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The Form of the Solutions of System of Rational Difference Equation

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Abstract

In this article, we study the form of the solutions of the system of difference equations $x_{n+1} = ((y_{n-8})/(1+y_{n-2}x_{n-5}y_{n-8}))$, $y_{n+1} = ((x_{n-8})/(1x_{n-2}y_{n-5}x_{n-8}))$, with the initial conditions are real numbers. Also, we give the numerical examples of some of difference equations and got some related graphs and figures using by Matlab.

1. Introduction

Our aim in this studying to get the techniques of solutions of the system of rational difference equations

$$x_{n+1} = \frac{y_{n-8}}{1+y_{n-2}x_{n-5}y_{n-8}}, \qquad y_{n+1} = \frac{x_{n-8}}{\pm 1 \pm x_{n-2}y_{n-5}x_{n-8}},$$

with real number's initial conditions $x_{-8}, x_{-7}, x_{-6}, x_{-5}, x_{-4}, x_{-3}, x_{-2}, x_{-1}, x_0, y_{-8}, y_{-7}, y_{-6}, y_{-5}, y_{-4}, y_{-3}, y_{-2}, y_{-1}, y_0$.
Lately, difference equations appear as discrete analogues of discovered evolution because most analysis of time evolving variables are discrete. Also, there has been an increasing interest in the study of qualitative analysis of system of rational difference equations. Discrete systems can be described as operators acting on functions with countable domains. These functions are also called discrete functions or sequences. Although difference equations looks simple in form, but it is highly difficult to understand thoroughly the behaviors of their solutions, see [1]-[44] and the references cited therein. There are many papers with related to the difference equations system for example, Ahmed and Elsayed [1] has got the expressions of solutions of some rational difference equations systems

$$x_{n+1} = \frac{x_{n-1}y_{n-2}}{y_n(-1 \pm x_{n-1}y_{n-2})}, \quad y_{n+1} = \frac{y_{n-1}x_{n-2}}{x_n(\pm 1 \pm y_{n-1}x_{n-2})}.$$

Din investigated the boundedness character, the local asymptotic stability of equilibrium points and global of the unique positive equilibrium point of a discrete perdator-pery model given by

$$x_{n+1} = \frac{\alpha x_n - \beta x_n y_n}{1+\gamma x_n}, \quad y_{n+1} = \frac{\delta x_n y_n}{x_n + \eta y_n}.$$

El-Dessoky [2] obtained the solutions and periodicity for some systems of third-order rational difference equations

$$x_{n+1} = \frac{y_{n-1}y_{n-2}}{x_n(\pm 1 \pm y_{n-1}y_{n-2})}, \qquad y_{n+1} = \frac{x_{n-1}x_{n-2}}{y_n(\pm 1 \pm x_{n-1}x_{n-2})}.$$

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In [3], El-Dessoky and Elsayed studied the solution and periodic nature of some systems of rational difference equations

$$x_{n+1}=\frac{x_ny_{n-1}}{y_{n-1}\pm y_n},\qquad y_{n+1}=\frac{y_nx_{n-1}}{x_{n-1}\pm x_n}.$$

El-Dessoky et al. [4] obtained the rational system of difference equations

$$x_{n+1}=\frac{x_{n-3}y_{n-4}}{y_n\left(\pm1\pm x_{n-3}y_{n-4}\right)},\quad y_{n+1}=\frac{y_{n-3}x_{n-4}}{x_n\left(\pm1\pm y_{n-3}x_{n-4}\right)}.$$

Elsayed and Ibrahim [5] solved solutions for some systems of nonlinear rational difference equations

$$x_{n+1}=\frac{x_{n-2}y_{n-1}}{y_n(\pm1\pm x_{n-2}y_{n-1})},\qquad y_{n+1}=\frac{y_{n-2}x_{n-1}}{x_n(\pm1\pm y_{n-2}x_{n-1})}.$$

Elsayed and Alghamdi [6] solved the form of the solution of nonlinear difference equation systems

$$x_{n+1}=\frac{x_{n-7}}{1+x_{n-7}y_{n-3}},\quad y_{n+1}=\frac{y_{n-7}}{\pm1\pm y_{n-7}x_{n-3}}.$$

Haddad et al [7] obtained solution form of a higher-order system of difference equations and dynamical behavior of its special case

$$x_{n+1}=\frac{x^p_{n-k+1}y_n}{ay^p_{n-k}+by_n},\quad y_{n+1}=\frac{y^p_{n-k+1}x_n}{\alpha x^p_{n-k}+\beta x_n}.$$

In [8] Kurbanli studied the behavior of solutions of the following systems of difference equations

$$x_{n+1}=\frac{x_{n-1}}{y_nx_{n-1}-1},\quad y_{n+1}=\frac{y_{n-1}}{x_ny_{n-1}-1}.$$

Kurbanli et al. [9, 10] obtained the solutions of following problems

$$\begin{aligned} x_{n+1}&=\tfrac{x_{n-1}+y_n}{y_nx_{n-1}-1},\quad y_{n+1}=\tfrac{y_{n-1}+x_n}{x_ny_{n-1}-1}.\\ x_{n+1}&=\tfrac{x_{n-1}}{y_nx_{n-1}+1},\quad y_{n+1}=\tfrac{y_{n-1}}{x_ny_{n-1}+1}.\end{aligned}$$

Mansour et al. [11] investigated the solutions and periodicity of some system of difference equations

$$x_{n+1}=\frac{x_{n-5}}{-1+x_{n-5}y_{n-2}},\quad y_{n+1}=\frac{y_{n-5}}{\pm1\pm y_{n-5}x_{n-2}}.$$

Touafek and Elsayed [12] gave the solutions of following systems of difference equations

$$x_{n+1}=\frac{x_{n-3}}{\pm1\pm x_{n-3}y_{n-1}},\quad y_{n+1}=\frac{y_{n-3}}{\pm1\pm y_{n-3}x_{n-1}}.$$

Definition 1.1. *A sequence $\{x_n\}_{n=-k}^{\infty}$ is said to be periodic with period p if $x_{n+p}=x_n$ for all $n\geq -k$.*

2. The main results

2.1. The first system: $x_{n+1}=\frac{y_{n-8}}{1+y_{n-2}x_{n-5}y_{n-8}}, y_{n+1}=\frac{x_{n-8}}{1+x_{n-2}y_{n-5}x_{n-8}}$

In this part, we study the solutions of the system of difference equations

$$x_{n+1}=\frac{y_{n-8}}{1+y_{n-2}x_{n-5}y_{n-8}},y_{n+1}=\frac{x_{n-8}}{1+x_{n-2}y_{n-5}x_{n-8}},$$

with a real number‘s initial conditions.

