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SP-Fuzzy Soft Ideals in Semigroups

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Abstract. In this paper, the definition of a new concept which is a member of the class $(\tilde{U,P})$ and which is referred to as UP-fuzzy soft subset of a soft set on the class (U, P) is introduced, where $(\tilde{U,P})$ denotes the fuzzy soft class and (U, P) denotes the soft class with the universal set U and the set of parameters P. We give the definitions of the complement and α-level soft set of a UP-fuzzy soft subset of a soft set. It is demonstrated that UP-fuzzy soft subsets provide De Morgan rules for restricted union and restricted intersection. Furthermore, considering a semigroup S as an universal set, this paper presents some new algebraic notions which are called SP-fuzzy soft subsemigroup and SP-fuzzy soft left (right, bi-, quasi, interior) ideal of a soft semigroup. We examine some basic properties such as restricted union, extended union, restricted intersection, extended intersection and product of the families of SP-fuzzy soft subsemigroups and SP-fuzzy soft left (right, bi-, quasi, interior) ideals. It is obtained that the restricted intersection of the family of SP-fuzzy soft subsemigroups is a SP-fuzzy soft subsemigroup of the restricted intersection of the family of soft sets. Moreover it is indicated that an α-level soft set of a SP-fuzzy soft subset is a soft subsemigroup for all $\alpha \in [0, 1]$ if and only if the SP-fuzzy soft subset is a SP-fuzzy soft subsemigroup.

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1. Introduction

The soft set theory originally is proposed by Molodtsov as a mathematical method to deal with uncertainties [26]. In [24], Maji et. al. present some certain soft binary operations and some basic definitions on soft sets. In [12] Ali et. al. discuss some results in [24] and they give some new notions. In [25] Maji et. al. and in [32] Yang et. al. study on fuzzy soft sets. Ahmad and Kharal discuss their studies and define arbitrary fuzzy soft operations [1]. Applications of soft sets to semigroups have been attracted the attention of many researchers. Ali et. al. derive some properties of soft ideals, soft quasi-ideals and soft bi-ideals over a semigroup S [13]. Feng et al. investigate the application of soft binary relations in semigroup theory [8]. The notions of fuzzy soft semigroup and fuzzy soft (left, right) ideal are given by Yang [31]. Kazancı and Yılmaz introduce the concepts of ϵ-soft set and q-soft set [16].

In this paper, based on the sense of the definition of fuzzy sets, we propose a fuzzy soft set of a crisp soft set as new

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concept. We introduce some new algebraic notions which are called SP-fuzzy soft subsemigroup and SP-fuzzy soft left (right, bi-, quasi, interior) ideal of a soft semigroup.

2. Preliminaries

Let S be a semigroups. For any non-empty subsets A and B of the semigroup S, the set AB is defined by the set $\{ab \mid a \in A, b \in B\}$. If $A = \emptyset$ or $B = \emptyset$, then $AB = \emptyset$. By a subsemigroup of S, we mean a non-empty subset A of S such that $A^2 \subseteq A$, and by a left (right) ideal of S, we mean a non-empty subset A of S such that $SA \subseteq A$ ($AS \subseteq A$). By two-sided ideal (or ideal) of S, we mean a non-empty subset of S which is both a left and right ideal of S. A non-empty subset A of S is called a bi-ideal of S if $ASA \subseteq A$. A subsemigoup A of S is called a bi-ideal of S if $ASA \subseteq A$. If A is a left (right, two-sided or quasi) ideal of S, then A is a bi-ideal of S. A non-empty subset A of S is called an interior ideal if $SAS \subseteq A$. A non-empty subset A of S is called a quasi ideal of S if $SA \cap AS \subseteq A$. If any subset A of S is a quasi ideal, then A is a subsemigroup of S. The reader will find more information about semigroups in [11, 22, 23].

2.1. **Fuzzy Subsets of Semigroups.** A function f from a nonempty set S to the unit interval $[0, 1]$ is called a fuzzy subset of S [33]. From the date the fuzzy set theory was introduced, it has found wide repercussions all over the world. Fuzzy set theory and its extensions are still subject to many researches [1–3, 5–7, 10, 14, 15, 18, 20, 21, 25, 27, 29–32].

