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ON LOCALLY UNIT REGULARITY CONDITIONS FOR ARBITRARY LEAVITT PATH ALGEBRAS

TUFAN ÖZDIN

Abstract. Let Γ be a graph, K be any field and S be the endomorphism ring of $L := L_K(\Gamma)$ considered as a right L-module. In this paper, we give defination of the left locally unit regular ring. We show that (1) if S is locally unit regular, then L is locally unit regular, (2) if L is morphic and image projective then S is left morphic, (3) S is a directly finite ring then L is directly finite, (4) if S is an exchange ring then L is directly finite and if L is a directly finite ring then L is an exchange ring.

1. Introduction

Throughout this article Γ will denote a directed graph, K will denote an arbitrary field and the Leavitt path algebras (shortly LPAs) of Γ with coefficients in K will denoted $L := L_K(\Gamma)$.

LPAs can be regarded as the algebraic counterparts of the graph C^*-algebras, the descendants from the algebras investigated by Cuntz in [6]. LPAs can be viewed as a broad generalization of the algebras constructed by Leavitt in [11] to produce rings without the Invariant Basis Number property. LPAs associated to directed graphs were introduced in [4, 1]. These $L_K(\Gamma)$ are algebras associated to directed graphs and are the algebraic analogs of the Cuntz-Krieger graph C^*-algebras [15].

Let Γ be a graph and K a field. In [3], G. Abrams and K. M. Rangaswamy showed how definition of von Neumann regular ring (recall that a ring R is von Neumann regular if for every $a \in R$ there exists $b \in R$ such that $a = aba$) is extended to locally unit regular ring and in [3, Theorem 2] if Γ is arbitrary graph, $L_K(\Gamma)$ is locally unit regular if and only if Γ is acyclic. This article is organized as follows. In Section 2, we recall some preliminaries about LPAs which we need in the next section. In Section 3, for the ring S of endomorphism ring of $L_K(\Gamma)$ (viewed as a right $L_K(\Gamma)$-module), we prove that: (1) if S is locally unit regular,

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then L is locally unit regular, (2) if L is morphic and image projective then S is left morphic, (3) if S is a directly finite ring then L is directly finite, (4) if S is an exchange ring then L is directly finite and if L is a directly finite ring then L is an exchange ring.

2. Definitions and Preliminaries

We recall some graph-theoretic concepts, the definition and standard examples of LPAs.

Definition 1. *A (directed) graph $\Gamma = (V, E, r, s)$ consist of two set V and E (with no restriction on their cardinals) together with maps $r, s : E \to V$. The elements of V are called vertices and the elements of E edges. For $e \in E$, the vertices $s(e)$ and $r(e)$ are called the source and range of e. If $s^{-1}(v)$ is a finite set for every $v \in V$, then the graph is called row-finite. If V is finite and Γ is row finite, then E must necessarily be finite as well; in this case we say simply that Γ is finite.*

A vertex which emits (receives) no edges is called a sink (source). A vertex v is called an infinite emitter if $s^{-1}(v)$ is an infinite set. A vertex v is a bifurcation if $s^{-1}(v)$ has at least two elements. A path p in a graph Γ is a finite sequence of edges $p = e_1 ... e_n$ such that $r(e_i) = s(e_{i+1})$ for $1 \leq i \leq n-1$. In this case, $s(p) = s(e_1)$ and $r(p) = r(e_n)$ are the source and range of p, respectively, and n is the length of p. We view the elements of V as paths of length 0.

A path $p = e_1 ... e_n$ is said to be closed path based at v if $s(p) = v = r(p)$. If p is an closed path in Γ and $s(e_i) \neq s(e_i)$ for all $i \neq j$, then p is said to be a cycle. A cycle consisting of just one edge is called a loop. A graph which contains no cycles is called acyclic. A graph Γ is said to be no-exit if no vertex of any cycle is a bifurcation.

