Proceedings of International Conference on Mathematics
and Mathematics Education (ICMME 2018)
Turk. J. Math. Comput. Sci.
10(2018) 1–6

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School Success Ranking in Multi Criteria Decision Making

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Received: 17-07-2018 • Accepted: 20-11-2018

Abstract. The aim of this paper is to proposed an application multi criteria decision making in intuitionistic fuzzy sets. Multicriteria decision making is a well known concept that aims to select the best solution among several alternatives by evaluating multiple conflicting criteria, explicitly in decision making. In this paper; success ranking of schools has been researched in multi criteria decision making. Also the most successful school has been determined among these ranked schools. Success ranking of among the randomly selected schools in a city have been researched. This method could be applicable to all schools. For this paper have been benefitted from similarity measure for intuitionistic fuzzy sets in multi criteria decision making problem. Each option have been compared with both the positive-ideal solution and the negative-ideal solution.

2010 AMS Classification: 03E72,90B50.

Keywords: intuitionistic fuzzy sets, distance measure, decision making, multi criteria.

1. Introduction

The notion of fuzzy logic was firstly defined by L.A.Zadeh in 1965 [10]. Then, intuitionistic fuzzy set (shortly IFS) was defined by K.Atanassov in 1986 [1]. Intuitionistic fuzzy sets form a generalization of the notion of fuzzy sets. The intuitionistic fuzzy set theory is useful in various application areas, such as algebraic structures, robotics, control systems, agriculture areas, computer, irrigation, economy and various engineering fields. The knowledge and semantic representation of intuitionistic fuzzy set become more meaningful, resourceful and applicable since it includes the membership degree, the non-membership degree and the hesitation margin [2] Various applications of intuitionistic fuzzy set have been carried out through distance measures approach. Many researchers have explored various applications of intuitionistic fuzzy set such as medical diagnosis, medical application, career determination, real life situations, education, artifical intelligence, networking.

Decision making is the action of choosing between two or more options. Multicriteria decision making is a well known concept that aims to select the best solution among several alternatives. The main goal of multi criteria decision making methods is to solve complex problems by selecting, comparing and ranking different attributes of multiple alternatives in a flexible manner [3].The basic working principle of any MCDM method is same: Selection of Criteria, Selection of Alternatives, Selection of Aggregation Methods and ultimately Selection of Alternatives based on weights or outranking [6]. MCDM is concerned with structuring and solving decision and planning problems involving multiple criteria.

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The purpose is to support decision makers facing such problems. Some of the multi criteria decision making methods are as follows: Analytical Hierarchy Process (AHP), Fuzzy Multi Criteria Decision Making Process, Elimination and Choice Expressing Reality (ELECTRE), Preference Ranking Organization Method for Enrichment of Evaluations, The TOPSIS Method. The subjective characteristics of the alternatives are generally uncertain and need to be evaluated based on the decision maker's insufficient knowledge and judgments [9]. The nature of this vagueness and uncertainty is fuzzy, rather than random, especially when subjective assessments are involved in the decision making process. Fuzzy set theory offers a possibility for handing these sorts of data and information involving the subjective characteristics of human nature in the decision making process. Since the theory of fuzzy sets was proposed in 1965, it has been used for handling fuzzy decision making problems. Bellman and Zadeh the firstly introduced decision making in a fuzzy environment. Kickert pointed out that the multicriteria decision making problem is a kind of problem that all the alternatives in the choice set can be evaluated according to a number of criteria; he also pointed out that the problem is to construct an evaluation procedure to rank the set of alternatives in order of preference. Kickert has discussed the field of fuzzy multicriteria decision making [5]. Multi criteria fuzzy decision making has been one of the fastest growing area during the last decades on account of its practicality. In multi criteria decision making problems, usually we must choose the optimal alternative from multiple alternatives according to some criteria. Because of the complex and ambiguity based on the nature of the problems, the problem of research in multi criteria decision making given in the form of interval criterion values becomes an attention [7]. Applications of multi criteria decision making problem have increased in intuitionistic fuzzy set. Multi criteria fuzzy decision making methods based on intuitionistic fuzzy sets were studied in Li(2005), Lin, Yuan and Xia (2007), Liu and Wang(2007) and Xu(2007). Szmidt et al. provided a solution to a multicriteria decision making problem by using similarity measures for IFSs [8]. Later, Szmidt et al. proposed a new method of IFSs which takes into account not only the amount of information related to an alternative (expressed by a distance from an ideal positive alternative) but also the reliability of information represented by an alternative meant as how sure the information is and Szmidt et al. presented some of the extended decision making are presented. Many researcher have introduced this field: Liu, Wang, Chen, Ye, Zhang, Xu, etc.

