

## Soft k-Uni Ideals of Semirings and its Algebraic Applications

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**ABSTRACT:** In this paper, firstly, information about soft set introduced by Molodtsov is given. Then, the operations on the soft set are introduced. After then a definition of soft uni-k-ideal of a semiring by using the union operation of sets is given. Finally, some algebraic applications by using soft uni-k-ideal are investigated.

**Keywords:** Soft sets, k-ideals, soft uni-ideals, soft k-uni ideals.

## Yarıhalkaların Esnek k-Birleşimsel İdealleri ve Cebirsel Uygulamaları

**ÖZET:** Bu çalışmada, ilk olarak Molodtsov tarafından ortaya atılan esnek küme teorisi hakkında bilgi verildi. Sonra esnek küme üzerindeki işlemler tanımlandı. Daha sonra kümelerin birleşim işlemi kullanılarak bir yarıhalkanın esnek birleşimsel k-ideal tanımı verildi. Son olarak, esnek birleşimsel k-ideali kullanılarak bazı cebirsel uygulamalar araştırıldı.

**Anahtar Kelimeler:** Esnek kümeler, k-idealler, esnek birleşimsel idealler, esnek k-birleşimsel idealler

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## INTRODUCTION

Recently, soft set theory have been investigated intensively. Firstly, Molodtsov (Molodtsov, 1999) suggested soft set theory. Operations of soft sets are investigated by Maji et al. (Maji et al., 2003). Publications on soft set theory are continuing rapidly. Then, Çağman and Enginoğlu redefined definition of the soft set and investigated some properties (Çağman and Enginoğlu, 2010). Firstly, Aktaş and Çağman studied on soft algebra (Aktaş and Çağman, 2007). They defined a soft group and investigated its algebraic property. Acar et al. studied on the soft ring (Acar et al., 2010). Feng et al. defined the soft semiring and soft ideal (Feng et al., 2008). The concept of idealistic soft BCK/BCI-algebras was introduced by Jun and Park (Jun and Park, 2008). Sun et al. studied the soft modules (Sun et al., 2008). Çağman et al. studied the concept of soft int-groups and investigated algebraic properties of soft int-group (Çağman et al., 2012). Then Çıtak and Çağman gave a

definition of soft int-ring (Çıtak and Çağman, 2015). Sezgin et al. expressed a definition of soft uni-group and investigated some properties of soft uni-group (Sezgin et al., 2015). Soft BL-algebras are introduced by Zhan and Jun (Zhan and Jun, 2010).

Then, Zhan and Xu introduced the soft lattice implication algebras (Zhan and Xu, 2011). Atagün and Sezgin studied the int-soft substructures of groups and semirings (Atagün and Sezgin, 2017). In recent years, soft algebraic structures have been studied quickly (Ali et al. 2009, Ali et al. 2011, Lee et al. 2014, Sezgin et al. 2014, Jana and Pal 2016, Sezgin 2017, Ma et al. 2017).

In this paper, we define the concept of soft uni-ideal of a semiring. We also introduce the concept of soft  $k$ -uni-ideal of a semiring by using intersection operation of the set and investigate the basic properties of soft  $k$ -uni-ideal. Moreover, we define a soft  $k$ -product of two soft left  $k$ -uni-ideals and works on their algebraic structures in detail.

## MATERIALS AND METHODS

Throughout this work,  $U$  is a universal set,  $E$  is a set of parameters,  $X \subseteq E$  and  $P(U)$  is the power set of  $U$ .

**Definition 1.** A nonempty set  $S$  together with a binary operation  $*$  is a semigroup if  $*$  is associative in  $S$ , that is, for all  $a, b, c \in S$ ,  $a*(b*c) = (a*b)*c$ .

A semigroup is commutative, if  $*$  is commutative in  $S$ , that is, for all  $a, b \in S$ ,

$$a * b = b * a$$

(Golan, 1992).

