Conference Paper

MATDER Journal of Mathematics Education

Vol. 3 No.1 (2018), 32-40

Analysis of Defining and Drawing Skills of Secondary School Students: Parallelogram Example *

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Received: 07.05.2018; **Accepted:** 28.08.2018; **Published**: 15.09.2018

Abstract: The aim of this study is to comparatively examine the students' ability to define and draw parallelogram for each class level. General survey model was chosen as the methodology of this study and the working group of the study consists of 120 middle school students from a state middle school in Turkey. Two open-ended questions were used to gather data. One of the questions was taken from the study of Fujita (2012) and the other question was prepared by researchers based on the relevant literature, mathematics curricula and textbooks. The document analysis method was used to analyze data. As a result of the research, it was seen that students at all class levels drawn prototype-parallelogram, and had difficulty in defining parallelograms. It has been determined that students at all grade levels cannot consider a rhombus as a special form of parallelogram, and do not prefer it in their drawings.

Keywords: parallelogram, define- and drawing skills, secondary school students

INTRODUCTION

Geometry is an important branch of mathematics to teach. The study of geometry contributes to helping students develop the skills of visualisation, critical thinking, intuition, perspective, problem-solving, conjecturing, deductive reasoning, logical argument and proof (Jones, 2002). Despite the great importance placed on geometry education included in the mathematics curriculum, much research shows that geometry perception levels of students are not at the expected level (Clements & Battissa, 1992; Carroll, 1998). The topic of quadrilaterals, which holds an important place in primary and secondary school mathematics program, are able to develop some mathematical skills such as defining, classifying geometric shapes, drawing, relational understanding, logical deduction, deductive and inductive thinking (MEB, 2013; 2015). Despite this importance,when the literature is examined, it is seen that the students have some difficulties with the quadrilaterals.

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* This study was presented as an oral presentation at the International Conference on Mathematics and Education (ICMME-2018) on June 27-29.

Citation: Biber, A. Ç., Tuna, A., Korkmaz, S., & Aliustaoğlu, F. (2018). The Investigation of the Relationship Between Mathematical Connection Skills and Self-Efficacy Beliefs. *MATDER Journal of Mathematics Education, 3(1),* 32-40.

It has been revealed that students have problems in defining quadrilaterals (de Villiers, 1994; Fujita & Jones, 2006; 2007; Okazaki & Fujita, 2007; Ergün, 2010; Berkün, 2011; Aktaş & Aktaş, 2012; Fujita, 2012; Türnüklü, Alaylı & Akkaş, 2013; Aktaş, 2016; Karakuş & Erşen, 2016; Ayaz, 2016), drawing quadrilaterals Berkün, 2011; Erşen & Karakuş, 2013; Türnüklü, Alaylı & Akkaş, 2013), hierarchical classification of quadrilaterals (de Villiers, 1994; Fujita & Jones, 2006; Akuysal, 2007; Okazaki & Fujita, 2007; Berkün, 2011; Türnüklü, Alaylı & Akkaş, 2013; Karakuş & Erşen, 2016), and so on. In these studies, Fujita (2012) determined that students often recognize prototypes of quadrilaterals and that they are not aware of the hierarchical relationship between quadrilaterals. In his work, Fujita (2012) identified four developmental levels that revealed levels of understanding quadrilaterals:

“Level 0”: The student has no basic knowledge of parallelogram

"Prototype Level" where the student has limited parallelogram knowledge

"Partially Prototype Level" in which the student has expanded the limited knowledge of parallelogram, for example, the student accepts equilateral triangles as parallelogram, but can not fully explain the relation between them.

"Hierarchical Level" where the student can determine the relation between the parallelogram and some other special quadrilaterals and can explain the relation between them mathematically.

