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Coding Matrices for $GL(2,q)$

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Abstract

We use the BN-pair structure for the general linear group to write a suitable listing of the elements of the finite group $GL(2,q)$ which is then used to determine its ring of matrices. This approach of identifying finite group ring with ring of matrices has been used effectively to construct linear codes, benefiting from the ring-theoretic structure of both group rings and the ring of matrices.

1. Introduction

Group rings of finite groups became a rich source for constructing error-correcting codes and investigation, their properties since F.J. MacWilliams [1] and S.D Berman [2] considered cyclic codes as ideals in the group algebra of finite cyclic groups. R. Ferraz and Polcino Milies [3] brought the techniques and deep structure of the group algebras into play by studying idempotents which generate codes. In [4] T. Hurley proved that the group ring RG of a finite group G of order n over a ring R is isomorphic to a ring of G-matrices of size $n\times n$ over R, and this was used in many later papers to construct and analyse codes from units and zero-divisors. When R has an identity and no zero-divisors (e.g. when R is a field), Hurley used this identification to describe the unit group $U(RG)$ and zero-divisors of RG in terms of the properties of their corresponding matrices. The first (and main) step towards getting codes from a group ring RG is to choose an appropriate listing for the elements of G upon which depend other steps namely; finding the matrix of G (relative to the listing), the ring of matrices of RG and constructing unit-type and zero-divisor-type codes (all this steps are explained in [5]). The types of matrices have been determined for several classes of finite groups such as cyclic, elementary abelian and dihedral groups [4]. Matrices which appear in this identification include several types such as circulant, Toeplitz, Walsh-Toeplitz and Hankel Matrices. In this paper the linear group $G = GL(2,q)$ is considered; we shall use the BN-pair structure of G (see [6], section 69) to choose a listing for its elements suitable for determining the matrix of G and hence the ring of matrices of RG. Being the first linear group to be considered in this manner we hope this will lead to constructing new linear (unit-type and zero-divisor-type) codes.

2. The ring of matrices of a group

Let G be a finite group of order n with a given listing $G = \{g_1, g_2, \ldots, g_n\}$, and let R be a ring. Consider the matrix of the group G relative to its listing, say $M(G)$, which has the following form:

$$M(G) = \begin{pmatrix} g_1^{-1}g_1 & g_1^{-1}g_2 & \cdots & g_1^{-1}g_n \\ g_2^{-1}g_1 & g_2^{-1}g_2 & \cdots & g_2^{-1}g_n \\ \vdots & \vdots & \vdots & \vdots \\ g_n^{-1}g_1 & g_n^{-1}g_2 & \cdots & g_n^{-1}g_n \end{pmatrix}$$

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Now let $u = \sum_{i=1}^{n} \alpha_{g_i} g_i$ be an element in the group ring RG. Then the RG-matrix which corresponds to u in $R_{(n\times n)}$, the ring of $(n\times n)$-matrices, is given by:

$$M(RG,u) = \begin{pmatrix} \alpha_{g_1^{-1}g_1} & \alpha_{g_1^{-1}g_2} & \cdots & \alpha_{g_1^{-1}g_n} \\ \alpha_{g_2^{-1}g_1} & \alpha_{g_2^{-1}g_2} & \cdots & \alpha_{g_2^{-1}g_n} \\ \vdots & \vdots & \vdots & \vdots \\ g_n^{-1}g_1 & g_n^{-1}g_2 & \cdots & g_n^{-1}g_n \end{pmatrix}_{n\times n}$$

Theorem 2.1. *[4] Given a listing of the elements of a group G of order n. There is a bijective ring homomorphism $\sigma : u \longmapsto M(RG,u)$ between the group ring RG and the ring of $(n\times n)$ RG-matrices over R.*

There are several types of the RG-matrix which appear as isomorphic to a certain group rings. These types include Toeplitz-type matrices, Walsh-Toeplitz matrices, circulant matrices, Toeplitz combined with Hankel-type matrices and block-type circulant matrices; see [4] for more specifics and examples.

3. BN-pair structure of $G = GL(2,q)$

Definition 3.1. *[6] A finite group $G = (G,B,N,U,R,W)$ is said to have a split BN-pair of rank n if the following conditions are satisfied:*

- *G has a BN-pair of rank n, such that:*
  - $G = <B,N>$,
  - $B\cap N = H \trianglelefteq N$,
  - *$W = N/H$, is the corresponding Coxeter (Weyl) group which is generated by involutions, $W = < w_1, w_2, \ldots, w_r >$.*
- *There exist a normal subgroup $U \trianglelefteq B$ such that $B = U \rtimes H$ (semidirect product),*
- *$U = O_p(G)$, and H is an abelian p'-group.*

We have the *Bruhat decomposition* ,see[6]

$$G = \dot{\bigcup_{w\in W}} BwB,$$

On the other hand $U = U_w^+ . U_w^-$, where $U_w^+ = U\cap U^w$ and $U_w^- = U\cap U^{w_o w}$, where w_o is the unique element of the coxeter group W of maximal length.
Also we have,

$$BwB = BwU_w^-, \quad for\ each\ w\in W.$$

Therefore,

$$G = \dot{\bigcup_{w\in W}} BwB = \dot{\bigcup_{w\in W}} BwU_w^-,$$

and each element in G can be written uniquely in the form bnu, where $b\in B$, $u\in U_w^-$, n is the coset representative of an element $w\in W$.
Now, for example, if $G = GL(n,q)$ then G has the structure of split BN-pair, where B is the subgroup of an upper triangular matrices, N is the subgroup of monomial matrices, and H is the subgroup of the diagonal matrices. In fact, the Coxeter group W of $G = GL(n,q)$ is isomorphic to the symmetric group S_n.

$$W = N/H \cong S_n.$$

We shall concentrate on the case when $n = 2$. From the split BN-pair setting, we have
$GL(2,q) = B\bigcup BwB$, where

$$U = \left\{ \begin{pmatrix} 1 & \lambda \\ 0 & 1 \end{pmatrix} | \ \lambda\in\mathbb{F}_q \right\},\ H = \left\{ \begin{pmatrix} x & 0 \\ 0 & y \end{pmatrix} | \ x,y\in\mathbb{F}_q^* \right\}.$$

Since we have $BwB = BwU_w^-$, $\forall w\in W$, where $U_w^- = U\cap U^{w_o w}$, and since the Coxeter group W in this case is isomorphic to $S_2 = \{e\ ,\ (12)\}$. Then $w = w_o = (12)$, and $U_w^- = U\cap U^{w_o w_o} = U$. Thus,

$$G = B\cup Bw_oU = B\cup Bn_oU,$$

where $n_oH = Hn_o = w_o$. The monomial subgroup N in our case have the form;

$$N = \begin{pmatrix} * & 0 \\ 0 & * \end{pmatrix} \bigcup \begin{pmatrix} 0 & * \\ * & 0 \end{pmatrix},$$

and for $n_o\in N$, take $n_o = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$ which corresponds to the permutation (12) in S_2.
Therefore,

$$G = GL(2,q) = HU\cup HUn_oU$$

$$= \left\{ \begin{pmatrix} x & 0 \\ 0 & y \end{pmatrix}\begin{pmatrix} 1 & \lambda \\ 0 & 1 \end{pmatrix} \cup \begin{pmatrix} x' & 0 \\ 0 & y' \end{pmatrix}\begin{pmatrix} 1 & \alpha \\ 0 & 1 \end{pmatrix}\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}\begin{pmatrix} 1 & \beta \\ 0 & 1 \end{pmatrix} | \lambda,\alpha,\beta\in\mathbb{F}_q, and\ x,y\in\mathbb{F}_q^* \right\},$$

Counting the elements we have,

$$\left|\left\{\begin{pmatrix} x & 0\\ 0 & y\end{pmatrix}\begin{pmatrix} 1 & \lambda\\ 0 & 1\end{pmatrix}\cup\begin{pmatrix} x' & 0\\ 0 & y'\end{pmatrix}\begin{pmatrix} 1 & \alpha\\ 0 & 1\end{pmatrix}\begin{pmatrix} 0 & 1\\ 1 & 0\end{pmatrix}\begin{pmatrix} 1 & \beta\\ 0 & 1\end{pmatrix}|\lambda,\alpha,\beta\in\mathbb{F}_q, and\ x,y\in\mathbb{F}_q^*\right\}\right|$$

$$= (q-1)^2.\,q + (q-1)^2.\,q^2 = (q-1)^2.\,(q+q^2) = |GL(2,q)|.$$

Notation :
We write $h(x\ ,\ y) = \begin{pmatrix} x & 0\\ 0 & y\end{pmatrix}\in H;\, x,y\in\mathbb{F}_q^*$ and $u(\lambda) = \begin{pmatrix} 1 & \lambda\\ 0 & 1\end{pmatrix}\in U;\,\lambda\in\mathbb{F}_q$.

4. Multiplications

In the light of the coset decomposition of $GL(2,q) = B\cup Bn_oU = HU\cup HUn_oU$, we shall discuss four cases of element multiplication in $G = GL(2,q)$:

$*$	HU	HUn_oU
HU	CASE 1	CASE 2
HUn_oU	CASE 3	CASE 4

Proposition 4.1. *For each $x,x',y,y'\in\mathbb{F}_q^*$ and $\lambda,\lambda',\beta'\beta'\in\mathbb{F}_q$ we define the multiplication as the following:*
Case 1:
$h(x'\ ,\ y')\ u(\lambda')\ \cdot h(x\ ,\ y)\ u(\lambda) = h(x'x\ ,\ y'y)\ u(\lambda + x^{-1}\lambda'y)$
Case 2:
$h(x'\ ,\ y')\ u(\lambda')\ \cdot h(x\ ,\ y)\ u(\lambda)\ n_o\ u(\beta) = h(x'x\ ,\ y'y)\ u(\lambda + x^{-1}\lambda'y)\ n_o\ u(\beta)$
Case 3:
$h(x'\ ,\ y')\ u(\lambda')\ n_o\ u(\beta')\ \cdot h(x\ ,\ y)\ u(\lambda) = h(x'y\ ,\ y'x)\ u(y^{-1}\lambda'x)\ n_o\ u(\lambda + x^{-1}\beta'y)$
Case 4:
$h(x',y')u(\lambda')\ n_o\ u(\beta')\ \cdot h(x,y)\ u(\lambda)\ n_o\ u(\beta) = h(-x'y\alpha^{-1},y'x\alpha)\ u(-\alpha-\alpha^2y^{-1}\lambda'x)\ n_o\ u(\beta+\alpha^{-1})$
where, $\alpha = \lambda + x^{-1}\beta'y$.

Special case *:*
If $\alpha = \lambda + x^{-1}\beta'y = 0$ the multiplication will be as :
$h(x',y')u(\lambda')\ n_o\ u(\beta')\ \cdot h(x,y)\ u(\lambda)\ n_o\ u(\beta) = h(x'y\ ,\ y'x)\ u(\beta + y^{-1}\lambda'x)$

5. Inverses

The following proposition gives the rule for getting the inverses of the elements of $GL(2,q)$.

Proposition 5.1. *For each $x,y\in\mathbb{F}_q^*$ and $\lambda,\beta\in\mathbb{F}_q$ there are two cases for getting the inverse:*
*1- **The element of the form** $h(x\ ,\ y)\ u(\lambda)$:*
$[h(x\ ,\ y)\ u(\lambda)]^{-1} = h(x^{-1},y^{-1})\ u(-x\lambda y^{-1})$
*2- **The element of the form** $h(x\ ,\ y)\ u(\lambda)\ n_o\ u(\beta)$:*
$[h(x\ ,\ y)\ u(\lambda)\ n_o\ u(\beta)]^{-1} = h(y^{-1},x^{-1})\ u(-x^{-1}\beta y)\ n_o\ u(-x\lambda y^{-1})$

6. Elements listing of $G = GL(2,q)$

The listing of this group depends on the number q and we discuss two cases:
Case1: when $q = p$ is an odd-prime ($q = p > 3$):
In this case, the linear group $G = GL(2,p)$ has (p^2-1) blocks, each consists of $\left(\frac{p-1}{2}\right)$ matrices each of size $(2p)$, (note that $(p^2-1)(\frac{p-1}{2})(2p) = (p-1)^2(p^2+p) = |GL(2,p)|$) obtained by the following listing:
Type (x,x); $x\in\mathbb{Z}_p^*$ gives $(p+1)$ blocks:

(1) THE BLOCK $B(x,x)$; $x\in\mathbb{Z}_p^*$, obtained from the following listing subset:
$T(x,x): h(x,x),\ h(p-x,p-x)u(p-1),\ h(x,x)u(p-2), h(p-x,p-x)u(p-3),\ \cdots,\ \cdots,\ h(x,x)u(1),\ \cdots,\ \cdots,\ \cdots,\ h(x,x)u(2),$
$h(p-x,p-x)u(1)$.
such that, $x = 1,2,\cdots,\frac{p-1}{2}$.
THE BLOCKS $B(x,x)(\lambda)$**;** $\lambda = 0,1,2,\cdots,p-1$, obtained from the listing subsets:
$T(x,x)n_ou(\lambda);\ \lambda = 0,1,2,\cdots,p-1$.

