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Soft Separation Axioms First Results

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Abstract. In this paper, we study soft separation axioms in first sections and then we give the definition of Complemental Soft Topological Spaces. Here, some new theorems and their proofs are given. Further the following relation $ST_0 \iff ST_1 \iff ST_2 \iff ST_3 \iff ST_4$ is written as a natural result of the given theorems.

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Keywords: Soft set, Soft point, Soft function, Soft topology, Soft topological space, Soft $T_i, (i = 0, 1, 2, 3, 4)$ spaces, Soft complemental spaces, Soft separation axioms, Complemental $T_i, (i = 0, 1, 2, 3, 4)$ spaces.

1. Introduction

In this paper we give some new results of separation axioms in soft topological spaces.After Zadeh [18] hundreds of papers have been written on fuzzy sets and soft sets theory. Some papers on soft set theory in which we interested are of Aktas [1], Aygunoglu [2], Cagman [4], Hazra [8], Hussain [9], Maji [11], Min [12] and Zorlutuna [19]. Also we must mention here on fuzzy set theory papers are of Roy [14], Tanay [16] and Yang [17]. There are some latest works studied by Gocur and Kopuzlu with Peyghan [6], [7] , Samadi and Tayebi with Hussain and Ahmad [10] on separation axioms in soft topological spaces by taking into account of Shabir and Naz [15]'s paper. As mentioned before Shabir and Naz [15] and Cagman et. al [3] made valuable contributions to the separation axioms in soft topological spaces. In one of the latest works, Hussain and Ahmad [10] made an extended study on notations and soft sets of a soft topology. It is worth mentioning now that we define and discuss a new soft topological space named complemental soft topological spaces. Also it is investigated that the relation $ST_0 \iff ST_1 \iff ST_2 \iff ST_3 \iff ST_4$ holds in complemental soft topological spaces while for every $e \in E$ there exists soft T_i where $i = 0, 1, 2, 3, 4$ which corresponds to T_i where $i = 0, 1, 2, 3, 4$ respectively.

2. Methods

The main purpose of this study is to be useful for the future studies on soft topology and to carry out general framework for the practical applications and to solve the complicated problems containing uncertainties in economics, engineering, medical, environment and in general man-machine systems of various types. After a comprehensive literature search, we noticed that separation axioms vary according to the established space structure. After mentioning previous works, we developed complemental soft topological space and gave some theorems and finally wrote the corollaries by the contributions of assistant profesors at Ordu University.

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3. Preliminaries

Definition 3.1. Let X be an initial universe and E be a set of parameters. Let $P(X)$ denote the power set of X. Then the function F defined from E to $P(X)$ is called a soft set over X and denoted by (F, E) [13].

A soft set can be seen as a set of ordered pairs, given as $F = \{(e, F(e)) : e \in E\}$. If for any $e \in E$, $F(e) = \varnothing$ then ordered pair $(e, \varnothing)$ may not be shown in a soft set. Also the set of all the soft sets defined from E to $P(X)$ over X is shown by $S_E(X)$ [3].

Definition 3.2. Let $F \in S_E(X)$, if for all $e' \in E \setminus \{e\}$, $F(e') = \varnothing$ where for all $e \in E$ and $F(e) = \varnothing$, then F is said to be a soft point and denoted by e_F. If $F(e) \subseteq G(e)$ for all $e \in E$ then e_F is said to be an element of G and denoted by $e_F \tilde{\in} G$ [3] , [10] , [15].

Some basic set operations on soft sets and definition of different soft sets are given in [1], [5]. Also the definition of a soft function is given in [3].

Definition 3.3. Let $\tau \subseteq S_E(X)$ then τ is said to be a soft topology over X if the following condition is satisfied
i. $ST1$. $\tilde{X}, \Phi \in \tau$
ii. $ST2$. For any two $F, G \in \tau$ we have $F \sqcap G \in \tau$
iii. $ST3$. For any $\{F_i : i \in I\} \subseteq \tau$ we have $\bigsqcup_{i \in I} F_i \in \tau$.
If τ is a soft topology then the triplet (X, τ, E) is called a soft topological space and an element of τ is called a soft open set [3], [15].