Aktas and Aktas (2012), who conducted a study based on Fujita's (2012) study, found that 9th grade students were not at the expected level of achievement in defining a parallelogram, and that students who correctly defined them remembered parallelogram with its typical image. They also found no inferences that could reveal the hierarchical relationship between quadrilaterals. Berkün (2011) conducted his research on 5th and 7th grade students and found that students were unaware of the hierarchical relationship between the quadrilaterals. He claims also that students think that it is a uniform drawing belonging to each special quadrant, and those who have made more than one drawing have only changed the position or size of the drawing. In their work with 4th grade students under the NAEP (The National Assessment of Educational Progress) Walcott, Mohr and Kastberg (2009), found that students use a non-mathematical language when describing parallelogram and that students use names of “oblique rectangles or rectangles with oblique edge” instead of parallelogram names.

When the above explanations and studies are evaluated, it is important to find out how secondary school students define geometric concepts, how they draw shapes, how they classify geometric shapes and objects, and how they determine their relations with each other. In this context, it is thought that it is important to determine the conceptual learning of the geometric concepts of the secondary school students (5th, 6th, 7th, and 8th grade students). As a matter of fact, research is needed to determine whether students' polygonal perception, identification, and classification patterns change according to the class level. In this study, from special quadrangles only parallelograms are used, in order to gain in-depth knowledge of the students' conceptual learning in the field of geometry. Parallelograms contain the most hierarchical relationships within the family of special quadrilateral. As a matter of fact, rhombuses, rectangles and squares are also a parallelogram. In addition, the concept of parallelograms serves as a bridge to understanding other lower- and upper-level geometric concepts (Ulusoy & Çakıroğlu, 2017).

In this study, students from every grade level of secondary school are involved. The ability to define and drawing skills of students at all class levels has been examined. The study is also based on the evaluation framework of Fujita (2012). It can be said that the research from these directions is different from the other researches. In this research, it is aimed to comparatively examine the students' ability to describe and draw parallelogram for each class level. For these purposes, research questions are identified as follows:

1. What is the level of definition of the parallelogram of the secondary school students?
2. How are the parallelogram drawings of the secondary school students?

METHODOLOGY

In this study, it is aimed to comparatively examine the students' ability to describe and draw parallelogram for each class level. Therefore, a general survey model is conducted. Karasar (2008) describes the general screening models as; screening operations to reach some general judgments about a universe or a set of samples taken from the universe which compose of multitude of elements.

Study Group

The study group consists of 120 middle school students from a state middle school in Samsun. Since one of the researchers is a mathematics teacher in the middle school, the convenience sampling method is preferred. Convenience sampling method is practical and gives researchers time (Yıldırım & Şimşek, 2008). The demographic properties of working group is given in the following Table1.

Table1: Demographic properties of working group

Grade	5th	6th	7th	8th	Total
Number of students	30	30	28	32	120

Analysis of Data

The document analysis method is used to analyze data. Document analysis is a systematic procedure for reviewing or evaluating documents—both printed and electronic (computer-based and Internet-transmitted) material. Like other analytical methods in qualitative research, document analysis requires that data be examined and interpreted in order to elicit meaning, gain understanding, and develop empirical knowledge (Bowen, 2009). Students' written answers to two open-ended questions are considered as documents in this study.

For the analysis of students' answers to question 1, Fujita's (2012) assessment criteria are used. These criteria are given in the following Table 2 and Table 3.

Table 2: Students' level of understanding parallelogram (Fujita, 2012)

Level	Description
D-P-Hierarchical	Learners can accept squares, rectangles and rhombi are also parallelograms. 'The opposing direction inclusion relationship' of definitions and attributes is understood
D-P-Partial Prototypical	Learners have begun to extend their figural concepts. For example, they accept rhombi are also parallelograms but not squares and rectangles. Their judgement would be likely to be prototypical type 2
D-P-Prototypical	Learners who have their own limited personal figural concepts. Their judgement would be either prototypical type 1 or 2
Level 0	Learners do not have basic knowledge of parallelograms

Table 3: Evaluation criteria for question1

Question	D-P- Hierarchical	D-P-Partial Prototypical	D-PPrototypical	Level 0
Q 1	correct definition (rectangle with opposite edges parallel to each other)	writing different features of parallelogram	define according to external appearance of the parallelogram (oblique rectangle etc.)	empty or other misconceptions

In question 2 students were asked to draw three different parallelograms at the dotted partitions. The main purpose of using the dotted partition is to see exactly which quadrangle the students draw and to determine whether students are paying attention to critical features of parallelograms. In addition, the suggestions in the secondary school mathematics curriculum for the use of square or dotted paper on teaching basic geometric concepts have been taken into consideration (MEB, 2013).