Theorem 2.1. *Suppose that $x_{-8}=a$, $x_{-7}=b$, $x_{-6}=c$, $x_{-5}=d$, $x_{-4}=e$, $x_{-3}=f$, $x_{-2}=g$, $x_{-1}=h$, $x_0=k$, $y_{-8}=l$, $y_{-7}=m$, $y_{-6}=p$, $y_{-5}=q$, $y_{-4}=r$, $y_{-3}=s$, $y_{-2}=t$, $y_{-1}=u$, $y_0=v$ are arbitrary real numbers and let $\{x_n,y_n\}$ be solutions of the system 2.1. Then all solutions of 2.1 are given by*

$$x_{18n-8}=a\prod_{i=0}^{n-1}\frac{(1+(6i)agq)(1+(6i+3)agq)}{(1+(6i+1)agq)(1+(6i+4)agq)},$$

$$x_{18n-7}=b\prod_{i=0}^{n-1}\frac{(1+6ibhr)(1+(6i+3)bhr)}{(1+(6i+1)bhr)(1+(6i+4)bhr)},$$

$$x_{18n-6}=c\prod_{i=0}^{n-1}\frac{(1+6icks)(1+(6i+3)cks)}{(1+(6i+1)cks)(1+(6i+4)cks)},$$

$$x_{18n-5}=d\prod_{i=0}^{n-1}\frac{(1+(6i+1)dlt)(1+(6i+4)dlt)}{(1+(6i+2)dlt)(1+(6i+5)dlt)},$$

$$x_{18n-4}=e\prod_{i=0}^{n-1}\frac{(1+(6i+1)emu)(1+(6i+4)emu)}{(1+(6i+2)emu)(1+(6i+5)emu)},$$

$$x_{18n-3}=f\prod_{i=0}^{n-1}\frac{(1+(6i+1)fpv)(1+(6i+4)fpv)}{(1+(6i+2)fpv)(1+(6i+5)fpv)},$$

$$x_{18n-2}=g\prod_{i=0}^{n-1}\frac{(1+(6i+2)agq)(1+(6i+5)agq)}{(1+(6i+3)agq)(1+(6i+6)agq)},$$

$$x_{18n-1}=h\prod_{i=0}^{n-1}\frac{(1+(6i+2)bhr)(1+(6i+5)bhr)}{(1+(6i+3)bhr)(1+(6i+6)bhr)},$$

$$x_{18n}=k\prod_{i=0}^{n-1}\frac{(1+(6i+2)cks)(1+(6i+5)cks)}{(1+(6i+3)cks)(1+(6i+6)cks)},$$

$$x_{18n+1}=\frac{l}{1+dlt}\prod_{i=0}^{n-1}\frac{(1+(6i+3)dlt)(1+(6i+6)dlt)}{(1+(6i+4)dlt)(1+(6i+7)dlt)},$$

$$x_{18n+2}=\frac{m}{1+emu}\prod_{i=0}^{n-1}\frac{(1+(6i+3)emu)(1+(6i+6)emu)}{(1+(6i+4)emu)(1+(6i+7)emu)},$$

$$x_{18n+3}=\frac{p}{1+fpv}\prod_{i=0}^{n-1}\frac{(1+(6i+3)fpv)(1+(6i+6)fpv)}{(1+(6i+4)fpv)(1+(6i+7)fpv)},$$

$$x_{18n+4}=\frac{q(1+agq)}{(1+2agq)}\prod_{i=0}^{n-1}\frac{(1+(6i+4)agq)(1+(6i+7)agq)}{(1+(6i+5)agq)(1+(6i+8)agq)},$$

$$x_{18n+5}=\frac{r(1+bhr)}{(1+2bhr)}\prod_{i=0}^{n-1}\frac{(1+(6i+4)bhr)(1+(6i+7)bhr)}{(1+(6i+5)bhr)(1+(6i+8)bhr)},$$

$$x_{18n+6}=\frac{s(1+cks)}{(1+2cks)}\prod_{i=0}^{n-1}\frac{(1+(6i+4)cks)(1+(6i+7)cks)}{(1+(6i+5)cks)(1+(6i+8)cks)},$$

$$x_{18n+7}=\frac{t(1+2dlt)}{(1+3dlt)}\prod_{i=0}^{n-1}\frac{(1+(6i+5)dlt)(1+(6i+8)dlt)}{(1+(6i+6)dlt)(1+(6i+9)dlt)},$$

$$x_{18n+8}=\frac{u(1+2emu)}{(1+3emu)}\prod_{i=0}^{n-1}\frac{(1+(6i+5)emu)(1+(6i+8)emu)}{(1+(6i+6)emu)(1+(6i+9)emu)},$$

$$x_{18n+9}=\frac{v(1+2fpv)}{(1+3fpv)}\prod_{i=0}^{n-1}\frac{(1+(6i+5)fpv)(1+(6i+8)fpv)}{(1+(6i+6)fpv)(1+(6i+9)fpv)},$$

$$y_{18n-8}=l\prod_{i=0}^{n-1}\frac{(1+6idlt)(1+(6i+3)dlt)}{(1+(6i+1)dlt)(1+(6i+4)dlt)},$$

$$y_{18n-7}=m\prod_{i=0}^{n-1}\frac{(1+6iemu)(1+(6i+3)emu)}{(1+(6i+1)emu)(1+(6i+4)emu)},$$

$$y_{18n-6}=p\prod_{i=0}^{n-1}\frac{(1+6fpv)(1+(6i+3)fpv)}{(1+(6i+1)fpv)(1+(6i+4)fpv)},$$

$$y_{18n-5}=q\prod_{i=0}^{n-1}\frac{(1+(6i+1)agq)(1+(6i+4)agq)}{(1+(6i+2)agq)(1+(6i+5)agq)},$$

$$y_{18n-4}=r\prod_{i=0}^{n-1}\frac{(1+(6i+1)bhr)(1+(6i+4)bhr)}{(1+(6i+2)bhr)(1+(6i+5)bhr)},$$

$$y_{18n-3} = s\prod_{i=0}^{n-1}\frac{(1+(6i+1)cks)(1+(6i+4)cks)}{(1+(6i+2)cks)(1+(6i+5)cks)},$$

$$y_{18n-2} = t\prod_{i=0}^{n-1}\frac{(1+(6i+2)dlt)(1+(6i+5)dlt)}{(1+(6i+3)dlt)(1+(6i+6)dlt)},$$