Definition 2. *(LPAs of Arbitrary Graph)*

For an arbitrary graph Γ and a field K, the Leavitt path K-algebra of Γ, denoted by $L_K(\Gamma)$, is the K-algebra generated by the set $V \cup E \cup \{e^* | e \in E\}$ with the following relations,

1. $v_i v_j = \delta_{v_i, v_j} v_i$ *for every* $v_i, v_j \in V$
2. $s(e)e = e = er(e)$ *for all* $e \in E$.
3. $r(e)e^* = e^* = e^* s(e)$ *for all* $e \in E$.
4. *(CK1)* $e^* f = \delta_{e,f} r(e)$ *for all* $e, f \in E$.
5. *(CK2)*$v = \sum_{\{e \in E, s(e) = v\}} ee^*$ *for every* $v \in V$ *that is neither a sink nor an infinite emitter.*

The first three relations are the path algebra relations. The last two are the so-called Cuntz-Krieger relations. We let $r(e^*)$ denote $s(e)$, and we let $s(e^*)$ denote $r(e)$. If $p = e_1 ... e_n$ is a path in Γ, we write p^* for the element $e_n^* ... e_1^*$ of $L_K(\Gamma)$. With this notation, the LPA $L_K(\Gamma)$ can be viewed as a $K-$vector space span of $\{pq^* \mid p, q$ are paths in $\Gamma\}$. (Recall that the elements of V are viewed as paths of length 0, so that this set includes elements of the form v with $v \in V$.)

If Γ is a finite graph, then $L_K(\Gamma)$ is unital with $\sum_{v\in V} v = 1_{L_K(\Gamma)}$; otherwise, $L_K(\Gamma)$ is a ring with a set of local units consisting of sums of distinct vertices of the graph.

Many well-known algebras can be realized as the LPAs of a graph. The most basic graph configuration is shown below (the isomorphism for can be found in [1]).

Example 1. *The ring of Laurent polynomials $K[x, x^{-1}]$ is the LPA of the graph given by a single loop graph.*

We will now outline some easily derivable basic facts about the endomorphism ring S of $L := L_K(\Gamma)$. Let Γ be any graph and K be any field. Denote by S the unital ring $End(L_L)$. Then we may identify L with subring of S, concretely, the following is a monomorphism of rings:

$$\phi : L \to End(L_L)$$
$$x \mapsto \lambda_x$$

where $\lambda_x : L \to L$ is the left multiplication by x, i.e., for every $y \in L$, $\lambda_x(y) = xy$ which is a homomorphism of right $L-$module. The map ϕ is also a monomorphism because given a nonzero $x \in L$ there exists an idempotent $u \in L$ such that $xu = x$, hence $0 \neq x = \lambda_x(u)$.

3. Results

According to Abrams and Rangaswamy [3]:

• A (possibly nonunital) ring R is called a *ring with local units* if, for each finite subset S of R, there exists an idempotent e of R such that $S \subseteq eRe$;

• If R is a ring with local units then R is called *locally unit regular* if for each $a \in R$ there is an idempotent (a local unit) v and local inverses u, u' such that $uu' = v = u'u$, $va = a = av$ and $aua = a$ (see [3, Definition 6]).

Theorem 1. *Let Γ be an arbitrary graph, K be any field and S be the endomorphism ring of $L := L_K(\Gamma)$.*

1. *If S is locally unit regular, then L is locally unit regular. Moreover L is regular.*
2. *If L is locally unit regular, then vLv is locally unit regular for every non zero idempotent v of L.*