The answers for each question are independently analyzed by two different researchers, and necessary subcategories were created. The obtained data are also checked by a third researcher. Discrepancies between them are reviewed again and data analysis is finalized. In these comparisons, the percentage of incompatibility that Miles and Huberman (1994) suggested, reliability (Reliability = Opinion Unity / (Opinion Unity + Opinion Separation)) is calculated for each category separately. The percentage of Question 1 is % 83 and Question 2 is % 94. All calculated percentages are higher than 70% and therefore analysis in the study can be considered as reliable (Miles & Huberman, 1994).

FINDINGS

The data in this study is investigated under the two following categories: “defining a parallelogram” and “parallelogram drawings”.

Defining a Parallelogram

In the first question, students are asked to describe the parallelogram. The level of definition of the parallelograms of the students is given in Table 4.

Table 4: Students’ level of definitions of a parallelogram

Grades	5th		6th		7th		8th	
Levels	**f**	**%**	**f**	**%**	**f**	**%**	**f**	**%**
D-P-Hierarchical	2	7	-	0	5	17.8	3	9.3
D-P-Partial Prototypical	4	13	1	3.3	6	21.4	5	15.6
D-P-Prototypical	4	13	18	60.0	4	14.3	10	31.3
Level 0	20	67	11	36.7	13	46.5	14	43.8

When Table 4 is examined, it can be seen that 6th grade students can not define parallelograms, 7% of 5th grade students, 17.8% of 7th grade students and 9.3% of 8th grade students can describe a parallelogram at hierarchical level. 13% of Grade 5 students, 3.3% of Grade 6 students, 21.4% of Grade 7 students, and 15.6% of Grade 8 students can define parallelograms at D-P-Partial

Prototypical level,that is in the definitions given by the students, they can list all the features of parallelograms. It can be seen that 13% of 5th grade students, 60% of 6th grade students, 14.3% of 7th grade students and 31.3% of 8th grade students can define parallelograms at D-P-Prototypical level, that is, students are more likely to describe parallelograms according to the external appearance of parallelograms. 67% of Grade 5 students, 36.7% of Grade 6 students, 46.5% of Grade 7 students, and 43.8% of Grade 8 students were assigned to level 0 because they did not correctly define the parallelograms.

Some examples of parallelogram definitions for each level are shown in Table 5 below.

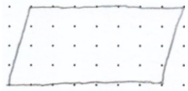
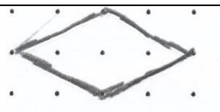
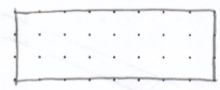
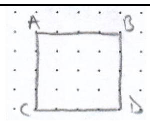
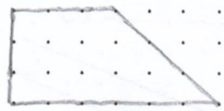
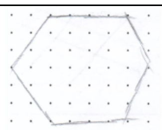
Table 5: Some examples from student answers

Levels	**Sample student answers**
D-P-Hierarchical	Karşılıklı kenarları paralel olan dörtgenlere paralel kenar denir. (Quadrangles with parallel sides are called parallelograms.)
D-P-Partial Prototypical	Karşılıklı kenarları eşit olan ve iç açıları toplamı 360 derece olan bir dörtgendir. Karşılıklı kenarları paralel ve uzunlukları eşittir. (A rectangle whose opposite sides are equal and whose sum of inner angles is 360 degrees. Opposite sides are parallel and equal in length.)
D-P-Prototypical	Kare veya dikdörtgenin vb. yana yatmışı. (shapes such as squares or rectangles are tilted to the side)
Level 0	İki kenarın birbirine olan paralelliği (two sides parallel to each other)

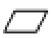
Drawings

In question 2 students were asked to draw three different parallelograms in dotted sections. As a result of the examination, two categories were determined as the correct drawing and the wrong drawing. Then the correct drawings are divided into subcategories as prototype parallelograms, non-prototype parallelograms, rhombus, rectangles, and squares. Wrong drawings are divided into subcategories, such as trapezoids, rectangles that are not parallel to each other's edges, and those that are empty or irrelevant. These findings are shown in Table6.