Let f, g be fuzzy subsets of S, then $f \subseteq g$ means that $f(a) \leq g(a)$ for all $a, \in S$. For $t \in [0, 1]$, the set $f_t = \{a \in S \mid f(a) \geq t\}$ is called the level set of f. Let f and g be two fuzzy subsets of S, then the followings are the operations on fuzzy subsets: For all $(f_i)_{i\in\Lambda}, (g_i)_{i\in\Lambda}, f, g \in \mathcal{F}(S)$ and $x \in S$

$$(\bigwedge_{i\in\Lambda} f_i)(x) = \bigwedge_{i\in\Lambda} f_i(x),$$
$$(\bigvee_{i\in\Lambda} f_i)(x) = \bigvee_{i\in\Lambda} f_i(x),$$

Let A be a subset of S, then χ_A is denoted the characteristic function of A and defined as

$$\chi_A(x) = \begin{cases} 1 & , \quad x \in A \\ 0 & , \quad x \notin A \end{cases}$$

$$(f \cdot g)(x) = \begin{cases} \bigvee_{x=a.b} f(a) \wedge g(b) & , \quad \exists a, b \in \mathrm{S} \quad \text{such that} \quad x = a.b \\ 0 & , \quad \text{otherwise.} \end{cases}$$

for all $x \in S$, respectively.

Definition 2.1 ([20, 21, 27]). *Let S be an semigroup and f be a fuzzy subset of S.*

(i) *f is called a fuzzy subsemigroup of S if $f(ab) \geq f(a) \wedge f(b)$ for all $a, b \in S$.*
(ii) *f is called a fuzzy left (right) ideal of S if $f(ab) \geq f(b)$ ($f(ab) \geq f(a)$) for all $a, b \in S$. f is called a fuzzy two-sided ideal S if it is both a fuzzy left and fuzzy right ideal of S.*
(iii) *f is called a fuzzy generalized bi-ideal of S if $f((ab)c) \geq f(a) \wedge f(c)$ for all $a, b, c \in S$.*
(iv) *A fuzzy subsemigroup f is called a fuzzy bi-ideal of S if $f((ab)c) \geq f(a) \wedge f(c)$ for all $a, b, c \in S$.*
(v) *f is called a fuzzy quasi ideal of S if $(f \circ 1) \wedge (1 \circ f) \leq f$.*
(vi) *f is called a fuzzy interior ideal of S if $f((ax)b) \geq f(x)$ for all $a, b, x \in S$.*

2.2. ***Soft Sets.*** We give some known and useful definitions and notations on soft sets studied in references [19, 24–26, 31].

Definition 2.2 ([26]). *Let U be an initial universe set and P be a set of parameters. The power set of U is denoted by $\mathcal{P}(U)$ and A is a subset of P. A pair (F, A) is called a soft set over U where F is a mapping given by $F : A \to \mathcal{P}(U)$. The pair (U, P) denotes the collection of all soft sets on U with the attributes from P and is called a soft class [19]. In this paper, we consider a soft class (S, P) with a semigroup S as the initial universe.*

Definition 2.3 ([26]). *Let (F, A) and (G, B) be two soft sets over S, (F, A) is called a soft subset of (G, B), denoted by $(F, A) \subseteq (G, B)$, if (i) $A \subseteq B$, (ii)$F(x) \subseteq G(x)$ for each $x \in A$.*

Definition 2.4 ([4, 9, 12, 17]). *Let $\{(F_i, A_i) \mid i \in \Lambda\}$ be a family of soft sets in a soft class (S, P). Then*

(i) *The restricted intersection of the family* $\{(F_i, A_i) \mid i \in \Lambda\}$*, denoted by* $(\bigcap_r)_{i\in\Lambda}(F_i, A_i)$*, is the soft set* (F, A) *defined as:* $A = \bigcap_{i\in\Lambda} A_i$*,* $F(x) = \bigcap_{i\in\Lambda} F_i(x)$ $(\forall x \in A)$*,*

(ii) *The extended intersection of the family* $\{(F_i, A_i) \mid i \in \Lambda\}$*, denoted by* $(\bigcap_e)_{i\in\Lambda}(F_i, A_i)$*, is the soft set* (F, A) *defined as:* $A = \bigcup_{i\in\Lambda} A_i$*,* $F(x) = \bigcap_{i\in\Lambda(x)} F_i(x)$ $(\forall x \in A)$ *where* $\Lambda(x) = \{i \mid x \in A_i\}$*,*

(iii) *The restricted union of the family* $\{(F_i, A_i) \mid i \in \Lambda\}$*, denoted by* $(\bigcap_r)_{i\in\Lambda}(F_i, A_i)$*, is the soft set* (F, A) *defined as:* $A = \bigcap_{i\in\Lambda} A_i$*,* $F(x) = \bigcup_{i\in\Lambda} F_i(x)$ $(\forall x \in A)$*.*

