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ECONOMIC AND ECONOMIC-STATISTICAL DESIGN OF FRS BAYESIAN CONTROL CHART USING MONTE CARLO METHOD AND ARTIFICIAL BEE COLONY ALGORITHM (ABC)

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Abstract: In this paper, instead of traditional statistics, the Bayesian statistic has been considered in control charts based on economic and economic-statistical design. Due to the fact that this statistic does not have any specified distribution, the Monte Carlo method and artificial bee colony (ABC) algorithm have been utilized in order to obtain design optimum parameters (sample size, sampling interval and control limit). The study indicates that the Bayesian statistic performance of control charts can be effective in this optimization problem. In addition, Numerical and comparison section are presented to support this proposition.

Key words: Artificial Bee colony (ABC) Algorithm; Bayesian Control Chart; Economic-statistical Design; Monte Carlo Method.
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1. Introduction

The X-bar chart is a proven statistical process control tool that is applied to reduce the variability of a process. Parameters that must be selected by control chart designer are the sample size n, the sampling interval h, and the width of the control limit k.

The statistical design strategy is to choose design parameters in order to maximize the time between false alarms and minimize the time to detect an off-target process. Statistically designed control charts have desirable statistical properties, but the operating cost can be quite high.

To reduce the total costs involved, many researchers have considered an economic criterion in design of a control chart ([4], [5], [13] and [1]). The economic approach involves developing a cost function that considers all cost components related to monitoring and controlling a process. The design parameters are chosen such that the cost function could be minimized. This method is called The economic design of control charts in the quality control literature. Focusing on a single factor while ignoring others, can be contributed to keep us away from optimization.

In order to solve this problem, [17] applied some statistical constraints in order to minimize average cost in per unit of time, and proposed economic-statistical designs. Regards to the designers needs, these constraints are based upon different statistical performance measures (For example, Type I error, power of the chart, adjusted average time to signal (AATS) and average number of false alarm (ANF)). As a matter of fact, economic-statistical designs are economic designs that some statistical limitations have been applied on them. Several authors have followed this scheme ([24], [15], [23] and [7].

Traditional non-Bayesian Control charts are widely used to establish and maintain statistical control of a process. However, [21, 22] has shown that these control techniques are not optimal.

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Initial efforts to Bayesian process control (not directly to a control chart) was raised by [8], [6] and [16]. More recent endeavors on Bayesian control chart are done by [18, 19], [2], [20] and [14].

In these papers, first of all, it has been presented to the economic design without statistical considerations. Second, since the Bayesian statistic does not have specified distribution, design parameters have been obtained based on some special constraints and not in the general case. Finally, production run has been considered finite.

In this paper, based on economic and economic-statistical design, parameters of Bayesian control chart have been obtained far from the above limitations. In numerical section, the newer and faster ABC algorithm has been utilized which is also more accurate than the most commonly used one (Genetic algorithm). The paper is organized as follows: Next section has a brief review of Bayesian control chart. The economic and economic-statistical designs of Bayesian control chart are described in the section 3. In Section 4, the optimization procedure has been discussed by using ABC algorithm. In section 5, design optimum parameters are obtained based on two above designs by numerical method and economic and economic-statistical design of Bayesian chart is compared with traditional non-Bayesian approaches. Also to look at the effects of statistical properties, the two discussed designs were compared. The last section provides some concluding remarks.

2. Bayesian scheme

A machine that produces some items, is possessing a measurable quality characteristic that can be in one of two states: An in-control state (0) when only common causes of variation are presented, and out-of control state (1) when an assignable cause is presented. Random samples of size n are taken from the process at any h time units. The length of time the process stays in control is exponentially distributed with mean $\frac{1}{\lambda}$. The process starts in control and while it remains in that state, is normally distributed with mean μ^0 and the standard deviation σ.Without loss of generality, in this paper is considered $\mu^0 = 0$ and $\sigma = 1$. The process is subject to the one assignable cause that shifts the mean of the quality characteristic (process mean) to $\mu^1 = \mu^0 + \delta\sigma$, without affecting the standard deviation (without loss of generality $\delta > 0$).