Table 6: Students' parallelogram drawing skills and some examples of drawings *

Categories	Subcategories	Example Drawings	5th grade		6thgrade		7th grade		8th grade	
			f	%	f	%	f	%	f	%
Correct Drawings	Prototype Parallelogram Drawings		9	30	54	80	15	54	29	91
	Non-Prototype Parallelogram Drawings		6	20	7	24	6	21	9	28
	Rhombus Drawings		2	6	4	13	1	3	3	9
	Rectangles Drawings		16	53	3	10	19	68	12	37
	Squares Drawings		4	13	2	7	12	43	13	40
Incorrect Drawings	Trapezoids drawings		9	30	4	13	5	18	5	15
	Rectangles drawings that are not parallel to each other's edges		12	40	11	37	10	36	5	15
	Empty or irrelevant drawings		4	13	4	13	1	3	2	6

*Since a student draws three different rectangles, the sum of the percentage of correct drawings and those who make the wrong drawings at each class level does not give 100%.

Table 6 shows that 30% of the 5th grade students, 80% of the 6th grade students, 54% of the 7th grade students and 91% of the 8th grade students draw a typical parallelogram (). It was determined that 20% of 5th grade students, 24% of 6th grade students, 21% of 7th grade students and

28% of 8th grade students were drawing unusual parallelograms. It was also found that 53% of 5th grade students draw a rectangle, 68% of 7th grade students draw a rectangle, 43% draw a square, and 37% of 8th grade students draw a rectangle and 40% square. 30% of 5th grade students, 13% of 6th grade students, 18% of 7th grade students and 15% of 8th grade students draw trapezoids. It is also seen that 40% of Grade 5 students, 37% of Grade 6 students, 36% of Grade 7 students and 15% of Grade 8 students draw quadrangles that are not parallel to each other's edges.

DISCUSSION AND CONCLUSION

When the parallelogram definitions were evaluated, it was found out that students' ability of defining the parallelograms at all class levels were inadequate in general. Especially, it has been determined that most 5th grade students can not define a parallelogram; and 6th grade students were more likely to define a parallelogram according to their external appearances of parallelogram. It was observed that 7th grade students were partially more successful than others in defining parallelogram. It is seen that there are fewer students (% 17.8) who are aware of the necessary and sufficient conditions for definition at the hierarchical level in 7th grade. When we look at the level of definition of the 8th grade, it is seen that there are students at all levels but most of them are at Prototype or Level 0. It can be said that 8th grade students are very inadequate in defining a parallelogram when class level is taken into consideration. This indicates that there is no significant increase in the ability of students to define a parallelogram, despite the increase of the class level. As a matter of fact, Özdemir, Erdoğan and Dur (2014) determined that the quadratic definitions of teacher candidates were at the prototype level, that is, the level of middle school students, in the study conducted by the university with the elementary mathematics teacher candidates in the fourth grade. This result supports the result of our research that the students' ability to define despite the increase of the class level has not changed. In addition, according to other studies, it has been revealed that it is difficult for students to define and it has been seen that the students try to make definitions according to the prototype they have created in their minds (De Villiers, 1998; Fujita & Jones, 2007; Aktaş & Aktaş, 2012; Fujita, 2012; Erşen & Karakuş, 2013; Türnüklü, Alaylı & Akkaş, 2013; Akkaş & Türnüklü, 2015).