Type (x,y); $x\neq y$ gives the following (p^2-p-2) blocks:

THE BLOCKS $B_i(x,y)$**;** $i = 1,2,\cdots,p-2$, obtained from the listing subsets:

$T_i(x,y)(j);\ 1 \le i \le p-2,\ 1 \le j \le \frac{p-1}{2}$; where,
$T_i(x,y)(j):\ h(j,j(p-i)),\ h(j(p-1),ji)u(i),\ h(j,j(p-i))u(2i), h(j(p-1),ji)u(3i),\ \cdots,\ \cdots,\ h(j(p-1),ji)u((p-3)i),$
$h(j,j(p-i))u((p-2)i),\ h(j(p-1),ji)u((p-1)i).$

THE BLOCKS $B_i(x,y)(\lambda)$**;** $x \ne y,\ i = 1,2,\cdots,p-2,\ 0 \le \lambda \le p-1$, obtained from the listing subsets:
$T_i(x,y)(j)n_o u(\lambda);\ 1 \le i \le p-2,\ 1 \le j \le \frac{p-1}{2}.$
The total number of blocks in this case:
$(p+1)+(p^2-p-2) = p^2-1.$

Case2, when q **is a power of p** ($q = p^n$)**, and** ($n \ge 2,\ p \ge 2$) :
In this case we take $\mathbb{F}_q^* = <a|a^{q-1} = 1>$, then the matrix of $G = GL(2,q)$ has (q^2-1) blocks each block consists of $\binom{q}{p}$ matrices each of size $p(q-1)$ (note that $(q^2-1)(\frac{q}{p})p(q-1) = (q-1)^2(q^2+q) = |GL(2,q)|$) obtained by the following listing:

Type (a^i,a^i) **gives the following** $(q+1)$ **blocks:**

THE BLOCK $B(a^i,a^i)$; $i = 1,2,\cdots,q-1$ obtained by the following listing subset:
$T(a^i,a^i),\ T(a^i,a^i)(1),\ T(a^i,a^i)(2),\ \cdots,\ T(a^i,a^i)((q/p)-1).$
Where,
$T(a^i,a^i):\ h(1,1),\ h(a^{q-2}\ ,\ a^{q-2})\ u(a^{q-2})\ ,\ h(a^{q-3}\ ,\ a^{q-3})\ u(2(a^{q-2}))\ ,\ \cdots,\ \cdots,\ \cdots,\ h(a^{q-p}\ ,\ a^{q-p})\ u((p-1)(a^{q-2}))\ ,$
$h(a^{q-(p+1)}\ ,\ a^{q-(p+1)})\ ,\ h(a^{q-(p+2)}\ ,\ a^{q-(p+2)})\ u(a^{q-2})\ ,\ \cdots,\ \cdots,\ h(a\ ,\ a)\ u((p-2)(a^{q-2}))\ ,\ h(1\ ,\ 1)\ u((p-1)(a^{q-2}))\ ,$
$h(a^{q-2},a^{q-2})\ ,\cdots,\ \cdots,\ h(a,a)u((p-1)(a^{q-2})).$
Such that, $T(a^i,a^i)(s) = T(a^i,a^i)u(a^{k_s});\ s = 1,2,\cdots,(q/p)-1,\ \mathit{and}\ a^{k_s} \in \{a,a^2,\cdots,a^{q-3}\}.$

THE BLOCKS $B(a^i,a^i)(\lambda)$; $\lambda = 0,1,2,\cdots,a^{q-2}$, obtained by the following listing subsets:
$T(a^i,a^i)n_o u(\lambda),\ T(a^i,a^i)(1)n_o u(\lambda),\ \cdots,\ T(a^i,a^i)((q/p)-1)n_o u(\lambda).$

Type (a^i,a^j)**;** $i \ne j$ **gives the following** (q^2-q-2) **blocks:**

THE BLOCKS $B_r(a^i,a^j)$**;** $r = 1,2,\cdots,q-2$, obtained from the listing subsets:
$T_r(a^i,a^j),\ T_r(a^i,a^j)(1),\ T_r(a^i,a^j)(2),\ \cdots,\ T_r(a^i,a^j)((q/p)-1)$, such that:
$T_r(a^i,a^j)(s) = T_r(a^i,a^j)u(a^{k_s});\ s = 1,2,\cdots,(q/p)-1.$
Where:
$T_r(a^i,a^j):\ h(1,a^r)\ ,\ h(a^{q-2},a^{r-1})u(a^{r-1})\ ,\ h(a^{q-3},a^{r-2})u(2a^{r-1})\ ,\ h(a^{q-4},a^{r-3})u(3a^{r-1})\ ,\ \cdots\ ,\ \cdots\ ,\ h(a^{q-p},a^{(q-p)+r})u((p-1)a^{r-1})\ ,$
$h(a^{(q-p)-1},a^{(q-p)+(r-1)})\ ,\ h(a^{(q-p)-2},a^{(q-p)+(r-2)})u(a^{r-1})\ ,\cdots\cdots,\ \cdots\ ,\ h(a,a^{r+1})u((p-1)a^{r-1}).$

THE BLOCKS $B_r(a^i,a^j)(\lambda)$**;** $r = 1,2,\cdots,q-2,\ 0 \le \lambda \le a^{q-2}$, obtained from the listing subsets:
$T_r(a^i,a^j)n_o u(\lambda),\ T_r(a^i,a^j)(1)n_o u(\lambda),\ T_r(a^i,a^j)(2)n_o u(\lambda),\ \cdots\cdots,\ T_r(a^i,a^j)((q/p)-1)n_o u(\lambda).$
The total number of blocks in this case:
$(q+1)+(q^2-q-2) = q^2-1.$

Example 6.1. ***The special case when*** $(q = 3,\ G = GL(2,3))$ *:*
From the general theory, the matrix of this group has $3^2-1 = 8$ *blocks each block consists of* $\frac{3-1}{2} = 1$ *matrix of size* $2 \times 3 = 6$ *obtained from the following listing:*
Block $B(x,x)$***:***
$h(1,1),\ h(2,2)u(2),\ h(1,1)u(1),\ h(2,2),\ h(1,1)u(2),\ h(2,2)u(1),$
Block $B(x,x)(\lambda = 0)$***:***
$h(1,1)n_o,\ h(2,2)u(2)n_o,\ h(1,1)u(1)n_o,\ h(2,2)n_o,\ h(1,1)u(2)n_o,\ h(2,2)u(1)n_o,$
Block $B(x,x)(\lambda = 1)$***:***
$h(1,1)n_o u(1),\ h(2,2)u(2)n_o u(1),\ h(1,1)u(1)n_o u(1),\ h(2,2)n_o u(1),\ h(1,1)u(2)n_o u(1),$
$h(2,2)u(1)n_o u(1),$
Block $B(x,x)(\lambda = 2)$***:***
$h(1,1)n_o u(2),\ h(2,2)u(2)n_o u(2),\ h(1,1)u(1)n_o u(2),\ h(2,2)n_o u(2),\ h(1,1)u(2)n_o u(2),$
$h(2,2)u(1)n_o u(2),$
Block $B_1(x,y)$***:***
$h(1,2),\ h(2,1)u(1),\ h(1,2)u(2),\ h(2,1),\ h(1,2)u(1),\ h(2,1)u(2),$
Block $B_1(x,y)(\lambda = 0)$***:***
$h(1,2)n_o,\ h(2,1)u(1)n_o,\ h(1,2)u(2)n_o,\ h(2,1)n_o,\ h(1,2)u(1)n_o,\ h(2,1)u(2)n_o,$
Block $B_1(x,y)(\lambda = 1)$***:***
$h(1,2)n_o u(1),\ h(1,2)u(1)n_o u(1),\ h(2,1)u(2)n_o u(1),\ h(2,1)n_o u(1),\ h(2,1)u(1)n_o u(1),\ h(1,2)u(2)n_o u(1),$
Block $B_1(x,y)(\lambda = 2)$***:***
$h(1,2)n_o u(2),\ h(1,2)u(1)n_o u(2),\ h(2,1)u(2)n_o u(2),\ h(2,1)n_o u(2),\ h(2,1)u(1)n_o u(2),\ h(1,2)u(2)n_o u(2).$

7. The matrix of $G = GL(2,q)$

Now we shall determine the G-matrix of the group $GL(2,q)$ with respect to the listing which obtained in the previous section:
We start with the first case when $q = p$. Consider $T(1,1)$ in the first block $B(x,x)$ which has the elements $g_1, g_2, \cdots, \cdots, g_{2p-2}, g_{2p-1}, g_{2p}$ with their inverses. Then the resulting matrix will have the following form:

$$\begin{pmatrix}
g_1 & g_2 & g_3 & g_4 & g_5 & \cdots & g_{2p-3} & g_{2p-2} & g_{2p-1} & g_{2p} \\
g_{2p} & g_1 & g_2 & g_3 & g_4 & \cdots & g_{2p-4} & g_{2p-3} & g_{2p-2} & g_{2p-1} \\
g_{2p-1} & g_{2p} & g_1 & g_2 & g_3 & \cdots & \cdots & g_{2p-4} & g_{2p-3} & g_{2p-2} \\
g_{2p-2} & g_{2p-1} & g_{2p} & g_1 & g_2 & \cdots & \cdots & \cdots & g_{2p-4} & g_{2p-3} \\
g_{2p-3} & g_{2p-2} & g_{2p-1} & g_{2p} & g_1 & \cdots & \cdots & \cdots & \cdots & g_{2p-4} \\
\ddots & \ddots & \ddots & \ddots & \ddots & \ddots & \ddots & \ddots & \ddots & \ddots \\
g_5 & \cdots & \cdots & \cdots & \cdots & \cdots & g_1 & g_2 & g_3 & g_4 \\
g_4 & g_5 & \cdots & \cdots & \cdots & \cdots & g_{2p} & g_1 & g_2 & g_3 \\
g_3 & g_4 & g_5 & \cdots & \cdots & \cdots & g_{2p-1} & g_{2p} & g_1 & g_2 \\
g_2 & g_3 & g_4 & g_5 & \cdots & \cdots & g_{2p-2} & g_{2p-1} & g_{2p} & g_1
\end{pmatrix}$$

This is a circulant matrix type.
Now for any subset in the the block $B(x,x)$. Consider the elements of $T(t,t)$: $g_{t_1}, g_{t_2}, \cdots, \cdots, g_{t_{2p-1}}, g_{t_{2p}}$ with the inverses of the elements in $T(1,1)$, then we will get the following matrix:

$$\begin{pmatrix}
g_{t_1} & g_{t_2} & g_{t_3} & g_{t_4} & g_{t_5} & \cdots & g_{t_{2p-3}} & g_{t_{2p-2}} & g_{t_{2p-1}} & g_{t_{2p}} \\
g_{t_{2p}} & g_{t_1} & g_{t_2} & g_{t_3} & g_{t_4} & \cdots & g_{t_{2p-4}} & g_{t_{2p-3}} & g_{t_{2p-2}} & g_{t_{2p-1}} \\
g_{t_{2p-1}} & g_{t_{2p}} & g_{t_1} & g_{t_2} & g_{t_3} & \cdots & \cdots & g_{t_{2p-4}} & g_{t_{2p-3}} & g_{t_{2p-2}} \\
g_{t_{2p-2}} & g_{t_{2p-1}} & g_{t_{2p}} & g_{t_1} & g_{t_2} & \cdots & \cdots & \cdots & g_{t_{2p-4}} & g_{t_{2p-3}} \\
g_{t_{2p-3}} & g_{t_{2p-2}} & g_{t_{2p-1}} & g_{t_{2p}} & g_{t_1} & \cdots & \cdots & \cdots & \cdots & g_{t_{2p-4}} \\
\ddots & \ddots & \ddots & \ddots & \ddots & \ddots & \ddots & \ddots & \ddots & \ddots \\
g_{t_5} & \cdots & \cdots & \cdots & \cdots & \cdots & g_{t_1} & g_{t_2} & g_{t_3} & g_{t_4} \\
g_{t_4} & g_{t_5} & \cdots & \cdots & \cdots & \cdots & g_{t_{2p}} & g_{t_1} & g_{t_2} & g_{t_3} \\
g_{t_3} & g_{t_4} & g_{t_5} & \cdots & \cdots & \cdots & g_{t_{2p-1}} & g_{t_{2p}} & g_{t_1} & g_{t_2} \\
g_{t_2} & g_{t_3} & g_{t_4} & g_{t_5} & \cdots & \cdots & g_{t_{2p-2}} & g_{t_{2p-1}} & g_{t_{2p}} & g_{t_1}
\end{pmatrix}$$