Theorem 3.4. *Let (X, τ, E) be a soft topological space over X for all $e \in E$. Then the collection of sets $\tau_e = \{F_i(e) : i \in I\}$ is a topology on X [15].*

Theorem 3.5. *Let (X, τ_i, E) be a collection of soft topological spaces over X for all $e \in E$. Then the triplet $(X, \cap_{i \in I} \tau_i, E)$ is a soft topology on X [15].*

Theorem 3.6. *Let (X, τ, E) be a soft topological space over X for all $e \in E$. Let A be a non-empty subset of X, where $\tilde{A} = \{(e, A) : e \in E\}$ then the collection of soft sets $\tau_A = \{F \sqcap \tilde{A} : F \in \tau\}$ is a soft topology on A [3], [10], [15].*

We know that a soft subspace, a soft hereditary property and a soft homeomorphism can be defined on soft sets [1], [2], [5].

Definition 3.7. Let (X, τ, E) be a soft topological space over X. Let $a_F, b_G \in S_E(X)$ be given, where $a_F \neq b_G$. If there exists soft open sets $H_1, H_2 \in \tau$ such that $a_F \tilde{\in} H_1$ and $b_G \tilde{\notin} H_1$ or $a_F \tilde{\notin} H_2$ and $b_G \tilde{\in} H_2$ then the soft topological space (X, τ, E) is said to be a soft T_0 space and denoted by ST_0 [3], [10], [15].

Being a ST_i space, where $i = 0, 1, 2, 3$ in a soft topological space is both a soft hereditary property and a soft topological property [15].

Definition 3.8. Let (X, τ, E) be a soft topological space over X. Let $e_F, e'_G \in S_E(X)$ be given, where $e_F \neq e'_G$. If there exists soft open sets $H_1, H_2 \in \tau$ such that $e_F \tilde{\in} H_1$, $e'_G \tilde{\notin} H_1$ and $e_F \tilde{\in} H_2$, $e'_G \tilde{\notin} H_2$ then the soft topological space (X, τ, E) is said to be a soft T_1 space and denoted by ST_1 [3], [10], [15].

Theorem 3.9. *Let (X, τ, E) be a soft topological space over X. If every element of τ is soft closed then τ is said to be a ST_1 [15].*

Definition 3.10. Let (X, τ, E) be a soft topological space over X. Let $e_F, e'_G \in S_E(X)$ be given, where $e_F \neq e'_G$. If there exists soft open sets $H_1, H_2 \in \tau$ such that $H_1 \sqcap H_2 = \Phi$ then the soft topological space (X, τ, E) is said to be a soft T_2 space (or a soft Hausdorff space) and denoted by ST_2 [3], [10], [15].

Theorem 3.11. *Let (X, τ, E) be a ST_2 space over X. Then the topological space (X, τ_e) is a T_2 space for all $e \in E$ [15].*

Definition 3.12. Let (X, τ, E) be a soft topological space over X and a soft closed $G \in S_E(X)$ be given. If there exist soft open sets $H_1, H_2 \in \tau$ such that $H_1 \sqcap H_2 = \Phi$ and $G \sqsubseteq H_2$, where $e_F \in S_E(X)$ provided that $e_F \notin G$. Then the soft topological space (X, τ, E) is called a soft regular space [3], [10], [15].

Definition 3.13. Let (X, τ, E) be a soft topological space over X for all $e \in E$. If the soft topological space is both a soft regular and a soft T_1 space then it is said to be a soft T_3 space and denoted by ST_3 [3], [10], [15].

Theorem 3.14. *Every ST_3 space may not be a ST_2 space [15].*

Definition 3.15. Let (X, τ, E) be a soft topological space over X and soft closed $H_1, H_2 \in S_E(X)$ be given, where $H_1 \sqcap H_2 = \Phi$. If there exist soft open $G_1, G_2 \in \tau$ such that $H_1 \sqsubseteq G_1$ and $H_2 \sqsubseteq G_2$, provided that $G_1 \sqcap G_2 = \Phi$. Then the soft topological space (X, τ, E) is called a soft normal space [3], [10], [15].

Definition 3.16. Let (X, τ, E) be a soft topological space over X for all $e \in E$. If the soft topological space is both a soft normal and a soft T_1 space then it is said to be a soft T_4 space and denoted by ST_4 [3], [10], [15].

Theorem 3.17. *Every ST_4 space may not be a ST_3 space [15].*

4. Results and Discussions

Following results may inspire many researchers to make thorough study on this area.

Definition 4.1. Let (X, τ, E) be a soft topological space over X. If for every $G \in \tau$ there exist $G^{\tilde{c}} \in \tau$ then the soft set G is called complemental soft set and the soft space (X, τ, E) is called a complemental soft topological space.