In the second question, students are asked to draw three different parallelograms at the dotted sections. When we look at the results, it is seen that 5th grade students mostly draw prototype parallelograms and rectangles. This shows that 5th grade students generally draw typical parallelograms. 5th grade students draw a rectangle, which is a special parallelogram, more than the other quadrants. It is thought that this is so because the students likened the rectangle and the parallelogram formally to each other. As a matter of fact, 5th grade students do not prefer rhombus and square drawings very much, and this supports the previous result. In addition, in the study conducted with the mathematics teacher candidates, Türnüklü (2014) stated that the teacher candidates related rectangle to parallelogram, which supports the findings in our research. Most students in grades 6, 7, and 8 have drawn parallel the prototype of parallelogram (). This suggests that students prefer the typical parallelogram model they are accustomed to, even if the class level increases. Students often see the typical parallelogram in their lessons. Therefore, it can be said that they created this model as a concept image in their minds. As a matter of fact, studies have shown that teachers frequently use the typical parallelogram model in mathematics lessons (Akuysal, 2007; Ergün, 2010; Erşen & Karakuş, 2013; Türnüklü, Alaylı & Akkaş, 2013; Akkaş & Türnüklü, 2015). 5th and 7th graders' parallelogram drawing preferences are close together. In the mathematics program of the 5th and 7th grades, while the quadrangles are processed in the lessons, the rectangles and parallelograms are given at the same time. Moreover, according to the students, these two rectangles are very similar to each other. Because of these reasons, it can be said that at both grade levels, students mostly drawn rectangles instead of parallelograms. When the secondary school mathematics program is examined; in the 5th grade, other special squares are trained outside the trapezoid (square, rectangular, rhombus, and parallelogram). In the 6th grade, only the concepts of the height of the parallelogram and the domain relation are discussed. For the 7th class, all special quadrangles (square, rectangle, rhombus, parallax and trapezoid) are handled together. In the 8th grade, no special quadrants are included.

As a result of the research, it was seen that students at all class levels drawn prototype-parallelograms, and students at all class levels had difficulty in defining a parallelogram. It has been determined that students at all grade levels cannot consider the rhombus as a special form of a parallelogram, and do not prefer it in their drawings. As a matter of fact, Aktaş and Aktaş (2012) stated that 8th grade students could not establish a relation between rhombus and parallelogram, and similarly, in his work with students in the 9-13 age group, Nakahara (1995) stated that it is more difficult for students to establish a parallelogram-rhombus relationship. In general, it was determined that the 7th grade was more successful and the 6th grade was more unsuccessful in all the questions.

As a result of our research it can be said that students do not prefer non-prototype parallelogram drawings, and that the prototypes were drawn by students only by changing the size and stance. It has been seen that students draw trapezoids and polygons with two edges parallel to each other such as hexagons and pentagons instead of parallelograms. It can be said that the students perceive geometrical shapes, with two edges parallel to each other as parallelograms and that they draw such geometric shapes. As a matter of fact, Ulusoy and Çakıroğlu (2017) in their study with 7th grade students reached the conclusion that the students focused on the concept of "parallel edges" from the concept of "parallelograms" by taking the direction of this syntactical similarity and seeing parallel shapes as parallelograms.

From these results, it may be advisable to include special prototype images as well as special forms in lessons in the teaching of the parallelogram. Instead of giving the definitions directly, students should be offered opportunities to explore them. In teaching quadrangles, suitable learning environments should be provided by considering van Hiele geometry thinking levels. Activities such as concept maps can be prepared to reveal the hierarchical relationships of the parallelogram with some other special quadrilaterals. Special teaching methods such as realistic mathematics education or problem based learning, can be applied for better understanding the quadrants. In addition, concrete materials (geometry strips, geometry boards, etc.), dynamic geometry software (Geoegebra, Cabri etc.) and origami (paper folding) activities can provide a better understanding of the parallelograms of students. It should also be emphasized that the concepts of parallelogram and parallel edges are not the same for students and it will be appropriate to include examples of this difference.

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