$$y_{18n-1} = u\prod_{i=0}^{n-1}\frac{(1+(6i+2)emu)(1+(6i+5)emu)}{(1+(6i+3)emu)(1+(6i+6)emu)},$$

$$y_{18n} = v\prod_{i=0}^{n-1}\frac{(1+(6i+2)fpv)(1+(6i+5)fpv)}{(1+(6i+3)fpv)(1+(6i+6)fpv)},$$

$$y_{18n+1} = \frac{a}{1+agq}\prod_{i=0}^{n-1}\frac{(1+(6i+3)agq)(1+(6i+6)agq)}{(1+(6i+4)agq)(1+(6i+7)agq)},$$

$$y_{18n+2} = \frac{b}{1+bhr}\prod_{i=0}^{n-1}\frac{(1+(6i+3)bhr)(1+(6i+6)bhr)}{(1+(6i+4)bhr)(1+(6i+7)bhr)},$$

$$y_{18n+3} = \frac{c}{1+cks}\prod_{i=0}^{n-1}\frac{(1+(6i+3)cks)(1+(6i+6)cks)}{(1+(6i+4)cks)(1+(6i+7)cks)},$$

$$y_{18n+4} = \frac{d(1+dlt)}{(1+2dlt)}\prod_{i=0}^{n-1}\frac{(1+(6i+4)dlt)(1+(6i+7)dlt)}{(1+(6i+5)dlt)(1+(6i+8)dlt)},$$

$$y_{18n+5} = \frac{e(1+emu)}{(1+2emu)}\prod_{i=0}^{n-1}\frac{(1+(6i+4)emu)(1+(6i+7)emu)}{(1+(6i+5)emu)(1+(6i+8)emu)},$$

$$y_{18n+6} = \frac{f(1+fpv)}{(1+2fpv)}\prod_{i=0}^{n-1}\frac{(1+(6i+4)fpv)(1+(6i+7)fpv)}{(1+(6i+5)fpv)(1+(6i+8)fpv)},$$

$$y_{18n+7} = \frac{g(1+2agq)}{(1+3agq)}\prod_{i=0}^{n-1}\frac{(1+(6i+5)agq)(1+(6i+8)agq)}{(1+(6i+6)agq)(1+(6i+9)agq)},$$

$$y_{18n+8} = \frac{h(1+2bhr)}{(1+3bhr)}\prod_{i=0}^{n-1}\frac{(1+(6i+5)bhr)(1+(6i+8)bhr)}{(1+(6i+6)bhr)(1+(6i+9)bhr)},$$

$$y_{18n+9} = \frac{k(1+2cks)}{(1+3cks)}\prod_{i=0}^{n-1}\frac{(1+(6i+5)cks)(1+(6i+8)cks)}{(1+(6i+6)cks)(1+(6i+9)cks)}.$$

Proof. For $n=0$, the result holds. Now, assume that $n>0$ and that our assumption holds for $n-1$. That is,

$$x_{18n-17} = \frac{l}{1+dlt}\prod_{i=0}^{n-2}\frac{(1+(6i+3)dlt)(1+(6i+6)dlt)}{(1+(6i+4)dlt)(1+(6i+7)dlt)},$$

$$x_{18n-16} = \frac{m}{1+emu}\prod_{i=0}^{n-2}\frac{(1+(6i+3)emu)(1+(6i+6)emu)}{(1+(6i+4)emu)(1+(6i+7)emu)},$$

$$x_{18n-15} = \frac{p}{1+fpv}\prod_{i=0}^{n-2}\frac{(1+(6i+3)fpv)(1+(6i+6)fpv)}{(1+(6i+4)fpv)(1+(6i+7)fpv)},$$

$$x_{18n-14} = \frac{q(1+agq)}{(1+2agq)}\prod_{i=0}^{n-2}\frac{(1+(6i+4)agq)(1+(6i+7)agq)}{(1+(6i+5)agq)(1+(6i+8)agq)},$$

$$x_{18n-13} = \frac{r(1+bhr)}{(1+2bhr)}\prod_{i=0}^{n-2}\frac{(1+(6i+4)bhr)(1+(6i+7)bhr)}{(1+(6i+5)bhr)(1+(6i+8)bhr)},$$

$$x_{18n-12} = \frac{s(1+cks)}{(1+2cks)}\prod_{i=0}^{n-2}\frac{(1+(6i+4)cks)(1+(6i+7)cks)}{(1+(6i+5)cks)(1+(6i+8)cks)},$$

$$x_{18n-11} = \frac{t(1+2dlt)}{(1+3dlt)}\prod_{i=0}^{n-2}\frac{(1+(6i+5)dlt)(1+(6i+8)dlt)}{(1+(6i+6)dlt)(1+(6i+9)dlt)},$$

$$x_{18n-10} = \frac{u(1+2emu)}{(1+3emu)}\prod_{i=0}^{n-2}\frac{(1+(6i+5)emu)(1+(6i+8)emu)}{(1+(6i+6)emu)(1+(6i+9)emu)},$$

$$x_{18n-9} = \frac{v(1+2fpv)}{(1+3fpv)}\prod_{i=0}^{n-2}\frac{(1+(6i+5)fpv)(1+(6i+8)fpv)}{(1+(6i+6)fpv)(1+(6i+9)fpv)}$$

$$\begin{aligned}
y_{18n-17} &= \frac{a}{1+agq}\prod_{i=0}^{n-2}\frac{(1+(6i+3)agq)(1+(6i+6)agq)}{(1+(6i+4)agq)(1+(6i+7)agq)},\\
y_{18n-16} &= \frac{b}{1+bhr}\prod_{i=0}^{n-2}\frac{(1+(6i+3)bhr)(1+(6i+6)bhr)}{(1+(6i+4)bhr)(1+(6i+7)bhr)},\\
y_{18n-15} &= \frac{c}{1+cks}\prod_{i=0}^{n-2}\frac{(1+(6i+3)cks)(1+(6i+6)cks)}{(1+(6i+4)cks)(1+(6i+7)cks)},\\
y_{18n-14} &= \frac{d(1+dlt)}{(1+2dlt)}\prod_{i=0}^{n-2}\frac{(1+(6i+4)dlt)(1+(6i+7)dlt)}{(1+(6i+5)dlt)(1+(6i+8)dlt)},\\
y_{18n-13} &= \frac{e(1+emu)}{(1+2emu)}\prod_{i=0}^{n-2}\frac{(1+(6i+4)emu)(1+(6i+7)emu)}{(1+(6i+5)emu)(1+(6i+8)emu)},\\
y_{18n-12} &= \frac{f(1+fpv)}{(1+2fpv)}\prod_{i=0}^{n-2}\frac{(1+(6i+4)fpv)(1+(6i+7)fpv)}{(1+(6i+5)fpv)(1+(6i+8)fpv)},\\
y_{18n-11} &= \frac{g(1+2agq)}{(1+3agq)}\prod_{i=0}^{n-2}\frac{(1+(6i+5)agq)(1+(6i+8)agq)}{(1+(6i+6)agq)(1+(6i+9)agq)},\\
y_{18n-10} &= \frac{h(1+2bhr)}{(1+3bhr)}\prod_{i=0}^{n-2}\frac{(1+(6i+5)bhr)(1+(6i+8)bhr)}{(1+(6i+6)bhr)(1+(6i+9)bhr)},\\
y_{18n-9} &= \frac{k(1+2cks)}{(1+3cks)}\prod_{i=0}^{n-2}\frac{(1+(6i+5)cks)(1+(6i+8)cks)}{(1+(6i+6)cks)(1+(6i+9)cks)}.
\end{aligned}$$