Proof. (1) Take $x \in L$. Since S is local unit regular, there exists an idempotent $e \in S$ such that $\lambda_x \in eSe$ and elements $f, g \in eSe$ such that $fg = e = gf$ and $\lambda_x f \lambda_x = \lambda_x$. Choose an idempotent $u \in L$ such that $x\lambda_{e(u)} = x = \lambda_{e(u)}x$ so $x \in \lambda_{e(u)} L \lambda_{e(u)}$. Note that,

$$\lambda_{f(u)}\lambda_{g(u)} = \lambda_{e(u)} = \lambda_{g(u)}\lambda_{f(u)}$$

and

$$\lambda_x = \lambda_x\lambda_{f(x)} = \lambda_x\lambda_{f(ux)} = \lambda_x\lambda_{f(u)}\lambda_x.$$

Since $f \in eSe$, there exists $h \in S$ such that $f = ehe$. Then $f(u) = e(u)h(u)e(u)$, so

$$\lambda_{f(u)} = \lambda_{e(u)h(u)e(u)} = \lambda_{e(u)}\lambda_{h(u)}\lambda_{e(u)}$$

and we get $\lambda_{f(u)} \in \lambda_{e(u)}L\lambda_{e(u)}$. Similarly $\lambda_{g(u)} \in \lambda_{e(u)}L\lambda_{e(u)}$. Hence L is locally unit regular.

(2) Take any $a \in vLv$. Since L is locally unit regular, there exist an idempotent e and local inverses u, u' such that $ea = a = ae$, $uu' = e = u'u$ and $aua = a$. As $ea = a$ and $av = a$ which imply $vea = va = a = ea$ and $eav = ea$ respectively, we get $ea = eav = vea$. Now $ea \in vLv$, which implies $e \in vLv$. Then $ve = e = ev$. Let $e^* = vev$, $h = vuv$ and $h' = veu'ev$. Note that

$$e^*e^* = (vev)(vev) = vevev = veevv = vev = e^* \in vLv$$

$$hh' = (vuv)(veu'ev) = vuveu'ev = vueu'ev = veuu'^*$$

$$h'h = (veu'ev)(vuv) = veu'evuv = veu'euv = veu'^*$$

$$aha = a(vuv)a = vauav = vav = a,$$

which imply vLv is locally unit regular. □

Definition 3. *A ring R is dependent if, for each $a, b \in R$, there are $s, t \in R$, not both zero, such that $sa + tb = 0$.*

Let Γ be an arbitrary graph, K be any field and S be the endomorphism ring of $L := L_K(\Gamma)$ considered as a right L-module. If S is dependent so is L. In fact, suppose S is dependent and $a, b \in L$. Then there are elements $f, g \in S$, not both zero, such that $f\lambda_a + g\lambda_b = 0$. If u_1 and u_2 are local units in L satisfying $u_1a = a = au_1$ and $u_2b = b = bu_2$, then

$$f\lambda_a = f\lambda_{u_1a} = f\lambda_{u_1}\lambda_a = \lambda_{f(u_1)}\lambda_a$$

and

$$g\lambda_b = g\lambda_{u_2b} = g\lambda_{u_2}\lambda_b = \lambda_{g(u_2)}\lambda_b.$$

Now

$$\begin{aligned} 0 &= f\lambda_a + g\lambda_b \\ &= \lambda_{f(u_1)}\lambda_a + \lambda_{g(u_2)}\lambda_b, \end{aligned}$$

and hence L is dependent.

In the literature on von-Neumann regular rings, various conditions have been shown to characterize the subclass of unit regular rings. In [8, Theorem 6], Ehrlich showed that every unit regular ring R is dependent. In [10, Corollary 10], Henriksen shows that not all dependent regular rings are unit regular. The following observation gives one more such condition for dependent rings.