(iv) *The extended union of the family* $\{(F_i, A_i) \mid i \in \Lambda\}$*, denoted by* $(\bigcap_e)_{i\in\Lambda}(F_i, A_i)$*, is the soft set* (F, A) *defined as:* $A = \bigcup_{i\in\Lambda} A_i$*,* $F(x) = \bigcup_{i\in\Lambda(x)} F_i(x)$ $(\forall x \in A)$ *where* $\Lambda(x) = \{i \mid x \in A_i\}$*,*

Definition 2.5 ([4, 9, 17, 24])**.** *Let* $\{(F_i, A_i) \mid i \in \Lambda\}$ *be a family of soft sets in a soft class* (S, P)*. Then*

(i) *The* $\wedge$*-intersection of the family* $\{(F_i, A_i) \mid i \in \Lambda\}$*, denoted by* $\bigwedge_{i\in\Lambda}(F_i, A_i)$*, is the soft set* (F, A) *defined as:* $A = \prod_{i\in\Lambda} A_i$*,* $F((x_i)_{i\in\Lambda}) = \bigcap_{i\in\Lambda} F_i(x_i)$ $(\forall (x_i)_{i\in\Lambda} \in A)$*,*

(ii) *The* $\vee$*-union of the family* $\{(F_i, A_i) \mid i \in \Lambda\}$*, denoted by* $\bigvee_{i\in\Lambda}(F_i, A_i)$*, is the soft set* (F, A) *defined as:* $A = \prod_{i\in\Lambda} A_i$*,* $F((x_i)_{i\in\Lambda}) = \bigcup_{i\in\Lambda} F_i(x_i)$ $(\forall (x_i)_{i\in\Lambda} \in A)$*,*

(iii) *The product of the family* $\{(F_i, A_i) \mid i \in \Lambda\}$*, denoted by* $\prod_{i\in\Lambda}(F_i, A_i)$*, is the soft set* (F, A) *defined as:* $A = \prod_{i\in\Lambda} A_i$*,* $F((x_i)_{i\in\Lambda}) = \prod_{i\in\Lambda} F_i(x_i)$ $(\forall (x_i)_{i\in\Lambda} \in A)$*.*

Definition 2.6 ([31])**.** *Let* (F, A) *be a soft set in a soft class* (S, P)*.*

(i) (F, A) *is said to be a soft semigroup over* S *if and only if* $F(x)$ *is a subsemigroup of* S *for all* $x \in A$*.*

(ii) (F, A) *is said to be a soft left (right) ideal over* S *if and only if* $F(x)$ *is a left (right) ideal of* S *for all* $x \in A$*.* (F, A) *is said to be a soft ideal over* S *if and only if* $F(x)$ *is both a left and a right ideal of* S *for all* $x \in A$*.*

(iii) (F, A) *is said to be a soft (generalized) bi-ideal over* S *if and only if* $F(x)$ *is a (generalized) bi-ideal of* S *for all* $x \in A$*.*

(iv) (F, A) *is said to be a soft interior ideal over* S *if and only if* $F(x)$ *is an interior ideal of* S *for all* $x \in A$*.*

(v) (F, A) *is said to be a soft quasi ideal over* S *if and only if* $F(x)$ *is a quasi ideal of* S *for all* $x \in A$*.*

2.3. ***Fuzzy Soft Sets.*** We give some known and useful definitions and notations on fuzzy soft sets studied in references [1, 2, 5, 24, 25, 28, 31].

Definition 2.7 ([25])**.** *Let* U *be an initial universe set and* P *be a set of parameters. A pair* (f, E) *is called a fuzzy soft set over* U*, where* $f : E \to F(U)$ *is a mapping, where* $F(U)$ *is denote the set of all fuzzy sets of* U*. The pair* $(\tilde{U,P})$ *denotes the collection of all fuzzy soft sets on* U *with the attributes from* P *and is called a fuzzy soft class [1]. In this paper, we consider a fuzzy soft class* $(\tilde{U,P})$ *with a semigroup* S *as the initial universe.*

Definition 2.8 ([2])**.** *Let* (f, E) *be a fuzzy soft set over* S*. For each* $\alpha \in [0, 1]$*, the set* $(f, E)_\alpha = (f_\alpha, E)$ *is called an* α*-level set of* (f, E)*, where* $f_\alpha(a) = \{x \in S \mid f(a)(x) \geq \alpha\}$ *for each* $a \in E$*. Obviously,* $(f, E)_\alpha$ *is a soft set over* S*.*