Furthermore, if we get the elements of $T(t,t)$ as above with inverses of an arbitrary subset in $B(x,x)$, say $T(j,j)$, we will get the following matrix:

$$\begin{pmatrix}
g_{k_1} & g_{k_2} & g_{k_3} & g_{k_4} & g_{k_5} & \cdots & g_{k_{2p-3}} & g_{k_{2p-2}} & g_{k_{2p-1}} & g_{k_{2p}} \\
g_{k_{2p}} & g_{k_1} & g_{k_2} & g_{k_3} & g_{k_4} & \cdots & g_{k_{2p-4}} & g_{k_{2p-3}} & g_{k_{2p-2}} & g_{k_{2p-1}} \\
g_{k_{2p-1}} & g_{k_{2p}} & g_{k_1} & g_{k_2} & g_{k_3} & \cdots & \cdots & g_{k_{2p-4}} & g_{k_{2p-3}} & g_{k_{2p-2}} \\
g_{k_{2p-2}} & g_{k_{2p-1}} & g_{k_{2p}} & g_{k_1} & g_{k_2} & \cdots & \cdots & \cdots & g_{k_{2p-4}} & g_{k_{2p-3}} \\
g_{k_{2p-3}} & g_{k_{2p-2}} & g_{k_{2p-1}} & g_{k_{2p}} & g_{k_1} & \cdots & \cdots & \cdots & \cdots & g_{k_{2p-4}} \\
\ddots & \ddots & \ddots & \ddots & \ddots & \ddots & \ddots & \ddots & \ddots & \ddots \\
g_{k_5} & \cdots & \cdots & \cdots & \cdots & \cdots & g_{k_1} & g_{k_2} & g_{k_3} & g_{k_4} \\
g_{k_4} & g_{k_5} & \cdots & \cdots & \cdots & \cdots & g_{k_{2p}} & g_{k_1} & g_{k_2} & g_{k_3} \\
g_{k_3} & g_{k_4} & g_{k_5} & \cdots & \cdots & \cdots & g_{k_{2p-1}} & g_{k_{2p}} & g_{k_1} & g_{k_2} \\
g_{k_2} & g_{k_3} & g_{k_4} & g_{k_5} & \cdots & \cdots & g_{k_{2p-2}} & g_{k_{2p-1}} & g_{k_{2p}} & g_{k_1}
\end{pmatrix}$$

Where, $g_{t_i} * {g_{j_i}}^{-1} = g_{k_i}$, $1 \le i \le 2p$, such that:
$g_{k_1} = h(j^{-1}t, j^{-1}t)$, $g_{k_2} = h(j^{-1}(p-t), j^{-1}(p-t))\, u(p-1)$, $g_{k_3} = h(j^{-1}t, j^{-1}t)\, u(p-2)$,
$g_{k_4} = h(j^{-1}(p-t), j^{-1}(p-t))\, u(p-3)$, $g_{k_5} = h(j^{-1}t, j^{-1}t)\, u(p-4)$,
$\vdots$
, $g_{k_{2p-4}} = h(j^{-1}(p-t), j^{-1}(p-t))\, u(5)$, $g_{k_{2p-3}} = h(j^{-1}t, j^{-1}t)\, u(4)$,
$g_{k_{2p-2}} = h(j^{-1}(p-t), j^{-1}(p-t))\, u(3)$, $g_{k_{2p-1}} = h(j^{-1}t, j^{-1}t)\, u(2)$,
$g_{k_{2p}} = h(j^{-1}(p-t), j^{-1}(p-t))\, u(1)$,

This obtained using these tow following equations :
(1) $(p-j)^{-1}t = j^{-1}(p-t)$
(2) $(p-j)^{-1}(p-t) = j^{-1}t$.

Now, if we take the elements of $T_i(x,y)(1)(\lambda = w)$ from the block $B_i(x,y)(\lambda = w)$, $g_{i_1}, g_{i_2}, \cdots, \cdots, g_{i_{2p-1}}, g_{i_{2p}}$, with the inverses of elements of $T_j(x,y)(1)(\lambda = k)$ from block $B_j(x,y)(\lambda = k)$, $g_{j_1}^{-1}, g_{j_2}^{-1}, \cdots, \cdots, g_{j_{s-1}}^{-1}, g_{j_s}^{-1}$. The resulting matrix will have the following form:

$$\begin{pmatrix} l_1 & l_2 & l_3 & l_4 & l_5 & \dots & l_{2p-3} & l_{2p-2} & l_{2p-1} & l_{2p} \\ l_{2p} & l_1 & l_2 & l_3 & l_4 & \dots & \cdots & l_{2p-3} & l_{2p-2} & l_{2p-1} \\ l_{2p-1} & l_{2p} & l_1 & l_2 & l_3 & \dots & \dots & \cdots & l_{2p-3} & l_{2p-2} \\ l_{2p-2} & l_{2p-1} & l_{2p} & l_1 & l_2 & \dots & \dots & \dots & \cdots & l_{2p-3} \\ l_{2p-3} & l_{2p-2} & l_{2p-1} & l_{2p} & l_1 & \dots & \dots & \dots & \dots & \cdots \\ \ddots & \ddots & \ddots & \ddots & \ddots & \ddots & \ddots & \ddots & \ddots & \ddots \\ l_5 & \dots & \dots & \dots & \dots & \dots & l_1 & l_2 & l_3 & l_4 \\ l_4 & l_5 & \dots & \dots & \dots & \dots & l_{2p} & l_1 & l_2 & l_3 \\ l_3 & l_4 & l_5 & \dots & \dots & \dots & l_{2p-1} & l_{2p} & l_1 & l_2 \\ l_2 & l_3 & l_4 & l_5 & \dots & \dots & l_{2p-2} & l_{2p-1} & l_{2p} & l_1 \end{pmatrix}$$

Where, $g_{i_s} * g_{j_s}^{-1} = l_s$, $1 \le s \le 2p$.

Also, for another matrix in the same Block. We can get the elements of $T_i(x,y)(n)(\lambda = w)$ from the Block $B_i(x,y)(\lambda = w)$ with the inverses of the elements of $T_j(x,y)(n)(\lambda = k)$ in the block $B_j(x,y)(\lambda = k)$. Then we will get the same circulant matrix which we obtained when we take the elements from the first subsets of these certain blocks. We find in this case the $G-matrix$ of the group $GL(2,q)$ with respect to the above proposed listing form $\left(q^2-1 \times q^2-1\right)$- Block Circulant matrix.

Now, for the other case when q is a power of p :
For the elements from the coset(B) consider $T_b(a^i,a^j)$ from the block $B_b(a^i,a^j)$, $b_1,b_2,\cdots,\cdots,b_{s-2},b_{s-1},b_s$, where $s = p(q-1)$ is the number of the elements in each subsets in this type, and a is the generator of the multiplicative group of the field $\mathbb{F}_q$). Together with the inverses of elements in $T(a^i,a^i)$ from the block $B(a^i,a^i)$. The resulting matrix will have the following form:

$$\begin{pmatrix} b_1 & b_2 & b_3 & b_4 & b_5 & \dots & b_{s-3} & b_{s-2} & b_{s-1} & b_s \\ b_s & b_1 & b_2 & b_3 & b_4 & \dots & \cdots & b_{s-3} & b_{s-2} & b_{s-1} \\ b_{s-1} & b_s & b_1 & b_2 & b_3 & \dots & \dots & \cdots & b_{s-3} & b_{s-2} \\ b_{s-2} & b_{s-1} & b_s & b_1 & b_2 & \dots & \dots & \dots & \cdots & b_{s-3} \\ b_{s-3} & b_{s-2} & b_{s-1} & b_s & b_1 & \dots & \dots & \dots & \dots & \cdots \\ \ddots & \ddots & \ddots & \ddots & \ddots & \ddots & \ddots & \ddots & \ddots & \ddots \\ b_5 & \dots & \dots & \dots & \dots & \dots & b_1 & b_2 & b_3 & b_4 \\ b_4 & b_5 & \dots & \dots & \dots & \dots & b_s & b_1 & b_2 & b_3 \\ b_3 & b_4 & b_5 & \dots & \dots & \dots & b_{s-1} & b_s & b_1 & b_2 \\ b_2 & b_3 & b_4 & b_5 & \dots & \dots & b_{s-2} & b_{s-1} & b_s & b_1 \end{pmatrix}$$

The same matrix will appear when we take the elements of $T_b(a^i,a^j)(n)$ from the block $B_b(a^i,a^j)$, with the inverses of elements in $T(a^i,a^i)(n)$ from the block $B(a^i,a^i)$.
Finally for the elements from the double coset (Bn_oU) consider $T_x(a^i,a^j)(\lambda = a^z)$ from the block $B_x(a^i,a^j)(\lambda = a^z)$, $c_1,c_2,\cdots,\cdots,c_{s-1},c_s$, with the inverses of elements of $T_y(a^i,a^j)(\lambda = a^\beta)$ from the block $B_y(a^i,a^j)(\lambda = a^\beta)$ $d_1^{-1},d_2^{-1},\cdots,\cdots,d_{s-1}^{-1},d_s^{-1}$. Then the matrix will be as the following :

$$\begin{pmatrix} z_1 & z_2 & z_3 & z_4 & z_5 & \dots & z_{s-3} & z_{s-2} & z_{s-1} & z_s \\ z_s & z_1 & z_2 & z_3 & z_4 & \dots & \cdots & z_{s-3} & z_{s-2} & z_{s-1} \\ z_{s-1} & z_s & z_1 & z_2 & z_3 & \dots & \dots & \cdots & z_{s-3} & z_{s-2} \\ z_{s-2} & z_{s-1} & z_s & z_1 & z_2 & \dots & \dots & \dots & \cdots & z_{s-3} \\ z_{s-3} & z_{s-2} & z_{s-1} & z_s & z_1 & \dots & \dots & \dots & \dots & \cdots \\ \ddots & \ddots & \ddots & \ddots & \ddots & \ddots & \ddots & \ddots & \ddots & \ddots \\ z_5 & \dots & \dots & \dots & \dots & \dots & z_1 & z_2 & z_3 & z_4 \\ z_4 & z_5 & \dots & \dots & \dots & \dots & z_s & z_1 & z_2 & z_3 \\ z_3 & z_4 & z_5 & \dots & \dots & \dots & z_{s-1} & z_s & z_1 & z_2 \\ z_2 & z_3 & z_4 & z_5 & \dots & \dots & z_{s-2} & z_{s-1} & z_s & z_1 \end{pmatrix}$$

Where, $c_k * d_k^{-1} = z_k$, $1 \le k \le s$. The same matrix will obtained when we take the elements of $T_x(a^i,a^j)(m)(\lambda = a^z)$ from the block $B_x(a^i,a^j)(\lambda = a^z)$, with the inverses of elements of $T_y(a^i,a^j)(m)(\lambda = a^\beta)$ from the block $B_y(a^i,a^j)(\lambda = a^\beta)$.

Note that :
When the matrix is appear again in a certain block (with the same elements) the order of the element maybe change (they permute) not always the same (as in the example of $GL(2,5)$) .
We find the G-matrix of the group $GL(2,q)$ with respect to the above proposed listing using the multiplication table as well as the rule for getting the inverses explained above. It turns out that the G-matrix (and hence the coding matrices) is a $\left(q^2-1 \times q^2-1\right)$-block circulant matrix.

Remark 7.1. *When $q = 2$, the general linear group $G = GL(2,q) \cong S_3$ and the matrix of this group is actually a block circulant matrix as well. Also, when $q = 3$, the matrix of $GL(2,3)$ with respect to the listing given in [example (6.1)] is a block circulant matrix.*

Summarizing we have the following,

Theorem 7.2. *: With respect to the elements listing for the group $G = GL(2,q)$, the G-matrix has the form of block circulant matrix.*

8. Examples of the G-matrices

Here we have two examples of the matrix of $G = GL(2,q)$:
1- The listing of elements of $GL(2,5)$:
$|Gl(2,5)|$= 480. This will divide in to 24 blocks with 20 elements in each block .