Example 4.2. Let $E = \{a, b\}$, $X = \{x, y\}$ and $F_1, F_2, ..., F_{14} \in S_E(X)$ where

$$F_1 = \{(a, \{x\}), (b, \{y\})\}, F_2 = \{(a, \{y\}), (b, \{x\})\}, F_3 = \{(a, \{x\}), (b, \{x\})\}, F_4 = \{(a, \{y\}), (b, \{y\})\}$$
$$F_5 = \{(a, \{x\}), (b, X)\}, F_6 = \{(a, X), (b, \{x\})\}, F_7 = \{(a, \{y\}), (b, X)\}, F_8 = \{(a, X), (b, \{y\})\}$$
$$F_9 = \{(a, \{x\})\}, F_{10} = \{(b, \{x\})\}, F_{11} = \{(a, \{y\})\}, F_{12} = \{(b, \{y\})\}, F_{13} = \{(a, X)\}, F_{14} = \{(b, X)\}$$

then the soft sets family

$$\tau = \{\Phi, \tilde{X}, F_1, F_2, ..., F_{14}\}$$

is a soft topology and since for every soft set $F_i \in \tau$ there exists $F_i^{\tilde{c}} \in \tau$ where $i = 1, 2, ..., 14$ and therefore (X, τ, E) is a complemental soft topological space.

Theorem 4.3. *Let (X, τ, E) be a complemental soft topological space over X. Then for each $e_F \in S_E(X)$ there exists $H \in \tau$ such that $e_F \tilde{\in} H$.*

Proof. Let (X, τ, E) be a complemental soft topological space over X and any $a_F \in S_E(X)$ be given. Then there must be a soft open $H \in S_E(X)$ such that $a_F \tilde{\in} H$. Because of being a complemental soft topological space, for each soft open $H \in S_E(X)$, there exists a soft closed $H^{\tilde{c}} \in S_E(X)$. Therefore for each $a_F \tilde{\in} H$, we have $H \in \tau$ and this completes the proof . □

Lemma 4.4. *Let any (X, τ, E) be given as in Theorem 4.3 . Then for each soft complemental $H \in \tau$ there exists $H^{\tilde{c}} \in \tau$.*

Theorem 4.5. *Every complemental soft topological space is a ST_0 space.*

Proof. Let (X, τ, E) be a complemental soft topological space over X and any two $a_F, b_F \in S_E(X)$, where $a_F \neq b_F$, be given. From Theorem 4.3 there exists a soft open $a_F \tilde{\in} H$ such that $H \in \tau$. Since $a_F \neq b_F$, then $b_F \tilde{\notin} H$, which means (X, τ, E) is a ST_0 space . □

Theorem 4.6. *Let (X, τ, E) be a complemental ST_0 space for each $e \in E$, then the topological space (X, τ_e) is a T_0 space.*

Proof. Let (X, τ, E) be a ST_0 space and soft open F, G from this space be given. Take two soft points $a_F \tilde{\in} F$ and $b_G \tilde{\in} G$ where $a_F \neq b_G$, if $a_F \tilde{\in} F$ then $a_F \tilde{\notin} F^{\tilde{c}}(e)$, where $e \in E$. Since for each $e \in E$, $b_G \tilde{\notin} F(e)$ so we get $b_G \tilde{\notin} F$. Similarly when $b_G \tilde{\in} G$ for each $e \in E$, we have $b_G \tilde{\notin} G^{\tilde{c}}(e)$. Thus for each $e \in E$, $a_F \tilde{\notin} G(e)$, so we get $a_F \tilde{\notin} G$. Therefore (X, τ, E) is a ST_0 space. For any $e \in E$, (X, τ_e) is a topological space and $a_F \in F$ and $b_G \in F^{\tilde{c}}$ or $b_G \in G$ and $a_F \in G^{\tilde{c}}$. This indicates that $a_F \in F(e)$ and $b_G \notin F(e)$ or similarly we get $b_G \in G(e)$ and $a_F \notin G(e)$. Thus for each $e \in E$, (X, τ_e) is a T_0 space. □