Theorem 2. *If $L_K(\Gamma)$ is locally unit regular, then it is dependent.*

Proof. Let $L_K(\Gamma)$ be locally unit regular and let some elements provide locally unit regular condition in the definition. Take $a, b \in L_K(\Gamma)$. If both a and b have local inverses in $L_K(\Gamma)$, then there exist u_1 and u_2 in $L_K(\Gamma)$ such that $u_1 a = v$ and $u_2 b = v$ for local unit v in $L_K(\Gamma)$. So, we get $sa + tb = 0$, where $s = u_1$ and $t = -u_2$. If one of the elements, say a, has no local inverse in $L_K(\Gamma)$, by definition of locally unit regularity, then we can write $aua = a \Rightarrow aua = va \Rightarrow (au - v)a = 0$. Now we get $au - v \neq 0$. Assume $au - v = 0$. So $au = v$, it is a contradiction. Then, for $s = (au - v) \neq 0$ and $t = 0$, which implies $sa + tb = 0$. □

Definition 4. *Let R be a ring with local units. We call R left (right) locally unit regular ring if for each $a \in R$ there exist an idempotent $v \in R$ and left (right) local inverses u, u' such that $u'u = v$ $(uu' = v)$, $va = a$ $(av = a)$ and $aua = a$.*

Definition 5. *(*[12]*) Let M be a right R-module, and let $S = End_R(M)$. Then M is called is a d-Rickart (or dual Rickart) module if the image in M of any single element of S is a direct summand of M. Clearly, R_R a d-Rickart module iff R is a regular ring.*

Definition 6. *Given paths $p, q \in \Gamma$, we say that q is an initial segment of p if $p = qm$ for some path $m \in \Gamma$. It is well known that, given non-zero paths pq^* and mn^* in $L_K(\Gamma)$, q is an initial segment of m if and only if $(pq^*)(mn^*) \neq 0$.*

Theorem 3. *Let Γ be a graph, K be any field and S be the endomorphism ring of $L := L_K(\Gamma)$ considered as a right L-module. The following conditions are equivalent.*

1. *S is left locally unit regular.*
2. *S is regular and, for all paths $x, y \in L$, $Sx = Sy$ implies x is an initial segment of y.*
3. *L is dual-Rickart and, for all paths $x, y \in L$, $Sx = Sy$ implies x is an initial segment of y.*

Proof. (1) $\Rightarrow$ (2) Assume that S is left locally unit regular. Hence S is regular and L is left locally unit regular by Theorem 1. Let $x, y \in L$ be two paths. Then there exist an idempotent $v \in L$ and left local inverses $v_1, v_2 \in L$ such that $vy = y$, $v_2 v_1 = v$ and $y = y v_1 y$. If $Sx = Sy$, then $x = f(y)$ for some $f \in S$. Now $y = y v_1 y$ implies $f(y) = f(y v_1 y)$, and so $x = \underbrace{f(y v_1)}_{\in L} y$. Hence x is an initial segment of y.

(2) $\Rightarrow$ (3) This follows from [17, Corollary 3.2].

(3) $\Rightarrow$ (1) Assume that L is dual-Rickart. Then $f(L)$ is a direct summand of L, where $f \in S$. Let e be an idempotent in S with $f(L) = eL$. Let $x \in L$. Then there exists $y \in L$ such that $f(x) = e(y)$. Now

$$(ef)(x) = e(f(x)) = e(e(y)) = e(y) = f(x),$$

which implies $ef = f$. Let h be the left inverse of f and $g = fe$. Then $gh = e$ and $fhf = f$. □

Definition 7. *([13]) An endomorphism α of a module M is called morphic if $M/M\alpha \cong \mathrm{Ker}(\alpha)$, equivalently there exists $\beta \in \mathrm{End}(M)$ such that $M\beta = \mathrm{Ker}(\alpha)$ and $\mathrm{Ker}(\beta) = M\alpha$ by [13, Lemma 1]. The module M is called a morphic module if every endomorphism is morphic. If R is a ring, an element a in R is called left morphic if right multiplication $\cdot a :_R R \to_R R$ is a morphic endomorphism, that is if $R/Ra \cong l(a)$. The ring itself is called a left morphic ring if every element is left morphic, that is if ${}_RR$ is a morphic module.*

Note that if S is dependent then $L_K(\Gamma)$ is morphic by [14, Corollary 3.5].