**Definition 2.** A semiring is a nonempty set  $S$  together with two binary operations addition and multiplication denoted by “+”, “.” respectively, satisfying

- i.  $(S, +)$  is a commutative semigroup,
- ii.  $(S, \cdot)$  is a semigroup,
- iii. distributive law holds, that is, for all  $a, b, c \in S$ ,  $a \cdot (b + c) = a \cdot b + a \cdot c$  and  $(a + b) \cdot c = a \cdot c + b \cdot c$

(Golan, 1992).

Henceforth,  $S$  and  $R$  is a semiring.

**Definition 3.** A subset  $I$  of  $S$  is a left (right) ideal of  $S$ , if

- i.  $a + b \in I$  for all  $a, b \in I$
- ii.  $b \cdot a \in I (a \cdot b \in I)$  for any  $a \in I$  and  $b \in S$

If  $I$  is both a left and right ideal, then  $I$  is an ideal (Golan, 1992).

**Definition 4.** Let  $I$  be a left (right) ideal of  $S$ . A left (right) ideal of  $S$  is a left (right)  $k$ -ideal of  $S$  if  $b, c \in I, a \in S, a + b = c$  implies  $a \in I$  (La Torre, 1965).

**Definition 5.**  $F_X$  is a soft set over  $U$  where  $F_X : E \rightarrow P(U)$  is a function such that  $F_X(a) = \emptyset$  if  $a \notin X$  (Molodtsov, 1999).

The set of soft sets is symbolized by  $S(U)$ .

**Definition 6.** Let  $F_X \in S(U)$ .  $F_X$  is an empty soft set if  $F_X(a) = \emptyset$  for each  $a \in X$ . An

empty soft set is symbolized by  $\tilde{\emptyset}$ .

$F_X$  is said to be a universal soft set if  $F_X(a) = U$  for each  $a \in X$ . A universal soft set is symbolized by  $\tilde{A}$  (Maji et al., 2003).

**Definition 7.** Let  $F_X, G_X \in S(U)$ .  $F_X$  is said to be a soft subset of  $G_X$ , if  $F_X(a) \subseteq G_X(a)$  for each  $a \in X$ . A soft subset is symbolized by  $F_X \tilde{\subseteq} G_X$ .

$F_X$  and  $G_X$  said to equal soft sets if  $F_X(a) = G_X(a)$  for each  $a \in X$ . Equal soft sets are symbolized by  $F_X \tilde{=} G_X$  (Çağman and Enginoğlu, 2010).

**Definition 8.** Let  $F_X, G_X \in S(U)$ . The union of  $F_X$  and  $G_X$  is defined by  $(F_X \cup G_X)(a) = F_X(a) \cup G_X(a)$  for each  $a \in X$ .

The intersection of  $F_X$  and  $G_X$  is defined by  $(F_X \cap G_X)(a) = F_X(a) \cap G_X(a)$  for each  $a \in X$ . They are symbolized by  $F_X \tilde{\cup} G_X$  and  $F_X \tilde{\cap} G_X$ , respectively (Çağman and Enginoğlu, 2010).

**Definition 9.** Let  $F_X, G_X \in S(U)$ .  $\wedge$ -product and  $\vee$ -product of  $F_X$  and  $G_X$  are defined by  $(F_X \wedge G_X)(a, b) = F_X(a) \cap G_X(b)$  and  $(F_X \vee G_X)(a, b) = F_X(a) \cup G_X(b)$  for each  $a, b \in X$ , respectively. They are symbolized by  $F_X \wedge G_X$  and  $F_X \vee G_X$ , respectively (Çağman and Enginoğlu, 2010).

