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Research Article

Calculation of the diffusion lengths for one-speed neutrons in a slab with forward and backward scattering

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ABSTRACT

The diffusion lengths for one-speed neutrons in a slab are calculated using the first kind of Chebyshev polynomials approximation (T_N) method. The scattering models are constituted in place of the scattering function with an argument of the cosine of the neutron scattering angle. Therefore, the forward-backward-isotropic (FBI) scattering model is used as the scattering function in transport equation which describes the interaction and the conservation of the neutrons throughout a system. In the solution algorithm, first the neutron angular flux is expanded in terms of the Chebyshev polynomials of first kind. After inserting this expansion in the transport equation, the coupled differential equations are derived using the properties of the Chebyshev polynomials of first kind. These equations are solved together and then the diffusion equation is obtained by applying the first order approximation (N = 1) which is known as the diffusion approximation. Finally, the diffusion lengths for one-speed neutrons are calculated for selected values of the collision, backward and forward scattering parameters. The calculated diffusion lengths are given in the tables together with the ones already obtained in literature in order to indicate the applicability of the present method. The convenience and rapid convergence of the present method with its easily executable equations can be observed from the derived equations and the results in tables.

1. Introduction

The neutrons are interacted with all materials inside a reactor system. The transport equation is describes their behaviors with an integro-differential form. Therefore, It is not very easy to find an exact solution for the transport equation. As well known, the neutrons are neutral particles and they are not affected by electric or magnetic fields and thus their path through the material is in zigzag. Therefore, it is difficult to determine how they interact with the materials inside the system and to guess the distribution of them through the boundaries. In order to solve this equation, some detailed knowledge about nuclear cross sections and energy dependence of the neutrons should be known. When a polynomial approximation technique is established to solve the problems stated in different fields of engineering and science especially in transport theory, first approximation of the method is tried and then its higher orders are applied to the problems. The first order approximation of these techniques is known as the diffusion approximation. The first estimates of a nuclear reactor is still done by using diffusion approximation in which many properties about the neutrons such as transport and energy spectrum of them can be predicted convincingly. This theory works well for values close to unity in c, the number of secondary neutrons per collision [1].

Many stochastic and deterministic methods have been developed for the problems about particle or photon transports. Among the deterministic methods, the spherical harmonics (P_N) is one of the most commonly preferred ones because of its accepted application convenience for the problems in transport theory. The first order P_1 approximation is easy in derivation of the equations for many geometries and the results obtained from it can be accepted as accurate in many studies [1-3]. However, this does not mean that the spherical harmonics method is always true and valid for all computations. Therefore, the Chebyshev polynomials instead of Legendre polynomials are used by Aspelund, Conkie and Yabushita in series expansion of the neutron angular flux and they reported the applicability of the method for the

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problems of transport theory [4-6].

In the last decade, the first kind of Chebyshev polynomials, i.e. T_N method is used effectively for the solutions of the problems related with transport theory such as criticality, eigenvalue spectrum and diffusion lengths of the neutrons in bare and reflected slabs and spheres [7-10]. However, in any of those studies, T_1 the lowest order approximation of the Chebyshev polynomials of first kind is not used for the calculation of the diffusion lengths calculations in the case of forward and backward scattering. Therefore in this study, apart from the previous studies T_1 approximation is firstly applied one-dimensional transport equation to determine diffusion lengths of neutrons in the case of forward and backward scattering. It is aimed to show that applicability and thus the accuracy of the T_1 approximation against the traditional P_1 approximation in diffusion length calculations. In other words, the present method independent from the problem under consideration is thought to be useful in the long run as an alternative method. Hence, using various values of the collision and the scattering parameters in the mathematical derivations of the method, the diffusion lengths of neutrons are calculated numerically. At the end, the numerical results are obtained from the present method and they are tabulated with ones already presented in literature to indicate the accordance of the methods.

In this study, an alternative deterministic method based on the polynomial expansion of the Chebyshev polynomials of first kind is presented other than the traditional methods. This is important for the literature since this alternative method can be applied to other problems to get better results or new results for the problems in science and engineering. This method can also be improved to solve various problems in many areas.