Theorem 4.7. *Every complemental soft topological space is a ST_1 space.*

Proof. Let (X, τ, E) be a complemental soft topological space over X and any two $a_F, b_G \in S_E(X)$, where $a_F \neq b_G$, be given. From Theorem 4.3 there exists a soft open $a_F \tilde{\in} H_1$ and $b_G \tilde{\notin} H_1$ or in a similar way $a_F \tilde{\in} H_2$ and $b_G \tilde{\notin} H_2$ such that $H_1, H_2 \in \tau$. So we get (X, τ, E) is a ST_1 space. □

Theorem 4.8. *Let (X, τ, E) be a complemental ST_1 space for each $e \in E$, then the topological space (X, τ_e) is a T_1 space.*

Proof. Let (X, τ, E) be a ST_1 space and soft open F, G taken from this space be given. It is also given that $a_F \neq b_G$ where $a_F \tilde{\in} F$ and $b_G \tilde{\in} G$. If $a_F \tilde{\in} F$ for every $e \in E$, then $a_F \tilde{\notin} F(e)^{\tilde{c}}$. So we get $b_G \tilde{\notin} F(e)$ for every $e \in E$ and thus we have $b_G \tilde{\notin} F$. In a similar way, if $b_G \tilde{\in} G$ for every $e \in E$, then $b_G \tilde{\notin} G(e)^{\tilde{c}}$. So we get $a_F \tilde{\notin} G(e)$ for every $e \in E$ and thus we have $a_F \tilde{\notin} G$. Therefore (X, τ, E) is a ST_1 space. For any $e \in E$, if (X, τ_e) is a topological space then we have $a_F \in F$ and $b_G \in F^{\tilde{c}}$ while $b_G \in G$ and $a_F \in G^{\tilde{c}}$. So we get $a_F \in F(e)$ and $b_G \notin F(e)$ when $b_G \in G(e)$ and $a_F \notin G(e)$. □

Theorem 4.9. *Every complemental soft topological space is a ST_2 space.*

Proof. Let (X, τ, E) be a complemental soft topological space over X and any two $a_F, b_G \in S_E(X)$, where $a_F \neq b_G$, be given. From Theorem 4.3 and Lemma 4.4 $a_F \tilde{\in} H_1$ and $b_G \tilde{\notin} H_1$ or in a similar way $a_F \tilde{\in} H_2$ and $b_G \tilde{\notin} H_2$, when $H_1 \sqcap H_2 = \Phi$ such that $H_1, H_2 \in \tau$. So we get (X, τ, E) is a ST_2 space. □

Theorem 4.10. *Let (X, τ, E) be a complemental ST_2 space for each $e \in E$, then the topological space (X, τ_e) is a T_2 space.*

Proof. Let (X, τ, E) be a ST_2 space and soft open F, G taken from this space be given. It is also given that $a_F \neq b_G$ where $a_F \tilde{\in} F$ and $b_G \tilde{\in} G$. If $a_F \tilde{\in} F$ for every $e \in E$, then $a_F \tilde{\notin} F(e)^{\tilde{c}}$. So we get $b_G \tilde{\notin} F(e)$ for every $e \in E$ and thus we have $b_G \tilde{\notin} F$. In a similar way, if $b_G \tilde{\in} G$ for every $e \in E$, then $b_G \tilde{\notin} G(e)^{\tilde{c}}$. So we get $a_F \tilde{\notin} G(e)$ for every $e \in E$ and thus we have $a_F \tilde{\notin} G$. Since F and G are complemental soft set pairs in a common soft space so we have $F \sqcap G = \Phi$. It follows that for every $e \in E$ we get $a_F \tilde{\notin} G(e)$ and thus $a_F \tilde{\notin} G$. Therefore (X, τ, E) is a ST_2 space. For any $e \in E$, if (X, τ_e) is a topological space then we have $a_F \in F$ and $b_G \in F^{\tilde{c}}$ while $b_G \in G$ and $a_F \in G^{\tilde{c}}$,while $F \sqcap G = \Phi$. So we get $a_F \in F(e)$ and $b_G \notin F(e)$ when $b_G \in G(e)$ and $a_F \notin G(e)$. That is, (X, τ_e) is a T_2 space. □

Theorem 4.11. *Every complemental soft topological space is a soft regular space.*

Proof. Let (X, τ, E) be a complemental soft topological space over X and a non-empty soft open $H \in S_E(X)$, where $a_H \tilde{\in} H$ and soft open $F, G \in S_E(X)$ where $F \sqsubseteq G$ be given. From Theorem 4.3 we have $a_H \tilde{\notin} F$. From the given theorem we also have $G \sqcap H = \Phi$. From the Lemma 4.4 F is a complemental soft set, so F is also a soft closed and this completes the proof. □