Type $(x\ ,\ x)$:
Block $B(x,x)$**:**
T (1,1):
$l_1 = h(1,1)$, $l_2 = h(4,4)\ u(4)$, $l_3 = h(1,1)\ u(3)$, $l_4 = h(4,4)\ u(2)$, $l_5 = h(1,1)\ u(1)$,
$l_6 = h(4,4)$, $l_7 = h(1,1)\ u(4)$, $l_8 = h(4,4)\ u(3)$, $l_9 = h(1,1)\ u(2)$, $l_{10} = h(4,4)\ u(1)$.
T (2,2):
$l_{11} = h(2,2)$, $l_{12} = h(3,3)\ u(4)$, $l_{13} = h(2,2)\ u(3)$, $l_{14} = h(3,3)\ u(2)$, $l_{15} = h(2,2)\ u(1)$,
$l_{16} = h(3,3)$, $l_{17} = h(2,2)\ u(4)$, $l_{18} = h(3,3)\ u(3)$, $l_{19} = h(2,2)\ u(2)$, $l_{20} = h(3,3)\ u(1)$.
Block $B(x,x)(\lambda = 0)$**:**
T (1,1)$(\lambda = 0)$**:**
$l_{21} = h(1,1)\ n_o$, $l_{22} = h(4,4)\ u(4)\ n_o$, $l_{23} = h(1,1)\ u(3)\ n_o$, $l_{24} = h(4,4)\ u(2)\ n_o$,
$l_{25} = h(1,1)\ u(1)\ n_o$, $l_{26} = h(4,4)\ n_o$, $l_{27} = h(1,1)\ u(4)\ n_o$, $l_{28} = h(4,4)\ u(3)\ n_o$,
$l_{29} = h(1,1)\ u(2)\ n_o$, $l_{30} = h(4,4)\ u(1)\ n_o$.
T (2,2)$(\lambda = 0)$**:**
$l_{31} = h(2,2)n_o$, $l_{32} = h(3,3)u(4)n_o$, $l_{33} = h(2,2)u(3)n_o$, $l_{34} = h(3,3)u(2)n_o$, $l_{35} = h(2,2)u(1)n_o$,
$l_{36} = h(3,3)n_o$, $l_{37} = h(2,2)u(4)n_o$, $l_{38} = h(3,3)u(3)n_o$, $l_{39} = h(2,2)u(2)n_o$, $l_{40} = h(3,3)u(1)n_o$, .
Block $B(x,x)(\lambda = 1)$**:**
T (1,1)$(\lambda = 1)$**:**
$l_{41} = h(1,1)n_ou(1)$, $l_{42} = h(4,4)u(4)n_ou(1)$, $l_{43} = h(1,1)u(3)n_ou(1)$, $l_{44} = h(4,4)u(2)n_ou(1)$,
$l_{45} = h(1,1)u(1)n_ou(1)$, $l_{46} = h(4,4)n_ou(1)$, $l_{47} = h(1,1)u(4)n_ou(1)$, $l_{48} = h(4,4)u(3)n_ou(1)$,
$l_{49} = h(1,1)u(2)n_ou(1)$, $l_{50} = h(4,4)u(1)n_ou(1)$.
T (2,2)$(\lambda = 1)$**:**
$l_{51} = h(2,2)n_ou(1)$, $l_{52} = h(3,3)u(4)n_ou(1)$, $l_{53} = h(2,2)u(3)n_ou(1)$, $l_{54} = h(3,3)u(2)n_ou(1)$,
$l_{55} = h(2,2)u(1)n_ou(1)$, $l_{56} = h(3,3)n_ou(1)$, $l_{57} = h(2,2)u(4)n_ou(1)$, $l_{58} = h(3,3)u(3)n_ou(1)$,
$l_{59} = h(2,2)u(2)n_ou(1)$, $l_{60} = h(3,3)u(1)n_ou(1)$.
Block $B(x,x)(\lambda = 2)$:
T (1,1)$(\lambda = 2)$**:**
$l_{61} = h(1,1)n_ou(2)$, $l_{62} = h(4,4)u(4)n_ou(2)$, $l_{63} = h(1,1)u(3)n_ou(2)$, $l_{64} = h(4,4)u(2)n_ou(2)$,
$l_{65} = h(1,1)u(1)n_ou(2)$, $l_{66} = h(4,4)n_ou(2)$, $l_{67} = h(1,1)u(4)n_ou(2)$, $l_{68} = h(4,4)u(3)n_ou(2)$,
$l_{69} = h(1,1)u(2)n_ou(2)$, $l_{70} = h(4,4)u(1)n_ou(2)$.
T (2,2)$(\lambda = 2)$**:**
$l_{71} = h(2,2)n_ou(2)$, $l_{72} = h(3,3)u(4)n_ou(2)$, $l_{73} = h(2,2)u(3)n_ou(2)$, $l_{74} = h(3,3)u(2)n_ou(2)$,
$l_{75} = h(2,2)u(1)n_ou(2)$, $l_{76} = h(3,3)n_ou(2)$, $l_{77} = h(2,2)u(4)n_ou(2)$, $l_{78} = h(3,3)u(3)n_ou(2)$,
$l_{79} = h(2,2)u(2)n_ou(2)$, $l_{80} = h(3,3)u(1)n_ou(2)$.
Block $B(x,x)(\lambda = 3)$:
T (1,1)$(\lambda = 3)$**:**
$l_{81} = h(1,1)n_ou(3)$, $l_{82} = h(4,4)u(4)n_ou(3)$, $l_{83} = h(1,1)u(3)n_ou(3)$, $l_{84} = h(4,4)u(2)n_ou(3)$,
$l_{85} = h(1,1)u(1)n_ou(3)$, $l_{86} = h(4,4)n_ou(3)$, $l_{87} = h(1,1)u(4)n_ou(3)$, $l_{88} = h(4,4)u(3)n_ou(3)$,
$l_{89} = h(1,1)u(2)n_ou(3)$, $l_{90} = h(4,4)u(1)n_ou(3)$.
T (2,2)$(\lambda = 3)$**:**
$l_{91} = h(2,2)n_ou(3)$, $l_{92} = h(3,3)u(4)n_ou(3)$, $l_{93} = h(2,2)u(3)n_ou(3)$, $l_{94} = h(3,3)u(2)n_ou(3)$,
$l_{95} = h(2,2)u(1)n_ou(3)$, $l_{96} = h(3,3)n_ou(3)$, $l_{97} = h(2,2)u(4)n_ou(3)$, $l_{98} = h(3,3)u(3)n_ou(3)$,
$l_{99} = h(2,2)u(2)n_ou(3)$, $l_{100} = h(3,3)u(1)n_ou(3)$.
Block $B(x,x)(\lambda = 4)$:
T (1,1)$(\lambda = 4)$**:**
$l_{101} = h(1,1)n_ou(4)$, $l_{102} = h(4,4)u(4)n_ou(4)$, $l_{103} = h(1,1)u(3)n_ou(4)$, $l_{104} = h(4,4)u(2)n_ou(4)$,
$l_{105} = h(1,1)u(1)n_ou(4)$, $l_{106} = h(4,4)n_ou(4)$, $l_{107} = h(1,1)u(4)n_ou(4)$, $l_{108} = h(4,4)u(3)n_ou(4)$,
$l_{109} = h(1,1)u(2)n_ou(4)$, $l_{110} = h(4,4)u(1)n_ou(4)$.
T (2,2)$(\lambda = 4)$**:**
$l_{111} = h(2,2)n_ou(4)$, $l_{112} = h(3,3)u(4)n_ou(4)$, $l_{113} = h(2,2)u(3)n_ou(4)$, $l_{114} = h(3,3)u(2)n_ou(4)$,
$l_{115} = h(2,2)u(1)n_ou(4)$, $l_{116} = h(3,3)n_ou(4)$, $l_{117} = h(2,2)u(4)n_ou(4)$, $l_{118} = h(3,3)u(3)n_ou(4)$,
$l_{119} = h(2,2)u(2)n_ou(4)$, $l_{120} = h(3,3)u(1)n_ou(4)$, .

Type $(x\ ,\ y)$:
Block $B_1(x,y)$:
$T_1(x,y)(1)$**:**
$l_{121} = h(1,4)$, $l_{122} = h(4,1)u(1)$, $l_{123} = h(1,4)u(2)$, $l_{124} = h(4,1)u(3)$, $l_{125} = h(1,4)u(4)$,
$l_{126} = h(4,1)$, $l_{127} = h(1,4)u(1)$, $l_{128} = h(4,1)u(2)$, $l_{129} = h(1,4)u(3)$, $l_{130} = h(4,1)u(4)$.
$T_1(x,y)(2)$**:**
$l_{131} = h(2,3)$, $l_{132} = h(3,2)u(1)$, $l_{133} = h(2,3)u(2)$, $l_{134} = h(3,2)u(3)$, $l_{135} = h(2,3)u(4)$,