2. Theory

The one-dimensional linear transport equation derived with forward-backward-isotropic scattering (FBI) model for monoenergetic neutrons is given as,

$$\mu\frac{\partial\psi(x,\mu)}{\partial x}+\sigma_T(1-\alpha c)\psi(x,\mu) = \frac{c\sigma_T}{2}(1-\alpha-\beta)\int_{-1}^{1}\psi(x,\mu')\mathrm{d}\mu'+\beta c\sigma_T\psi(x,-\mu) \quad (1)$$

where $\psi(x,\mu)$ is the angular neutron flux at point x and direction μ, cosine of the scattering angel between the neutron velocity vector and the positive x-axis. α and β are the parameters of forward and backward scattering probabilities in a collision and σ_T is the total macroscopic cross section [2].

In this study, a successfully applied angular neutron flux [7] is chosen because of its easily executable derivations and advantages,

$$\psi(x,\mu)=\frac{\Phi_0(x)T_0(x)}{\pi\sqrt{1-\mu^2}}+\frac{2}{\pi\sqrt{1-\mu^2}}\sum_{n=1}^{\infty}\Phi_n(x)T_n(x) \quad (2)$$

$$-a\le x\le a,\qquad -1\le\mu\le 1$$

Later this angular flux will be used in Eq. (1) to obtain the moment equations. But before it, the orthogonality and recurrence relations of the Chebyshev polynomials of first kind are needed to derive the equations [11],

$$\int_{-1}^{1}\frac{T_m(x)T_n(x)}{\sqrt{1-\mu^2}}\mathrm{d}\mu=\begin{cases}0; & m\ne n\\ \dfrac{\pi}{2}; & m=n\ne 0\\ \pi; & m=n=0\end{cases} \quad (3)$$

$$T_{n+1}(\mu)-2\mu T_n(\mu)+T_{n-1}(\mu)=0\,. \quad (4)$$

First, Eq. (2) is inserted into Eq. (1), then the resultant equation is multiplied by $T_0(\mu)$ and $T_1(\mu)$ and integrated over $\mu\in[-1, 1]$. During these operations, Eq. (3) and Eq. (4) are auxiliary for the derivations in the integrals. At the end of this application, T_N moments of equations for n = 0 and n = 1 are obtained, respectively;

$$\frac{\mathrm{d}\Phi_1(x)}{\mathrm{d}x}+\sigma_T(1-c)\Phi_0(x)=0 \quad (5)$$

$$\frac{\mathrm{d}\Phi_2(x)}{\mathrm{d}x}+\frac{\mathrm{d}\Phi_0(x)}{\mathrm{d}x}+2\sigma_T\left[1-c(\alpha-\beta)\right]\Phi_1(x)=0 \quad (6)$$

These equations are known as T_1 equations. As well known, $\Phi_{N+1}(x)=0$ and $\mathrm{d}\Phi_{N+1}(x)/\mathrm{d}x=0$ are used in spherical harmonics (P_N) method [2,3]. The same approximation technique is also valid for T_N method. Therefore, for T_1 approximation, $\mathrm{d}\Phi_2(x)/\mathrm{d}x=0$ is used in Eq. (6) and $\Phi_1(x)$ is obtained as,

$$\Phi_1(x)=-\frac{1}{2\sigma_T\left[1-c(\alpha-\beta)\right]}\frac{\mathrm{d}\Phi_0(x)}{\mathrm{d}x} \quad (7)$$

Eq. (7) is very familiar with the equation known as Fick's law of diffusion which defines the flow of atoms from high concentration to low concentration. Then, the diffusion equation can be obtained by inserting Eq. (7) into Eq. (5),

$$\frac{\mathrm{d}^2\Phi_0(x)}{\mathrm{d}x^2} - 2\sigma_T^2\left[1-c(\alpha-\beta)\right](1-c)\Phi_0 = 0 \tag{8}$$

The same procedure developed for the traditional P_1 approximation is followed to obtain Eq. (8). Then, from the definition of the diffusion length, the square root of the inverse of the coefficient of the second term of Eq. (8) can be referred as the diffusion length (L) in T_1 approximation,

$$L = \frac{1}{\sigma_T\sqrt{2\left[1-c(\alpha-\beta)\right](1-c)}} \tag{9}$$

In addition, Eq. (8) is a homogeneous differential equation with constant coefficient. When this differential equation is solved, the scalar neutron flux for $c < 1$ can be obtained,

$$\Phi_0(x) = Ae^{\sigma_T\sqrt{2[1-c(\alpha-\beta)](1-c)}\,x} + Be^{-\sigma_T\sqrt{2[1-c(\alpha-\beta)](1-c)}\,x}, \tag{10}$$

where A and B are the constants which can be found from boundary conditions.