Definition 2.9 ([25])**.** *Let* (f, E) *and* (g, H) *be two fuzzy soft sets over* S*,* (f, E) *is called a fuzzy soft subset of* (g, H)*, denoted by* $(f, E)\tilde{\subseteq}(g, H)$*, if (i)* $E \subseteq H$*, (ii) for each* $a \in E$*,* $f(a) \leq g(a)$*.*

Definition 2.10 ([1, 24, 25])**.** *Let* $\{(f_i, E_i) \mid i \in \Lambda\}$ *be a family of fuzzy soft sets in a fuzzy soft class* $(\tilde{S,P})$*. Then*

(i) *The restricted intersection of the family* $\{(f_i, E_i) \mid i \in \Lambda\}$*, denoted by* $(\tilde{\bigwedge}_r)_{i\in\Lambda}(f_i, E_i)$*, is a fuzzy soft set* (f, E)*,* $E = \bigcap_{i\in\Lambda} E_i$ *and for all* $x \in E$*,* $f(x) = \bigwedge_{i\in\Lambda} f_i(x)$*.*

(ii) *The extended intersection of the family* $\{(f_i, E_i) \mid i \in \Lambda\}$*, denoted by* $(\tilde{\bigwedge}_e)_{i\in\Lambda}(f_i, E_i)$*, is a fuzzy soft set* (f, E)*,* $E = \bigcup_{i\in\Lambda} E_i$ *and for all* $x \in E$*,* $f(x) = \bigwedge_{i\in\Lambda(x)} f_i(x)$ *where* $\Lambda(x) = \{i | x \in E_i\}$*,*

(iii) *The restricted union of the family* $\{(f_i, E_i) \mid i \in \Lambda\}$*, denoted by* $(\tilde{\bigvee}_r)_{i\in\Lambda}(f_i, E_i)$*, is a fuzzy soft set* (f, E)*,* $E = \bigcap_{i\in\Lambda} E_i$ *and for all* $x \in E$*,* $f(x) = \bigvee_{i\in\Lambda} f_i(x)$*.*

(iv) *The extended union of the family* $\{(f_i, E_i) \mid i \in \Lambda\}$*, denoted by* $(\tilde{\bigvee}_e)_{i\in\Lambda}(f_i, E_i)$*, is a fuzzy soft set* (f, E)*,* $E = \bigcup_{i\in\Lambda} E_i$ *and for all* $x \in E$*,* $f(x) = \bigvee_{i\in\Lambda(x)} f_i(x)$ *where* $\Lambda(x) = \{i | x \in E_i\}$*.*

Definition 2.11 ([5, 25])**.** *Let* $\{(f_i, E_i) \mid i \in \Lambda\}$ *be a family of fuzzy soft sets in a fuzzy soft class* $(\tilde{S,P})$*. Then*

(i) *The fuzzy* $\wedge$*-intersection of the family* $\{(f_i, E_i) \mid i \in \Lambda\}$*, denoted by* $\tilde{\bigwedge}_{i\in\Lambda}(f_i, E_i)$*, is the soft set* (f, E) *defined as:* $E = \prod_{i\in\Lambda} E_i$*,* $f((x_i)_{i\in\Lambda}) = \bigwedge_{i\in\Lambda} f_i(x_i)$ $(\forall (x_i)_{i\in\Lambda} \in E)$*,*

(ii) *The fuzzy* $\vee$*-union of the family* $\{(f_i, E_i) \mid i \in \Lambda\}$*, denoted by* $\tilde{\bigvee}_{i\in\Lambda}(f_i, E_i)$*, is the soft set* (f, E) *defined as:* $E = \prod_{i\in\Lambda} E_i$*,* $f((x_i)_{i\in\Lambda}) = \bigvee_{i\in\Lambda} f_i(x_i)$ $(\forall (x_i)_{i\in\Lambda} \in E)$*.*

(iii) *The product of the family* $\{(f_i, E_i) \mid i \in \Lambda\}$*, denoted by* $\tilde{\prod}_{i\in\Lambda}(f_i, E_i)$*, is a fuzzy soft set* (f, E)*,* $E = \prod_{i\in\Lambda} E_i$ *and,* $f((x_i)_{i\in\Lambda}) = \bigvee_{\substack{J\subseteq\Lambda \\ J \text{ is finite}}} (\bigwedge_{j\in J} f_j(x_j))$*.*