Theorem 4.12. *Let (X, τ, E) be a soft topological space over X. Then the space is said to be a regular space if and only if every soft set belongs to this space is both open and closed.*

Proof. Let (X, τ, E) be a complemental soft topological space over X. Let a soft closed $F \sqsubseteq S_E(X)$ and a soft point $a_H \tilde{\in} F^{\tilde{c}}$ be given. When we take the Theorem 4.3 and the Lemma 4.4 into consideration we obviously have $F \sqsubseteq F$ and $F^{\tilde{c}} \sqsubseteq F^{\tilde{c}}$ so the proof is completed. □

Theorem 4.13. *Let (X, τ, E) be a complemental soft topological space over X , then the topological space (X, τ_e) is a regular space for every $e \in E$.*

Proof. Let (X, τ, E) be a complemental soft topological space over X. From Theorem 4.3 every soft set of the given space is both open and closed. Hence for every parameter of $e \in E$ we have a soft closed $F \in S_E(X)$ and a soft point $a_G \tilde{\in} F^{\tilde{c}}$. In this case we obtain $F \sqsubseteq F$ and $a_G \tilde{\in} F^{\tilde{c}}$ that is what we want to. □

Example 4.14. Let $E = \{a, b\}$ and $X = \{x, y\}$ and $F_1, F_2, ..., F_{14} \in S_E(X)$ be given as in Example 4.2. Then for each $e \in E$ topological spaces

$$\tau_a = \{\phi, X, \{x\}, \{y\}\}$$

and

$$\tau_b = \{\phi, X, \{x\}, \{y\}\}$$

are both regular spaces. Because when we take the definition of a regular space into consideration for the topological space τ_a it is clear to say that we have a closed $F = \{x\}$ while $y \notin F$, also an open $H = \{y\}$, and an open $G = \{x\}$ where $G \cap H = \phi$. By the similar way it is also possible for the topological space τ_b that we have a closed $F = \{x\}$ while $y \notin F$, also an open $H = \{y\}$ and an open $G = \{x\}$ where $G \cap H = \phi$.

Theorem 4.15. *Every complemental soft topological space is a ST_3 space.*

Proof. Let (X, τ, E) be a complemental soft topological space over X and a non-empty soft open $H \in S_E(X)$, where $a_H \tilde{\in} H$ and soft open $F, G \in S_E(X)$ where $F \sqsubseteq G$ be given. From Theorem 4.3 we have $a_H \tilde{\notin} F$. From the given theorem we also have $G \sqcap H = \Phi$. From the Lemma 4.4 F is a complemental soft set, so F is also a soft closed. Moreover let any two $a_F, b_G \in S_E(X)$, where $a_F \neq b_G$, be given. From Theorem 4.3 there exists a soft open $a_F \tilde{\in} H_1$ and $b_G \tilde{\notin} H_1$ or in a similar way $a_F \tilde{\in} H_2$ and $b_G \tilde{\notin} H_2$ such that $H_1, H_2 \in \tau$. So we get (X, τ, E) is a ST_1 space and this completes the proof. □

Theorem 4.16. *Let (X, τ, E) be a complemental ST_3 space for each $e \in E$, then the topological space (X, τ_e) is a T_3 space.*

Proof. Let (X, τ, E) be a complemental soft topological space over X. Every soft set of the space $S_E(X)$ is a soft open and soft closed because of Definition 4.1 . Therefore there must be a soft closed $H \in S_E(X)$ and a soft $a_G \tilde{\in} F^{\tilde{c}}$ for every parameter of $e \in E$. In this case we have $H \sqsubseteq H$ and $a_F \tilde{\in} H$. Moreover, let soft points $a_F \tilde{\in} H_1$ and $b_G \tilde{\in} H_2$ be given such that $a_F \neq b_G$. If $a_F \tilde{\in} H_1$ for every $e \in E$, then $G(b) \subseteq H_1(e)^c$. So we get $G(b) \tilde{\notin} H_1(e)$ and $b_G \tilde{\notin} H_1$ for every $e \in E$. In a similar way, if $b_G \tilde{\in} H_2$ for every $e \in E$, then $F(a) \subseteq H_2(e)^c$. So we get $F(a) \tilde{\notin} H_2(e)$ and $a_F \tilde{\notin} H_2$ for every $e \in E$. Hence (X, τ, E) is a ST_1 space. For any $e \in E$ if (X, τ_e) is a topological space, then $a_F \tilde{\in} H_1$ and $b_G \tilde{\in} H_1^{\tilde{c}}$ and also $b_G \tilde{\in} H_2$ and $a_F \tilde{\in} H_2^{\tilde{c}}$, so we get $F(a) \subseteq H_1(e)$, $G(b) \tilde{\notin} H_1(e)$ and $G(b) \subseteq H_2(e)$, hence $F(a) \tilde{\notin} H_2(e)$. Thus for every $e \in E$, topological space (X, τ_e) is T_1 and so the proof is completed. □