$l_{136}=h(3,2),\ l_{137}=h(2,3)u(1),\ l_{138}=h(3,2)u(2),\ l_{139}=h(2,3)u(3),\ l_{140}=h(3,2)u(4).$
Block $B_1(x,y)(\lambda=0)$:
$T_1(x,y)(1)(\lambda=0)$**:**
$l_{141}=h(1,4)n_o,\ l_{142}=h(4,1)u(1)n_o,\ l_{143}=h(1,4)u(2)n_o,\ l_{144}=h(4,1)u(3)n_o,\ l_{145}=h(1,4)u(4)n_o,$
$l_{146}=h(4,1)n_o,\ l_{147}=h(1,4)u(1)n_o,\ l_{148}=h(4,1)u(2)n_o,\ l_{149}=h(1,4)u(3)n_o,\ l_{150}=h(4,1)u(4)n_o.$
$T_1(x,y)(2)(\lambda=0)$**:**
$l_{151}=h(2,3)n_o,\ l_{152}=h(3,2)u(1)n_o,\ l_{153}=h(2,3)u(2)n_o,\ l_{154}=h(3,2)u(3)n_o,\ l_{155}=h(2,3)u(4)n_o,$
$l_{156}=h(3,2)n_o,\ l_{157}=h(2,3)u(1)n_o,\ l_{158}=h(3,2)u(2)n_o,\ l_{159}=h(2,3)u(3)n_o,\ l_{160}=h(3,2)u(4)n_o.$
Block $B_1(x,y)(\lambda=1)$:
$T_1(x,y)(1)(\lambda=1)$**:**
$l_{161}=h(1,4)n_ou(1),\ l_{162}=h(4,1)u(1)n_ou(1),\ l_{163}=h(1,4)u(2)n_ou(1),\ l_{164}=h(4,1)u(3)n_ou(1),$
$l_{165}=h(1,4)u(4)n_ou(1),\ l_{166}=h(4,1)n_ou(1),\ l_{167}=h(1,4)u(1)n_ou(1),\ l_{168}=h(4,1)u(2)n_ou(1),$
$l_{169}=h(1,4)u(3)n_ou(1),\ l_{170}=h(4,1)u(4)n_ou(1).$
$T_1(x,y)(2)(\lambda=1)$**:**
$l_{171}=h(2,3)n_ou(1),\ l_{172}=h(3,2)u(1)n_ou(1),\ l_{173}=h(2,3)u(2)n_ou(1),\ l_{174}=h(3,2)u(3)n_ou(1),$
$l_{175}=h(2,3)u(4)n_ou(1),\ l_{176}=h(3,2)n_ou(1),\ l_{177}=h(2,3)u(1)n_ou(1),\ l_{178}=h(3,2)u(2)n_ou(1),$
$l_{179}=h(2,3)u(3)n_ou(1),\ l_{180}=h(3,2)u(4)n_ou(1).$
Block $B_1(x,y)(\lambda=2)$:
$T_1(x,y)(1)(\lambda=2)$**:**
$l_{181}=h(1,4)n_ou(2),\ l_{182}=h(4,1)u(1)n_ou(2),\ l_{183}=h(1,4)u(2)n_ou(2),\ l_{184}=h(4,1)u(3)n_ou(2),$
$l_{185}=h(1,4)u(4)n_ou(2),\ l_{186}=h(4,1)n_ou(2),\ l_{187}=h(1,4)u(1)n_ou(2),\ l_{188}=h(4,1)u(2)n_ou(2),$
$l_{189}=h(1,4)u(3)n_ou(2),\ l_{190}=h(4,1)u(4)n_ou(2).$
$T_1(x,y)(2)(\lambda=2)$**:**
$l_{191}=h(2,3)n_ou(2),\ l_{192}=h(3,2)u(1)n_ou(2),\ l_{193}=h(2,3)u(2)n_ou(2),\ l_{194}=h(3,2)u(3)n_ou(2),$
$l_{195}=h(2,3)u(4)n_ou(2),\ l_{196}=h(3,2)n_ou(2),\ l_{197}=h(2,3)u(1)n_ou(2),\ l_{198}=h(3,2)u(2)n_ou(2),$
$l_{199}=h(2,3)u(3)n_ou(2),\ l_{200}=h(3,2)u(4)n_ou(2).$
Block $B_1(x,y)(\lambda=3)$:
$T_1(x,y)(1)(\lambda=3)$**:**
$l_{201}=h(1,4)n_ou(3),\ l_{202}=h(4,1)u(1)n_ou(3),\ l_{203}=h(1,4)u(2)n_ou(3),\ l_{204}=h(4,1)u(3)n_ou(3),$
$l_{205}=h(1,4)u(4)n_ou(3),\ l_{206}=h(4,1)n_ou(3),\ l_{207}=h(1,4)u(1)n_ou(3),\ l_{208}=h(4,1)u(2)n_ou(3),$
$l_{209}=h(1,4)u(3)n_ou(3),\ l_{210}=h(4,1)u(4)n_ou(3).$
$T_1(x,y)(2)(\lambda=3)$**:**
$l_{211}=h(2,3)n_ou(3),\ l_{212}=h(3,2)u(1)n_ou(3),\ l_{213}=h(2,3)u(2)n_ou(3),\ l_{214}=h(3,2)u(3)n_ou(3),$
$l_{215}=h(2,3)u(4)n_ou(3),\ l_{216}=h(3,2)n_ou(3),\ l_{217}=h(2,3)u(1)n_ou(3),\ l_{218}=h(3,2)u(2)n_ou(3),$
$l_{219}=h(2,3)u(3)n_ou(3),\ l_{220}=h(3,2)u(4)n_ou(3).$
Block $B_1(x,y)(\lambda=4)$:
$T_1(x,y)(1)(\lambda=4)$**:**
$l_{221}=h(1,4)n_ou(4),\ l_{222}=h(4,1)u(1)n_ou(4),\ l_{223}=h(1,4)u(2)n_ou(4),\ l_{224}=h(4,1)u(3)n_ou(4),$
$l_{225}=h(1,4)u(4)n_ou(4),\ l_{226}=h(4,1)n_ou(4),\ l_{227}=h(1,4)u(1)n_ou(4),\ l_{228}=h(4,1)u(2)n_ou(4),$
$l_{229}=h(1,4)u(3)n_ou(4),\ l_{230}=h(4,1)u(4)n_ou(4).$
$T_1(x,y)(2)(\lambda=4)$**:**
$l_{231}=h(2,3)n_ou(4),\ l_{232}=h(3,2)u(1)n_ou(4),\ l_{233}=h(2,3)u(2)n_ou(4),\ l_{234}=h(3,2)u(3)n_ou(4),$
$l_{235}=h(2,3)u(4)n_ou(4),\ l_{236}=h(3,2)n_ou(4),\ l_{237}=h(2,3)u(1)n_ou(4),\ l_{238}=h(3,2)u(2)n_ou(4),$
$l_{239}=h(2,3)u(3)n_ou(4),\ l_{240}=h(3,2)u(4)n_ou(4).$
Block $B_2(x,y)$:
$T_2(x,y)(1)$**:**
$l_{241}=h(1,3),\ l_{242}=h(4,2)u(2),\ l_{243}=h(1,3)u(4),\ l_{244}=h(4,2)u(1),\ l_{245}=h(1,3)u(3),$
$l_{246}=h(4,2)\ ,\ l_{247}=h(1,3)u(2),\ l_{248}=h(4,2)u(4),\ l_{249}=h(1,3)u(1),\ l_{250}=h(4,2)u(3).$
$T_2(x,y)(2)$**:**
$l_{251}=h(2,1),\ l_{252}=h(3,4)u(2),\ l_{253}=h(2,1)u(4),\ l_{254}=h(3,4)u(1),\ l_{255}=h(2,1)u(3),$
$l_{256}=h(3,4),\ l_{257}=h(2,1)u(2),\ l_{258}=h(3,4)u(4),\ l_{259}=h(2,1)u(1),\ l_{260}=h(3,4)u(3).$
Block $B_2(x,y)(\lambda=0)$:
$T_2(x,y)(1)(\lambda=0)$**:**
$l_{261}=h(1,3)n_o,\ l_{262}=h(4,2)u(2)n_o,\ l_{263}=h(1,3)u(4)n_o,\ l_{264}=h(4,2)u(1)n_o,\ l_{265}=h(1,3)u(3)n_o,$
$l_{266}=h(4,2)n_o,\ l_{267}=h(1,3)u(2)n_o,\ l_{268}=h(4,2)u(4)n_o,\ l_{269}=h(1,3)u(1)n_o,\ l_{270}=h(4,2)u(3)n_o.$
$T_2(x,y)(2)(\lambda=0)$**:**
$l_{271}=h(2,1)n_o,\ l_{272}=h(3,4)u(2)n_o,\ l_{273}=h(2,1)\ u(4)n_o,\ l_{274}=h(3,4)u(1)n_o,\ l_{275}=h(2,1)u(3)n_o,$
$l_{276}=h(3,4)n_o,\ l_{277}=h(2,1)u(2)n_o,\ l_{278}=h(3,4)u(4)n_o,\ l_{279}=h(2,1)u(1)n_o,\ l_{280}=h(3,4)u(3)n_o.$
Block $B_2(x,y)(\lambda=1)$:
$T_2(x,y)(1)(\lambda=1)$**:**
$l_{281}=h(1,3)n_ou(1),\ l_{282}=h(4,2)u(2)n_ou(1),\ l_{283}=h(1,3)u(4)n_ou(1),\ l_{284}=h(4,2)u(1)n_ou(1),$
$l_{285}=h(1,3)u(3)n_ou(1),\ l_{286}=h(4,2)n_ou(1),\ l_{287}=h(1,3)u(2)n_ou(1),\ l_{288}=h(4,2)u(4)n_ou(1),$
$l_{289}=h(1,3)u(1)n_ou(1),\ l_{290}=h(4,2)u(3)n_ou(1).$
$T_2(x,y)(2)(\lambda=1)$**:**
$l_{291}=h(2,1)n_ou(1),\ l_{292}=h(3,4)u(2)n_ou(1),\ l_{293}=h(2,1)u(4)n_ou(1),\ l_{294}=h(3,4)u(1)n_ou(1),$
$l_{295}=h(2,1)u(3)n_ou(1),\ l_{296}=h(3,4)n_ou(1),\ l_{297}=h(2,1)u(2)n_ou(1),\ l_{298}=h(3,4)u(4)n_ou(1),$

$l_{299} = h(2,1)u(1)n_o u(1),\ l_{300} = h(3,4)u(3)n_o u(1).$
Block $B_2(x,y)(\lambda = 2)$:
$T_2(x,y)(1)(\lambda = 2)$**:**
$l_{301} = h(1,3)n_o u(2),\ l_{302} = h(4,2)u(2)n_o u(2),\ l_{303} = h(1,3)u(4)n_o u(2),\ l_{304} = h(4,2)u(1)n_o u(2),$
$l_{305} = h(1,3)u(3)n_o u(2),\ l_{306} = h(4,2)n_o u(2),\ l_{307} = h(1,3)u(2)n_o u(2),\ l_{308} = h(4,2)u(4)n_o u(2),$
$l_{309} = h(1,3)u(1)n_o u(2),\ l_{310} = h(4,2)u(3)n_o u(2).$
$T_2(x,y)(2)(\lambda = 2)$**:**
$l_{311} = h(2,1)n_o u(2),\ l_{312} = h(3,4)u(2)n_o u(2),\ l_{313} = h(2,1)u(4)n_o u(2),\ l_{314} = h(3,4)u(1)n_o u(2),$
$l_{315} = h(2,1)u(3)n_o u(2),\ l_{316} = h(3,4)n_o u(2),\ l_{317} = h(2,1)u(2)n_o u(2),\ l_{318} = h(3,4)u(4)n_o u(2),$
$l_{319} = h(2,1)u(1)n_o u(2),\ l_{320} = h(3,4)u(3)n_o u(2).$
Block $B_2(x,y)(\lambda = 3)$:
$T_2(x,y)(1)(\lambda = 3)$**:**
$l_{321} = h(1,3)n_o u(3),\ l_{322} = h(4,2)u(2)n_o u(3),\ l_{323} = h(1,3)u(4)n_o u(3),\ l_{324} = h(4,2)u(1)n_o u(3),$
$l_{325} = h(1,3)u(3)n_o u(3),\ l_{326} = h(4,2)n_o u(3),\ l_{327} = h(1,3)u(2)n_o u(3),\ l_{328} = h(4,2)u(4)n_o u(3),$
$l_{329} = h(1,3)u(1)n_o u(3),\ l_{330} = h(4,2)u(3)n_o u(3).$
$T_2(x,y)(2)(\lambda = 3)$**:**
$l_{331} = h(2,1)n_o u(3),\ l_{332} = h(3,4)u(2)n_o u(3),\ l_{333} = h(2,1)u(4)n_o u(3),\ l_{334} = h(3,4)u(1)n_o u(3),$
$l_{335} = h(2,1)u(3)n_o u(3),\ l_{336} = h(3,4)n_o u(3),\ l_{337} = h(2,1)u(2)n_o u(3),\ l_{338} = h(3,4)u(4)n_o u(3),$
$l_{339} = h(2,1)u(1)n_o u(3),\ l_{340} = h(3,4)u(3)n_o u(3).$
Block $B_2(x,y)(\lambda = 4)$:
$T_2(x,y)(1)(\lambda = 4)$**:**
$l_{341} = h(1,3)n_o u(4),\ l_{342} = h(4,2)u(2)n_o u(4),\ l_{343} = h(1,3)u(4)n_o u(4),\ l_{344} = h(4,2)u(1)n_o u(4),$
$l_{345} = h(1,3)u(3)n_o u(4),\ l_{346} = h(4,2)n_o u(4),\ l_{347} = h(1,3)u(2)n_o u(4),\ l_{348} = h(4,2)u(4)n_o u(4),$
$l_{349} = h(1,3)u(1)n_o u(4),\ l_{350} = h(4,2)u(3)n_o u(4).$
$T_2(x,y)(2)(\lambda = 4)$**:**
$l_{351} = h(2,1)n_o u(4),\ l_{352} = h(3,4)u(2)n_o u(4),\ l_{353} = h(2,1)u(4)n_o u(4),\ l_{354} = h(3,4)u(1)n_o u(4),$
$l_{355} = h(2,1)u(3)n_o u(4),\ l_{356} = h(3,4)n_o u(4),\ l_{357} = h(2,1)u(2)n_o u(4),\ l_{358} = h(3,4)u(4)n_o u(4),$
$l_{359} = h(2,1)u(1)n_o u(4),\ l_{360} = h(3,4)u(3)n_o u(4).$
Block $B_3(x,y)$:
$T_3(x,y)(1)$**:**
$l_{361} = h(1,2),\ l_{362} = h(4,3)u(3),\ l_{363} = h(1,2)u(1),\ l_{364} = h(4,3)u(4),\ l_{365} = h(1,2)u(2),$
$l_{366} = h(4,3),\ l_{367} = h(1,2)u(3),\ l_{368} = h(4,3)u(1),\ l_{369} = h(1,2)u(4),\ l_{370} = h(4,3)u(2).$
$T_3(x,y)(2)$**:**
$l_{371} = h(2,4),\ l_{372} = h(3,1)u(3),\ l_{373} = h(2,4)u(1),\ l_{374} = h(3,1)u(4),\ l_{375} = h(2,4)u(2),$
$l_{376} = h(3,1),\ l_{377} = h(2,4)u(3),\ l_{378} = h(3,1)u(1),\ l_{379} = h(2,4)u(4),\ l_{380} = h(3,1)u(2).$
Block $B_3(x,y)(\lambda = 0)$:
$T_3(x,y)(1)(\lambda = 0)$**:**
$l_{381} = h(1,2)n_o,\ l_{382} = h(4,3)u(3)n_o,\ l_{383} = h(1,2)u(1)n_o,\ l_{384} = h(4,3)u(4)n_o,\ l_{385} = h(1,2)u(2)n_o,$
$l_{386} = h(4,3)n_o,\ l_{387} = h(1,2)u(3)n_o,\ l_{388} = h(4,3)u(1)n_o,\ l_{389} = h(1,2)u(4)n_o,\ l_{390} = h(4,3)u(2)n_o.$
$T_3(x,y)(2)(\lambda = 0)$**:**
$l_{391} = h(2,4)n_o,\ l_{392} = h(3,1)u(3)n_o,\ l_{393} = h(2,4)u(1)n_o,\ l_{394} = h(3,1)u(4)n_o,\ l_{395} = h(2,4)u(2)n_o,$
$l_{396} = h(3,1)n_o,\ l_{397} = h(2,4)u(3)n_o,\ l_{398} = h(3,1)u(1)n_o,\ l_{399} = h(2,4)u(4)n_o,\ l_{400} = h(3,1)u(2)n_o.$
Block $B_3(x,y)(\lambda = 1)$:
$T_3(x,y)(1)(\lambda = 1)$**:**
$l_{401} = h(1,2)n_o u(1),\ l_{402} = h(4,3)u(3)n_o u(1),\ l_{403} = h(1,2)u(1)n_o u(1),\ l_{404} = h(4,3)u(4)n_o u(1),$
$l_{405} = h(1,2)u(2)n_o u(1),\ l_{406} = h(4,3)n_o u(1),\ l_{407} = h(1,2)u(3)n_o u(1),\ l_{408} = h(4,3)u(1)n_o u(1),$
$l_{409} = h(1,2)u(4)n_o u(1),\ l_{410} = h(4,3)u(2)n_o u(1).$
$T_3(x,y)(2)(\lambda = 1)$**:**
$l_{411} = h(2,4)n_o u(1),\ l_{412} = h(3,1)u(3)n_o u(1),\ l_{413} = h(2,4)u(1)n_o u(1),\ l_{414} = h(3,1)u(4)n_o u(1),$
$l_{415} = h(2,4)u(2)n_o u(1),\ l_{416} = h(3,1)n_o u(1),\ l_{417} = h(2,4)u(3)n_o u(1),\ l_{418} = h(3,1)u(1)n_o u(1),$
$l_{419} = h(2,4)u(4)n_o u(1),\ l_{420} = h(3,1)u(2)n_o u(1).$
Block $B_3(x,y)(\lambda = 2)$:
$T_3(x,y)(1)(\lambda = 2)$**:**
$l_{421} = h(1,2)n_o u(2),\ l_{422} = h(4,3)u(3)n_o u(2),\ l_{423} = h(1,2)u(1)n_o u(2),\ l_{424} = h(4,3)u(4)n_o u(2),$
$l_{425} = h(1,2)u(2)n_o u(2),\ l_{426} = h(4,3)n_o u(2),\ l_{427} = h(1,2)u(3)n_o u(2),\ l_{428} = h(4,3)u(1)n_o u(2),$
$l_{429} = h(1,2)u(4)n_o u(2),\ l_{430} = h(4,3)u(2)n_o u(2).$
$T_3(x,y)(2)(\lambda = 2)$**:**
$l_{431} = h(2,4)n_o u(2),\ l_{432} = h(3,1)u(3)n_o u(2),\ l_{433} = h(2,4)u(1)n_o u(2),\ l_{434} = h(3,1)u(4)n_o u(2),$
$l_{435} = h(2,4)u(2)n_o u(2),\ l_{436} = h(3,1)n_o u(2),\ l_{437} = h(2,4)u(3)n_o u(2),\ l_{438} = h(3,1)u(1)n_o u(2),$
$l_{439} = h(2,4)u(4)n_o u(2),\ l_{440} = h(3,1)u(2)n_o u(2).$
Block $B_3(x,y)(\lambda = 3)$:
$T_3(x,y)(1)(\lambda = 3)$**:**
$l_{441} = h(1,2)n_o u(3),\ l_{442} = h(4,3)u(3)n_o u(3),\ l_{443} = h(1,2)u(1)n_o u(3),\ l_{444} = h(4,3)u(4)n_o u(3),$
$l_{445} = h(1,2)u(2)n_o u(3),\ l_{446} = h(4,3)n_o u(3),\ l_{447} = h(1,2)u(3)n_o u(3),\ l_{448} = h(4,3)u(1)n_o u(3),$
$l_{449} = h(1,2)u(4)n_o u(3),\ l_{450} = h(4,3)u(2)n_o u(3).$
$T_3(x,y)(2)(\lambda = 3)$**:**