It is not hard to think that this process on obtaining the diffusion length can be done with other methods. The diffusion length for isotropic scattering from the method of separation of variables is given as

$$1 = cL\tanh^{-1}\frac{1}{L}, \tag{11}$$

and it is called as the asymptotic relaxation length in that method.

The diffusion lengths of the neutrons obtained by the traditional P_1 (diffusion) approximation can be given as,

$$L = \frac{1}{\sigma_T\sqrt{3\left[1-c(\alpha-\beta)\right](1-c)}}\,. \tag{12}$$

This result is derived for a slab with forward and backward scattering. Other results from various methods can also be found from the literature. However, there is no need to exceed them here. Because it is accepted as to be necessary to compare the results obtained from the present methods with ones obtained from one or two references.

These methods are discussed in detail in many references, therefore more explanations about them can be got by following them [2,3].

3. Results

An analytic study for the diffusion length of neutrons in a homogeneous slab with forward and backward scattering is studied through equations (1) to (9). In this analysis, first the angular neutron flux is expanded in terms of the first kind of Chebyshev polynomials. After applying the first order approximation of the method, the equations of moments are obtained in Eqs. (5) and (6). Then, these coupled differential equations are solved together to obtain an analytic expression for the diffusion length. Finally, the numerical results for the present study are calculated from Eq. (9). The diffusion lengths are also calculated from Eqs. (11) and (12) and they are given in the tables and figures side by side for comparison. Therefore, one can easily see the efficiency or the performance of the present method from tables and figures. These calculations are done for various values mediums related with the collision parameter c, forward scattering parameter α and backward scattering parameter β. In all cases, the calculations can be carried out using a simple calculator or a mathematical software and the normalized value of the total macroscopic cross section is taken as, $\sigma_T = 1$ cm^{-1}.

During the numerical calculations, special softwares may be used. However, since this study is a first step for further studies, there is no need to use any software to calculate the results from equations (9), (11) and (12). A simple calculator could be enough for this kind of applications.

Table 1. Diffusion lengths L obtained from T_1 approximation for ($\alpha = 0.0$ and $\beta = 0.0$) and comparison with literature values, (cm).

c	T_1(present method, Eq. (9))	P_1 (Eq. (12))	Exact
0.99	7.07107	5.77350	5.79673
0.98	5.00000	4.08248	4.11552
0.95	3.16228	2.58199	2.63515
0.90	2.23607	1.82574	1.90320
0.80	1.58114	1.29099	1.40763
0.50	1.00000	0.81650	1.04438
0	0.70711	0.57735	1.00000

Table 2. Diffusion lengths L obtained from T_1 approximation for ($\alpha = 0.3$ and $\beta = 0.0$) and comparison with literature values, (cm).

c	T_1(present method, Eq. (13))	P_1 (Eq. (12))
0.99	8.43349	6.88592
0.98	5.95069	4.85872
0.95	3.73979	3.05352
0.90	2.61712	2.13687
0.80	1.81369	1.48087
0.50	1.08465	0.88561
0	0.70711	0.57735

Table 3. Diffusion lengths L obtained from T_1 approximation for (α = 0.0 and β = 0.3) and comparison with literature values, (cm).

c	***T*₁(present method, Eq. (9))**	***P*₁ (Eq. (12))**
0.99	6.20890	5.06955
0.98	4.39545	3.58887
0.95	2.78964	2.27773
0.90	1.98419	1.62008
0.80	1.41990	1.15935
0.50	0.93250	0.76139
0	0.70711	0.57735

In order to see visually the general behavior of the diffusion lengths of neutrons with respect to the collision forward and backward scattering parameters the following figures are plotted.

