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Modeling of daily maximum and minimum temperature changes in Bitlis province using Copula Method

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Abstract

This paper aims to examine the relationship between daily maximum and minimum temperatures of Bitlis in Turkey between 2012-2017 years with Copula method. To present the relationship between the variables, we use copula families such as; Gumbel, Clayton, Frank, Joe, Gaussian and Survival Clayton copula. To explain dependence structures of the data set and to determine parameters of Gumbel, Clayton, Frank, Joe, Gaussian and Survival Clayton copula families, we calculate Kendall Tau and Spearman Rho values which are nonparametric. With the help of Kolmogorov Smirnov, Cramer Von Mises which are goodness of fit test, Maximum likelihood method, Akaike information Criteria ad Bayes information criteria, we find the suitable copula family for this data set. The results show that there is a strong dependence between daily maximum and minimum temperatures of Bitlis between 2012-2017 years.

Keywords: Copula functions, Kendall Tau, Spearman Rho, Goodness of fit test, Akaike information criteria, Bayes information criteria.

Copula Metodu Kullanarak Bitlis İlindeki Günlük Maksimum ve Minumum Sıcaklık Değişimlerinin Modellenmesi

Özet

Bu makalenin amacı Bitlis'in 2012-2017 yılları arasındaki günlük maksimum ve minumum sıcaklıkları arasındaki ilişkiyi Copula methodu ile açıklamaktır. İlişkiyi açıklamak için çeşitli copula aileleri kullanılmıştır. Bunlar; Gumbel, Clayton, Frank, Joe, Gaussian ve Survival Claytondur. Bağımlılık yapısını açıklamak ve Gumbel, Clayton, Frank, Joe, Gaussian ve Survival Clayton copula ailelerinin parametrelerini belirlemek için parametrik olmayan metod olan Kendall Tau ve Spearman Rho değerleri hesaplanmıştır. Uyum iyiliği testleri Kolmogorov Smirnov, Cramer Von Mises, maksimum olabilirlik metodu, Akaike bilgi kriteri ve Bayes bilgi kriteri yardımıyla veri seti için uygun copula ailesi bulunmuştur. Sonuçlar Bitlis'in 2012 yılı ile 2017 yılları arasında günlük maksimum ve minumum sıcaklık değişimleri için güçlü bir bağımlılık olduğunu göstermiştir.

Anahtar Kelimeler: Copula Fonksiyonları, Kendall Tau, Spearman Rho, Uyum iyiliği Testi, Akaike bilgi kriteri, Bayes bilgi kriteri.

1. Introduction

The daily movements of the world are also influential in the formation of temperature differences. Horton [1] and Easterling, et. al [2] said that daily temperature differences are global and regional. Due to the temperature differences between night and day, weather events come to the fore and live life is also affected. Because of the daily temperature differences, weather events are usually characterized by the characteristics of the water that undergoes a change of state. It is the climate element that closely controls temperature, geographical conditions and other atmospheric phenomena. It determines the intensity and distribution of the influence of the outer climate elements. Temperature also has a direct

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effect on human life. It affects people's living spaces, their settlement, their cultural, social and economic activities, nutrition, dressing and warming.

Climate elements are called atmospheric phenomena such as temperature, pressure, wind, humidity, precipitation and cloudiness. Climate characteristics that are effective in a region are under the control of climate elements. Temperature has the greatest influence in climate elements. Temperature controls climate elements, determines their distribution and activities. Climate elements must be thoroughly examined to determine the climate that is effective in any location. For this, arithmetic averages of the values obtained by observation of daily atmospheric events are taken. Factors that cause surface temperature distribution are; glancing angle of solar rays, hours of sunshine, elevation, moisture, ocean currents, winds, land and sea distribution and plant cover.