$l_{451} = h(2,4)n_o u(3)$, $l_{452} = h(3,1)u(3)n_o u(3)$, $l_{453} = h(2,4)u(1)n_o u(3)$, $l_{454} = h(3,1)u(4)n_o u(3)$,
$l_{455} = h(2,4)u(2)n_o u(3)$, $l_{456} = h(3,1)n_o u(3)$, $l_{457} = h(2,4)u(3)n_o u(3)$, $l_{458} = h(3,1)u(1)n_o u(3)$,
$l_{459} = h(2,4)u(4)n_o u(3)$, $l_{460} = h(3,1)u(2)n_o u(3)$.

Block $B_3(x,y)(\lambda = 4)$:

$T_3(x,y)(1)(\lambda = 4)$**:**

$l_{461} = h(1,2)n_o u(4)$, $l_{462} = h(4,3)u(3)n_o u(4)$, $l_{463} = h(1,2)u(1)n_o u(4)$, $l_{464} = h(4,3)u(4)n_o u(4)$,
$l_{465} = h(1,2)u(2)n_o u(4)$, $l_{466} = h(4,3)n_o u(4)$, $l_{467} = h(1,2)u(3)n_o u(4)$, $l_{468} = h(4,3)u(1)n_o u(4)$,
$l_{469} = h(1,2)u(4)n_o u(4)$, $l_{470} = h(4,3)u(2)n_o u(4)$.

$T_3(x,y)(2)(\lambda = 4)$**:**

$l_{471} = h(2,4)n_o u(4)$, $l_{472} = h(3,1)u(3)n_o u(4)$, $l_{473} = h(2,4)u(1)n_o u(4)$, $l_{474} = h(3,1)u(4)n_o u(4)$,
$l_{475} = h(2,4)u(2)n_o u(4)$, $l_{476} = h(3,1)n_o u(4)$, $l_{477} = h(2,4)u(3)n_o u(4)$, $l_{478} = h(3,1)u(1)n_o u(4)$,
$l_{479} = h(2,4)u(4)n_o u(4)$, $l_{480} = h(3,1)u(2)n_o u(4)$.

The matrix of $GL(2,5)$:

We will discuss some blocks of the $G-matrix$ for $G = GL(2,5)$.
First we take the first block $B(x,x)$. That is :

$$\left(\begin{array}{ccccccccc||ccccccccc}
l_1 & l_2 & l_3 & l_4 & l_5 & l_6 & l_7 & l_8 & \cdots & l_{11} & l_{12} & l_{13} & l_{14} & l_{15} & l_{16} & l_{17} & l_{18} & \cdots \\
l_{10} & l_1 & l_2 & l_3 & l_4 & l_5 & l_6 & l_7 & \cdots & l_{20} & l_{11} & l_{12} & l_{13} & l_{14} & l_{15} & l_{16} & l_{17} & \cdots \\
l_9 & l_{10} & l_1 & l_2 & l_3 & l_4 & l_5 & l_6 & \cdots & l_{19} & l_{20} & l_{11} & l_{12} & l_{13} & l_{14} & l_{15} & l_{16} & \cdots \\
l_8 & l_9 & l_{10} & l_1 & l_2 & l_3 & l_4 & l_5 & \cdots & l_{18} & l_{19} & l_{20} & l_{11} & l_{12} & l_{13} & l_{14} & l_{15} & \cdots \\
l_7 & l_8 & l_9 & l_{10} & l_1 & l_2 & l_3 & l_4 & \cdots & l_{17} & l_{18} & l_{19} & l_{20} & l_{11} & l_{12} & l_{13} & l_{14} & \cdots \\
l_6 & l_7 & l_8 & l_9 & l_{10} & l_1 & l_2 & l_3 & \cdots & l_{16} & l_{17} & l_{18} & l_{19} & l_{20} & l_{11} & l_{12} & l_{13} & \cdots \\
l_5 & l_6 & l_7 & l_8 & l_9 & l_{10} & l_1 & l_2 & \cdots & l_{15} & l_{16} & l_{17} & l_{18} & l_{19} & l_{20} & l_{11} & l_{12} & \cdots \\
l_4 & l_5 & l_6 & l_7 & l_8 & l_9 & l_{10} & l_1 & \cdots & l_{14} & l_{15} & l_{16} & l_{17} & l_{18} & l_{19} & l_{20} & l_{11} & \cdots \\
\vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\
\hline\hline
l_{16} & l_{17} & l_{18} & l_{19} & l_{20} & l_{11} & l_{12} & l_{13} & \cdots & l_1 & l_2 & l_3 & l_4 & l_5 & l_6 & l_7 & l_8 & \cdots \\
l_{15} & l_{16} & l_{17} & l_{18} & l_{19} & l_{20} & l_{11} & l_{12} & \cdots & l_{10} & l_1 & l_2 & l_3 & l_4 & l_5 & l_6 & l_7 & \cdots \\
l_{14} & l_{15} & l_{16} & l_{17} & l_{18} & l_{19} & l_{20} & l_{11} & \cdots & l_9 & l_{10} & l_1 & l_2 & l_3 & l_4 & l_5 & l_6 & \cdots \\
l_{13} & l_{14} & l_{15} & l_{16} & l_{17} & l_{18} & l_{19} & l_{20} & \cdots & l_8 & l_9 & l_{10} & l_1 & l_2 & l_3 & l_4 & l_5 & \cdots \\
l_{12} & l_{13} & l_{14} & l_{15} & l_{16} & l_{17} & l_{18} & l_{19} & \cdots & l_7 & l_8 & l_9 & l_{10} & l_1 & l_2 & l_3 & l_4 & \cdots \\
l_{11} & l_{12} & l_{13} & l_{14} & l_{15} & l_{16} & l_{17} & l_{18} & \cdots & l_6 & l_7 & l_8 & l_9 & l_{10} & l_1 & l_2 & l_3 & \cdots \\
l_{20} & l_{11} & l_{12} & l_{13} & l_{14} & l_{15} & l_{16} & l_{17} & \cdots & l_5 & l_6 & l_7 & l_8 & l_9 & l_{10} & l_1 & l_2 & \cdots \\
l_{19} & l_{20} & l_{11} & l_{12} & l_{13} & l_{14} & l_{15} & l_{16} & \cdots & l_4 & l_5 & l_6 & l_7 & l_8 & l_9 & l_{10} & l_1 & \cdots \\
\vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots
\end{array}\right)$$

Now for another block we take the elements of block $B_2(x,y)(\lambda = 2)$ with inverses of the elements of the block $B(x,x)$. So, we get :

$$\left(\begin{array}{cccccccc||cccccccc}
l_{301} & l_{302} & l_{303} & l_{304} & l_{305} & l_{306} & l_{307} & \cdots & l_{311} & l_{312} & l_{313} & l_{314} & l_{315} & l_{316} & l_{317} & \cdots \\
l_{310} & l_{301} & l_{302} & l_{303} & l_{304} & l_{305} & l_{306} & \cdots & l_{320} & l_{311} & l_{312} & l_{313} & l_{314} & l_{315} & l_{316} & \cdots \\
l_{309} & l_{310} & l_{301} & l_{302} & l_{303} & l_{304} & l_{305} & \cdots & l_{319} & l_{320} & l_{311} & l_{312} & l_{313} & l_{314} & l_{315} & \cdots \\
l_{308} & l_{309} & l_{310} & l_{301} & l_{302} & l_{303} & l_{304} & \cdots & l_{318} & l_{319} & l_{320} & l_{311} & l_{312} & l_{313} & l_{314} & \cdots \\
l_{307} & l_{308} & l_{309} & l_{310} & l_{301} & l_{302} & l_{303} & \cdots & l_{317} & l_{318} & l_{319} & l_{320} & l_{311} & l_{312} & l_{313} & \cdots \\
l_{306} & l_{307} & l_{308} & l_{309} & l_{310} & l_{301} & l_{302} & \cdots & l_{316} & l_{317} & l_{318} & l_{319} & l_{320} & l_{311} & l_{312} & \cdots \\
l_{305} & l_{306} & l_{307} & l_{308} & l_{309} & l_{310} & l_{301} & \cdots & l_{315} & l_{316} & l_{317} & l_{318} & l_{319} & l_{320} & l_{311} & \cdots \\
\vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\
\hline\hline
l_{316} & l_{317} & l_{318} & l_{319} & l_{320} & l_{311} & l_{312} & \cdots & l_{301} & l_{302} & l_{303} & l_{304} & l_{305} & l_{306} & l_{307} & \cdots \\
l_{315} & l_{316} & l_{317} & l_{318} & l_{319} & l_{320} & l_{311} & \cdots & l_{310} & l_{301} & l_{302} & l_{303} & l_{304} & l_{305} & l_{306} & \cdots \\
l_{314} & l_{315} & l_{316} & l_{317} & l_{318} & l_{319} & l_{320} & \cdots & l_{309} & l_{310} & l_{301} & l_{302} & l_{303} & l_{304} & l_{305} & \cdots \\
l_{313} & l_{314} & l_{315} & l_{316} & l_{317} & l_{318} & l_{319} & \cdots & l_{308} & l_{309} & l_{310} & l_{301} & l_{302} & l_{303} & l_{304} & \cdots \\
l_{312} & l_{313} & l_{314} & l_{315} & l_{316} & l_{317} & l_{318} & \cdots & l_{307} & l_{308} & l_{309} & l_{310} & l_{301} & l_{302} & l_{303} & \cdots \\
l_{311} & l_{312} & l_{313} & l_{314} & l_{315} & l_{316} & l_{317} & \cdots & l_{306} & l_{307} & l_{308} & l_{309} & l_{310} & l_{301} & l_{302} & \cdots \\
l_{320} & l_{311} & l_{312} & l_{313} & l_{314} & l_{315} & l_{316} & \cdots & l_{305} & l_{306} & l_{307} & l_{308} & l_{309} & l_{310} & l_{301} & \cdots \\
\vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots
\end{array}\right)$$