Theorem 4. *Let Γ be any graph and let K be any field. If $L_K(\Gamma)$ is left morphic and regular ring then $L_K(\Gamma)$ is left locally unit regular ring.*

Proof. Let $L_K(\Gamma) = L$ be left morphic and regular ring. Then each $a \in L$ is both regular and morphic. So, there exist an $x \in L$ such that $a = axa$ and for some $b \in L$, $La = ann(b)$ and $Lb = ann(a)$. Let $u = xax + b$. Then $a = aua$. To see that u is left local inverse, since L has local units, choose an idempotent $v \in L$ such that $va = a$. Then we get, $0 = va - a = va - axa = (v - ax)a$, so $v - ax \in ann(a) = Lb$ and there exists an element $y \in L$ such that $v - ax = yb$. We take $u' = a + y(v - ax)$. We show that $u'u = v$:

$$\begin{aligned} u'u &= (a + y(v - ax))(xax + b) \\ &= axax + ab + y(v - ax)xax + y(v - ax)b \\ &= ax + ab + yvxax - yxaxax + yvb - yxab \\ &= ax + yb = v \end{aligned}$$

Hence $L = L_K(\Gamma)$ is left locally regular ring. □

Theorem 5. *Let Γ be a graph, K be any field and S be the endomorphism ring of $L := L_K(\Gamma)$ considered as a right L-module. If $L_K(\Gamma)$ is morphic and image projective then S is left morphic.*

Proof. Let $L := L_K(\Gamma)$ be morphic and image projective. Given any $\alpha \in S$, since L is morphic, we may choose an $\beta \in S$ such that, $L\alpha = ker(\beta)$ and $L\beta = ker(\alpha)$. Since $\alpha\beta = 0$, $S\alpha \subset ann^S(\beta)$. Conversely, if $\gamma \in ann^S(\beta)$ then $\gamma\beta = 0$ so $L\gamma \subset ker(\beta) = L\alpha$ and hence $\gamma \in S\alpha$ because L is image projective. Thus $S\alpha = ann^S(\beta)$. We may see $S\beta = ann^S(\alpha)$ in the same way. Hence S is left morphic. □

Definition 8. *([16, Definition 4.1]) If R is a ring with local units then R is called directly finite if for each $x, y \in R$ there is an idempotent u such that $xu = x = ux$ and $yu = y = uy$, we have that $xy = u$ implies $yx = u$.*

Theorem 6. *Let Γ be a graph, K be any field and S be the endomorphism ring of $L := L_K(\Gamma)$ considered as a right L-module. If S is a directly finite ring then $L_K(\Gamma)$ is directly finite.*

Proof. Take any x, y in $L_K(\Gamma)$. Since S is a direct finite ring, there is an idempotent ε in S such that $\lambda_x\varepsilon = \lambda_x = \varepsilon\lambda_x$ and $\lambda_y\varepsilon = \lambda_y = \varepsilon\lambda_y$, we have that $\lambda_x\lambda_y = \varepsilon$ implies $\lambda_y\lambda_x = \varepsilon$. For an idempotent u with $xu = x = ux$ and $yu = y = uy$,

$$\lambda_x\lambda_y = \varepsilon \Rightarrow \lambda_x\lambda_y\lambda_x = \varepsilon\lambda_x \Rightarrow \lambda_x = \varepsilon\lambda_{uv} \Rightarrow \lambda_x = \lambda_{\varepsilon(u)}\lambda_x$$

$$\lambda_x = \varepsilon\lambda_x = \lambda_{\varepsilon(x)} = \lambda_{\varepsilon(xu)} = \lambda_{\varepsilon(x)}\lambda_{\varepsilon(u)} = \varepsilon\lambda_x\lambda_{\varepsilon(u)}$$