Definition 2.12. *Let* $(f, E_1), (g, E_2)$ *be fuzzy soft sets in a fuzzy soft class* $(\tilde{S, P})$*. Then the fuzzy product of them , denoted by* $(f, E_1)\tilde{\times}(g, E_2)$*, is the soft set* (h, C) *defined as:* $C = E_2 \times E_1$*,* $h(a, b) = f(a) \cdot g(b)$ *for all* $a \in E_1, b \in E_2$*.*

Definition 2.13 ([31])**.** *Let* (f, E) *be a fuzzy soft set over* S*,* (f, E) *is called a fuzzy soft semigroup if and only if* $f(a)$ *is a fuzzy subsemigroup over* S *for each* $a \in E$*.*

Definition 2.14 ([31])**.** *Let* (f, E) *be a fuzzy soft set over* S*,* (f, E) *is called a fuzzy soft left (right) ideal if and only if* $f(a)$ *is a fuzzy left (right) ideal over* S *for each* $a \in E$*.*

Definition 2.15 ([31])**.** *Let* (f, E) *be a fuzzy soft set over* S*,* (f, E) *is called a fuzzy soft ideal if and only if* (f, E) *is both a fuzzy soft left and a fuzzy soft right ideal over* S*.*

Definition 2.16 ([28])**.** *Let* (f, E) *be a fuzzy soft set over* S*, then*

(i) (f, E) *is called a fuzzy soft (generalized) bi-ideal if and only if* (f, E) *is a fuzzy (generalized) bi-ideal over* S*.*
(ii) (f, E) *is called a fuzzy soft quasi ideal if and only if* (f, E) *is a fuzzy quasi ideal over* S*.*
(iii) (f, E) *is called a fuzzy soft interior ideal if and only if* (f, E) *is a fuzzy interior ideal over* S*.*

3. Main Results

Let μ be a membership function of A. Then A is a ”crisp” set and we sense μ as a ”fuzzy” concept. In the following definition, inspiration of that point, we propose a fuzzy soft set of a crisp soft set as new concept of a member of the class $(\tilde{U, P})$.

Definition 3.1. *Let* (F, A) *be a soft set in a soft class* (U, P) *and* (f, E) *be a fuzzy soft set in the fuzzy soft class* $(\tilde{U, P})$*. Then* (f, E) *is said to be a* UP*-fuzzy soft subset of* (F, A)*, denoted by* $(f, E) \subseteq_{UP} (F, A)$*, if* $E \subseteq A$ *and* $f(x)$ *is a fuzzy subset of* $F(x)$ *for all* $x \in E$*.*

Example 3.2. *Let* U *be the set of houses* h_1, h_2, h_3, h_4, h_5*. Suppose that Suppose that* $P = \{e_1 = \text{beautiful}, e_2 = \text{cheap}, e_3 = \text{with a garden}, e_4 = \text{in green surroundings}, e_5 = \text{with a sea view}, e_6 = \text{in a quiet neighborhood}, e_7 = \text{wooden}, e_8 = \text{within the city center}\}$ *and* $E = \{e_1, e_2, e_6\}, A = \{e_1, e_2, e_3, e_4, e_6\}$*.*
$F(e_1) = \{h_1, h_2, h_3, h_4, h_5\}$,
$F(e_2) = \{h_1, h_3, h_5\}$,
$F(e_3) = \{h_1, h_2, h_4\}$,
$F(e_4) = \{h_2, h_3, h_4, h_5\}$,
$F(e_6) = \{h_1, h_2, h_3, h_4, h_5\}$.
$f(e_1) = \{(h_1, 0, 7), (h_2, 0, 6), (h_3, 0, 5), (h_4, 0, 8), (h_5, 1)\}$,
$f(e_2) = \{(h_1, 1), (h_3, 0, 5), (h_5, 0, 7)\}$,
$f(e_6) = \{(h_1, 0, 5), (h_2, 0, 6), (h_3, 0, 8), (h_4, 0, 8), (h_5, 0, 7)\}$.
Then $(f, E) = \{(\text{beautiful houses}, f(e_1)), (\text{cheap houses}, f(e_2)), (\text{houses in a quiet neighborhood}, f(e_6))\}$*.*