The statistic of Bayesian control chart is defined as the posterior probability that process be in-control which denoted by ξ_t at the tth stage of sampling. Thus, $\xi_t = P(\mu_t = \mu^0 \big| X^{(t)})$, where $X^{(t)}$ is all observation till time t. According to properties of Markov chain, it is simply to show that:

$$\xi_t = \frac{\xi_{t-1} e^{-\lambda h} f(X_t \big| \mu_t = \mu^0)}{\xi_{t-1} e^{-\lambda h} f(X_t \big| \mu_t = \mu^0) + (1 - e^{-\lambda h} \xi_{t-1}) f(X_t \big| \mu_t = \mu^1)}$$

where $f(X_t \big| \mu_t = \mu^0)$ and $f(X_t \big| \mu_t = \mu^1)$ are density functions of X_t given the process is in-control and out-of-control, respectively. It is obvious that $0 < \xi_t < 1$. Production continues as long as $\xi_t > k$, where k is control limit. Thus parameters of a Bayesian control chart are n, h and k that should be determined.

3. Economic and Economic-statistical design of Bayesian scheme

Economic-statistical design that used in this paper is economic model that was proposed by [3] with some statistical constraints in terms of AATS and ANF. According to this model, the process cycle consists of the following phases: in control (phases 1), out of control (phases 2), assignable cause detection (phases 3), and repair (phases 4). Let ATC be the average time from the start of production until the first signal after the process shift. Therefore, the expected length of a production cycle is given by

$$E(T) = ATC + T_0 \times ANF + T_1 \tag{3.1}$$

where T_0 is the average time taken for a false alarm, when the process is in control and T_1 is the average time to find and remove the assignable cause.

The expected net profit from a production cycle is given by

$$E(P) = \frac{V_0}{\lambda} + V_1 \times AATS - a_3 - a_3' \times ANF - a_2 \times ANI \tag{3.2}$$

where V_0 is the average profit per hour earned when the process is operating in control, V_1 is the average profit per hour earned when the process is operating out of control, AATS is the average time to signal after process shift, a_3 is the average cost to detect and remove the assignable cause, a'_3 is the average consequence cost of a false alarm, a_2 is the cost per inspected item and ANI is average number of inspected items per cycle. The loss function , or average production cycle cost per hour is given by

$$E(L) = V_0 - \frac{E(C)}{E(T)} = V_0 - \frac{\frac{V_0}{\lambda} + V_1 \times AATS - a_3 - a'_3 \times ANF - a_2 \times ANI}{\frac{1}{\lambda} + T_0 \times ANF + AATS + T_1}. \tag{3.3}$$

On the ED, we look for minimizing without any limitation on . But on the ESD some statistical constraints apply as follows:

$$AATS \leqslant AATS_0$$
$$ANF \leqslant ANF_0.$$

We fix $AATS_0 = 7$ and $ANF_0 = 0.5$. To evaluate object function , it is necessary to determine AATS, ATC, ANI and ANF. Computation common methods of these parameters require knowing the distribution of control chart statistic. For example in X-bar chart, when quality characteristic is normally distributed, the distribution of control chart statistic, i.e. $\bar{X}$, is normal too and so we can apply Markov chain approach to compute these parameters and transition probability matrix.

Since the Bayesian statistic does not have any specified distribution, the Monte Carlo method has been used to obtain the mentioned parameters. According to the definition of AATS and ATC it is obvious that

$$AATS = ATC - \frac{1}{\lambda}$$

Therefore it is sufficient to compute three parameters ATC, ANF and ANI.

Let S, be the time from the start of production until the first signal after the process shift. Thus, $ATC = E(S)$. If we simulate process cycle N times, then for large values of N, a good approximation of ATC is $\frac{\sum_i^N S_i}{N}$, where S_i is value of in ith cycle. Other parameters can be computed in such a way. In this paper we considered .