So, $\lambda_x\lambda_{\varepsilon(u)} = \lambda_x = \lambda_{\varepsilon(u)}\lambda_x$. Similarly $\lambda_y\lambda_{\varepsilon(u)} = \lambda_y = \lambda_{\varepsilon(u)}\lambda_y$. Assume that, $\lambda_x\lambda_y = \lambda_{\varepsilon(u)}$. We then see that $\lambda_y\lambda_x = \lambda_{\varepsilon(u)}$.

$$\lambda_y\lambda_x = \lambda_y\lambda_{\varepsilon(u)}\lambda_x = \lambda_y\lambda_x\lambda_{\varepsilon(u)} = \varepsilon\lambda_{\varepsilon(u)} = \lambda_{\varepsilon^2(u)} = \lambda_{\varepsilon(u)},$$

as desired. □

Ones hopes that if $L_K(\Gamma)$ is directly finite then $L_K(\Gamma)$ is locally unit regular but this is not true. Because $K[x, x^{-1}]$ is a commutative Leavitt path algebra (of the graph with one vertex and one loop) clearly directly finite. But it is not von Neumann regular ring.

Corollary 1. *Let Γ be a graph, K be any field and S be the endomorphism ring of $L := L_K(\Gamma)$ considered as a right L-module. If S is a directly finite ring, then Γ is no exit.*

Proof. Let S be a directly finite ring. Then $L_K(\Gamma)$ is a directly finite ring. So, by [16, Proposition 4.3], Γ is no exit. □

Definition 9. *R is said to be a (left) exchange ring if for any direct decomposition $A = M \oplus N = \oplus_{i\in I} A_i$ of any left R-module A, where $R \cong M$ as left R-modules and I is a finite set, there always exist submodules B_i of A_i such that $A = M \oplus (\oplus_{i\in I} B_i)$.*

Theorem 7. *Let Γ be an infinite graph, K be any field and S be the endomorphism ring of $L := L_K(\Gamma)$ considered as a right L-module. Then*

(1) *If S is an exchange ring then L is directly finite.*
(2) *If L is a directly finite ring then L is an exchange ring.*

Proof. (1) Let S be an exchange ring. Then, by [5, Proposition 2.10], $L_K(\Gamma)$ is an exchange ring. For every $x, y \in L$ and an idempotent $u \in L$ such that $xu = x = ux$ and $yu = y = uy$ we have that $xy = u$. We show that $yx = u$. Since L is an exchange ring, there exist $r, s \in L$ such that $u = rx = s + x - sx$. So, $u = rx \Rightarrow uy = rxy \Rightarrow y = ru \Rightarrow yx = rux = rx = u$, as desired.

(2) Let L be a directly finite ring. For any $x, y \in L$ and an idempotent $u \in L$ such that $xu = x = ux$ and $yu = y = uy$ we have that $xy = u$ implies $yx = u$. We show that L is an exchange ring. For any $x \in L$ taking $r = y$ and $s = u$, we get $u = rx = s + x - sx$. So, L is an exchange ring. □

Corollary 2. *Let Γ be infinite graph, K be any field and S be the endomorphism ring of $L := L_K(\Gamma)$ considered as a right L-module. Then the following conditions are equivalent.*

(1) S *is an exchange ring.*
(2) $L_K(\Gamma)$ *is an exchange ring.*
(3) $L_K(\Gamma)$ *is a directly finite ring.*
(4) Γ *is no exit*

Proof. (1) $\Leftrightarrow$ (2) This is [5, Proposition 2.10].
(2) $\Leftrightarrow$ (3) This follows from Theorem 7 (1) and Theorem 7 (2).
(3) $\Leftrightarrow$ (4) This is [16, Teorem 4.12]. □

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Current address: Tufan ÖZDIN: Department of Mathematics, Faculty of Science and Art, Erzincan University, 24100 Erzincan, TURKEY.
E-mail address: `tozdin@erzincan.edu.tr; tufan.ozdin@hotmail.com`
ORCID Address: `http://orcid.org/0000-0001-8081-1871`