Copulas are multivariate uniform distributions. They represent a way of trying to extract the dependence structure from the joint distribution function and to separate dependence and marginal behavior. Copulas represent a useful approach to understanding and modeling dependent random variables. The copula theory is slightly new to meteorology and hydro climatology but has already established itself to be highly potential in frequency analysis, multivariate modeling, simulation and prediction. They allow us to focus explicitly on the dependence structure. Sklar [3] formed structure and properties of a copulas their connection with random variables. Genest and Mackay [4] showed how copulas can be used the existence of distribution with singular components and to give a relation with Kendall Tau. Genest and Rivest [5] proposed the problem of selecting Archimedean copula providing suitable representation of the dependence structure between two variables. Ana Tustel et. al. [6] obtained distribution-free multivariate Kolmogorov –Smirnov goodness of fit test. Nelsen [7] examined copula and properties of copula theory. Bouye et. al. [8] were used extensively copulas in finance. Frey et. al. [9] modelled credit portfolio loses with copula. Silvapulle and Paramsoty [10] worked ML and IFM methods compared with SP method. Genest and Favre [11] worked to inference for copulas based on rank methods and Salvadori et. al. [12] used copula to model extremes in nature. Maity, R. [13] was given the background theory and look for application of copula theory and example of MATLAB codes. In this study, we are selected suitable copula function for temperature measurement data set that is daily maximum and minimum temperatures of Bitlis between 2012-2017 years. To explain the relationship between the variables copula families were used; Gumbel, Clayton, Frank, Joe, Gaussian and Survival Clayton copula families. With the help of nonparametric estimation of copula parameters, Kolmogorov Smirnov and Cramer Von Mises which are goodness of fit test, Maximum likelihood method and Akaike information Criteria, Bayes information criteria, we find the suitable Archimedean copula family for this data set.

2. Materials and Methods

2.1. Copula Theory

The copula is defined by a $C:[0,1]^2 \rightarrow [0,1]$ which holds the following conditions

- ✓ $C(u,0)=C(0,u)=0$ and $C(u,1)=C(1,u)=u, \forall u \in [0,1]$.
- ✓ $(u_1,u_2,v_1,v_2) \in [0,1]^4$, such that $u_1 \le u_2, v_1 \le v_2$
  $C(u_2,v_2)-C(u_2,v_1)-C(u_1,v_2)+C(u_1,v_1) \ge 0.$

Ultimately, for twice differentiable and 2-increasing property can be replaced by the condition

$$c(u,v)=\frac{\partial^2 C(u,v)}{\partial u \partial v} \ge 0 \tag{1}$$

where $c(u,v)$ is the copula density. Accordingly, C is defined by

$$C(u_1,u_2,\ldots,u_n,\theta)=P(U_1 \le u_1, U_2 \le u_2, \ldots U_n \le u_n).$$

Here θ is dependence parameter [3, 4, 5, 8, 9, 10, 13, 14, 15, 17, 18, 19, 20, 21].

The copula C for (X,Y) is the joint distribution function for the pair $F_X(X)$, $F_Y(Y)$ provided F_X and F_Y continuous.

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The joint probability density of the variables X and Y is obtained from the copula density $(u,v)=\frac{\partial^2 C(u,v)}{\partial u \partial v}$, as follows:

$$f_{xy}(x,y)=c(u,v)\,f_x(\mathrm{x})\,f_y(y), \tag{2}$$

where $f_x(\mathrm{x})$ and $f_y(y)$ are the marginal densities of the random variables X and Y. According to Sklar [22] an n-dimensional joint distribution can be decomposed into its n-univariate marginal distributions and an n-dimensional copula. In the extension of Sklar's theorem to continuous conditional distributions, Patton [23] shows that the lower (left) and upper (right) tail dependence of two random variables is given for the copula as:

$$\lambda_l = \lim_{u\to 0} P(\mathrm{F}_X(x)\le u \mid F_Y(x)\le u) = \lim_{u\to 0} C(u,u)/u \tag{3}$$

$$\lambda_u = \lim_{u\to 1} P(\mathrm{F}_X(x) > \mathrm{u} \mid F_Y(x) > u) = \lim_{u\to 1} 1-2u-C(u,u)/1-u \tag{4}$$

where λ_l and $\lambda_u \in [0,1]$ [7].