4. Optimization method

The purpose of the economic design of Bayesian control chart is to find the value of design parameters by minimizing the equation 3 unconditionally on it. But based on economic-statistical design, the optimization problem can be written as

$$\begin{array}{ll} \min & \mathrm{E}(L) \\ \text{s.t.}: & 0.1 \leqslant h \leqslant 8 \\ & n \in \mathbb{Z}^+ \\ & 0 < k < 1 \\ & AATS \leqslant 7 \\ & ANF \leqslant 0.5. \end{array}$$

This nonlinear constrained optimization problem is addressed through the ABC algorithm to obtain the optimum design parameters of the Bayesian control chart.

In 2005, Karaboga proposed an artificial bee colony, which is based on a particular intelligent behavior of honeybee swarms. ABC is developed based on inspecting the behaviors of real bees on finding nectar and sharing the information of food sources to the bees in the hive. Agents in ABC are the Employed bee, the Onlooker bee and the Scout bee. The employed bee stays on a food source and keeps neighborhood of the source in its memory. The onlooker bee gets information of food sources from the employed bees in the hive and select one food source to gather the nectar. The Scout is responsible for finding new food, new nectar and sources. Procedures of ABC are as follows:

- Initialize (Move the scouts)
- Move the onlookers
- Move the scouts only if the counters of the employed bees hit the limit
- Update the memory
- Check the termination condition.

Probability of selecting a nectar source is:

$$P_i = \frac{F(\theta_i)}{\sum_{K=1}^{S} F(\theta_j)}$$

where P_i is the probability of selecting the ith employed bee, S is the number of employed bees, θ_i is the position of the ith employed bee and $F(\theta_i)$ is fitness value. In fact, $F(\theta_i)$ is the value of response variable by ith independent variable. The new position, calculate as follow:

$$x_{ij}(t+1) = \theta_{ij}(t) + \varphi\big(\theta_{ij}(t) - \theta_{kj}(t)\big)$$

where x_i is position of the onlooker bee, t is the iteration number, θ_k is the randomly chosen employed bee, j is the dimension of the solution and $\varphi(.)$ is a series of random variable in the range . And the movement of the scout bees follows equation

$$\theta_{ij} = \theta_{j\,\min} + r(\theta_{j\,\max} - \theta_{j\,\min}) \qquad (4.1)$$

where $r \in [0,1]$ is a random number. For more knowledge about this, reader can refer to [9], [10], [11] and [12].

5. Numerical Result

The optimization and simulation algorithm for FRS Bayesian scheme based on two designs separately are coded in MATLAB. Input variables that have been considered for simulation are V_0,a_3, a_3', a_2, λ and δ which are given in Table 1. These values are proposed by Costa and Rahim (2001). For each row, simulation program was run and optimum design parameters and the minimum

Table 1. Values of a_2, a_3', a_3, V_0, V_1, T_0, T_1, λ and δ

Case	a_2	a_3'	a_3	V_0	V_1	T_0	T_1	λ	δ
1	5	500	500	500	0	5	1	0.01	1
2	10	500	500	500	0	5	1	0.01	1
3	5	250	500	500	0	5	1	0.01	1
4	5	500	500	250	0	5	1	0.01	1
5	5	500	500	500	0	2.5	1	0.01	1
6	5	500	500	500	0	5	1	0.01	1.5
7	5	500	50	500	0	5	1	0.01	1
8	5	500	500	500	0	5	10	0.01	1
9	5	500	500	500	0	5	1	0.01	0.75
10	5	500	500	500	0	5	1	0.01	0.5
11	5	500	500	500	0	5	1	0.05	1

costs for Bayesian approach based on economic and economic-statistical designs were determined in Table2 and Table3 respectively. Also the design parameters values for ED X-bar approach which has come in [3] are shown in Table 4. In that paper, some statistical properties such as AATS, ATC and ANI, had not reported. For better comparison, according to relation between AATS, ATC, ANI and LOSS and ANF, we have been computed them and has been shown in the Table 4. Also, the optimal values for X-bar scheme based on ESD was computed and presented in Table 5.