Now, it follows from system 2.1 that

$$x_{18n-8} = \frac{y_{18n-17}}{1+y_{18n-11}x_{18n-14}y_{18n-17}}$$

$$= \frac{\frac{a}{1+agq}\prod_{i=0}^{n-2}\frac{(1+(6i+3)agq)(1+(6i+6)agq)}{(1+(6i+4)agq)(1+(6i+7)agq)}}{1+\left(\begin{array}{c}\frac{g(1+2agq)}{(1+3agq)}\prod_{i=0}^{n-2}\frac{(1+(6i+5)agq)(1+(6i+8)agq)}{(1+(6i+6)agq)(1+(6i+9)agq)}\prod_{i=0}^{n-2}\frac{(1+(6i+4)agq)(1+(6i+7)agq)}{(1+(6i+5)agq)(1+(6i+8)agq)}\\ \frac{a}{1+agq}\prod_{i=0}^{n-2}\frac{(1+(6i+3)agq)(1+(6i+6)agq)}{(1+(6i+4)agq)(1+(6i+7)agq)}\end{array}\right)}$$

$$\begin{aligned}
&= \frac{\frac{a}{1+agq}\prod_{i=0}^{n-2}\frac{(1+(6i+3)agq)(1+(6i+6)agq)}{(1+(6i+4)agq)(1+(6i+7)agq)}}{1+\frac{agq}{(1+3agq)}\prod_{i=0}^{n-2}\frac{(1+(6i+3)agq)(1+(6i+6)agq)}{(1+(6i+6)agq)(1+(6i+9)agq)}} = \frac{\frac{a}{1+agq}\prod_{i=0}^{n-2}\frac{(1+(6i+3)agq)(1+(6i+6)agq)}{(1+(6i+4)agq)(1+(6i+7)agq)}}{1+\frac{agq(1+3agq)}{(1+3agq)(1+(6n-3)agq}}\\
&= \frac{\frac{a}{1+agq}\prod_{i=0}^{n-2}\frac{(1+(6i+3)agq)(1+(6i+6)agq)}{(1+(6i+4)agq)(1+(6i+7)agq)}}{\frac{(1+(6n-2)agq}{(1+(6n-3)agq}}\\
&= \frac{a}{1+agq}\prod_{i=0}^{n-2}\frac{(1+(6i+3)agq)(1+(6i+6)agq)}{(1+(6i+4)agq)(1+(6i+7)agq)}\frac{(1+(6n-3)agq}{(1+(6n-2)agq}.
\end{aligned}$$

Hence, we have

$$x_{18n-8} = a\prod_{i=0}^{n-1}\frac{(1+6iagq)(1+(6i+3)agq)}{(1+(6i+1)agq)(1+(6i+4)agq)},$$

and

$$y_{18n-8} = \frac{x_{18n-17}}{1+x_{18n-11}y_{18n-14}x_{18n-17}}$$

$$= \frac{\frac{l}{1+dlt}\prod_{i=0}^{n-2}\frac{(1+(6i+3)dlt)(1+(6i+6)dlt)}{(1+(6i+4)dlt)(1+(6i+7)dlt)}}{1+\left(\begin{array}{c}\frac{t(1+2dlt)}{(1+3dlt)}\prod_{i=0}^{n-2}\frac{(1+(6i+5)dlt)(1+(6i+8)dlt)}{(1+(6i+6)dlt)(1+(6i+9)dlt)}\frac{l}{1+dlt}\prod_{i=0}^{n-2}\frac{(1+(6i+3)dlt)(1+(6i+6)dlt)}{(1+(6i+4)dlt)(1+(6i+7)dlt)}\\ \frac{d(1+dlt)}{(1+2dlt)}\prod_{i=0}^{n-2}\frac{(1+(6i+4)dlt)(1+(6i+7)dlt)}{(1+(6i+5)dlt)(1+(6i+8)dlt)}\end{array}\right)}$$

$$\begin{aligned}
&= \frac{\frac{l}{1+dlt}\prod_{i=0}^{n-2}\frac{(1+(6i+3)dlt)(1+(6i+6)dlt)}{(1+(6i+4)dlt)(1+(6i+7)dlt)}}{1+\frac{dlt}{(1+3dlt)}\prod_{i=0}^{n-2}\frac{(1+(6i+3)dlt)(1+(6i+6)dlt)}{(1+(6i+6)dlt)(1+(6i+9)dlt)}} == \frac{\frac{l}{1+dlt}\prod_{i=0}^{n-2}\frac{(1+(6i+3)dlt)(1+(6i+6)dlt)}{(1+(6i+4)dlt)(1+(6i+7)dlt)}}{1+\frac{dlt(1+3dlt)}{(1+3dlt)(1+(6n-3)dlt}}\\
&= \frac{\frac{l}{1+dlt}\prod_{i=0}^{n-2}\frac{(1+(6i+3)dlt)(1+(6i+6)dlt)}{(1+(6i+4)dlt)(1+(6i+7)dlt)}}{\frac{(1+(6n-2)dlt}{(1+(6n-3)dlt}}\\
&= \frac{l}{1+dlt}\prod_{i=0}^{n-2}\frac{(1+(6i+3)dlt)(1+(6i+6)dlt)}{(1+(6i+4)dlt)(1+(6i+7)dlt)}\frac{(1+(6n-3)dlt}{(1+(6n-2)dlt}.
\end{aligned}$$

Therefore, we have

$$y_{18n-8}=l\prod_{i=0}^{n-1}\frac{(1+6idlt)(1+(6i+3)dlt)}{(1+(6i+1)dlt)(1+(6i+4)dlt)}.$$