In this paper, an application of multi criteria decision making has been introduced in success ranking of middle schools using similarity measures in intuitionistic fuzzy sets. For this paper; middle schools in Kahramanmaras city in Turkey have been researched. Each middle school point has been calculated depending on average of student examination score (over 100 marks total). Intuitionistic fuzzy sets have been used as a tool since it incorporates the membership degree(the marks of the questions that have been correctly answered by the student, the non-membership degree (the marks of the questions that have been wrongly answered by the student) and the hesitation degree (the marks of the questions that are free from any answer). In this research; official data has been utilized that were obtained from the Ministry of Education and student examination score have been researched for $2016-2017$ academic year.

In many countries, many institutions make decisions based on a single criterion in the selection of staff. But a single criterion may not always give accurate results. This application could be used in situations that are not dependent on a single criterion. The options are middle schools in this paper. Criteria that determine the success of middle school are lessons. The criteria in this study have been determined as the basic lessons in middle school. Average of student examination score have been determined as criteria. Each criterion represents a lesson. Lessons are Turkish, Mathematics, Science, Social, English, Religion. The aim of this paper is to determination success ranking of middle school according to these criteria. Also the most successful school has been determined among these ranked schools. Multi criteria decision making has many application areas. For this paper have been benefitted from similarity measures for intuitionistic fuzzy sets that proposed new solution by Szmidt and Kacprzyk [8]. The advantage of this method; options have been compared to the positive-ideal solution and negative-ideal solution. The best considered option should be as close as possible to the positive-ideal solution and as far as possible to the negative-ideal solution. The best option taking into account only positive-ideal solution can be misleading.

2. PRELIMINARIES

Definition 2.1 ([10]). Let X be a nonempty set. A fuzzy set A drawn from X is defined as $A = \{\langle x, \mu_A(x)\rangle |\ x \in X\}$, where $\mu_A(x) : X \to [0, 1]$ is the membership function of the fuzzy set A.

Definition 2.2 ([1]). Let X be a nonempty set. An intuitionistic fuzzy set A in X is an object having the form

$$A = \{\langle x, \mu_A(x), \nu_A(x)\rangle |\ x \in X\},$$

where the function

$$\mu_A(x), \nu_A(x) : X \to [0, 1]$$

define respectively, the degree of membership and degree of nonmembership of the element $x \in X$, to the set A, which is a subset of X, and for every element $x \in X$,

$$0 \leq \mu_A(x) + \nu_A(x) \leq 1.$$

According to Fuzzy Set Theory, if the membership degree of an element x is $\mu(x)$, if the nonmembership degree of an element x is $1 - \mu(x)$

Furthermore, we have

$$\pi_A(x) = 1 - \mu_A(x) - \nu_A(x)$$

called the intuitionistic fuzzy set index or hesitation on margin of x in A. $\pi_A(x)$ is degree of indeterminacy of $x \in X$ to the IFS A and $\pi_A(x) \in [0, 1]$ i.e.,

$$\pi_A : X \to [0, 1]$$

for every $x \in X$. $\pi_A(x)$ expresses the lack of knowledge of whether x belongs to IFS A or not.