Example 4.17. Let $E = \{a, b\}$ and $X = \{x, y\}$ and $F_1, F_2, ..., F_{14} \in S_E(X)$ be given as in Example 4.2. Then for each $e \in E$ topological spaces

$$\tau_a = \{\phi, X, \{x\}, \{y\}\}$$

and

$$\tau_b = \{\phi, X, \{x\}, \{y\}\}$$

are both T_3 spaces. It is clear when we take the Theorems 4.8 and 4.16 into consideration.

Theorem 4.18. *Every complemental soft topological space is a soft normal space.*

Proof. Let (X, τ, E) be a complemental soft topological space over X and non-empty soft open $F_1, F_2 \in S_E(X)$, where $F_1 \sqcap F_2 = \Phi$ be given. From Theorem 4.3 and Lemma 4.4 we have soft open $G_1, G_2 \in S_E(X)$, where $F_1 \sqsubseteq G_1$ and $F_2 \sqsubseteq G_2$ such that $G_1 \sqcap G_2 = \Phi$ □

Theorem 4.19. *Let (X, τ, E) be a soft topological space over X and soft sets $F_1, F_2, ..., F_n \in S_E(X)$ be soft open and soft closed, then the topological space (X, τ, E) is a soft normal space.*

Proof. The proof is clear from Theorem 4.11. □

Theorem 4.20. *Let (X, τ, E) be a complemental soft topological space over X for each $e \in E$, then the topological space (X, τ_e) is a normal space.*

Proof. Let (X, τ, E) be a complemental soft topological space over X for each $e \in E$ and soft closed $F_1, F_2 \in S_E(X)$ be given such that $F_1 \sqcap F_2 = \Phi$. From Theorem 4.3 and Lemma 4.4 there must be soft open sets $G_1, G_2 \in S_E(X)$ such that $F_1 \sqsubseteq G_1$, $F_2 \sqsubseteq G_2$ and $G_1 \sqcap G_2 = \Phi$. Since the soft closed sets F_1, F_2 and the soft open sets G_1, G_2 are complemental soft sets therefore we have $F_1 \cap F_2 = \phi$ and so $G_1 \cap G_2 = \phi$. It follows that we have discoint F_i and discoint G_i satisfying the conditions given above in τ_{e_i} for every $e \in E$. Hence the topological space (X, τ_e) is a normal space. □

Example 4.21. Let $E = \{a, b\}$ and $X = \{x, y\}$ and $F_1, F_2, ..., F_{14} \in S_E(X)$ be given as in Example 4.2. Then for each $e \in E$ topological spaces

$$\tau_a = \{\phi, X, \{x\}, \{y\}\}$$

and

$$\tau_b = \{\phi, X, \{x\}, \{y\}\}$$

are both normal spaces. Because when we take the definition of a normal space into consideration for the topological space τ_a it is clear to say that we have closed sets $F_1 = \{x\}$, $F_2 = \{y\}$ where $F_1 \cap F_2 = \phi$ and also open sets $G_1 = \{x\}$, $G_2 = \{y\}$ where $G_1 \cap G_2 = \phi$ such that $F_1 \subseteq G_1$ and $F_2 \subseteq G_2$. It is also the case for the topological space τ_b that we have closed sets $F_1 = \{x\}$, $F_2 = \{y\}$ where $F_1 \cap F_2 = \phi$ and also open sets $G_1 = \{x\}$, $G_2 = \{y\}$ where $G_1 \cap G_2 = \phi$ such that $F_1 \subseteq G_1$ and $F_2 \subseteq G_2$.