**Definition 10.** Let  $F_X \in S(U)$  and  $\varphi$  be a function from a set  $X$  to a set  $Y$ . For all

$$b \in Y, \varphi(F_X): Y \rightarrow P(U), \varphi(F_X)(b) = \begin{cases} \cup\{F_X(a) : a \in X, \varphi(a) = b\} & \text{if } b \in \varphi(X) \\ \emptyset, & \text{if } b \notin \varphi(X) \end{cases}$$

is a soft image of  $F_X$  under  $\varphi$ . For all  $a \in X$ ,

$\varphi^{-1}(F_X): X \rightarrow P(U)$ ,  $\varphi^{-1}(F_X)(a) = F_X(\varphi(a))$  is a soft preimage (or soft inverse image) of  $F_X$  under  $\varphi$  (Çağman et al., 2012).

**Definition 11.** Let  $F_X \in S(U)$  and  $A \in P(U)$ . A set  $F_X^{\subseteq A} = \{a \in X : F_X(a) \subseteq A\}$  is a lower  $A$ -inclusion of  $F_X$  (Sezgin et al., 2014).

**Definition 12.** The relative complement of the soft set  $F_X$  over  $U$  is denoted by  $F_X^r$ , where  $F_X^r: X \rightarrow P(U)$  is a mapping given as  $F_X^r(a) = U - F_X(a)$  for all  $a \in X$  (Sezgin et al., 2014).

**Definition 13.** Let  $S$  be a ring and  $F_S \in S(U)$ .  $F_S$  is called a soft left (right) int-ideal over  $U$ , if it satisfies the following axioms:

- i.  $F_S(a + b) \supseteq F_S(a) \cap F_S(b)$
- ii.  $F_S(ab) \supseteq F_S(b)$  ( $F_S(ab) \supseteq F_S(a)$ )

for all  $a, b \in S$ .

$F_S$  is a soft int-ideal over  $U$ , if it is both soft left int-ideal and soft right int-ideal over  $U$  (Çıtak and Çağman, 2015).

**Definition 14.** Let  $S$  be a semiring and  $F_S \in S(U)$ .  $F_S$  is called a soft left (right) int-ideal over  $U$ , if it satisfies the following axioms:

- (i)  $F_S(a + b) \supseteq F_S(a) \cap F_S(b)$
- (ii)  $F_S(ab) \supseteq F_S(b)$  ( $F_S(ab) \supseteq F_S(a)$ )

for all  $a, b \in S$ .

$F_S$  is a soft int-ideal over  $U$ , if it is both soft left int-ideal and soft right int-ideal over  $U$  (Çıtak, 2017).

## RESULTS AND DISCUSSION

In this section, we first introduce a concept of soft uni-ideal of a semiring. Then, we give the concept of soft k-uni-ideal of a semiring by using union operation of set and investigate the basic properties of soft k-uni-ideal. Finally, we define a soft k-product of two soft left k-uni-ideals and study their algebraic structures.

**Definition 1.** Let  $F_S \in S(U)$ .  $F_S$  is called a soft left (right) uni-ideal over  $U$ , if it satisfies the following axioms:

$$(i) F_S(a + b) \subseteq F_S(a) \cup F_S(b)$$

$$(ii) F_S(ab) \subseteq F_S(b) \quad (F_S(ab) \subseteq F_S(a))$$

for all  $a, b \in S$ .

$F_S$  is a soft uni-ideal over  $U$ , if it is both soft left uni-ideal and soft right uni-ideal over  $U$ .

**Definition 2.** Let  $F_S$  be a soft left uni-ideal over  $U$ .  $F_S$  is called a soft left k-uni-ideal over  $U$ , if for each  $a, b, c \in S$ ,  $a + b = c$  implies  $F_S(a) \subseteq F_S(b) \cup F_S(c)$ .

A soft right k-uni-ideal can be defined similarly.

**Theorem 3.** Let  $F_S$  and  $G_S$  be two soft left (right) k-uni-ideals over  $U$ . Then,  $F_S \tilde{\cup} G_S$  is

a soft left (right) k-uni-ideal over  $U$ .

a soft left (right) k-uni-ideal over  $U$ .