Similarly, we can prove the other relations. □

Lemma 2.2. *If x_i, y_i, since $i=-8,-7,-6,\ldots,-1,0$ are arbitrary real numbers and let $\{x_i, y_i\}$ be solutions of system 2.1, then the following statements are true*

(i) *If* $x_{-8}=a=0$ *then we get*

$$x_{18n-8}=y_{18n+1}=0,\ x_{18n-2}=y_{18n+7}=g,\ x_{18n+4}=y_{18n-5}=q.$$

(ii) *If* $x_{-7}=b=0$ *then we obtain*

$$x_{18n-7}=y_{18n+2}=0,\ x_{18n-1}=y_{18n+8}=h,\ x_{18n+5}=y_{18n-4}=r.$$

(iii) *If* $x_{-6}=c=0$ *then*

$$x_{18n-6}=y_{18n+3}=0,\ x_{18n}=y_{18n+9}=k,\ x_{18n+6}=y_{18n-3}=s.$$

(iv) *If* $x_{-5}=d=0$ *then*

$$x_{18n-5}=y_{18n+4}=0,\ x_{18n+1}=y_{18n-8}=l,\ x_{18n+7}=y_{18n-2}=t.$$

(v) *If* $x_{-4}=e=0$ *then*

$$x_{18n-4}=y_{18n+5}=0,\ x_{18n+2}=y_{18n-7}=m,\ x_{18n+8}=y_{18n-1}=u.$$

(vi) *If* $x_{-3}=f=0$ *then we see that*

$$x_{18n-3}=y_{18n+6}=0,\ x_{18n+3}=y_{18n-6}=p,\ x_{18n+9}=y_{18n}=v.$$

(vii) *If* $x_{-2}=g=0$ *then we have*

$$x_{18n-2}=y_{18n+7}=0,\ x_{18n+4}=y_{18n-5}=q,\ x_{18n-8}=y_{18n+1}=a.$$

$(viii)$ *If* $x_{-1}=h=0$ *then*

$$x_{18n-1}=y_{18n+8}=0,\ x_{18n+5}=y_{18n-4}=r,\ x_{18n-7}=y_{18n+2}=b.$$

(ix) *If* $x_{0}=k=0$ *then we get*

$$x_{18n}=y_{18n+9}=0,\ x_{18n+6}=y_{18n-3}=s,\ x_{18n-6}=y_{18n+3}=c.$$

(x) *If* $y_{-8}=l=0$ *then*

$$y_{18n-8}=x_{18n+1}=0,\ y_{18n-2}=x_{18n+7}=t,\ y_{18n+4}=x_{18n-5}=d.$$

(xi) *If* $y_{-7}=m=0$ *then we get*

$$y_{18n-7}=x_{18n+2}=0,\ y_{18n-1}=x_{18n+8}=u,\ y_{18n+5}=x_{18n-4}=e.$$

(xii) *If* $y_{-6}=p=0$ *then we have*

$$y_{18n-6}=x_{18n+3}=0,\ y_{18n}=x_{18n+9}=v,\ y_{18n+6}=x_{18n-3}=f.$$

$(xiii)$ *If* $y_{-5}=q=0$ *then*

$$y_{18n-5}=x_{18n+4}=0,\ y_{18n+1}=x_{18n-8}=a,\ y_{18n+7}=x_{18n-2}=g.$$

(xiv) *If* $y_{-4}=r=0$ *then we see*

$$y_{18n-4}=x_{18n+5}=0,\ y_{18n+2}=x_{18n-7}=b,\ y_{18n+8}=x_{18n-1}=h.$$

(xv) *If* $y_{-3}=s=0$ *then*

$$y_{18n-3}=x_{18n+6}=0,\ y_{18n+3}=x_{18n-6}=c,\ y_{18n+9}=x_{18n}=k.$$

(xvi) *If* $y_{-2}=t=0$ *then*

$$y_{18n-2}=x_{18n+7}=0,\ y_{18n+4}=x_{18n-5}=d,\ y_{18n-8}=x_{18n+1}=l.$$

$(xvii)$ *If* $y_{-1}=u=0$ *then we get*

$$y_{18n-1}=x_{18n+8}=0,\ y_{18n+5}=x_{18n-4}=e,\ y_{18n-7}=x_{18n+2}=m.$$

$(xviii)$ *If* $y_{0}=v=0$ *then we obtain*

$$y_{18n}=x_{18n+9}=0,\ y_{18n+6}=x_{18n-3}=f,\ y_{18n-6}=x_{18n+3}=p.$$

Proof. The proof follows from the form of the solutions of system 2.1. □

Lemma 2.3. *Let $\{x_n, y_n\}$ be a positive solution of System 2.1, then $\{x_n\},\{y_n\}$ are bounded and converges to zero.*

Proof. It follows from System 2.1 that

$$x_{n+1}=\frac{y_{n-8}}{1+y_{n-2}x_{n-5}y_{n-8}}\leq y_{n-8},\quad y_{n+1}=\frac{x_{n-8}}{1+x_{n-2}y_{n-5}x_{n-8}}\leq x_{n-8}.$$

Then we have

$$x_{n+10}=\frac{y_{n+1}}{1+y_{n+7}x_{n+4}y_{n+1}}\leq y_{n+1}\leq x_{n-8},\ y_{n+10}=\frac{x_{n+1}}{1+x_{n+7}y_{n+4}x_{n+1}}\leq x_{n+1}\leq y_{n-8}.$$

Then the subsequences $\{x_{18n-8}\}_{n=0}^{\infty}$, $\{x_{18n-7}\}_{n=0}^{\infty}$, ..., $\{x_{18n+9}\}_{n=0}^{\infty}$ are decreasing and so are bounded from above by $M=\max\{x_{-8},x_{-7},\ldots,x_{8},x_{9}\}$. Also, the subsequences $\{y_{18n-8}\}_{n=0}^{\infty}$, $\{y_{18n-7}\}_{n=0}^{\infty}$,..., $\{y_{18n+9}\}_{n=0}^{\infty}$ are decreasing and so are bounded from above by $L=\max\{y_{-8},y_{-7},\ldots,y_{8},y_{9}\}$. □

2.2. The second system: $x_{n+1}=\frac{y_{n-8}}{1+y_{n-2}x_{n-5}y_{n-8}}, y_{n+1}=\frac{x_{n-8}}{1-x_{n-2}y_{n-5}x_{n-8}}$

In this subsection, we get the solutions of the following system of the difference equations

$$x_{n+1}=\frac{y_{n-8}}{1+y_{n-2}x_{n-5}y_{n-8}}, y_{n+1}=\frac{x_{n-8}}{1-x_{n-2}y_{n-5}x_{n-8}}, \tag{2.1}$$

where the initial conditions are arbitrary real numbers with $y_{-2}x_{-5}y_{-8},\ y_{-1}x_{-4}y_{-7},\ y_0x_{-3}y_{-6}\neq -1$ and $x_{-2}y_{-5}x_{-8},\ x_{-1}y_{-4}x_{-7},\ x_0y_{-3}x_{-6}\neq 1$.

