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SEMI-SLANT SUBMANIFOLDS OF (k, μ)- CONTACT MANIFOLD

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ABSTRACT. In the present paper, we study semi-slant submanifolds of (k, μ)-contact manifold and give conditions for the integrability of invariant and slant distributions which are involved in the definition of semi-slant submanifold. Further, we show the totally geodesicity of such distributions.

1. **Introduction**

The geometry of slant submanifolds was initiated by Chen [6] as a natural generalization of both holomorphic and totally real submanifolds. Since then many geometers have studied such slant immersions in almost Hermitian manifolds. The contact version of slant immersions was introduced by Lotta [11]. Latter, Cabrerizo et al., [3] studied and characterized slant submanifolds of K-contact and Sasakian manifolds and have given several examples of such immersions.

In 1994, Papaghiuc [12] has introduced the notion of semi-slant submanifolds of almost Hermitian manifolds. Cabrerizo et al., [4] extended the study of semi-slant submanifolds to the setting of almost contact metric manifolds. They worked out the integrability conditions of the distributions involved on these submanifolds and studied the geometrical significance of these distributions. Motivated by these studies of the above authors [4, 9, 12], in the present paper we extend the study of the semi-slant submanifolds of (k, μ)-contact manifold, which consist of both Sasakian as well as non-Sasakian cases and are introduced in 1995 by Blair, Koufogiorgos and Papantoniou [2]. Hence it is worth studying and is a generalization of [4].

The paper is organized as follows: In section-2, we recall the notion of (k, μ)-contact manifold and some basic results of submanifolds, which are used for further study. Section-3 is devoted to study semi-slant submanifolds of (k, μ)-contact manifold. Lastly, in section-4 we consider totally umbilical and totally contact umbilical semi-slant submanifolds of (k, μ)-contact manifold and find the necessary conditions to be totally geodesic.

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2. Preliminaries

A contact manifold is a $C^\infty-(2n+1)$ manifold $\tilde{M}^{2n+1}$ equipped with a global 1-form η such that $\eta\wedge(d\eta)^n\neq 0$ everywhere on $\tilde{M}^{2n+1}$. Given a contact form η it is well known that there exists a unique vector field ξ, called the characteristic vector field of η, such that $\eta(\xi)=1$ and $d\eta(X,\xi)=0$ for every vector field X on $\tilde{M}^{2n+1}$. A Riemannian metric g is said to be associated metric if there exists a tensor field ϕ of type (1,1) such that

$$\phi^2 = -I+\eta\otimes\xi,\quad \eta(\xi)=1,\quad \phi\xi=0,\quad \eta\cdot\phi=0, \tag{2.1}$$

$$g(\phi X,\phi Y) = g(X,Y)-\eta(X)\eta(Y),\quad g(X,\xi)=\eta(X), \tag{2.2}$$

$$g(X,\phi Y) = -g(\phi X,Y), \tag{2.3}$$

for all vector fields $X,Y\in T\tilde{M}$. Then the structure (ϕ,ξ,η,g) on $\tilde{M}^{2n+1}$ is called a contact metric structure and the manifold $\tilde{M}^{2n+1}$ equipped with such a structure is called a contact metric manifold [1].

Now we define a $(1,1)$ tensor field h by $h=\frac{1}{2}\mathcal{L}_\xi\phi$, where $\mathcal{L}$ denotes the Lie differentiation, then h is symmetric and satisfies $h\phi=-\phi h$. Further, a q-dimensional distribution on a manifold M is defined as a mapping D on M which assigns to each point $p\in M$, a q-dimensional subspace D_p of T_pM.
The (k,μ)-nullity distribution of a contact metric manifold $\tilde{M}(\phi,\xi,\eta,g)$ is a distribution

$$\begin{aligned} N(k,\mu): p\to N_p(k,\mu) &= \{Z\in T_pM : \tilde{R}(X,Y)Z \\ &= k[g(Y,Z)X-g(X,Z)Y]+\mu[g(Y,Z)hX-g(X,Z)hY]\}, \end{aligned}$$

for all $X,Y\in T\tilde{M}$. Hence if the characteristic vector field ξ belongs to the (k,μ) nullity distribution, then we have