Now, if we take the elements of block $B_1(x,y)$ with the inverses of the elements of block $B_2(x,y)(\lambda=0)$ we get the following :

$$\left(\begin{array}{cccccccc||cccccccc}
l_{396} & l_{411} & l_{436} & l_{451} & l_{476} & l_{391} & l_{416} & \cdots & l_{381} & l_{406} & l_{421} & l_{446} & l_{461} & l_{386} & l_{401} & \cdots \\
l_{471} & l_{396} & l_{411} & l_{436} & l_{451} & l_{476} & l_{391} & \cdots & l_{466} & l_{381} & l_{406} & l_{421} & l_{446} & l_{461} & l_{386} & \cdots \\
l_{456} & l_{471} & l_{396} & l_{411} & l_{436} & l_{451} & l_{476} & \cdots & l_{441} & l_{466} & l_{381} & l_{406} & l_{421} & l_{446} & l_{461} & \cdots \\
l_{431} & l_{456} & l_{471} & l_{396} & l_{411} & l_{436} & l_{451} & \cdots & l_{426} & l_{441} & l_{466} & l_{381} & l_{406} & l_{421} & l_{446} & \cdots \\
l_{416} & l_{431} & l_{456} & l_{471} & l_{396} & l_{411} & l_{436} & \cdots & l_{401} & l_{426} & l_{441} & l_{466} & l_{381} & l_{406} & l_{421} & \cdots \\
l_{391} & l_{416} & l_{431} & l_{456} & l_{471} & l_{396} & l_{411} & \cdots & l_{386} & l_{401} & l_{426} & l_{441} & l_{466} & l_{381} & l_{406} & \cdots \\
l_{476} & l_{391} & l_{416} & l_{431} & l_{456} & l_{471} & l_{396} & \cdots & l_{461} & l_{386} & l_{401} & l_{426} & l_{441} & l_{466} & l_{381} & \cdots \\
\vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\
\hline\hline
l_{386} & l_{401} & l_{426} & l_{441} & l_{466} & l_{381} & l_{406} & \cdots & l_{396} & l_{411} & l_{436} & l_{451} & l_{476} & l_{391} & l_{416} & \cdots \\
l_{461} & l_{386} & l_{401} & l_{426} & l_{441} & l_{466} & l_{381} & \cdots & l_{471} & l_{396} & l_{411} & l_{436} & l_{451} & l_{476} & l_{391} & \cdots \\
l_{446} & l_{461} & l_{386} & l_{401} & l_{426} & l_{441} & l_{466} & \cdots & l_{456} & l_{471} & l_{396} & l_{411} & l_{436} & l_{451} & l_{476} & \cdots \\
l_{421} & l_{446} & l_{461} & l_{386} & l_{401} & l_{426} & l_{441} & \cdots & l_{431} & l_{456} & l_{471} & l_{396} & l_{411} & l_{436} & l_{451} & \cdots \\
l_{406} & l_{421} & l_{446} & l_{461} & l_{386} & l_{401} & l_{426} & \cdots & l_{416} & l_{431} & l_{456} & l_{471} & l_{396} & l_{411} & l_{436} & \cdots \\
l_{381} & l_{406} & l_{421} & l_{446} & l_{461} & l_{386} & l_{401} & \cdots & l_{391} & l_{416} & l_{431} & l_{456} & l_{471} & l_{396} & l_{411} & \cdots \\
l_{466} & l_{381} & l_{406} & l_{421} & l_{446} & l_{461} & l_{386} & \cdots & l_{476} & l_{391} & l_{416} & l_{431} & l_{456} & l_{471} & l_{396} & \cdots \\
\vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots
\end{array}\right)$$

2- The listing of elements of $GL(2,4)$:
$|Gl(2,4)|$=180, and this will divide into 15 blocks with 12 elements in each block .

Type $(a^i\ ,\ a^i)$:
Block $B(a^i,a^i)$**:**
$T(a^i,a^i)$**:**
$g_1=h(1,1),\ g_2=h(a^2,a^2)u(a^2),\ g_3=h(a,a),\ g_4=h(1,1)u(a^2),\ g_5=h(a^2,a^2),$
$g_6=h(a,a)u(a^2).$
$T(a^i,a^i)(1)$**:**
$g_7=h(1,1)u(a),\ g_8=h(a^2,a^2)u(1),\ g_9=h(a,a)u(a),\ g_{10}=h(1,1)u(1),$
$g_{11}=h(a^2,a^2)u(a),\ g_{12}=h(a,a)u(1).$
Block $B(a^i,a^i)(\lambda=0)$**:**
$T(a^i,a^i)(\lambda=0)$**:**
$g_{13}=h(1,1)n_o,\ g_{14}=h(a^2,a^2)u(a^2)n_o,\ g_{15}=h(a,a)n_o,\ g_{16}=h(1,1)u(a^2)n_o,$
$g_{17}=h(a^2,a^2)n_o,\ g_{18}=h(a,a)u(a^2)n_o.$
$T(a^i,a^i)(1)(\lambda=0)$**:**
$g_{19}=h(1,1)u(a)n_o,\ g_{20}=h(a^2,a^2)u(1)n_o,\ g_{21}=h(a,a)u(a)n_o,$
$g_{22}=h(1,1)u(1)n_o,\ g_{23}=h(a^2,a^2)u(a)n_o,\ g_{24}=h(a,a)u(1)n_o.$
Block $B(a^i,a^i)(\lambda=1)$**:**
$T(a^i,a^i)(\lambda=1)$**:**
$g_{25}=h(1,1)n_ou(1),\ g_{26}=h(a^2,a^2)u(a^2)n_ou(1),\ g_{27}=h(a,a)n_ou(1),$
$g_{28}=h(1,1)u(a^2)n_ou(1),\ g_{29}=h(a^2,a^2)n_ou(1),\ g_{30}=h(a,a)u(a^2)n_ou(1).$
$T(a^i,a^i)(1)(\lambda=1)$**:**
$g_{31}=h(1,1)u(a)n_ou(1),\ g_{32}=h(a^2,a^2)u(1)n_ou(1),\ g_{33}=h(a,a)u(a)n_ou(1),$
$g_{34}=h(1,1)u(1)n_ou(1),\ g_{35}=h(a^2,a^2)u(a)n_ou(1),\ g_{36}=h(a,a)u(1)n_ou(1).$
Block $B(a^i,a^i)(\lambda=a)$**:**
$T(a^i,a^i)(\lambda=a)$**:**
$g_{37}=h(1,1)n_ou(a),\ g_{38}=h(a^2,a^2)u(a^2)n_ou(a),\ g_{39}=h(a,a)n_ou(a),$
$g_{40}=h(1,1)u(a^2)n_ou(a),\ g_{41}=h(a^2,a^2)n_ou(a),\ g_{42}=h(a,a)u(a^2)n_ou(a).$
$T(a^i,a^i)(1)(\lambda=a)$**:**
$g_{43}=h(1,1)u(a)n_ou(a),\ g_{44}=h(a^2,a^2)u(1)n_ou(a),\ g_{45}=h(a,a)u(a)n_ou(a),$
$g_{46}=h(1,1)u(1)n_ou(a),\ g_{47}=h(a^2,a^2)u(a)n_ou(a),\ g_{48}=h(a,a)u(1)n_ou(a).$
Block $B(a^i,a^i)(\lambda=a^2)$**:**
$T(a^i,a^i)(\lambda=a^2)$**:**
$g_{49}=h(1,1)n_ou(a^2),\ g_{50}=h(a^2,a^2)u(a^2)n_ou(a^2),\ g_{51}=h(a,a)n_ou(a^2),$
$g_{52}=h(1,1)u(a^2)n_ou(a^2),\ g_{53}=h(a^2,a^2)n_ou(a^2),\ g_{54}=h(a,a)u(a^2)n_ou(a^2).$
$T(a^i,a^i)(1)(\lambda=a^2)$**:**
$g_{55}=h(1,1)u(a)n_ou(a^2),\ g_{56}=h(a^2,a^2)u(1)n_ou(a^2),\ g_{57}=h(a,a)u(a)n_ou(a^2),$
$g_{58}=h(1,1)u(1)n_ou(a^2),\ g_{59}=h(a^2,a^2)u(a)n_ou(a^2),\ g_{60}=h(a,a)u(1)n_ou(a^2).$

Type $h(a^i\ ,\ a^j)$:
Block $B_1(a^i,a^j)$**:**
$T_1(a^i,a^j)$**:**
$g_{61}=h(1,a),\ g_{62}=h(a^2,1)u(1),\ g_{63}=h(a,a^2),\ g_{64}=h(1,a)u(1),\ g_{65}=h(a^2,1),$
$g_{66}=h(a,a^2)u(1).$
$T_1(a^i,a^j)(1)$**:**
$g_{67}=h(1,a)u(a^2),\ g_{68}=h(a^2,1)u(a),\ g_{69}=h(a,a^2)u(a^2),\ g_{70}=h(1,a)u(a),$
$g_{71}=h(a^2,1)u(a^2),\ g_{72}=h(a,a^2)u(a).$