A set M of options fulfilling a set of criteria C;

Definition 2.3 ([4]). M is a set of options and C is a set of criteria

$$M = \{M_1, M_2, ..., M_m\}, C = \{C_1, C_2, ..., C_n\}$$

where each option M_i is expressed via intuitionistic fuzzy description, namely

$$M_i = \{(C_1, \mu_{i1}, \nu_{i1}), (C_2, \mu_{i2}, \nu_{i2}), ..., (C_n, \mu_{in}, \nu_{in})\}, i = 1, 2, ..., m$$

where μ_{ij} indicates the degree to which option M_i satisfies criterion C_j, ν_{ij} indicates the degree to which option M_i does not satisfy criterion C_j. Our goal is to point out the best option (to rank the considered options). The options should satisfy the criteria $C_j, C_k, ...,$ and C_p or criterion C_s, i.e.:

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$$(C_j \ \ and \ \ C_k \ \ and, ..., and \ \ C_p) \ \ or \ \ C_s \tag{2.1}$$

Szmidt and Kacprzyk proposed a general concept of a similarity measure for two elements of an intuitionistic fuzzy set (or sets). They give a modified measure of similarity that is meant for any criterion C_i and element A. Element $A(\mu_A = 1, \nu_A = 0$ and $\pi_A = 0)$ is their reference element representing the positive-ideal solution, i.e., a criterion which is fully satisfied $(\mu_A = 1)$. The proposed measure indicates if a criterion C_i is more similar to A (representing the positive-ideal solution, i.e. a fully satisfied criterion) or to $B(\mu_B = 0, \nu_B = 1$ and $\pi_B = 0)$ representing the negative-ideal solution, i.e. a fully dissatisfied criterion. In other words, it may indicate if the criterion considered is more satisfied or more dissatisfied.

Definition 2.4 ([8]).

$$Sim(C_i, A) = \frac{l_{IFS}(C_i, A)}{l_{IFS}(C_i, B)} \tag{2.2}$$

where: $l_{IFS}(C_i, A)$ is a distance from $C_i(\mu_{C_i}, \nu_{C_i}, \pi_{C_i})$ to $A(1, 0, 0)$, $l_{IFS}(C_i, B)$ is a distance from $C_i(\mu_{C_i}, \nu_{C_i}, \pi_{C_i})$ to $B(0, 1, 0)$. The distances $l_{IFS}(C_i, A)$ and $l_{IFS}(C_i, B)$ are calculated from

$$l_{IFS}(C_i, A) = \frac{1}{2}\sum_{i=1}^{n}(|1 - \mu_{C_i}| + |0 - \nu_{C_i}| + |0 - \pi_{C_i}|)$$

$$l_{IFS}(C_i, B) = \frac{1}{2}\sum_{i=1}^{n}(|0 - \mu_{C_i}| + |1 - \nu_{C_i}| + |0 - \pi_{C_i}|)$$

For, $0 \leq Sim(C_i, A) \leq \infty$.

The problem of finding an option M_i satisfying in the best way condition 2.1 can be solved by evaluating each option M_i

$$E(M_i) = Sim(A, M_i) = \min\{\max[Sim(A, C_j), Sim(A, C_k), ... \tag{2.3}$$
$$..., Sim(A, C_p)], Sim(A, C_s)\}$$

Condition (2.3) means that for each M_i we look for the worst satisfied criterion W_i among $C_j, C_k, ...,$ and C_p and next- we look for the better criterion between W_i and C_s. The worst means the least similar and the least similar and the best means the most similar.
The smallest value among $E(M_i), i = 1, ..., m$ (2.3) points out the option which best satisfies condition (2.1).