**Proof.** Let  $a, b, c \in S$  such that  $a + b = c$ . Then,

$$\begin{aligned} (F_S \cup G_S)(a) &= F_S(a) \cup G_S(a) \\ &\subseteq F_S(b) \cup F_S(c) \cup G_S(b) \cup G_S(c) \\ &= F_S(b) \cup G_S(b) \cup F_S(c) \cup G_S(c) \\ &= (F_S \cup G_S)(b) \cup (F_S \cup G_S)(c) \end{aligned}$$

Therefore,  $F_S \tilde{\cup} G_S$  is a soft left k-uni-ideal over  $U$ .

**Remark 4.** The following example shows that  $F_S \tilde{\cap} G_S$  is not a soft k-uni-ideal over  $U$ .

**Example 5.** Assume that set of the natural numbers  $\mathbb{N}$  is the universal set and  $S = \{x, y, z\}$  is the subset of set of parameters. The soft k-uni-ideals  $F_S$  and  $G_S$  over  $\mathbb{N}$  are defined as

+	x	y	z
x	x	y	y
y	y	x	x
z	y	x	x

and

.	x	y	z
x	z	z	z
y	z	z	z
z	z	z	z

$F_S(x) = \{2, 4, 6\}$ ,  $F_S(y) = \{1, 2, 3, 4, 5, 6, 8, 18\}$ ,  $F_S(z) = \{2, 4, 6\}$  and  $G_S(x) = \{2, 12, 22\}$ ,

$G_S(y) = \{2, 8, 12, 18, 22\}$ ,  $G_S(z) = \{2, 12, 22\}$ .

It shows that  $(F_S \cap G_S)(x+z) \not\subseteq (F_S \cap G_S)(x) \cup (F_S \cap G_S)(z)$ . Then,  $F_S \tilde{\cap} G_S$  is not a soft uni-ideal over  $\mathbb{N}$ . Therefore,  $F_S \tilde{\cap} G_S$  is not a soft k-uni-ideal over  $\mathbb{N}$ .

**Theorem 6.** Let  $F_S$  be a soft left (right) k-uni-ideal over  $U$  and  $G_R$  be a soft left (right) k-uni-ideal over  $U$ . Then,  $F_S \vee G_R$  is a soft left (right) k-uni-ideal over  $U$ .

**Proof.** Let  $(a_1, a_2), (b_1, b_2), (c_1, c_2) \in S \times R$  such that  $(a_1, a_2) + (b_1, b_2) = (c_1, c_2)$ . Hence,

$$\begin{aligned} (F_S \vee G_R)(a_1, a_2) &= F_S(a_1) \cup G_R(a_2) \\ &\subseteq F_S(b_1) \cup F_S(c_1) \cup G_R(b_2) \cup G_R(c_2) \\ &= F_S(b_1) \cup G_R(b_2) \cup F_S(c_1) \cup G_R(c_2) \\ &= (F_S \cup G_R)(b_1, b_2) \cup (F_S \cup G_R)(c_1, c_2) \end{aligned}$$

Therefore,  $F_S \vee G_R$  is a soft left k-uni-ideal over  $U$ .

**Remark 7.** The following example shows that  $F_S \wedge G_R$  is not a soft k-uni-ideal over  $U$ .

**Example 8.** Consider  $F_S$  in Example 3.6. Also let  $G_{\mathbb{Z}_4}$  be a soft k-uni-ideal over  $\mathbb{N}$  defined by  $G_{\mathbb{Z}_4}(0) = \{3, 5\}$ ,  $G_{\mathbb{Z}_4}(1) = \{1, 3, 5, 7, 9\}$ ,  $G_{\mathbb{Z}_4}(2) = \{1, 3, 5, 7\}$ ,  $G_{\mathbb{Z}_4}(3) = \{1, 3, 5, 7, 9\}$ .

It shows that  $(F_S \wedge G_{\mathbb{Z}_4})[(x, 2) + (z, 3)] \not\subseteq (F_S \wedge G_{\mathbb{Z}_4})(x, 2) \cup (F_S \wedge G_{\mathbb{Z}_4})(z, 3)$ . Then,  $F_S \wedge G_{\mathbb{Z}_4}$  is not a soft uni-ideal over  $\mathbb{N}$ . Therefore,  $F_S \wedge G_{\mathbb{Z}_4}$  is not a soft k-uni-ideal over  $\mathbb{N}$ .