Theorem 2.4. *System 2.1 has a periodic solution of period eighteen. Moreover $\{x_n,y_n\}_{n=-8}^{\infty}$ takes the form*

$$\begin{aligned}\{x_n\} &= \left\{\begin{array}{c} a,b,c,d,e,f,g,h,k,\frac{l}{1+dlt},\frac{m}{1+emu},\frac{p}{1+fpv},q-agq^2, \\ r-bhr^2,s-cks^2,\frac{t}{1+dlt},\frac{u}{1+emu},\frac{v}{1+fpv},a,b,c,\ldots\end{array}\right\},\\ \{y_n\} &= \left\{\begin{array}{c} l,m,p,q,r,s,t,u,v,\frac{a}{1-agq},\frac{b}{1-bhr},\frac{c}{1-cks},d(1+dlt), \\ e(1+emu),f(1+fpv),\frac{g}{1-agq},\frac{h}{1-bhr},\frac{k}{1-cks},l,m,p,\ldots\end{array}\right\}.\end{aligned}$$

or

$$\begin{aligned} x_{18n-8} &= a,\ x_{18n-7}=b,\ x_{18n-6}=c,\ x_{18n-5}=d,\ x_{18n-4}=e,\ x_{18n-3}=f,\\ x_{18n-2} &= g,\ x_{18n-1}=h,\ x_{18n}=k,\ x_{18n+1}=\frac{l}{1+dlt},\ x_{18n+2}=\frac{m}{1+emu},\\ x_{18n+3} &= \frac{p}{1+fpv},\ x_{18n+4}=q-agq^2,\ x_{18n+5}=r-bhr^2,\\ x_{18n+6} &= s-cks^2,\ x_{18n+7}=\frac{t}{1+dlt},\ x_{18n+8}=\frac{u}{1+emu},\ x_{18n+9}=\frac{v}{1+fpv},\end{aligned}$$

and

$$\begin{aligned} y_{18n-8} &= l,\ y_{18n-7}=m,\ y_{18n-6}=p,\ y_{18n-5}=q,\ y_{18n-4}=r,\ y_{18n-3}=s,\\ y_{18n-2} &= t,\ y_{18n-1}=u,\ y_{18n}=v,\ y_{18n+1}=\frac{a}{1-agq},\ y_{18n+2}=\frac{b}{1-bhr},\\ y_{18n+3} &= \frac{c}{1-cks}, y_{18n+4}=d(1+dlt),\ y_{18n+5}=e(1+emu),\\ y_{18n+6} &= f(1+fpv),\ y_{18n+7}=\frac{g}{1-agq},\ y_{18n+8}=\frac{h}{1-bhr}, y_{18n+9}=\frac{k}{1-cks}.\end{aligned}$$

Proof. For $n=0$, the result holds. Now, assume that $n>0$ and that our assumption holds for $n-1$. That is,

$$\begin{aligned} x_{18n-17} &= \frac{l}{1+dlt},\ x_{18n-16}=\frac{m}{1+emu},\ x_{18n-15}=\frac{p}{1+fpv},\\ x_{18n-14} &= q-agq^2,\ x_{18n-13}=r-bhr^2,\ x_{18n-12}=s-cks^2,\\ x_{18n-11} &= \frac{t}{1+dlt},\ x_{18n-10}=\frac{u}{1+emu},\ x_{18n-9}=\frac{v}{1+fpv},\end{aligned}$$

and

$$\begin{aligned} y_{18n-17} &= \frac{a}{1-agq},\ y_{18n-16}=\frac{b}{1-bhr},\ y_{18n-15}=\frac{c}{1-cks},\\ y_{18n-14} &= d(1+dlt),\ y_{18n-13}=e(1+emu),\ y_{18n-12}=f(1+fpv),\\ y_{18n-11} &= \frac{g}{1-agq},\ y_{18n-10}=\frac{h}{1-bhr}, y_{18n-9}=\frac{k}{1-cks}.\end{aligned}$$

Now, it follows from system 2.1 that

$$\begin{aligned} x_{18n-8} &= \frac{y_{18n-17}}{1+y_{18n-11}x_{18n-14}y_{18n-17}}=\frac{\frac{a}{1-agq}}{1+\frac{g}{1-agq}(q-agq^2)\frac{a}{1-agq}}\\ &= \frac{\frac{a}{1-agq}}{1+\frac{agq}{1-agq}}=\frac{a}{1-agq+agq}=a,\end{aligned}$$

also,

$$\begin{aligned} y_{18n-8} &= \frac{x_{18n-17}}{1-x_{18n-11}y_{18n-14}x_{18n-17}}=\frac{\frac{l}{1+dlt}}{1-\frac{t}{1+dlt}d(1+dlt)\frac{l}{1+dlt}}\\ &= \frac{\frac{l}{1+dlt}}{1-\frac{dlt}{1+dlt}}=l.\end{aligned}$$

The other relations can be proved by similar way. $\square$

The following cases can be proved similarly.

2.3. The third system: $x_{n+1}=\frac{y_{n-8}}{1+y_{n-2}x_{n-5}y_{n-8}}, y_{n+1}=\frac{x_{n-8}}{-1+x_{n-2}y_{n-5}x_{n-8}}$

In this part, we obtain the form of the solutions of the following system of the difference equations

$$x_{n+1}=\frac{y_{n-8}}{1+y_{n-2}x_{n-5}y_{n-8}}, y_{n+1}=\frac{x_{n-8}}{-1+x_{n-2}y_{n-5}x_{n-8}}, \tag{2.2}$$

where the initial conditions are arbitrary real numbers with $y_{-2}x_{-5}y_{-8}, y_{-1}x_{-4}y_{-7}, y_0x_{-3}y_{-6}\neq -1$ and $x_{-2}y_{-5}x_{-8}, x_{-1}y_{-4}x_{-7}, x_0y_{-3}x_{-6}\neq 1$.