Definition 3.3. *Let* (f, E) *be a* UP*-fuzzy soft subset of* (F, A)*.* (F_α, E) *called* α*-level soft subset of* (f, E)*, where* $F_\alpha : E \to \mathcal{P}(U)$ *is defined by* $F_\alpha(x) = \{a \in F(x) | f(x)(a) \geq \alpha\}$ *for all* $x \in E$*.*

Definition 3.4. *Let* (f, E) *be a* UP*-fuzzy soft subset of* (F, A)*. Then the* UP*-fuzzy soft subset* $(f, E)^c := (f^c, E)$ *of* (F, A) *is called the complement of* (f, E)*, where for any* $x \in E$*,* $f^c(x) : F(x) \longrightarrow [0, 1]$ *is defined by* $f^c(x)(a) = 1 - f(x)(a)$ *for all* $a \in F(x)$*.*

Theorem 3.5. Let $\{(f_i, E_i) \mid i \in \Lambda\}$ be the family of UP-fuzzy soft subsets of (F, A). Then De Morgan rules are provided for restricted intersection and union, i.e., $((\tilde{\bigwedge}_r)_{i\in\Lambda}(f_i, E_i))^c = (\tilde{\bigvee}_r)_{i\in\Lambda}(f_i, E_i)^c$ and $((\tilde{\bigvee}_r)_{i\in\Lambda}(f_i, E_i))^c = (\tilde{\bigwedge}_r)_{i\in\Lambda}(f_i, E_i)^c$.

Proof. *Let* $(\tilde{\bigwedge}_r)_{i\in\Lambda}(f_i, E_i) = (f, E)$ *and* $(\tilde{\bigvee}_r)_{i\in\Lambda}(f_i, E_i)^c = (g, B)$*. Then* $E = \bigcap_{i\in\Lambda} E_i = B$ *and* $f(x) = \bigwedge_{i\in\Lambda} f_i(x)$ *and* $g(x) = \bigvee_{i\in\Lambda} f_i^c(x)$ *for any* $x \in E$*. Thus for any* $x \in E$*,* $f^c(x)(a) = 1 - f(x)(a) = 1 - (\bigwedge_{i\in\Lambda} f_i(x))(a) = 1 - (\bigwedge_{i\in\Lambda} f_i(x)(a)) = \bigvee_{i\in\Lambda}(1 - f_i(x)(a)) = \bigvee_{i\in\Lambda}(f_i^c(x)(a)) = (\bigvee_{i\in\Lambda} f_i^c(x))(a) = g(x)(a)$ *for all* $a \in F(x)$*. Thus we obtain* $((\tilde{\bigwedge}_r)_{i\in\Lambda}(f_i, E_i))^c = (\tilde{\bigvee}_r)_{i\in\Lambda}(f_i, E_i)^c$*. The proof is similar for the equation* $((\tilde{\bigvee}_r)_{i\in\Lambda}(f_i, E_i))^c = (\tilde{\bigwedge}_r)_{i\in\Lambda}(f_i, E_i)^c$*.*

Definition 3.6. *Let S be a semigroup and let (F, A) be a soft subsemigroup in a soft class (S, P) and (f, E) be a fuzzy soft set in the fuzzy soft class $(\tilde{S}, P)$.*

(i) *(f, E) is called a SP-fuzzy soft subsemigroup of (F, A) if $E \subseteq A$ and $f(x)$ is a fuzzy subsemigroup of $F(x)$ for all $x \in E$.*
(ii) *(f, E) is called a SP-fuzzy soft left (right) ideal of (F, A) if $E \subseteq A$ and $f(x)$ is a fuzzy left (right) ideal of $F(x)$ for all $x \in E$.*
(iii) *(f, E) is called a SP-fuzzy soft (generalized) bi-ideal of (F, A) if $E \subseteq A$ and $f(x)$ is a (generalized) fuzzy bi-ideal of $F(x)$ for all $x \in E$.*
(iv) *(f, E) is called a SP-fuzzy soft quasi ideal of (F, A) if $E \subseteq A$ and $f(x)$ is a fuzzy quasi ideal of $F(x)$ for all $x \in E$.*
(v) *(f, E) is called a SP-fuzzy soft interior ideal of (F, A) if $E \subseteq A$ and $f(x)$ is a fuzzy interior ideal of $F(x)$ for all $x \in E$.*

Theorem 3.7. Let (F_1, A_1) and (F_2, A_2) be soft sets in a soft class (S, P) and $(F_1, A_1) \subseteq (F_2, A_2)$. If (f, E) is SP-fuzzy soft subsemigroup (left ideal, right ideal, bi-ideal, interior ideal, quasi ideal) of (F_2, A_2), then it is a SP-fuzzy soft subsemigroup (left ideal, right ideal, bi-ideal, interior ideal, quasi ideal) of (F_1, A_1).
Proof. It is straightforward.