Table 2. Optimal design and performance for the economic Bayesian approach

CASE	AATS	ANF	LOSS	n	h	k
1	0.83	0.03	31.09	4	1.21	0.42
2	0.07	0.01	34.80	2	0.81	0.56
3	0.27	0.00	14.39	2	3.10	0.07
4	0.93	0.18	20.56	6	3.73	0.54
5	0.04	0.00	14.82	1	1.05	0.27
6	0.08	0.07	21.06	2	1.13	0.51
7	0.26	0.17	19.97	3	1.76	0.68
8	0.57	0.01	59.69	5	3.18	0.14
9	0.35	0.08	22.45	4	2.31	0.62
10	0.14	0.00	16.77	2	1.60	0.06
11	0.18	0.00	99.20	1	0.10	0.76

TABLE 3. Optimal design and performance for the economic-statistical Bayesian approach

CASE	AATS	ANF	LOSS	n	h	k
1	0.04	0.00	15.67	1	0.89	0.12
2	0.02	0.00	35.65	2	0.50	0.69
3	0.17	0.00	19.27	1	0.57	0.72
4	0.27	0.08	27.79	1	0.57	0.91
5	0.03	0.00	28.48	3	0.81	0.38
6	0.07	0.00	19.24	3	6.66	0.01
7	0.10	0.00	23.54	4	1.13	0.18
8	0.14	0.00	59.88	1	0.50	0.11
9	0.33	0.05	13.06	1	2.87	0.50
10	0.11	0.01	49.82	7	0.89	0.66
11	0.07	0.00	99.57	1	0.10	0.51

TABLE 4. Optimal design and performance for the economic X-bar approach

CASE	AATS	ANF	LOSS	n	h	k
1	16.76	0.09	43.04	15	5.45	2.82
2	14.37	0.13	54.40	13	7.22	2.58
3	16.74	0.10	42.82	15	5.40	2.79
4	15.52	0.11	29.65	14	7.39	2.63
5	15.53	0.15	24.80	14	5.17	2.65
6	9.34	0.05	33.35	8	3.88	3.08
7	16.76	0.09	38.75	15	5.43	2.82
8	16.75	0.09	79.08	15	5.64	2.80
9	24.95	0.12	51.97	24	6.89	2.62
10	39.59	0.21	68.00	43	9.47	2.31
11	17.46	0.04	114.00	15	2.57	2.75

Table 5. Optimal design and performance for the economic-statistical X-bar approach

CASE	AATS	ANF	LOSS	n	h	k
1	1.02	0.00	19.66	1	1.02	12.61
2	1.45	0.00	23.67	1	1.45	25.73
3	1.02	0.00	17.21	1	1.02	20.78
4	1.46	0.00	14.28	1	1.45	16.87
5	1.02	0.00	19.66	1	1.02	23.25
6	1.02	0.00	19.66	1	1.02	26.17
7	1.02	0.00	19.66	1	1.02	22.96
8	1.06	0.00	58.59	1	1.06	27.67
9	1.02	0.00	19.66	1	1.02	27.03
10	1.02	0.00	19.66	1	1.02	25.14
11	0.48	0.00	67.68	1	0.48	10.94

6. Conclusion

In this paper we proposed an economic and economic-statistical design of Bayesian control chart and determined optimization design parameters based on them separately. The Monte Carlo method has been used based on the fact that finding the distribution of Bayesian statistic is difficult.

Design parameters, AATS, ANF and LOSS values show that although economic design based on Bayesian control chart can significantly improve cost in more rows of Table 1, its performance in statistical properties is poor in comparison with economic-statistical design. But in Bayesian approach, average time to signal after shifting the process to out of control state (AATS), average number of false alarm (ANF), sample size (n), the sampling interval (h) and cost of production have a good performance compared to X-bar approach in all rows.