Theorem 4.22. *Every complemental soft topological space is a ST_4 space.*

Proof. Let (X, τ, E) be a complemental soft topological space over X for each $e \in E$ and soft closed $F_1, F_2 \in S_E(X)$ be given such that $F_1 \sqcap F_2 = \Phi$. Because of Theorem 4.3 and Lemma 4.4 there must be soft open sets $G_1, G_2 \in S_E(X)$,where $F_1 \subseteq G_1$ and $F_2 \subseteq G_2$ such that $G_1 \sqcap G_2 = \Phi$. Moreover let any two $a_F, b_G \in S_E(X)$, where $a_F \neq b_G$, be given. From Theorem 4.3 there exists a soft open $a_F \tilde{\in} H_1$ and $b_G \tilde{\notin} H_1$ or in a similar way $a_F \tilde{\in} H_2$ and $b_G \tilde{\notin} H_2$ such that $H_1, H_2 \in \tau$. So we get (X, τ, E) is a ST_1 space and this completes the proof. □

Theorem 4.23. *Let (X, τ, E) be a complemental ST_4 space for each $e \in E$, then the topological space (X, τ_e) is a T_4 space.*

Proof. Let (X, τ, E) be a complemental soft topological space over X for each $e \in E$ and soft closed $F_1, F_2 \in S_E(X)$ where $F_1 \sqcap F_2 = \Phi$ be given. From Theorem 4.3 and Lemma 4.4 there must be soft open $G_1, G_2 \in S_E(X)$ such that $G_1 \sqcap G_2 = \Phi$, where $F_1 \sqsubseteq G_1$ and $F_2 \sqsubseteq G_2$. Since the soft closed F_1, F_2 and the soft open G_1, G_2 are all complemental soft sets so, $F_1 \cap F_2 = \phi$ and $G_1 \cap G_2 = \phi$. Hence we have closed sets $F_i \in \tau_{e_i}$ and open sets $G_i \in \tau_{e_i}$ satisfying the condition above for each $e \in E$.That is, the space (X, τ_e) is normal. Moreover let any two soft points $a_F \tilde{\in} F$ and $b_G \tilde{\in} G$, where $a_F \neq b_G$ be given. If $a_F \tilde{\in} F$ for every $e \in E$ then $a_F \tilde{\notin} F(e)^{\tilde{c}}$ and so we get $b_G \tilde{\notin} F(e)$ and then $b_G \tilde{\notin} F$.In a similar way, if $b_G \tilde{\in} G$ for every $e \in E$ then $b_G \tilde{\notin} G(e)^{\tilde{c}}$ and so we get $a_F \tilde{\notin} G(e)$ and then $a_F \tilde{\notin} G$. Hence the space (X, τ, E) is a ST_1 space. For any $e \in E$ when (X, τ_e) is a topological space and $a_F \in F$ and $b_G \in F^{\tilde{c}}$, we have $b_G \in F$ and $a_F \in G^{\tilde{c}}$ so we get $a_F \in F(e)$, $b_G \notin F(e)$, while $b_G \in G(e)$ and $a_F \notin G(e)$. So for every $e \in E$ the topological space (X, τ_e) is a T_1 space, which completes the proof. □

Example 4.24. Let $E = \{a, b\}$ and $X = \{x, y\}$ and $F_1, F_2, ..., F_{14} \in S_E(X)$ be given as in Example 4.2. Then for each $e \in E$ topological spaces

$$\tau_a = \{\phi, X, \{x\}, \{y\}\}$$

and

$$\tau_b = \{\phi, X, \{x\}, \{y\}\}$$

are both T_4 spaces. It is clear when we take the Theorems 4.8 and 4.20 into consideration.

Theorem 4.25. *Let (X, τ, E) be a complemental soft topological space over X then the followings are equivalent.*
i. The soft topological space (X, τ, E) is a soft normal space.
ii. If F is a soft closed and H is a soft open, where $F \sqsubseteq H$ then there exists a soft open G such that $F \sqsubseteq G \sqsubseteq \overline{G} \sqsubseteq H$.