$$\tilde{R}(X,Y)\xi = k[\eta(Y)X-\eta(X)Y]+\mu[\eta(Y)hX-\eta(X)hY]. \tag{2.4}$$

The contact metric manifold satisfying the relation (2.4) is called (k,μ) contact metric manifold [2]. It consists of both k-nullity distribution for $\mu=0$ and Sasakian for $k=1$. In (k,μ)-contact manifold the following relation holds:

$$(\tilde{\nabla}_X\phi)(Y) = g(X+hX,Y)\xi-\eta(Y)(X+hX), \tag{2.5}$$

for all $X,Y\in T\tilde{M}$, where $\tilde{\nabla}$ denotes the Levi-Civita connection on $\tilde{M}$. We also have on (k,μ)-contact manifold $\tilde{M}$

$$\tilde{\nabla}_X\xi = -\phi X-\phi hX. \tag{2.6}$$

Let M be a submanifold of a (k,μ)-contact manifold $\tilde{M}$, we denote by the same symbol g the induced metric on M. Let TM be the set of all vector fields tangent to M and $T^\perp M$ is the set of all vector fields normal to M. Then, the Gauss and Weingarten formulae are given by

$$\tilde{\nabla}_XY = \nabla_XY+\sigma(X,Y),\quad \tilde{\nabla}_XV = -A_VX+\nabla^\perp_XV, \tag{2.7}$$

for any $X, Y \in TM$, $V \in T^\perp M$, where ∇ (resp. $\nabla^\perp$) is the induced connection on the tangent bundle TM (resp. normal bundle $T^\perp M$) [7]. The shape operator A is related to the second fundamental form σ of M by

$$g(A_V X, Y) = g(\sigma(X, Y), V). \tag{2.8}$$

Now, for any $x \in M$, $X \in T_x M$ and $V \in T_x^\perp M$, we put

$$\phi X = TX + FX, \ \ \phi V = tV + fV, \tag{2.9}$$

where TX (resp. FX) is the tangential (resp. normal) component of ϕX, and tV (resp. fV) is the tangential (resp. normal) component of ϕV. The relation (2.9) gives rise to an endomorphism $T : T_x M \to T_x M$ whose square (T^2) will be denoted by Q. The tensor fields on M of type $(1, 1)$ determined by these endomorphisms will be denoted by the same letters T and Q respectively. From (2.3) and (2.9)

$$g(TX, Y) + g(X, TY) = 0, \tag{2.10}$$

for each $X, Y \in TM$. The covariant derivatives of the tensor fields T, Q and F are defined as

$$(\nabla_X T)Y = \nabla_X TY - T(\nabla_X Y), \tag{2.11}$$

$$(\nabla_X Q)Y = \nabla_X QY - Q(\nabla_X Y), \tag{2.12}$$

$$(\nabla_X F)Y = \nabla_X FY - F(\nabla_X Y). \tag{2.13}$$

Using (2.5), (2.6), (2.7), (2.9), (2.11), and (2.12), we obtain

$$(\nabla_X T)Y = A_{FY}X + t\sigma(X, Y) + g(X + hX, Y)\xi - \eta(Y)(X + hX), \tag{2.14}$$

$$(\nabla_X F)Y = -\sigma(X, TY) + f\sigma(X, Y). \tag{2.15}$$

3. Semi-slant submanifolds of a (k, μ)-contact manifold

As a generalization of slant and CR-submanifolds, Papaghiuc [12] introduced the notion of semi-slant submanifolds of an almost Hermitian manifolds. Cabrerizo et al., [4] gave the contact version of semi-slant submanifold and they obtained several interesting results. The purpose of the present section is to study semi-slant submanifolds of a (k, μ)-contact manifold.

A submanifold M of an almost contact metric manifold $\tilde{M}$ is said to be a slant submanifold if for any $x \in M$ and any $X \in T_x M$, the Wirtinger's angle, the angle between ϕX and $T_x M$, is constant $\theta \in [0, 2\pi]$. Here the constant angle θ is called the slant angle of M in $\tilde{M}$. The invariant submanifolds are slant submanifolds with slant angle 0 and anti-invariant submanifolds are slant submanifolds with slant angle $\frac{\pi}{2}$. A slant submanifold is called proper, if it is neither invariant nor anti-invariant. Recently, we have defined and studied slant submanifolds of a (k, μ)-contact manifold in [13].