**Definition 9.** Let  $F_S$  and  $G_S$  be two soft sets over  $U$ . Then,  $F_S \tilde{\odot}_k G_S$  is called a soft k-uni-product where

$$(F_S \tilde{\odot}_k G_S)(a) = \begin{cases} \bigcup F_S(a_i) \cup G_S(b_i) : i = 1, 2, & \text{if } a + a_1 b_1 = a_2 b_2 \\ \emptyset & \text{if } a \text{ cannot be predicated as } a + a_1 b_1 = a_2 b_2 \end{cases}$$



**Theorem 10** Let  $F_S$  be a soft right k-uni-ideal and  $G_S$  be a soft left k-uni-ideal over  $U$ .

Then,  $F_S \tilde{\odot}_k G_S \tilde{\supseteq} F_S \tilde{\cup} G_S$ .

**Proof.** Let  $a \in S$ . If  $(F_S \tilde{\odot}_k G_S)(a) = \emptyset$ , then it is a clear proof. Let  $(F_S \tilde{\odot}_k G_S)(a) \neq \emptyset$

. Since  $F_S$  is a soft right k-uni-ideal over  $U$ , then

$$\begin{aligned} F_S(a) &\subseteq F_S(a_1 b_1) \cup F_S(a_2 b_2) \\ &\subseteq F_S(a_1) \cup F_S(a_2) \end{aligned}$$

for all  $a_i, b_i \in S, i = 1, 2$ , satisfying  $a + a_1 b_1 = a_2 b_2$ . Similarly

$$G_S(a) \subseteq G_S(b_1) \cup G_S(b_2).$$

Thus,

$$\begin{aligned} (F_S \tilde{\odot}_k G_S)(a) &= \cup F_S(a_i) \cup G_S(b_i) : i = 1, 2, a + a_1 b_1 = a_2 b_2 \\ &\supseteq F_S(a) \cup G_S(a) \\ &= (F_S \cup G_S)(a) \end{aligned}$$

Therefore,  $F_S \tilde{\odot}_k G_S \tilde{\supseteq} F_S \tilde{\cup} G_S$ .

**Lemma 11.** Let  $F_S \in S(U)$ .  $F_S$  is a soft left (right) uni-ideal over  $U$  iff  $F_S^{\subseteq A}$  is a left

(right) ideal of  $S$ , for any  $A \in P(U)$  such that  $F_S^{\subseteq A} \neq \emptyset$ .

**Proof.** Assume that  $F_S$  is a soft left uni-ideal over  $U$ . Let  $a, b \in F_S^{\subseteq A}$ . Then,  $F_S(a) \subseteq A$

and  $F_S(b) \subseteq A$ . It follows that

$$\begin{aligned} F_S(a + b) &\subseteq F_S(a) \cup F_S(b) \\ &\subseteq A \end{aligned}$$

Thus,  $a + b \in F_S^{\subseteq A}$ . Let  $a \in F_S^{\subseteq A}$  and  $s \in S$ . It shows that  $F_S(a) \subseteq A$ . And then,

$$F_s(sa) \subseteq F_s(a) \\ \subseteq A$$

Thus,  $sa \in F_s^{\subseteq A}$ . Therefore,  $F_s^{\subseteq A}$  is a left ideal of  $S$ , for every  $A \in P(U)$  such that  $F_s^{\subseteq A} \neq \emptyset$ .