References

[1] Banerjee, P. K., and Rahim, M. A. (1988). Economic design of X-bar control charts under Weibull shock models, *Technometrics*, **30**, 407-414.

[2] Calabrese, J. M. (1995). Bayesian Process Control for Attributes,*Management Science*, **41**, 637-645.

[3] Costa, A. F. B., and Rahim, M. A. (2001). Economic design of X-bar charts with variable parameters: the Markov chain approach,*Journal of Applied Statistics*, **28/7**, 875-885.

[4] Duncan, A. J. (1956). The economic design of X-bar charts used to maintain current control of a process, *Journal of the American Statistical Association*, **51**, 228242.

[5] Duncan, A. J. (1971). The economic design of 2 charts where there is a multiplicity of assignable causes, *Journal of the American Statistical Association*, **66**, 107-121.

[6] Eckles, J. E. (1968). Optimum maintenance with incomplete information, *Operational Research*, **16**, 1058-1067.

[7] Faraz, A., Saniga, E., and Kazemzadeh, R. B. (2009). Economic and Economic Statistical Design of T^2 Control Chart with two-adaptive Sample Sizes, *Journal of Statistical Computation and Simulation*, **80**, 1299-1316.

[8] Girshick, M. A., and Rubin, H. (1952), A Bayes approach to a quality control model,*Annals of Mathematical Statistics*, **23**, 114–125.

[9] Karaboga, D. (2005). An Idea Based On Honey Bee Swarm For Numerical Optimization, TR-06, October 2005.

[10] Karaboga, D., and Akay, B. (2009). A comparative study of Artificial Bee Colony algorithm, *Applied Mathematics and Computation*, **214**, 108-132.

[11] Karaboga, D., and Basturk, B. (2007). A powerful and efficient algorithm for numerical function optimization: artificial bee colony (ABC) algorithm, *J. Glob Optim*, **39**, 459-471.

[12] Karaboga, D., and Basturk, B. (2008). On the performance of artificial bee colony (ABC) algorithm, *Applied Soft Computing*, **8**, 687-697.

[13] Lorenzen, T. J., and Vance, L. C. (1986). The economic design of control charts: a uni?ed approach, *Technometrics*, **28**, 3-10.

[14] Maikis, V. (2009). Multivariate Bayesian process control for a finite production run, *European Journal of Operational Research*, **194**, 795–806.

[15] Prabhu, S. S., Mongomery, D. C., and Runger, G. C. (1997).Economic-statistical design of an adaptive X-bar chart, *Int. J. Production Economics*, **49**, 1–15.

[16] Ross, S. M. (1970). *Applied Probability Models with Optimization Applications*, Holden-Day, San Francisco.

[17] Saniga, E. M. (1989). Economic statistical control chart designs with an application to X-bar and R charts, *Technometrics*, **31**, 313-320.

[18] Tagaras, G. (1994). A Dynamic Programming Approach to the Economic Design of X-bar Charts, *IIE Transactions*, **26/3**, 48–56.

[19] Tagaras, G. (1996). Dynamic Control Charts for Finite Production Runs, *European Journal of Operational Research*, **91**, 38-55.

[20] Tagaras, G., and Nikolaidis, Y. (2002). Comparing the Effectiveness of Various Bayesian X-bar Control Charts, *Operations Research*, **50**, 878-888.

[21] Taylor, H. M. (1965). Markovian sequential replacement processes, *Annals of Mathematical Statistics*, **36**, 13-21.

[22] Taylor, H. M. (1967). Statistical Control of a Gaussian Process, *Technometrics*, **9**, 29–41.

[23] Yang, S. F., and Rahim, M. A. (2005). Economic statistical process control for multivariate quality characteristics under Weibull shock model, *Int. J. Production Economics*, **98**, 215-226.

[24] Zhang, G., and Berardi, V. (1997). Economic statistical design of X-bar control charts for systems with Weibull in-control times, *Computers and Industrial Engineering*, **32**, 575-586.