3. Application of Success Ranking with Similarity Measure

$M = \{M_1, M_2, M_3, M_4, M_5, M_6, M_7, M_8, M_9, M_{10}\}$ be set of middle schools.
$C = \{C_1, C_2, C_3, C_4, C_5, C_6\}$ be set of criteria. Set of criteria respectively are
$C = \{Science, Mathematics, Turkish, Social, Religion, English\}$. The criteria for middle schools are basic lessons. Options and criteria are the following:

	C_1	C_2	C_3	C_4	C_5	C_6
M_1	(0.78,0.198,0.022)	(0.55,0.405,0.045)	(0.59,0.369,0.041)	(0.67,0.297,0.033)	(0.76,0.216,0.024)	(0.63,0.333,0.037)
M_2	(0.98,0.018,0.002)	(0.89,0.099,0.011)	(0.87,0.117,0.013)	(0.95,0.045,0.005)	(0.95,0.045,0.005)	(0.97,0.027,0.003)
M_3	(0.65,0.315,0.035)	(0.559,0.3969,0.0441)	(0.752,0.2232,0.0248)	(0.745,0.2295,0.0255)	(0.805,0.1755,0.0195)	(0.636,0.3276,0.0364)
M_4	(0.83,0.153,0.017)	(0.67,0.297,0.033)	(0.69,0.279,0.031)	(0.77,0.207,0.023)	(0.82,0.162,0.018)	(0.68,0.288,0.032)
M_5	(0.74,0.234,0.026)	(0.63,0.333,0.037)	(0.81,0.171,0.019)	(0.65,0.315,0.035)	(0.88,0.108,0.012)	(0.79,0.189,0.021)
M_6	(0.6,0.36,0.04)	(0.4,0.54,0.06)	(0.5,0.45,0.05)	(0.55,0.405,0.045)	(0.71,0.261,0.029)	(0.5,0.45,0.05)
M_7	(0.65,0.315,0.035)	(0.46,0.486,0.054)	(0.49,0.459,0.051)	(0.59,0.369,0.041)	(0.69,0.279,0.031)	(0.45,0.495,0.055)
M_8	(0.69,0.279,0.031)	(0.45,0.505,0.055)	(0.49,0.459,0.051)	(0.52,0.432,0.048)	(0.66,0.306,0.034)	(0.46,0.486,0.054)
M_9	(0.5,0.45,0.05)	(0.38,0.558,0.062)	(0.41,0.531,0.059)	(0.5,0.45,0.05)	(0.59,0.369,0.041)	(0.32,0.612,0.068)
M_{10}	(0.36,0.576,0.064)	(0.32,0.612,0.068)	(0.37,0.567,0.063)	(0.44,0.504,0.056)	(0.57,0.387,0.043)	(0.33,0603,0.067)

From(2.3),(2.2); calculations for M_1 are as follows:

M_1	$Sim(A, C_1) = 0.274$	$Sim(A, C_2) = 0.756$
	$Sim(A, C_3) = 0.649$	$Sim(A, C_4) = 0.469$
	$Sim(A, C_5) = 0.306$	$Sim(A, C_6) = 0.554$

$$E(M_1) = \min[0.756, 0.306] = 0.306$$

Calculations for M_2 are as follows:

M_2	$Sim(A, C_1) = 0.0203$	$Sim(A, C_2) = 0.122$
	$Sim(A, C_3) = 0.147$	$Sim(A, C_4) = 0.052$
	$Sim(A, C_5) = 0.052$	$Sim(A, C_6) = 0.0308$

$$E(M_2) = \min[0.147, 0.052] = 0.052$$

Calculations for M_3 are as follows:

M_3	$Sim(A, C_1) = 0.5109$	$Sim(A, C_2) = 0.7312$
	$Sim(A, C_3) = 0.3192$	$Sim(A, C_4) = 0.3309$
	$Sim(A, C_5) = 0.2365$	$Sim(A, C_6) = 0.5413$

$$E(M_3) = \min[0.7312, 0.2365] = 0.2365$$

Calculations for M_4 are as follows:

M_4	$Sim(A, C_1) = 0.2$	$Sim(A, C_2) = 0.469$
	$Sim(A, C_3) = 0.429$	$Sim(A, C_4) = 0.29$
	$Sim(A, C_5) = 0.214$	$Sim(A, C_6) = 0.449$

$$E(M_4) = \min[0.214, 0.469] = 0.214$$

Calculations for M_5 are as follows:

M_5	$Sim(A,C_1) = 0.339$	$Sim(A,C_2) = 0.554$
	$Sim(A,C_3) = 0.229$	$Sim(A,C_4) = 0.51$
	$Sim(A,C_5) = 0.134$	$Sim(A,C_6) = 0.258$

$$E(M_5) = \min[0.134, 0.554] = 0.134$$

Calculations for M_6 are as follows:

M_6	$Sim(A,C_1) = 0.625$	$Sim(A,C_2) = 1.304$
	$Sim(A,C_3) = 0.909$	$Sim(A,C_4) = 0.756$
	$Sim(A,C_5) = 0.392$	$Sim(A,C_6) = 0.909$

$$E(M_6) = \min[0.392, 1.304] = 0.392$$

Calculations for M_7 are as follows:

M_7	$Sim(A,C_1) = 0.51$	$Sim(A,C_2) = 1.0505$
	$Sim(A,C_3) = 0.942$	$Sim(A,C_4) = 0.649$
	$Sim(A,C_5) = 0.429$	$Sim(A,C_6) = 1.089$

$$E(M_7) = \min[0.429, 1.089] = 0.429$$

Calculations for M_8 are as follows:

M_8	$Sim(A,C_1) = 0.429$	$Sim(A,C_2) = 1.11$
	$Sim(A,C_3) = 0.942$	$Sim(A,C_4) = 0.845$
	$Sim(A,C_5) = 0.489$	$Sim(A,C_6) = 1.0505$

$$E(M_8) = \min[0.489, 1.11] = 0.489$$

Calculations for M_9 are as follows:

M_9	$Sim(A,C_1) = 0.909$	$Sim(A,C_2) = 1.402$
	$Sim(A,C_3) = 1.257$	$Sim(A,C_4) = 0.909$
	$Sim(A,C_5) = 0.649$	$Sim(A,C_6) = 1.752$

$$E(M_9) = \min[0.649, 1.752] = 0.649$$

Calculations for M_{10} are as follows:

M_{10}	$Sim(A,C_1) = 1.064$	$Sim(A,C_2) = 1.752$
	$Sim(A,C_3) = 1.454$	$Sim(A,C_4) = 1.129$
	$Sim(A,C_5) = 0.701$	$Sim(A,C_6) = 1.687$

$$E(M_{10}) = \min[0.701, 1.752] = 0.701$$

The smallest value among $E(M_i)$ points out the option which best satisfies condition. When the results of calculations are compared; the ranking of the options: $M_2, M_5, M_4, M_3, M_1, M_6, M_7, M_8, M_9, M_{10}$. M_2 is the best option among $M_1 - M_{10}$. Then according to the above calculations when success rankings are made between high school according to the above calculations, the most successful middle school is M_2. Also success rankings of middle schools:$M_2, M_5, M_4, M_3, M_1, M_6, M_7, M_8, M_9, M_{10}$.

4. Conclusion and Suggestions

For this paper have been benefitted from similarity measures for intuitionistic fuzzy sets that proposed new solution by Szmidt and Kacprzyk [8]. The advantage of this method; options have been compared to the positive-ideal solution and negative-ideal solution. The best considered option should be as close as possible to the positive-ideal solution and as far as possible to the negative-ideal solution. The best option taking into account only positive-ideal solution can be misleading. In this paper; success ranking of middle schools has been done by multi criteria method. Also the most successful school has been determined among these ranked schools. This method is suitable in order to achieve more sensible results. Applications could be made in different areas with this method. In many countries, many institutions make decisions based on a single criterion in the selection of staff. But a single criterion may not always give accurate results. This application could be used in situations that are not dependent on a single criterion.

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