2.1.1. Sklar Theorem

Let X and Y be random variables with continuous distribution functions F_X and F_Y, which are uniformly distributed on the interval $[0,1]$. Then, there is a copula such that for all $x,y\in R$

$$F_{XY}(X,Y)=C(F_X(X),F_Y(Y)\ . \tag{5}$$

The copula C for (X,Y) is the joint distribution function for the pair $F_X(X)$, $F_Y(Y)$ provided F_X and F_Y continuous [1, 2, 3, 5, 6, 7, 8, 9, 13, 14, 15, 17,18].

2.1.2. Gaussian Copula

The copula function can be written as;

$$C(u,v;\rho)=\int_{-\infty}^{\Phi^{-1}(u)}\int_{-\infty}^{\Phi^{-1}(v)} \frac{1}{2\pi\sqrt{1-\rho^2}} \exp\left(\frac{2\rho rs - r^2 - s^2}{2(1-\rho^2)}\right) drds \tag{6}$$

where $u=F_{Y_1}(\mathrm{y}_1)$, $v=F_{Y_2}(\mathrm{y}_2)$ is the inverse of the standart normal distribution and ρ is the general correlation coefficient [7].

2.1.3. Archimedean Copula

Let φ defines a function $\phi:[0,1]\rightarrow[0,\infty]$ which is continuous and provides the following conditions;

- ✓ $\phi(1)=0,\ \phi(0)=\infty.$
- ✓ For all $t\in(0,1)$, $\phi^{'}(t)<0$, φ is decreasing, for all $t\in(0,1)$ $\varphi''(t)\geq 0$, φ is convex.

φ has an inverse $\varphi^{-1}:[0,\infty]\rightarrow[0,1]$ which has the same properties out of $\phi^{(-1)}(0)=1$ and $\phi^{(-1)}(\infty)=0.$ The Archimedean Copula is defined by

$$C(u,v)=\phi^{(-1)}[\phi(u)+\phi(v)]. \tag{7}$$

[12].

2.1.4. Gumbel Copula

This Archimedean copula is defined with the help of generator function $\phi(t)=(-lnt)^{\theta}$, $\theta\geq 1$;

$$C_{\theta}(u,v)=\exp\left(-[(-\ln u)^{\theta}+(-\ln v)^{\theta}]^{1/\theta}\right) \tag{8}$$

where θ is the copula parameter restricted to $[1,\infty)$ [7].

2.1.5. Clayton Copula

This Archimedean copula is defined with the help of generator function $\phi(t)=\dfrac{t^{-\theta}-1}{\theta}$,

$$C_{\theta}(u,v)=(u^{-\theta}+v^{-\theta}-1). \tag{9}$$

where θ is the copula parameter restricted to $(0,\infty)$ [7].

2.1.6. Frank Copula

This Archimedean copula is defined with the help of generator function; $\phi(t)=-\ln\dfrac{-e^{-\theta t}-1}{e^{-\theta}-1}$;

$$C_{\theta}(u,v)=-\frac{1}{\theta}\ln\left(1+\frac{\left(e^{-\theta u}-1\right)\left(e^{-\theta v}-1\right)}{\left(e^{-\theta}-1\right)}\right) \tag{10}$$

where θ is the copula parameter restricted to $(0,\infty)$ [7].

2.1.7. Joe Copula

This Archimedean copula is defined with the help of generator function; $\varphi(t)=-\ln\left[1-(1-t)^{\theta}\right]$

$$C_{\theta}(u,v)=1-\left[(1-u)^{\theta}+(1-v)^{\theta}-((1-u)^{\theta}(1-v)^{\theta}\right]^{1/\theta} \tag{11}$$

where θ is the copula parameter restricted to $[1,\infty]$ [7].