Theorem 2.5. *System 2.2 has a periodic solution of period (36) which takes the form*

$$\{x_n\} = \left\{\begin{array}{c} a,b,c,d,e,f,g,h,k,\frac{l}{1+dlt},\frac{m}{1+emu},\frac{p}{1+fpv},\frac{q(-1+agq)}{-1+2agq},\\ \frac{r(-1+bhr)}{-1+2bhr},\frac{s(-1+cks)}{-1+2cks},\frac{t}{1-dlt},\frac{u}{1-emu},\frac{v}{1-fpv},\\ -a,-b,-c,-d,-e,-f,-g,-h,-k,\frac{-l}{1+dlt},\frac{-m}{1+emu},\frac{-p}{1+fpv},\\ \frac{-q(-1+agq)}{-1+2agq},\frac{-r(-1+bhr)}{-1+2bhr},\frac{-s(-1+cks)}{-1+2cks},\frac{t}{-1+dlt},\\ \frac{u}{-1+emu},\frac{v}{-1+fpv},a,b,c,d,\ldots \end{array}\right\},$$

$$\{y_n\} = \left\{\begin{array}{c} l,m,p,q,r,s,t,u,v,\frac{a}{-1+agq},\frac{b}{-1+bhr},\frac{c}{-1+cks},\\ -d(1+dlt),-e(1+emu),-f(1+fpv),\frac{g-2ag^2q}{-1+agq},\\ \frac{h-2bh^2r}{-1+bhr},\frac{k-2ck^2s}{-1+cks},\frac{l(-1+dlt)}{1+dlt},\frac{m(-1+emu)}{1+emu},\frac{p(-1+fpv)}{1+fpv},\frac{q}{-1+2agq},\\ \frac{r}{-1+2bhr},\frac{s}{-1+2cks},\frac{t(1+dlt)}{-1+dlt},\frac{u(1+emu)}{-1+emu},\frac{v(1+fpv)}{-1+fpv},\frac{a(-1+2agq)}{-1+agq},\\ \frac{b(-1+2bhr)}{-1+bhr},\frac{c(-1+2cks)}{-1+cks},d-d^2lt,e-e^2mu,f-f^2pv,\\ \frac{g}{1-agq},\frac{h}{1-bhr},\frac{k}{1-cks},l,m,p,\ldots \end{array}\right\}.$$

2.4. The fourth system: $x_{n+1}=\frac{y_{n-8}}{1+y_{n-2}x_{n-5}y_{n-8}}, y_{n+1}=\frac{x_{n-8}}{-1-x_{n-2}y_{n-5}x_{n-8}}$

In this case, we solve the form of the solutions of the following system of the difference equations

$$x_{n+1}=\frac{y_{n-8}}{1+y_{n-2}x_{n-5}y_{n-8}}, y_{n+1}=\frac{x_{n-8}}{-1-x_{n-2}y_{n-5}x_{n-8}}, \tag{2.3}$$

where the initial conditions are arbitrary real numbers with $y_{-2}x_{-5}y_{-8}$, $y_{-1}x_{-4}y_{-7}, y_0x_{-3}y_{-6}\neq -1$ and $x_{-2}y_{-5}x_{-8}, x_{-1}y_{-4}x_{-7}, x_0y_{-3}x_{-6}\neq -1$.

Theorem 2.6. *Every solutions of system 2.3 are periodic with period (36). Moreover $\{x_n,y_n\}_{n=-8}^{\infty}$ takes the form*

$$\{x_n\} = \left\{\begin{array}{c} a,b,c,d,e,f,g,h,k,\frac{l}{1+dlt},\frac{m}{1+emu},\frac{p}{1+fpv},q(1+agq),\\ r(1+bhr),s(1+cks),\frac{t(1+2dlt)}{1+dlt},\frac{u(1+2emu)}{1+emu},\frac{v(1+2fpv)}{1+fpv},\\ \frac{a(-1+agq)}{1+agq},\frac{b(-1+bhr)}{1+bhr},\frac{c(-1+cks)}{1+cks},\frac{-d}{1+2dlt},\frac{-e}{1+2emu},\\ \frac{-f}{1+2fpv},\frac{g(1+agq)}{-1+agq},\frac{h(1+bhr)}{-1+bhr},\frac{k(1+cks)}{-1+cks},\frac{-l(1+2dlt)}{1+dlt},\\ \frac{-m(1+2emu)}{1+emu},\frac{-p(1+2fpv)}{1+fpv},q(-1+agq),r(-1+bhr),\\ s(-1+cks),\frac{-t}{1+dlt},\frac{-u}{1+emu},\frac{-v}{1+fpv},a,b,c,\ldots \end{array}\right\},$$

$$\{y_n\} = \left\{\begin{array}{c} l,m,p,q,r,s,t,u,v,\frac{-a}{1+agq},\frac{-b}{1+bhr},\frac{-c}{1+cks},\\ \frac{-d(1+dlt)}{1+2dlt},\frac{-e(1+emu)}{1+2emu},\frac{-f(1+fpv)}{1+2fpv},\frac{g}{-1+agq},\frac{h}{-1+bhr},\\ \frac{k}{-1+cks},-l,-m,-p,-q-r,-s,-t,-u,-v,\\ \frac{a}{1+agq},\frac{b}{1+bhr},\frac{c}{1+cks},\frac{d(1+dlt)}{1+2dlt},\frac{e(1+emu)}{1+2emu)},\frac{f(1+fpv)}{1+2f[v)},\\ \frac{g}{1-agq},\frac{h}{1-bhr},\frac{k}{1-cks},l,m,p,\ldots \end{array}\right\}.$$

3. Numerical examples

Here we consider some numerical examples to illustrate the behavior of the solutions of the systems which we studied.

Example 3.1. *Consider the System 2.1 with the initial conditions* $x_{-8}=15$, $x_{-7}=-6.2$, $x_{-6}=-0.26$, $x_{-5}=-13$, $x_{-4}=12$, $x_{-3}=6$, $x_{-2}=9$, $x_{-1}=-2$, $x_0=-6$, $y_{-8}=7$, $y_{-7}=8$, $y_{-6}=-0.3$, $y_{-5}=11$, $y_{-4}=14$, $y_{-3}=2.5$, $y_{-2}=16$, $y_{-1}=-9$, $y_0=-0.3$. *See Figure 3.1 and see Figure 3.2 when we put* $x_{-8}=15$, $x_{-7}=6.2$, $x_{-6}=0.26$, $x_{-5}=13$, $x_{-4}=2$, $x_{-3}=16$, $x_{-2}=9$, $x_{-1}=2$, $x_0=6$, $y_{-8}=7$, $y_{-7}=18$, $y_{-6}=0.3$, $y_{-5}=11$, $y_{-4}=34$, $y_{-3}=2.5$, $y_{-2}=26$, $y_{-1}=9$, $y_0=0.3$.