Conversely, let  $F_s^{\subseteq A}$  be a left ideal of  $S$ , for every  $A \in P(U)$  such that  $F_s^{\subseteq A} \neq \emptyset$ .  $a, b \in F_s^{\subseteq B}$  such that  $B = F_s(a) \cup F_s(b)$  for each  $a, b \in S$ . Hence,  $a + b \in F_s^{\subseteq B}$ . Thus,

$$F_s(a + b) \subseteq B \\ = F_s(a) \cup F_s(b)$$

Also,  $a \in F_s^{\subseteq C}$  such that  $C = F_s(a)$ , we obtain  $sa \in F_s^{\subseteq C}$  for each  $s \in S$ . Then,  $F_s(sa) \subseteq F_s(a)$ . Therefore,  $F_s$  is a soft left uni-ideal over  $U$ .

**Theorem 12.** Let  $F_s \in S(U)$ .  $F_s$  is a soft left (right) k-uni-ideal over  $U$  iff  $F_s^{\subseteq A}$  is a left (right) k-ideal of  $S$  for any  $A \in P(U)$  such that  $F_s^{\subseteq A} \neq \emptyset$ .

**Proof.** Following Lemma 11, it was proved that a soft set  $F_s$  is a soft left uni-ideal iff  $G_s^{\subseteq A}$  is a left ideal of  $S$  for any  $A \in P(U)$  such that  $F_s^{\subseteq A} \neq \emptyset$ . Suppose that  $F_s$  is a soft left k-uni ideal over  $U$ . Let  $a, k \in F_s^{\subseteq A}$ ,  $s \in S$ ,  $s + a = k$ . Since  $a, k \in F_s^{\subseteq A}$ , we have  $F_s(a) \subseteq A$ ,  $F_s(k) \subseteq A$ . Also,  $F_s(s) \subseteq F_s(a) \cup F_s(k)$ . Thus,  $F_s(s) \subseteq A$ , and so  $s \in F_s^{\subseteq A}$ . Hence,  $F_s^{\subseteq A}$  is a left k-ideal of  $S$ .

Conversely, let  $F_s^{\subseteq A}$  be a left k-ideal of  $S$ , for any  $A \in P(U)$  such that  $F_s^{\subseteq A} \neq \emptyset$ . Let  $a, s, k \in S$  such that  $a + s = k$ . Suppose that  $F_s(s) = A_1$ ,  $F_s(k) = A_2$  ( $A_i \in P(U)$ ). Let  $A_1 \cup A_2 = A$ . Then,  $s \in F_s^{\subseteq A}$  and  $k \in F_s^{\subseteq A}$ .

Since  $F_S^{\subseteq A}$  is a left k-ideal of  $S$ , we have  $a \in F_S^{\subseteq A}$ , i.e.  $F_S(a) \subseteq F_S(s) \cup F_S(k)$ . Thus,  $F_S$  is a soft left k-uni-ideal over  $U$ .

**Lemma 13.** Let  $F_S \in S(U)$ .  $F_S$  is a soft left (right) uni-ideal set over  $U$  iff  $F_S^r$  is a left (right) int-ideal over  $U$ .

**Proof.** Let  $F_S$  is a soft left (right) uni-ideal over  $U$ . Then,

$$\begin{aligned} F_S^r(a+b) &= U - F_S(a+b) \\ &\supseteq U - F_S(a) \cup F_S(b) \\ &= U - F_S(a) \cap U - F_S(b) \\ &= F_S^r(a) \cap F_S^r(b) \end{aligned} \quad \text{and} \quad \begin{aligned} F_S^r(ab) &= U - F_S(ab) \\ &\supseteq U - F_S(a) \\ &= F_S^r(a) \end{aligned}$$

Therefore,  $F_S^r$  is a soft left int-ideal over  $U$ .

Conversely, let  $F_S^r$  be a soft left (right) int-ideal over  $U$ . Then,

$$\begin{aligned} F_S(a+b) &= U - F_S^r(a+b) \\ &\subseteq U - [F_S^r(a) \cap F_S^r(b)] \\ &= [U - F_S^r(a)] \cup [U - F_S^r(b)] \\ &= F_S(a) \cup F_S(b) \end{aligned} \quad \text{and} \quad \begin{aligned} F_S(ab) &= U - F_S^r(ab) \\ &\subseteq U - F_S^r(a) \\ &= F_S(a) \end{aligned}$$

Therefore,  $F_S$  is a soft left uni-ideal over  $U$ .