2.1.8. Survival Clayton Copula

A copula is survival Clayton copula;

$$C_{\theta}(u,v)=u+v-1+[(1-u)^{-\theta}+(1-v)^{-\theta}-1]^{-1/\theta} \tag{12}$$

and $\lambda_u=2^{-1/\theta}\lambda_l$ where λ_u is the upper tail dependence, λ_l is the lower tail dependence [7].

3. Results and Discussion

3.1. Data

In this study, we used data set daily maximum and minimum temperatures of Bitlis between 2012-2017 years. This data set was obtained from Elazığ meteorology directorate. There are 1828 observations. Table 1 summarizes statistics of series. For daily maximum and minimum temperatures of Bitlis, change interval of 1828 observations shown in Figure 1. In Table 1 Mean values of the data are different from each other and the corresponding standard deviations are fairly different. Skewness of the maximum and minimum temperatures is negative. It is indicated that the maximum and minimum temperatures are skewed left The kurtosis of the maximum and minimum temperatures is close. The Jarque-Bera (JB) test indicates that the normality of each return series distribution is strongly rejected at 0.05 level, which means all the two distributions are non-normal.

Table 1. Descriptive statistics of Minimum and Maximum temperatures

	Minimum temperatures	Maximum temperatures
Mean	3,785271	13,87482
Median	4,600000	14,30000
Maximum	21,40000	33,10000
Minimum	-24,10000	-15,90000
Std.dev.	9,323656	11,25311
Skewness	-0,342304	-0,056681
Kurtosis	2,354110	1,807151
Jarque-Bera	67,43626	109,2957
Probability	0,0000	0,0000