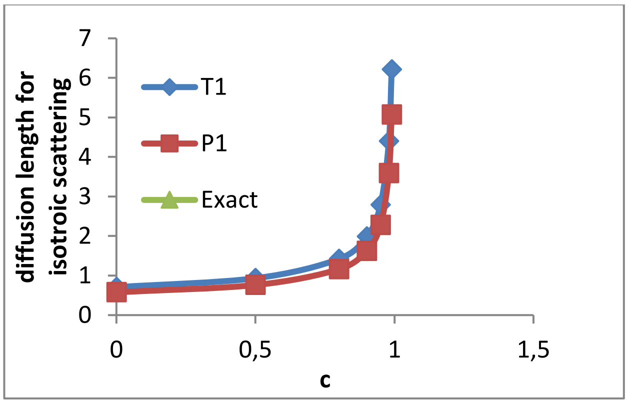

Figure 1. Diffusion lengths for isotropic scattering obtained from T_1, and P_1 approximations and exact results.

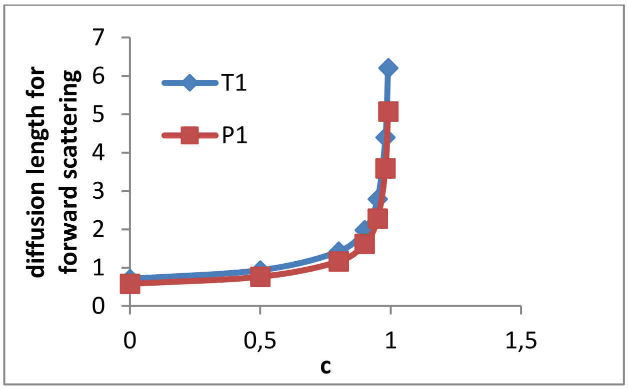

Figure 2. Diffusion lengths for forward scattering obtained from T_1 and P_1 approximations.

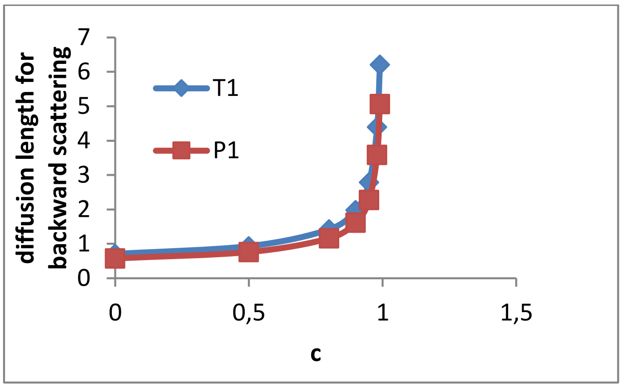

Figure 3. Diffusion lengths for backward scattering obtained from T_1 and P_1 approximations.

4. Conclusions

An analytic expression for the diffusion length is obtained from the present method and it is given in Eq. (9). The reference results tabulated in tables are quoted from Ref. [12]. One of them is the exact results obtained by Eq. (11) from transport theory and the other is the results obtained by Eq. (12) from traditional P_1 approximation. The numerical results for the diffusion length are calculated for the c values increasing from 0 to 1 and isotropic (α = 0.0 and β = 0.0), forward (α = 0.3 and β = 0.0) and backward (α = 0.3 and β = 0.0) scattering cases. While the lower values of c represents the weakly absorbing medium, the higher values of it represents highly scattering medium.

In Table 1, diffusion lengths for isotropic scattering obtained from the present method are given with the literature values. While the results obtained from the present method are given with the ones obtained from conventional P_1 approximation in the case of forward scattering in Table 2, they are given in the case of backward scattering in Table 3. By investigating the results given in the tables, one can first tend to insist on some inconsistencies between the results obtained from the present method and the ones obtained from the P_1 approximation and the exact ones. However, this assertion should not be taken into consideration since these are first order polynomial approximations. As well known, in a polynomial expansion based technique, it is not waited for good results in low order approximations. Hence, higher order approximations of the present method have been successfully practiced to related problems of the transport theory in previous studies [7-9]. However, this study should not to be evaluated with only the numerical results but also it should be evaluated with its easily executable equations and rapid convergence. It can be summarized that the present method is effective and alternative in the field of the science and engineering and it can be served to the literature as an impressive method.

In addition, in these figures given in this study, one can easily observe how the results obtained from the present method are in good accordance with the ones obtained from the traditional P_1 approximation and the exact ones. Therefore, this study can be a sign that the present method can be accepted as an alternative method for the problems of the transport theory and other applied sciences.

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