A submanifold M of an almost contact metric manifold $\tilde{M}$ is said to be a semi-slant submanifold of $\tilde{M}$ [4] if there exist two orthogonal distributions D_1 and D_2 on M such that:

(i) TM admits the orthogonal direct decomposition $TM = D_1 \oplus D_2 \oplus < \xi >$.
(ii) The distribution D_1 is an invariant distribution, i.e., $\phi(D_1) = D_1$.
(iii) The distribution D_2 is slant with slant angle $\theta \neq 0$.
In particular, if $\theta = \frac{\pi}{2}$, then a semi-slant submanifold reduces to a semi-invariant submanifold. On a semi-slant submanifold M, for any $X \in TM$, we write

$$X = P_1X + P_2X + \eta(X)\xi, \tag{3.1}$$

where $P_1X \in D_1$ and $P_2X \in D_2$. Now by equations (2.9) and (3.1)

$$\phi X = \phi P_1X + TP_2X + FP_2X. \tag{3.2}$$

Then, it is easy to see that

$$\phi P_1X = TP_1X, \;\; FP_1X = 0, \quad TP_2X \in D_2. \tag{3.3}$$

Thus

$$TX = \phi P_1X + TP_2X \;\; and \;\; FX = FP_2X. \tag{3.4}$$

Let ν denote the orthogonal complement of ϕD_2 in $T^{\perp}M$ i.e., $T^{\perp}M = \phi D_2 \oplus \nu$. Then it is easy to observe that ν is an invariant subbundle of $T^{\perp}M$.

Now, we are in a position to workout the integrability conditions of the distributions D_1 and D_2 on a semi-slant submanifold of a (k, μ)-contact manifold.

Lemma 3.1. *Let M be a semi-slant submanifold of a (k, μ)-contact manifold $\tilde{M}$, then*

$$g([X, Y], \xi) = 2g(\phi X, Y) + g(Y, \phi hX) - g(X, \phi hY), \tag{3.5}$$

for any $X, Y \in D_1 \oplus D_2$.

The assertion can be proved by using the fact that $\nabla_X\xi = -\phi X - \phi hX$ for $X \in D_1$ and (2.3). Since for any $X \in D_1$
$g([X, \phi X], \xi) \neq 0$, we have

Corollary 3.1. *Let M be a semi-slant submanifold of a (k, μ)-contact manifold $\tilde{M}$ such that dim$(D_1) \neq 0$. Then, the invariant distribution D_1 is not integrable.*

Now for the slant distribution, we have

Theorem 3.1. *Let M be a semi-slant submanifold of a (k, μ)-contact manifold $\tilde{M}$. Then the slant distribution D_2 is integrable if and only if slant angle of D_2 is $\frac{\pi}{2}$ i.e., M is semi-invariant submanifold.*

Proof. For any $Z, W \in D_2$, by (3.5) we have

$$g([Z, W], \xi) = 2g(TZ, W) + g(W, ThZ) - g(Z, ThW).$$

If D_2 is integrable, then $T \mid D_2 \equiv 0$ and so $\theta = \frac{\pi}{2}$. Hence M is a semi-invariant submanifold.
Conversely, if sla$(D_2) = \frac{\pi}{2}$, then $\phi Z = FZ$ for each $Z \in D_2$ and by equations (2.5) and (2.7)

$$\phi\nabla_Z W + \phi\sigma(Z, W) = -A_{FZ}W + \nabla^{\perp}_Z FW - g(Z + hZ, W)\xi,$$

for each $Z, W \in D_2$. Interchanging Z and W in the above equation and subtracting the obtained relation from the same, we obtain

$$\phi[Z,W] = A_{FZ}W - A_{FW}Z + \nabla^{\perp}_{Z}FW - \nabla^{\perp}_{W}FZ - g(hZ,W)\xi + g(hW,Z)\xi. \quad (3.6)$$