Theorem 3.8. Let (f, E) be a SP-fuzzy soft subset of (F, A). Then

(i) (F_α, E) is a soft subsemigroup for all $\alpha \in [0, 1]$ if and only if (f, E) is SP-fuzzy soft subsemigroup of a soft subsemigroup (F, A).
(ii) (F_α, E) is a soft left (right) ideal for all $\alpha \in [0, 1]$ if and only if (f, E) is SP-fuzzy soft left (right) ideal of a soft left (right) ideal (F, A).
(iii) (F_α, E) is a soft (generalized) bi-ideal for all $\alpha \in [0, 1]$ if and only if (f, E) is (generalized) SP-fuzzy soft bi-ideal of a (generalized) soft bi-ideal (F, A).
(iv) (F_α, E) is a soft interior ideal for all $\alpha \in [0, 1]$ if and only if (f, E) is SP-fuzzy soft interior ideal of a soft interior ideal (F, A).

Proof.

(i) *Suppose that (F_α, E) is a soft subsemigroup for all $\alpha \in [0, 1]$. Let $x \in E$ and $\alpha := f(x)(a) \wedge f(x)(b)$ for any $a, b \in F(x)$. So $f(x)(a) \geq \alpha$ and $f(x)(b) \geq \alpha$. Thus $a, b \in F_\alpha(x)$. $ab \in F_\alpha(x)$ since $F_\alpha(x)$ is subgroup of S for all $x \in E$. So $f(x)(ab) \geq \alpha$, i.e, $f(x)(ab) \geq f(x)(a) \wedge f(x)(b)$. Hence $f(x) : F(x) \to [0, 1]$ is fuzzy subsemigroup for all $x \in E$. Therefore (f, E) is a SP-fuzzy soft subsemigroup of (F, A). On the contrary, let $a, b \in F_\alpha(x)$ for any $\alpha \in [0, 1]$. Hence $f(x)(a) \geq \alpha$ and $f(x)(b) \geq \alpha$. Thus $f(x)(a) \wedge f(x)(b) \geq \alpha$. Thus $f(x)(ab) \geq \alpha$ since $f(x)$ is a fuzzy subsemigroup of $F(x)$ for all $x \in E$. So $ab \in F_\alpha(x)$. Therefore (F_α, E) is a soft subsemigroup for all $\alpha \in [0, 1]$.*
(ii) *Suppose that (F_α, E) is a soft left ideal for all $\alpha \in [0, 1]$. Let $x \in E$ and $\alpha := f(x)(b)$ for any $b \in F(x)$. Thus $b \in F_\alpha(x)$. $ab \in F_\alpha(x)$ for any $a \in F(x)$ since $F_\alpha(x)$ is left ideal of S for all $x \in E$. So $f(x)(ab) \geq \alpha$, i.e, $f(x)(ab) \geq f(x)(b)$. Hence $f(x) : F(x) \to [0, 1]$ is fuzzy left ideal for all $x \in E$. Therefore (f, E) is a SP-fuzzy soft left ideal of (F, A). On the contrary, let $b \in F_\alpha(x)$ for any $\alpha \in [0, 1]$. Hence $b \in F(x)$ and for any $s \in S$, $sb \in F(x)$ since (F, A) a soft left ideal of S. Thus $f(x)(sb) \geq f(x)(b) \geq \alpha$ since (f, E) is SP-fuzzy soft left (right) ideal of (F, A). So $sb \in F_\alpha(x)$. Therefore (F_α, E) is a soft left ideal for all $\alpha \in [0, 1]$. The proof is similar for the soft right ideals.*
(iii) *Suppose that (F_α, E) is a soft generalized bi-ideal for all $\alpha \in [0, 1]$. Let $x \in E$ and $\alpha := f(x)(a) \wedge f(x)(c) \wedge f(x)(b)$ for any $a, b, c \in F(x)$. So $f(x)(a) \geq \alpha$, $f(x)(b) \geq \alpha$ and $f(x)(c) \geq \alpha$. Thus $a, b, c \in F_\alpha(x)$. $acb \in F_\alpha(x)$ since $F_\alpha(x)$ is generalized bi ideal of S for all $x \in E$. So $f(x)(acb) \geq \alpha$, i.e, $f(x)(acb) \geq f(x)(a) \wedge f(x)(c) \wedge f(x)(b)$ for all $a, b, c \in F(x)$. Hence $f(x) : F(x) \to [0, 1]$ is fuzzy generalized bi-ideal for all $x \in E$. Therefore (f, E) is a SP-fuzzy soft generalized bi-ideal of (F, A). On the contrary, let $a, c \in F_\alpha(x)$ for any $\alpha \in [0, 1]$ and let $b \in S$. Hence $a, c \in F(x)$ and also for any $b \in S$, $abc \in F(x)$ since (F, A) a soft generalized bi-ideal of S. Thus $f(x)(abc) \geq f(x)(a) \wedge f(x)(c) \geq \alpha$ since (f, E) is SP-fuzzy soft generalized bi-ideal of (F, A). So $abc \in F_\alpha(x)$. Therefore (F_α, E) is a soft generalized bi-ideal for all $\alpha \in [0, 1]$. The proof is provided for soft bi-ideals with the case (i).*