Block $B_1(a^i,a^j)(\lambda=0)$**:**
$T_1(a^i,a^j)(\lambda=0)$**:**
$g_{73}=h(1,a)n_o,\ g_{74}=h(a^2,1)u(1)n_o,\ g_{75}=h(a,a^2)n_o,\ g_{76}=h(1,a)u(1)n_o,$
$g_{77}=h(a^2,1)n_o,\ g_{78}=h(a,a^2)u(1)n_o.$
$T_1(a^i,a^j)(1)(\lambda=0)$**:**
$g_{79}=h(1,a)u(a^2)n_o,\ g_{80}=h(a^2,1)u(a)n_o,\ g_{81}=h(a,a^2)u(a^2)n_o,$
$g_{82}=h(1,a)u(a)n_o,\ g_{83}=h(a^2,1)u(a^2)n_o,\ g_{84}=h(a,a^2)u(a)n_o.$
Block $B_1(a^i,a^j)(\lambda=1)$**:**
$T_1(a^i,a^j)(\lambda=1)$**:**
$g_{85}=h(1,a)n_ou(1),\ g_{86}=h(a^2,1)u(1)n_ou(1),\ g_{87}=h(a,a^2)n_ou(1),$
$g_{88}=h(1,a)u(1)n_ou(1),\ g_{89}=h(a^2,1)n_ou(1),\ g_{90}=h(a,a^2)u(1)n_ou(1).$
$T_1(a^i,a^j)(1)(\lambda=1)$**:**
$g_{91}=h(1,a)u(a^2)n_ou(1),\ g_{92}=h(a^2,1)u(a)n_ou(1),\ g_{93}=h(a,a^2)u(a^2)n_ou(1),$
$g_{94}=h(1,a)u(a)n_ou(1),\ g_{95}=h(a^2,1)u(a^2)n_ou(1),\ g_{96}=h(a,a^2)u(a)n_ou(1).$
Block $B_1(a^i,a^j)(\lambda=a)$**:**
$T_1(a^i,a^j)(\lambda=a)$**:**
$g_{97}=h(1,a)n_ou(a),\ g_{98}=h(a^2,1)u(1)n_ou(a),\ g_{99}=h(a,a^2)n_ou(a),$
$g_{100}=h(1,a)u(1)n_ou(a),\ g_{101}=h(a^2,1)n_ou(a),\ g_{102}=h(a,a^2)u(1)n_ou(a).$
$T_1(a^i,a^j)(1)(\lambda=a)$**:**
$g_{103}=h(1,a)u(a^2)n_ou(a),\ g_{104}=h(a^2,1)u(a)n_ou(a),\ g_{105}=h(a,a^2)u(a^2)n_ou(a),$
$g_{106}=h(1,a)u(a)n_ou(a),\ g_{107}=h(a^2,1)u(a^2)n_ou(a),\ g_{108}=h(a,a^2)u(a)n_ou(a).$
Block $B_1(a^i,a^j)(\lambda=a^2)$**:**
$T_1(a^i,a^j)(\lambda=a^2)$**:**
$g_{109}=h(1,a)n_ou(a^2),\ g_{110}=h(a^2,1)u(1)n_ou(a^2),\ g_{111}=h(a,a^2)n_ou(a^2),$
$g_{112}=h(1,a)u(1)n_ou(a^2),\ g_{113}=h(a^2,1)n_ou(a^2),\ g_{114}=h(a,a^2)u(1)n_ou(a^2).$
$T_1(a^i,a^j)(1)(\lambda=a^2)$**:**
$g_{115}=h(1,a)u(a^2)n_ou(a^2),\ g_{116}=h(a^2,1)u(a)n_ou(a^2),\ g_{117}=h(a,a^2)u(a^2)n_ou(a^2),$
$g_{118}=h(1,a)u(a)n_ou(a^2),\ g_{119}=h(a^2,1)u(a^2)n_ou(a^2),\ g_{120}=h(a,a^2)u(a)n_ou(a^2).$
Block $B_2(a^i,a^j)$**:**
$T_2(a^i,a^j)$**:**
$g_{121}=h(1,a^2),\ g_{122}=h(a^2,a)u(a),\ g_{123}=h(a,1),\ g_{124}=h(1,a^2)u(a),$
$g_{125}=h(a^2,a),\ g_{126}=h(a,1)u(a).$
$T_2(a^i,a^j)(1)$**:**
$g_{127}=h(1,a^2)u(1),\ g_{128}=h(a^2,a)u(a^2),\ g_{129}=h(a,1)u(1),\ g_{130}=h(1,a^2)u(a^2),$
$g_{131}=h(a^2,a)u(1),\ g_{132}h(a,1)u(a^2).$
Block $B_2(a^i,a^j)(\lambda=0)$**:**
$T_2(a^i,a^j)(\lambda=0)$**:**
$g_{133}=h(1,a^2)n_o,\ g_{134}=h(a^2,a)u(a)n_o,\ g_{135}=h(a,1)n_o,\ g_{136}=h(1,a^2)u(a)n_o,$
$g_{137}=h(a^2,a)n_o,\ g_{138}=h(a,1)u(a)n_o.$
$T_2(a^i,a^j)(1)(\lambda=0)$**:**
$g_{139}=h(1,a^2)u(1)n_o,\ g_{140}=h(a^2,a)u(a^2)n_o,\ g_{141}=h(a,1)u(1)n_o,$
$g_{142}=h(1,a^2)u(a^2)n_o,\ g_{143}=h(a^2,a)u(1)n_o,\ g_{144}=h(a,1)u(a^2)n_o.$
Block $B_2(a^i,a^j)(\lambda=1)$**:**
$T_2(a^i,a^j)(\lambda=1)$**:**
$g_{145}=h(1,a^2)n_ou(1), g_{146}=h(a^2,a)u(a)n_ou(1),\ g_{147}=h(a,1)n_ou(1),$
$g_{148}=h(1,a^2)u(a)n_ou(1),\ g_{149}=h(a^2,a)n_ou(1),\ g_{150}=h(a,1)u(a)n_ou(1).$
$T_2(a^i,a^j)(1)(\lambda=1)$**:**
$g_{151}=h(1,a^2)u(1)n_ou(1),\ g_{152}=h(a^2,a)u(a^2)n_ou(1),\ g_{153}=h(a,1)u(1)n_ou(1),$
$g_{154}=h(1,a^2)u(a^2)n_ou(1),\ g_{155}=h(a^2,a)u(1)n_ou(1),\ g_{156}=h(a,1)u(a^2)n_ou(1).$
Block $B_2(a^i,a^j)(\lambda=a)$**:**
$T_2(a^i,a^j)(\lambda=a)$**:**
$g_{157}=h(1,a^2)n_ou(a),\ g_{158}=h(a^2,a)u(a)n_ou(a),\ g_{159}=h(a,1)n_ou(a),$
$g_{160}=h(1,a^2)u(a)n_ou(a),\ g_{161}=h(a^2,a)n_ou(a),\ g_{162}=h(a,1)u(a)n_ou(a).$
$T_2(a^i,a^j)(1)(\lambda=a)$**:**
$g_{163}=h(1,a^2)u(1)n_ou(a),\ g_{164}=h(a^2,a)u(a^2)n_ou(a),\ g_{165}=h(a,1)u(1)n_ou(a),$
$g_{166}=h(1,a^2)u(a^2)n_ou(a),\ g_{167}=h(a^2,a)u(1)n_ou(a),\ g_{168}=h(a,1)u(a^2)n_ou(a).$
Block $B_2(a^i,a^j)(\lambda=a^2)$**:**
$T_2(a^i,a^j)(\lambda=a^2)$**:**
$g_{169}=h(1,a^2)n_ou(a^2),\ g_{170}=h(a^2,a)u(a)n_ou(a^2),\ g_{171}=h(a,1)n_ou(a^2),$
$g_{172}=h(1,a^2)u(a)n_ou(a^2),\ g_{173}=h(a^2,a)n_ou(a^2),\ g_{174}=h(a,1)u(a)n_ou(a^2).$
$T_2(a^i,a^j)(1)(\lambda=a^2)$**:**
$g_{175}=h(1,a^2)u(1)n_ou(a^2),\ g_{176}=h(a^2,a)u(a^2)n_ou(a^2),\ g_{177}=h(a,1)u(1)n_ou(a^2),$
$g_{178}=h(1,a^2)u(a^2)n_ou(a^2),\ g_{179}=h(a^2,a)u(1)n_ou(a^2),\ g_{180}=h(a,1)u(a^2)n_ou(a^2).$

Where, F_4 is the unique field (up to the isomorphism) with four elements such that;

$$F_4 \cong \frac{F_2[x]}{<x^2+x+1>},$$

where (x^2+x+1) is an irreducible polynomial in $F_2[x]$.
Now we examine some blocks of the $G-matrix$ of $GL(2,4)$. For the first block in the G-matrix we get the elements in the block $B(a^i,a^i)$ with its inverses and we get the following :

$$\left(\begin{array}{cccccc||cccccc}
g_1 & g_2 & g_3 & g_4 & g_5 & g_6 & g_7 & g_8 & g_9 & g_{10} & g_{11} & g_{12} \\
g_6 & g_1 & g_2 & g_3 & g_4 & g_5 & g_{12} & g_7 & g_8 & g_9 & g_{10} & g_{11} \\
g_5 & g_6 & g_1 & g_2 & g_3 & g_4 & g_{11} & g_{12} & g_7 & g_8 & g_9 & g_{10} \\
g_4 & g_5 & g_6 & g_1 & g_2 & g_3 & g_{10} & g_{11} & g_{12} & g_7 & g_8 & g_9 \\
g_3 & g_4 & g_5 & g_6 & g_1 & g_2 & g_9 & g_{10} & g_{11} & g_{12} & g_7 & g_8 \\
g_2 & g_3 & g_4 & g_5 & g_6 & g_1 & g_8 & g_9 & g_{10} & g_{11} & g_{12} & g_7 \\
\hline\hline
g_7 & g_8 & g_9 & g_{10} & g_{11} & g_{12} & g_1 & g_2 & g_3 & g_4 & g_5 & g_6 \\
g_{12} & g_7 & g_8 & g_9 & g_{10} & g_{11} & g_6 & g_1 & g_2 & g_3 & g_4 & g_5 \\
g_{11} & g_{12} & g_7 & g_8 & g_9 & g_{10} & g_5 & g_6 & g_1 & g_2 & g_3 & g_4 \\
g_{10} & g_{11} & g_{12} & g_7 & g_8 & g_9 & g_4 & g_5 & g_6 & g_1 & g_2 & g_3 \\
g_9 & g_{10} & g_{11} & g_{12} & g_7 & g_8 & g_3 & g_4 & g_5 & g_6 & g_1 & g_2 \\
g_8 & g_9 & g_{10} & g_{11} & g_{12} & g_7 & g_2 & g_3 & g_4 & g_5 & g_6 & g_1
\end{array}\right)$$

Now, if we consider the elements of block $B_2(a^i,a^j)(\lambda=0)$ with the inverses of the elements of block $B_1(a^i,a^j)$, we will get the following matrix:

$$\left(\begin{array}{cccccc||cccccc}
g_{73} & g_{80} & g_{75} & g_{82} & g_{77} & g_{84} & g_{76} & g_{83} & g_{78} & g_{79} & g_{74} & g_{81} \\
g_{84} & g_{73} & g_{80} & g_{75} & g_{82} & g_{77} & g_{81} & g_{76} & g_{83} & g_{78} & g_{79} & g_{74} \\
g_{77} & g_{84} & g_{73} & g_{80} & g_{75} & g_{82} & g_{74} & g_{81} & g_{76} & g_{83} & g_{78} & g_{79} \\
g_{82} & g_{77} & g_{84} & g_{73} & g_{80} & g_{75} & g_{79} & g_{74} & g_{81} & g_{76} & g_{83} & g_{78} \\
g_{75} & g_{82} & g_{77} & g_{84} & g_{73} & g_{80} & g_{78} & g_{79} & g_{74} & g_{81} & g_{76} & g_{83} \\
g_{80} & g_{75} & g_{82} & g_{77} & g_{84} & g_{73} & g_{83} & g_{78} & g_{79} & g_{74} & g_{81} & g_{76} \\
\hline\hline
g_{76} & g_{83} & g_{78} & g_{79} & g_{74} & g_{81} & g_{73} & g_{80} & g_{75} & g_{82} & g_{77} & g_{84} \\
g_{81} & g_{76} & g_{83} & g_{78} & g_{79} & g_{74} & g_{84} & g_{73} & g_{80} & g_{75} & g_{82} & g_{77} \\
g_{74} & g_{81} & g_{76} & g_{83} & g_{78} & g_{79} & g_{77} & g_{84} & g_{73} & g_{80} & g_{75} & g_{82} \\
g_{79} & g_{74} & g_{81} & g_{76} & g_{83} & g_{78} & g_{82} & g_{77} & g_{84} & g_{73} & g_{80} & g_{75} \\
g_{78} & g_{79} & g_{74} & g_{81} & g_{76} & g_{83} & g_{75} & g_{82} & g_{77} & g_{84} & g_{73} & g_{80} \\
g_{83} & g_{78} & g_{79} & g_{74} & g_{81} & g_{76} & g_{80} & g_{75} & g_{82} & g_{77} & g_{84} & g_{73}
\end{array}\right)$$

Also, if we get the elements of block $B_1(a^i,a^j)(\lambda=a)$ with the inverses of the elements of block $B_2(a^i,a^j)(\lambda=a^2)$, then the resulting matrix will have the following form:

$$\left(\begin{array}{cccccc||cccccc}
g_{71} & g_{111} & g_{67} & g_{113} & g_{69} & g_{109} & g_{139} & g_{34} & g_{141} & g_{36} & g_{143} & g_{32} \\
g_{109} & g_{71} & g_{111} & g_{67} & g_{113} & g_{69} & g_{32} & g_{139} & g_{34} & g_{141} & g_{36} & g_{143} \\
g_{69} & g_{109} & g_{71} & g_{111} & g_{67} & g_{113} & g_{143} & g_{32} & g_{139} & g_{34} & g_{141} & g_{36} \\
g_{113} & g_{69} & g_{109} & g_{71} & g_{111} & g_{67} & g_{36} & g_{143} & g_{32} & g_{139} & g_{34} & g_{141} \\
g_{67} & g_{113} & g_{69} & g_{109} & g_{71} & g_{111} & g_{141} & g_{36} & g_{143} & g_{32} & g_{139} & g_{34} \\
g_{111} & g_{67} & g_{113} & g_{69} & g_{109} & g_{71} & g_{34} & g_{141} & g_{36} & g_{143} & g_{32} & g_{139} \\
\hline\hline
g_{139} & g_{34} & g_{141} & g_{36} & g_{143} & g_{32} & g_{71} & g_{111} & g_{67} & g_{113} & g_{69} & g_{109} \\
g_{32} & g_{139} & g_{34} & g_{141} & g_{36} & g_{143} & g_{109} & g_{71} & g_{111} & g_{67} & g_{113} & g_{69} \\
g_{143} & g_{32} & g_{139} & g_{34} & g_{141} & g_{36} & g_{69} & g_{109} & g_{71} & g_{111} & g_{67} & g_{113} \\
g_{36} & g_{143} & g_{32} & g_{139} & g_{34} & g_{141} & g_{113} & g_{69} & g_{109} & g_{71} & g_{111} & g_{67} \\
g_{141} & g_{36} & g_{143} & g_{32} & g_{139} & g_{34} & g_{67} & g_{113} & g_{69} & g_{109} & g_{71} & g_{111} \\
g_{34} & g_{141} & g_{36} & g_{143} & g_{32} & g_{139} & g_{111} & g_{67} & g_{113} & g_{69} & g_{109} & g_{71}
\end{array}\right)$$

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