Further, by using equations (2.3), (2.7) and (2.8) in (2.5), it is easy to obtain that

$$A_{FZ}W = A_{FW}Z, \quad (3.7)$$

for each $Z, W \in D_2$. In view of (3.5), (2.1) and (3.7), equation (3.6) yields

$$[Z,W] = \phi(\nabla^{\perp}_{Z}FW - \nabla^{\perp}_{W}FZ). \quad (3.8)$$

The right hand side of the above lies in D_2 because on using equations (2.5), (2.7) and (2.10), we observe that

$$g(V, \nabla^{\perp}_{W}FZ) = -g(A_{\phi V}W, Z)$$

for all $V \in \nu$ and $Z, W \in D_2$. This shows that

$$g(\nabla^{\perp}_{Z}FW - \nabla^{\perp}_{W}FZ, V) = 0.$$

i.e., $\nabla^{\perp}_{Z}FW - \nabla^{\perp}_{W}FZ$ lies in FD_2 for each $Z, W \in D_2$, and thus from equation (3.8), $[Z,W] \in D_2$. □

Now, for $Y \in D_1 \oplus D_2$, by equation (2.5), we have

$$\tilde{\nabla}_{\xi}\phi Y = \phi\tilde{\nabla}_{\xi}Y.$$

In particular, for $Y \in D_1$, the above equation yields

$$\nabla_{\xi}\phi Y = \phi\nabla_{\xi}Y.$$

This implies $\nabla_{\xi}Y \in D_1$ for any $Y \in D_1$.

The above observation together with the fact that $\sigma(X,\xi) = 0$ for $X \in D_1$ yields

Lemma 3.2. *On a semi-slant submanifold M of a (k,μ)-contact manifold $\tilde{M}$,*

$$[X,\xi] \in D_1 \ \ \textit{and} \ \ [Z,\xi] \in D_2$$

for any $X \in D_1$ and $Z \in D_2$.

Lemma 3.3. *Let M be a semi-slant submanifold of a (k,μ)-contact manifold $\tilde{M}$. Then, for any $X, Y \in TM$, we have*

$$P_1(\nabla_X\phi P_1Y) + P_1(\nabla_X TP_2Y) = \phi P_1(\nabla_X Y) + P_1A_{FP_2Y}X - \eta(Y)P_1X \quad (3.9)$$

Proof. By using equations (2.1), (2.7), (3.1), (3.2) and (3.3) we obtain

$$\begin{aligned} &\nabla_X\phi P_1Y + \sigma(\phi P_1Y, X) + \nabla_X TP_2Y + \sigma(TP_2Y, X) - A_{FP_2Y}X + \nabla^{\perp}_{X}FP_2Y \\ &= \phi P_1\nabla_X Y + TP_2\nabla_X Y + FP_2\nabla_X Y + t\sigma(X,Y) + f\sigma(X,Y) \\ &+ g(X+hX,Y)\xi - \eta(Y)P_1(X+hX) - \eta(Y)P_2(X+hX) - \eta(Y)\eta(X)\xi. \end{aligned}$$

Equating the components of D_1 we get (3.9). □

Proposition 3.2. *Let M be a semi-slant submanifold of (k,μ)-contact manifold $\tilde{M}$. Then*
(i) $D_1\oplus <\xi>$ is integrable if and only if

$$\sigma(X,\phi Y)=\sigma(Y,\phi X); \tag{3.10}$$

(ii) $D_2\oplus <\xi>$ is integrable if and only if

$$P_1(\nabla_Z TW - A_{NW}Z - \nabla_W TZ + A_{NZ}W)=0; \tag{3.11}$$

for any $X,Y\in D_1$ and $Z,W\in D_2$.

Proof. Now, for any $X,Y\in D_1\oplus <\xi>$ and $V\in T^{\perp}M$

$$g(\tilde{\nabla}_X\phi Y-\tilde{\nabla}_Y\phi X,V)=g(\sigma(X,\phi Y)-\sigma(\phi X,Y),V),$$

after simplification, we get

$$g((\tilde{\nabla}_X\phi)Y-(\tilde{\nabla}_Y)\phi X+\phi[X,Y],V)=g(\sigma(X,\phi Y)-\sigma(\phi X,Y),V).$$