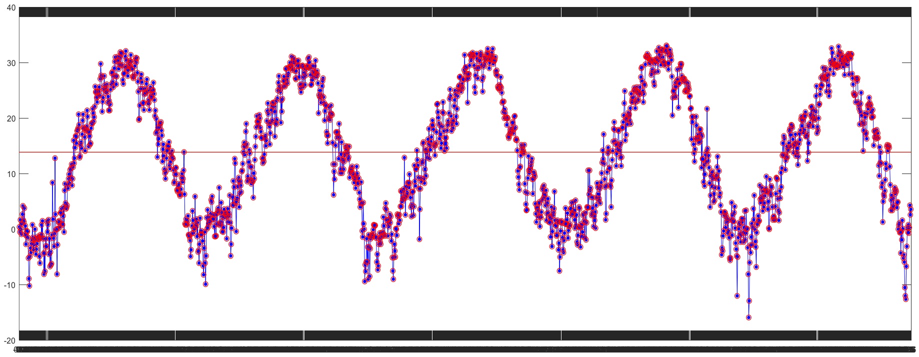

Figure 1. Change Graph Daily maximum temperatures of Bitlis

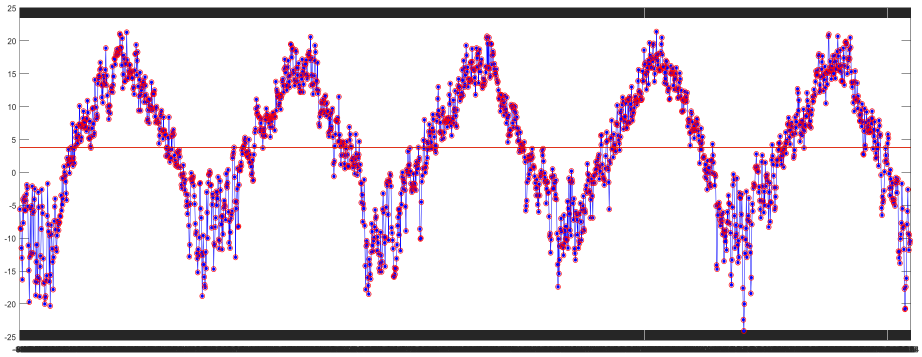

Figure 2. Change Graph Daily minimum temperatures of Bitlis

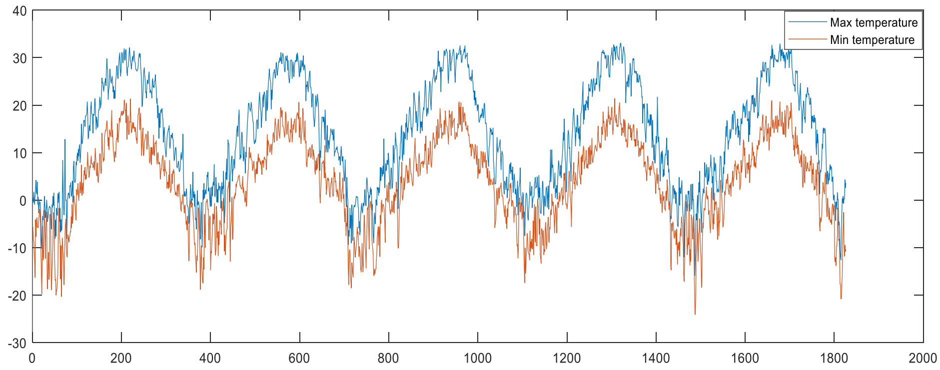

Figure 3. Change Graph Daily maximum and minimum temperatures of Bitlis

3.2. Copula modelling

In this paper, to model dependence, we present Gumbel, Clayton, Frank, Joe, Gaussian and Survival Clayton copula families. For this series, it has been found that Kendall Tau (τ) and Spearman Rho (ρ) are 0,825 and 0,962 respectively. Hence, it is seen that there is strong relationship between maximum and minumum temperatures. We give the emprical distrubution of maximum and minumum temperatures in Figure 4. Maximum Likelihood Estimation method is used for application for estimation of copula parameters. Accordingly, in Table 2 for copula families, parameter values, goodness of fit test test statistics and Logl**,** AIC, BIC values are calculated. According to the goodness of fit test statistics, we conclude that dependence structure of minimum temperatures and maximum temperatures series is modeled by Clayton, Gumbel, Frank, Gaussian and Survival Clayton copula. In the Table 2, take into account the AIC and BIC value, it is obvious that the Frank copula best fit for the pairs maximum and minumum temperatures. From Table 2 and Figure 3, the obtained tail dependence values for the pairs maximum and minumum temperatures when λ_l =0, λ_u = 0, symmetric tail dependency is observed in the tail of these pairs.

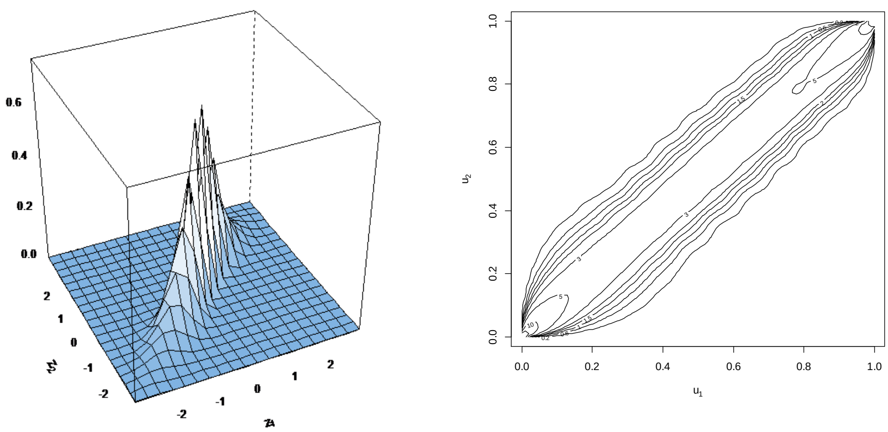

Figure 4. The maximum and minimum temperatures of Bitlis emprical distrubution function

