0005	0.0012	0.0005	0.0011
		MSE	0.0002	0.0415	0.0001	0.0434	0.0001	0.04
	$(0^{*^{24}},75)$	*Bias*	0.0104	−0.0517	0.0004	−0.0122	0.0005	−0.0117
		MSE	0.0002	0.0412	0.0001	0.042	0.0001	0.0387
	$(3^{*^{25}})$	*Bias*	0.0098	−0.0391	−0.0002	0.0009	−0.0002	0.0008
		MSE	0.00019	0.0377	$9.3e-05$	0.0393	$9.8e-05$	0.0362

3.2. Real Example

Proschan (1963) presented data on intervals between failures (in hours) of the airconditioning system of a fleet of 13 Boeing 720 jet airplanes. After analyzing the data, he observed that the failure distribution of the air-conditioning system for each of the planes was well approximated by exponential distributions. Here, for the purpose of illustration, the planes 7912 will be chosen and the corresponding failure time data are presented as follows:
23, 261, 87, 7, 120, 14, 62, 47, 225, 71, 246, 21, 42, 20, 5, 12, 120, 11, 3, 14, 71, 11, 14, 11, 16, 90, 1, 16, 52, 95.
For the illustration purpose, we add 5 to all of the data to get the two parameter exponential with $\eta = 5$ and $\theta = 59.6$. Different schemes are considered in the following table and the values of proposed estimators are obtained as:

TABLE 5. progressive censor real data with n=30

$\theta = 59.6, \eta = 5$		MLE		$PMLE$		$Bayes(\alpha = 2, \beta = 58)$
m	$r\ vector$	$\hat{\eta}$	$\hat{\theta}$	$\hat{\eta}$	$\hat{\theta}$	$\hat{\eta}$	$\hat{\theta}$
5	$(5*^5)$	6	46.8	4.05	58.5	4.38	48.67
	$(0*^3, 10, 15)$	6	46.4	4.067	58	4.39	48.33
10	$(2*^{10})$	6	44.1	4.37	49	4.48	45.36
	$(0*^8, 10, 10)$	6	33.9	4.74	37.66	4.79	36.09

It can seen that the PMLE of the scale parameter is better than the Bayes and MLE. Also the MLE of the location paramater over estimate the true value 5, while the Bayes and PMLE under estimate it, but it is very close to the true value 5.

4. Conclusion

This paper considered the estimation of the parameters of two parameters exponential distribution based on Type II progressive censored data. The MLE, PMLE and Bayes estimators is especially proposed for both location and scale parameters under SEL loss function. The results indicated that the PMLE is the same as uniformly minimum variance unbiased estimator (UMVUE). The simulation study, also revealed that the PMLE and Bayes estimators are better than the MLEs. So the use of these two estimators instead of MLE is recommended to the practitioners.

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