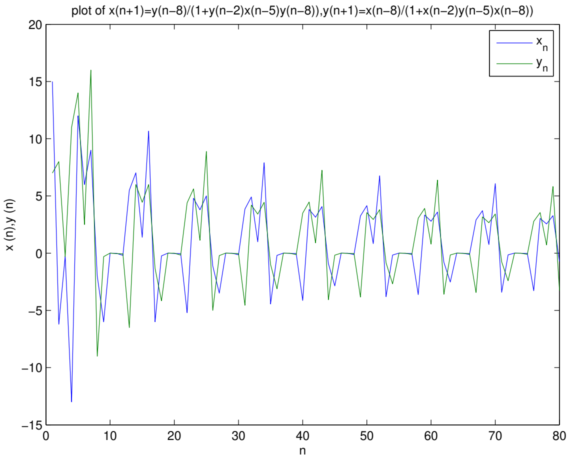

Figure 3.1

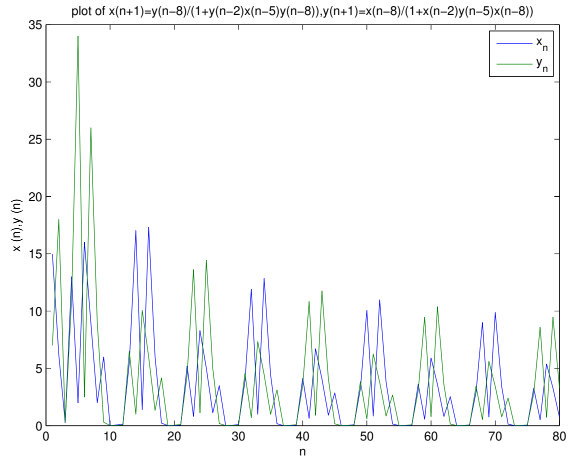

Figure 3.2

Example 3.2. *See Figure 3.3, when we take System 2.1 and put* $x_{-8} = -1.5$, $x_{-7} = 6.2$, $x_{-6} = 0.6$, $x_{-5} = 1.3$, $x_{-4} = 1.2$, $x_{-3} = 0.6$, $x_{-2} = 0.9$, $x_{-1} = 0.2$, $x_0 = 0.6$, $y_{-8} = 0.7$, $y_{-7} = -1.8$, $y_{-6} = 0.3$, $y_{-5} = 1.1$, $y_{-4} = 1.4$, $y_{-3} = 2.5$, $y_{-2} = 1.6$, $y_{-1} = 0.9$, $y_0 = 0.3$.

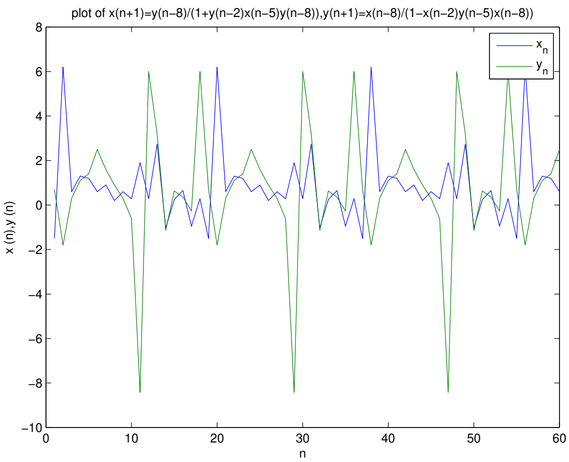

Figure 3.3

Example 3.3. *Figure 3.4 below describe the periodic solutions of System 2.2 when* $x_{-8} = -1.5$, $x_{-7} = -6.2$, $x_{-6} = 0.6$, $x_{-5} = 1.3$, $x_{-4} = 1.2$, $x_{-3} = -0.6$, $x_{-2} = 0.9$, $x_{-1} = 0.2$, $x_0 = 0.6$, $y_{-8} = 0.7$, $y_{-7} = -1.8$, $y_{-6} = 0.3$, $y_{-5} = 1.1$, $y_{-4} = 1.4$, $y_{-3} = 2.5$, $y_{-2} = 1.6$, $y_{-1} = 0.9$, $y_0 = 0.3$.

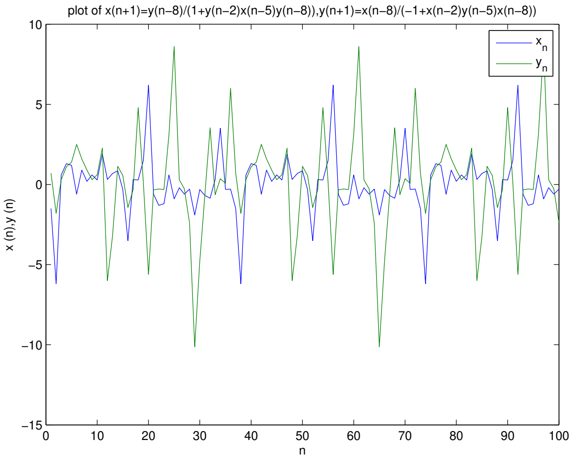

Figure 3.4

Example 3.4. *Consider the System* 2.3 *when* $x_{-8} = -1.5$, $x_{-7} = -6.2$, $x_{-6} = 0.6$, $x_{-5} = 1.3$, $x_{-4} = -1.2$, $x_{-3} = -0.6$, $x_{-2} = 0.9$, $x_{-1} = 0.2$, $x_0 = 0.6$, $y_{-8} = 0.7$, $y_{-7} = -1.8$, $y_{-6} = 0.3$, $y_{-5} = 1.1$, $y_{-4} = -1.4$, $y_{-3} = -2.5$, $y_{-2} = 1.6$, $y_{-1} = 0.9$, $y_0 = 0.3$. *See Figure 3.5.*

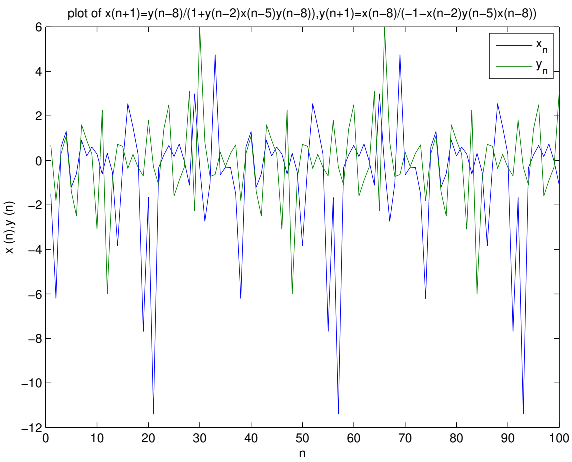

Figure 3.5

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