Now using (2.5) and (3.2), we obtain

$$g(FP_2[X,Y],V)=g(\sigma(X,\phi Y)-\sigma(\phi X,Y),V).$$

Removing inner product, we get

$$FP_2[X,Y]=\sigma(X,\phi Y)-\sigma(\phi X,Y). \tag{3.12}$$

Hence, if $D_1\oplus <\xi>$ is integrable then (3.10) holds directly from (3.12). Conversely, by using (3.10), it is easy to prove that

$$\sigma(X,\phi Y)-\sigma(Y,\phi X)=\sigma(P_1X,\phi P_1Y)-\sigma(P_1Y,\phi P_1X)=0,$$

for any $X,Y\in D_1\oplus <\xi>$. Thus, by applying (3.12) it follows that $FP_2[X,Y]=0$. So, we can easily deduce that $P_2[X,Y]$ must vanish. Since D_2 is a slant distribution with nonzero slant angle. Hence $[X,Y]\in D_1\oplus <\xi>$ and statement (i) holds. With regards to statement (ii), by virtue of (3.9) we have

$$\phi P_1[Z,W]=P_1(\nabla_Z TW-\nabla_W TZ-A_{FW}Z+A_{FZ}W).$$

for any $Z,W\in D_2\oplus <\xi>$. Hence (3.11) holds if and only if

$$\phi P_1[Z,W]=0, \tag{3.13}$$

for any $Z,W\in D_2\oplus <\xi>$. But it can be showed that (3.13) is equivalent to $D_2\oplus <\xi>$ being an integrable distribution. □

The Nijenhuis tensor field S of the tensor T is given by

$$S(X,Y)=[TX,TY]+T^2[X,Y]-T[TX,Y]-T[X,TY],$$

for $X,Y\in TM$. In particular, for $X\in D_1$ and $Z\in D_2$, the above equation on simplification takes the form

$$S(X,Z)=(\nabla_{TX}T)Z-(\nabla_{TZ}T)X+T(\nabla_Z T)X-T(\nabla_X T)Z.$$

Using (2.14) the above equation becomes

$$S(X,Z) = A_{FZ}TX + t\sigma(TX,Z) - t\sigma(TZ,X) - T(A_{FZ}X). \tag{3.14}$$

Theorem 3.3. *If the invariant distribution D_1 on a semi-slant submanifold M of a (k,μ)-contact manifold $\tilde{M}$ is integrable and its leaves are totally geodesic in M, then*
(i) $\sigma(D_1, D_1) \in \nu$,
(ii) $S(D_1, D_2) \in D_2$.

Proof. By hypothesis, for any X, Y in D_1 and Z in D_2

$$g(\nabla_X Y, Z) = 0,$$

and therefore by Gauss formula, we have

$$g(\phi\tilde{\nabla}_X Y, \phi Z) = 0.$$

The above equation on making use of equations (2.5), (2.7) and (2.9) yields

$$g(\sigma(X, \phi Y), FZ) = 0.$$

This proves statement (i). To prove statement (ii), use (3.14) to get

$$g(S(X,Z), Y) = g(A_{FZ}TX + t\sigma(TX,Z) - t\sigma(TZ,X) - TA_{FZ}X, Y).$$

The right hand side of the above equation is zero in view of statement (i) and thus (ii) is established. □

Next for the slant distribution, we have:

Theorem 3.4. *If the slant distribution D_2 on a semi-slant submanifold M of a (k,μ)-contact manifold $\tilde{M}$ is integrable and its leaves are totally geodesic in M, then*
(i) $\sigma(D_1, D_2) \in \nu$,
(ii) $S(D_1, D_2) \in D_1$.

Proof. By hypothesis,

$$g(\nabla_Z W, \phi X) = 0,$$

for any $Z, W \in D_2$ and $X \in D_1$. By applying (2.5), (2.7) and (2.9)

$$g(\sigma(X,Z), FW) = 0.$$

That proves (i). Now by using equation (3.14)

$$g(S(X,Z), W) = g(A_{FZ}TX + t\sigma(TX,Z) - t\sigma(TZ,X) - TA_{FZ}X, W),$$

for $X \in D_1$ and $Z, W \in D_2$. The right hand side of the above equation is zero by part (i). This proves (ii) and the theorem. □