Proof. $i \Rightarrow ii$: Let a soft normal (X, τ, E), a soft closed F and a soft open H be given, where $F \sqsubseteq H$. Since $H^{\tilde{c}}$ is soft closed and $F \sqcap H^{\tilde{c}} = \Phi$, so we have $F \sqsubseteq G$ and $H^{\tilde{c}} \sqsubseteq H_1$, where G, H_1 are soft open and discoint sets. Here we also have $G \sqsubseteq H_1^{\tilde{c}} \sqsubseteq H$ and since $H_1^{\tilde{c}}$ is soft closed, then it follows that $\overline{G} \sqsubseteq H_1^{\tilde{c}}$. So we get $F \sqsubseteq G \sqsubseteq \overline{G} \sqsubseteq H$.
$ii \Leftarrow i$:If the soft sets F_1 and F_2 are discoint then $F_1 \sqsubseteq F_2^{\tilde{c}}$ and $F_2^{\tilde{c}}$ is soft open, because of assumption we have a soft open G such that $F_1 \sqsubseteq G \sqsubseteq \overline{G} \sqsubseteq F_2^{\tilde{c}}$ and we obtain the discoint soft sets G and $(\overline{G})^{\tilde{c}}$ such that $F_1 \sqsubseteq G$ and $F_2 \sqsubseteq (\overline{G})^{\tilde{c}}$. Hence (X, τ, E) is a soft normal space. □

Theorem 4.26. *Let (X, τ, E) be a complemental soft topological space over X then it is a soft normal if and only if there exists at least one soft open H such that $F \sqsubseteq H \sqsubseteq \overline{H} \sqsubseteq G$, where F is soft closed, G is soft open and $F \sqsubseteq G$.*

Proof. Let (X, τ, E) be a complemental soft topological space over X and a soft closed $F \in S_E(X)$, a soft open $G \in S_E(X)$ where $F \sqsubseteq G$ be given. In this case $G^{\tilde{c}}$ is soft closed and $F \sqcap G^{\tilde{c}} = \Phi$. Because of assumption, when $F \sqsubseteq H$ and $G^{\tilde{c}} \sqsubseteq K$ we have $H \sqcap K = \Phi$. Since $H \sqcap K = \Phi$, then $H \sqsubseteq K^{\tilde{c}}$. But $K^{\tilde{c}}$ is soft closed so we have,

$$F \sqsubseteq H \sqsubseteq \overline{H} \sqsubseteq K^{\tilde{c}} \sqsubseteq G$$

then we get

$$F \sqsubseteq H \sqsubseteq \overline{H}$$

On the other hand let's suppose that for every soft closed F , soft open G and a soft open H we have $F \sqsubseteq G$ and

$$F \sqsubseteq H \sqsubseteq \overline{H} \sqsubseteq G$$

Let's say F_1 and F_2 are soft closed discoint sets, in this case $F_1 \sqsubseteq F_2^{\tilde{c}}$ when $F_2^{\tilde{c}}$ is soft open. Therefore there must be a soft open H such that

$$F_1 \sqsubseteq H \sqsubseteq \overline{H} \sqsubseteq F_2^{\tilde{c}}$$

But $F_2 \sqsubseteq (\overline{H})^{\tilde{c}}$ and $H \sqcap (\overline{H})^{\tilde{c}} = \Phi$. Thus we get $H \sqcap (\overline{H})^{\tilde{c}} = \Phi$ when $F_1 \sqsubseteq H$ and $F_2 \sqsubseteq (\overline{H})^{\tilde{c}}$ and this completes the proof. □

Corollary 4.27. *As a natural result of the Theorems 4.5, 4.7, 4.9 , 4.15 and 4.22 we have the following*

$$ST_0 \Longleftrightarrow ST_1 \Longleftrightarrow ST_2 \Longleftrightarrow ST_3 \Longleftrightarrow ST_4$$

Corollary 4.28. *As a natural result of the Theorems 4.6, 4.8, 4.10, 4.16 and 4.23 following statement is true. Let (X, τ, E) be a complemental ST_i where $i = 0, 1, 2, 3, 4$ then the space (X, τ_e) is T_i where $i = 0, 1, 2, 3, 4$ for each $e \in E$.*

5. Conclusion

Topology is an important area of a mathematics with many applications in the domains of computer science and physical sciences. Soft topology is a relatively new and promising domain which can lead to the development of new mathematical models and innovative approaches that will significantly contribute to the solutions of complex problems in natural sciences.

This paper continues the study of the theory of soft topological spaces and mainly soft separation axioms in complemental soft topological spaces. In the third section we gave the basic theorems of soft separation axioms in a soft topological spaces. We made some corrections of notations and also made the proof of some theorems by taking Cagman [3]'s work into account. In the forth section of the paper we investigate a new soft topological space called complemental soft topological spaces and we developed two statements named as Corollary 4.27 and Corollary 4.28.

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