**Theorem 14.** Let  $F_S \in S(U)$ .  $F_S$  is a soft left (right) k-uni-ideal over  $U$  iff  $F_S^r$  is a left (right) k-int-ideal over  $U$ .

**Proof.** By Lemma 13, we know that  $F_S$  is a soft left (right) uni-ideal over  $U$  iff  $F_S^r$  is a left (right) int-ideal over  $U$ .

Let  $F_S$  is a soft left (right) k-uni-ideal over  $U$ . Let  $a, b, c \in S$  such that  $a + b = c$ . Then,

$$\begin{aligned}
 F_S^r(a) &= U - F_S(a) \\
 &\supseteq U - F_S(b) \cup F_S(c) \\
 &= U - F_S(b) \cap U - F_S(c) \\
 &= F_S^r(b) \cap F_S^r(c)
 \end{aligned}$$

Therefore,  $F_S^r$  is a soft left k-int-ideal over  $U$ .

Conversely, let  $F_S^r$  is a soft left (right) k-int-ideal over  $U$ . Let  $a, b, c \in S$  such that  $a + b = c$ . Then,

$$\begin{aligned}
 F_S(a) &= U - F_S^r(a) \\
 &\subseteq U - [F_S^r(b) \cap F_S^r(c)] \\
 &= [U - F_S^r(b)] \cup [U - F_S^r(c)] \\
 &= F_S(b) \cup F_S(c)
 \end{aligned}$$

Therefore,  $F_S$  is a soft left k-uni-ideal over  $U$ .

**Theorem 15.** Let  $\varphi$  be a surjective homomorphism from  $R$  to  $S$ . Let  $F_R$  be a soft left (right) k-uni-ideal over  $U$ .  $\varphi^*(F_R)_S$  is a soft left (right) k-uni-ideal over  $U$ .

**Proof.** Let  $F_R$  be a soft left (right) k-uni-ideal over  $U$ . By Theorem 3.14,  $F_R^r$  is a soft left (right) k-int-ideal over  $U$ . By (Sezgin et al., 2015),  $\varphi(F_R^r)_S$  is a soft left (right) k-int-ideal over  $U$ .  $(\varphi^*(F_R)_S)^r$  is a soft left (right) k-int-ideal over  $U$  since  $\varphi(F_R^r)_S = (\varphi^*(F_R)_S)^r$  by (Sezgin et al., 2015). If  $(\varphi^*(F_R)_S)^r$  is a soft left (right) k-int-ideal over  $U$ , then  $\varphi^*(F_R)_S$  is a soft left (right) k-uni-ideal over  $U$ .

**Theorem 16.** Let  $\varphi$  be a surjective homomorphism from  $R$  to  $S$ . Let  $F_S$  be a soft left (right) k-uni-ideal over  $U$ .  $\varphi^{-1}(F_S)_R$  is a soft left(right) k-uni-ideal over  $U$ .

**Proof.** For any  $a, b, c \in S$  such that  $a + b = c$  ( $\varphi(a) + \varphi(b) = \varphi(c)$ ),

$$\begin{aligned}\varphi^{-1}(F_S)(a) &= F_S(\varphi(a)) \\ &\subseteq F_S(\varphi(b)) \cup F_S(\varphi(c)) \\ &= \varphi^{-1}(F_S)(b) \cup \varphi^{-1}(F_S)(c)\end{aligned}$$

proving that  $\varphi^{-1}(F_S)_R$  is a soft left (right) k-uni-ideal over  $U$ .

## CONCLUSION

Soft k-uni-ideal and their algebraic properties were researched by using soft sets and union of sets in

this paper. In the following studies, soft h-uni-ideal of a hemiring and their algebraic properties can be researched by a researcher.

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