Example: For any $\theta \in [0, \frac{\pi}{2}]$

$$x(u_1, u_2, u_3, u_4, u_5) = (u_1, 0, u_3, 0, u_2, 0, u_4 cos\theta, u_4 sin\theta, u_5)$$

defines a five dimensional semi-slant submanifold M, with slant angle θ, in R^9 with its usual (k,μ)-contact structure (ϕ_0,ξ,η,g) [13]. Further,

$$\begin{aligned} e_1 &= 2(\frac{\partial}{\partial x_1}+x_5\frac{\partial}{\partial t}); \qquad e_2=2\frac{\partial}{\partial x_5}; \qquad e_3=2(\frac{\partial}{\partial x_3}+x_7\frac{\partial}{\partial t});\\ e_4 &= cos\theta(2\frac{\partial}{\partial x_7}+sin\theta(2\frac{\partial}{\partial x_8}); \qquad e_5=\frac{\partial}{\partial t}=\xi, \end{aligned} \tag{3.15}$$

form a local orthonormal frame of TM. If we define the distribution $D_1=<e_1,e_2>$ and $D_2=<e_3,e_4>$, then it is easy to check that the distribution D_1 is invariant under ϕ and D_2 is slant with slant angle θ. That is M is semi-slant submanifold.

4. Totally umbilical submanifolds of (k,μ)-contact manifold

Definition 1. A submanifold M is said to be totally umbilical submanifold if its second fundamental form satisfies

$$\sigma(X,Y)=g(X,Y)H,$$

for all $X,Y\in TM$, where H is the mean curvature vector.

To investigate totally umbilical submanifolds of a (k,μ)-contact manifold, we first establish the following preliminary result.

Proposition 4.5. *Let M be a semi-slant submanifold of a (k,μ)-contact manifold $\tilde{M}$ with $\sigma(X,TX)=0$ for each $X\in D_1\oplus<\xi>$. If $D_1\oplus<\xi>$ is integrable then each of its leaves are totally geodesic in M as well as in $\tilde{M}$.*

Proof. For $X\in D_1\oplus<\xi>$, by equation (2.15)

$$(\nabla_X F)X=-\sigma(X,TX)+f\sigma(X,X),$$

by using (2.13) and the fact that $FX=0$ for each $X\in D_1$, we get

$$F\nabla_X X=f\sigma(X,X). \tag{4.1}$$

Now, making use of Proposition 3.2 and the assumption that $\sigma(X,TX)=0$, we obtain $\sigma(X,TY)=0$ i.e., $\sigma(X,Y)=0$ for each $X,Y\in D_1\oplus<\xi>$. This proves that the leaves of $D_1\oplus<\xi>$ are totally geodesic in $\tilde{M}$. Thus by (4.1), we obtain that $\nabla_X Y\in D_1\oplus<\xi>$ i.e., the leaves of $D_1\oplus<\xi>$ are totally geodesic in M. □

As an immediate consequence of the above, we have

Corollary 4.2. *Let M be a totally umbilical semi-slant submanifold of a (k,μ)-contact manifold $\tilde{M}$. If $D_1\oplus<\xi>$ is integrable, then each of its leaves are totally geodesic in M as well as in $\tilde{M}$.*

Definition 2. [10] A submanifold M of an almost contact metric manifold is said to be totally contact umbilical submanifold if

$$\sigma(X,Y)=g(\phi X,\phi Y)K+\eta(Y)\sigma(X,\xi)+\eta(X)\sigma(Y,\xi),$$

for all $X, Y \in TM$, where K is a normal vector field on M. If $K = 0$ then M is said to be a totally contact geodesic submanifold. For a submanifold of a (k, μ)-contact manifold, the condition for totally contact umbilicalness reduces to

$$\sigma(X, Y) = g(\phi X, \phi Y)K.$$

Theorem 4.6. *Let M be a totally contact umbilical semi-slant submanifold of a (k, μ)-contact manifold $\tilde{M}$, with $dim(D_1) \neq 0$. Then the mean curvature vector is a global section of FD_2.*

Proof. Let $X \in D_1$ be a unit vector field and $V \in \nu$, then

$$g(H, V) = g(\sigma(X, X), V) = g(\tilde{\nabla}_X \phi X, \phi V) = g(\sigma(X, \phi X, \phi V)) = 0$$

$\Longrightarrow H \in FD_2$. □

In view of Theorem 4.6, we have the following:

Theorem 4.7. *A totally contact umbilical semi-slant submanifold of a (k, μ)-contact manifold is totally contact geodesic if the invariant distribution D_1 is integrable.*

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