Table 2. Copula Modelling

	θ	**Logl**	**AIC**	**BIC**	λ_u	λ_l	**KSc**	**CvMc**
Clayton	9.28	730.6	-1459.2	-1453.69	0	0.93	4.058312 p=0.000	5.803535 p=0.000
Gumbel	5.64	1649.83	-3297.66	-3292.15	0.87	0	2.44345 p=0.000	1.23469 p=0.000
Frank	20.76	2156.2	-4310.4	-4304.89	0	0	0.879581 p=0.0000	0.151211 p=0.0000
Joe	10.04	732.16	-1462.31	-1456.8	0.93	0	2.947484 P=0.98	3.185518 p=0.98
Gaussian	0.96	1851.01	-3700.01	-3694.5	0	0	1.638694 p=0.000	0.7201704 p=0.000
Survival Clayton	9.28	708.17	-1414.34	-1408.83	0.93	0	4.190344 p=0.000	4.98722 p=0.000

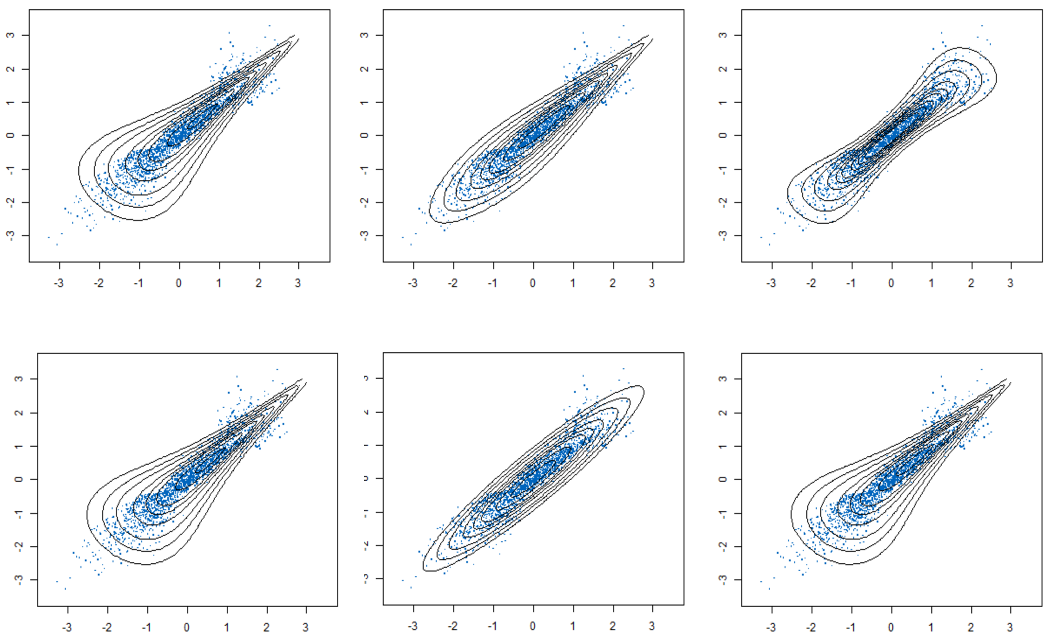

Figure 5. For maximum and minimum temperatures of Bitlis pairs Clayton, Gumbel, Frank, Joe, Gaussian and Survival Clayton copula scatter graph, respectively

4. Conclusions

In this paper, to model relationship between daily maximum and minimum temperatures of Bitlis between 2012-2017 years, it is used that copula function is nonparametric method. According to the results obtained, relationship between daily maximum and minimum temperatures of Bitlis between 2012-2017 years is modelled Frank copula (θ = 20.76). Looking at the tails of this pair, the Frank copula has zero tail dependence, so daily maximum and minimum temperatures of Bitlis pair has symmetric tail dependence. From Figure 5, it is shown that scatter is symmetric in tail. Under normal conditions, the night and day temperature difference is thought to be high. From this study, relationship between daily maximum and minimum temperatures of Bitlis is positive and strong. So, if the temperature is high during the day, it is high in the night and low during the day and low at night.

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