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EXISTENCE OF A SOLUTION OF A REACTION-DIFFUSION TYPE OF PROBLEM AND AN APPLİCATION

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Abstract: *We study the existence of solution of a reaction-diffusion type of problems. Then the technique used in the proof applied to get an approximate solution of a reaction-diffusion type of problem.*

Key words*: Monotone positive solutions, Second order nonlinear differential equations, reaction-diffusion process.*

1. Introduction

In this paper, we consider the solution of a reaction-diffusion process governed by the nonlinear second-order equation

$$x'' + x^p(t) = 0, 0 < t < L, p > -1,$$

where L is the length of the sample (heat conductor), p is the power of the reaction term (heat source), subject to the boundary conditions

$$x(0) = 0, x'(L) = a.$$

Negative integer, radical and decimal powers for the Dirichlet type of boundary conditions were considered in [1--4].

We use a new approach to show the existence of the positive solution. Then the approach in the proof is used to find a numerical (approximate) solution of a reaction-diffusion problem.

2. Main result

Without the loss of generality we take $L = 1$.

Theorem. The (reaction-diffusion) problem

$$x'' + x^p(t) = 0, \ t \in [0,1], p > -1, \quad (1)$$
$$x(0) = 0, x'(1) = a.$$

has a positive solution on (0,1) for all positive

$$a < ((p + 1)/2^p)^{1/(p-1)}.$$

Proof. Let $c > 0$ be a constant

$$c < ((p+1)/2^p)^{1/(p-1)}$$

and a be a fixed positive number with $a < c$. Consider the sequence

$$z_0(t) = 0,$$

$$z_n(t) = \int_0^t \tau(z_{n-1}(\tau) + a\tau)^p \, d\tau + t\int_t^1 (z_{n-1}(\tau) + a\tau)^p \, d\tau, n > 0. \tag{2}$$

We have

$$z_1'(t) = \int_t^1 (a\tau)^p \, d\tau < \frac{c^p}{p+1}(1 - t^{p+1}) \leq c,$$

$$0 \leq z_1(t) \leq ct, \dots$$

$$|z_n'(t)| = \int_t^1 (z_{n-1}(t) + a\tau)^p \, d\tau \leq \int_t^1 (2c\tau)^p \, d\tau \tag{3}$$

$$= \frac{2^p c^p}{p+1}(1 - t^{p+1}) < c$$

and therefore

$$z_n(t) \leq ct.$$

The sequence $\{z_n(t)\}_{n>0}$ is uniformly bounded and equiconvergent and it follows from Ascoli-Arzela lemma that $z_n(t) \to x_a(t)$ uniformly on [0,1] and

$$x_a(t) = \int_0^t \tau(x_a(\tau) + a\tau)^p \, d\tau + t\int_t^1 (x_a(\tau) + a\tau)^p \, d\tau, \tag{4}$$

$$x_a(t) + at = at + \int_0^t \tau(x_a(\tau) + a\tau)^p \, d\tau + t\int_t^1 (x_a(\tau) + a\tau)^p \, d\tau,$$

$$(x_a(\tau) + a\tau)'' + (x_a(\tau) + a\tau)^p = 0.$$

That is the function $y_a(t) = x_a(t) + at$ is the solution of the problem

$$x'' + x^p(t) = 0, \qquad t \in [0,1],$$

$$x(0) = 0, x'(1) = a.$$

Positivity of the solution for $t > 0$ easily follows from (2). Indeed

$$z_1(t) = \int_0^t \tau(a\tau)^p \, d\tau + t\int_t^1 (a\tau)^p \, d\tau > 0, \; t > 0 \tag{5}$$

$$z_2(t) = \int_0^t \tau(z_1(\tau) + a\tau)^p \, d\tau + t\int_t^1 (z_1(\tau) + a\tau)^p \, d\tau \geq z_1(t), \dots$$

The proof is complete.

3. Application

It is interesting that the technique used in the proof of the theorem can be used to find the approximate solution of the problem

Example 1. Consider the problem

$$x'' + x^2(t) = 0, \qquad t \in [0,1],$$

$$x(0) = 0, x'(1) = 1/2.$$

The iteration (2) gives

$$z_0(t) = 0$$

$$z_1(t) = \int_0^t \tau\left(\frac{1}{2}\tau\right)^2 d\tau + t\int_t^1 \left(\frac{1}{2}\tau\right)^2 d\tau = \frac{1}{12}t - \frac{1}{48}t^4$$

$$z_2(t) = \int_0^t \tau\left(\frac{1}{12}\tau - \frac{1}{48}\tau^4 + \frac{1}{2}\tau\right)^2 d\tau + t\int_t^1 \left(\frac{1}{12}\tau - \frac{1}{48}\tau^4 + \frac{1}{2}\tau\right)^2 d\tau$$

$$= \frac{1}{207360} t(-t^9 + 120t^6 - 5880t^3 + 22690)$$

$$z_3(t) = \int_0^t \tau(\frac{1}{207360}\tau(-\tau^9 + 120\tau^6 - 5880\tau^3 + 22690) + \frac{1}{2}\tau)^2\, d\tau$$

$$+ \int_t^1 (\frac{1}{207360}\tau(-\tau^9 + 120\tau^6 - 5880\tau^3 + 22690) + \frac{1}{2}\tau)^2\, d\tau$$

$$= -\frac{1}{19865154355200}t^{22} + \frac{1}{61272391680}t^{19} - \frac{109}{42998169600}t^{16}$$

$$+ \frac{83197}{335385722880}t^{13} - \frac{27043}{1612431360}t^{10} + \frac{88459}{107495424}t^7$$

$$- \frac{159696769}{5159780352}t^4 + \frac{2134656287}{18059231232}t.$$

The table below demonstrates that even three term iteration gives a good approximate solution of the problem.

Tab. 1. Some numerical values of the problem presented in Example 1.

t	Approx. solution $x(t) \approx z_3(t)+(1/2)t$	Error in the solution
0	0	0
0.1	0.0618	$1.0774644836472\times10^{-4}$
0.2	0.123591	$4.2994978275028\times10^{-4}$
0.3	0.18521	$9.6108060674743\times10^{-4}$
0.4	0.24649	$1.6869084214271\times10^{-3}$
0.5	0.30717	$2.5806338820972\times10^{-3}$
0.6	0.36693	$3.5998713616217\times10^{-3}$
0.7	0.425378	$4.6850364069459\times10^{-3}$
0.8	0.482056	$5.7597341589714\times10^{-3}$
0.9	0.53646	$6.7336730552093\times10^{-3}$
1.0	0.58806	$7.5084120709118\times10^{-3}$

From Table 1, it can be seen clearly that the error in the solution is negligible. Moreover, this error can be reduced more and more by considering more terms in the iteration (2).

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