(vi) *Suppose that (F_α, E) is a soft interior ideal for all $\alpha \in [0,1]$. Let $x \in E$ and $\alpha := f(x)(b)$ for any $b \in F(x)$. Thus $b \in F_\alpha(x)$. $abc \in F_\alpha(x)$ for any $a, c \in F(x)$ since $F_\alpha(x)$ is interior ideal of S for all $x \in E$. Hence $f(x)(abc) \geq \alpha$, i.e, $f(x)(abc) \geq f(x)(b)$. Thus $f(x) : F(x) \to [0,1]$ is fuzzy interior ideal for all $x \in E$. Therefore (f, E) is a SP-fuzzy soft interior ideal of (F, A). On the contrary, let $b \in F_\alpha(x)$ for any $\alpha \in [0,1]$ and let $a, c \in S$. Hence $b \in F(x)$ and also for any $a, c \in S$, $abc \in F(x)$ since (F, A) a soft interior ideal of S. Thus $f(x)(abc) \geq f(x)(b) \geq \alpha$ since (f, E) is SP-fuzzy soft interior ideal of (F, A). So $abc \in F_\alpha(x)$. Therefore (F_α, E) is a soft interior ideal for all $\alpha \in [0,1]$.*

Theorem 3.9. Let $(F, A_1), (G, A_2) \in (S, P)$ such that $F(a) \cdot G(b) = G(b) \cdot F(a)$ for any $a \in A_1$ and $b \in A_2$. If (f, E_1) is a SP-fuzzy soft right ideal of (F, A_1) and (g, E_2) is a SP-fuzzy soft left ideal of (G, A_2), then $(f, E_1)\tilde{\times}(g, E_2)$ is a SP-fuzzy soft subsemigroups of $(F, A_1) \times (G, A_2)$.
Proof. *$E_1 \times E_2 \subseteq A_1 \times A_2$. Let $u, v \in F(a) \cdot G(b)$ for any $(a, b) \in E_1 \times E_2$. Suppose that $(f, E_1)\tilde{\times}(g, E_2) = (h, C)$. Then*

$$\begin{aligned}
h(a,b)(u) \wedge h(a,b)(v) &= (f(a) \cdot g(b))(u) \wedge (f(a) \cdot g(b))(v) \\
&= \Big(\bigvee_{u=tk} (f(a)(t) \wedge g(b)(k))\Big) \wedge \Big(\bigvee_{u=sl} (f(a)(s) \wedge g(b)(l))\Big) \\
&\leq \Big(\bigvee_{\substack{u=tk \\ v=sl}} f(a)(ts) \wedge g(b)(k) \wedge f(a)(s) \wedge g(b)(kl)\Big) \\
&\leq \bigvee_{uv=tksl} (f(a)(ts) \wedge g(b)(kl)) \\
&= \bigvee_{uv=tskl} (f(a)(ts) \wedge g(b)(kl)) \\
&= h(a,b)(uv).
\end{aligned}$$

Thus (h, C) is a SP-fuzzy soft subsemigroups of $(F, A_1) \times (G, A_2)$ since $h(a, b)$ is a fuzzy subsemigroup of $F(a) \cdot G(b)$ for any $(a, b) \in E_1 \times E_2$.

Theorem 3.10. Let (f_i, E_i) be SP-fuzzy soft subsemigroup of (F_i, A_i) for all $i \in \Lambda$. Then

(i) $\tilde{\bigcap}^r_{i\in\Lambda}(f_i, E_i)$ is SP-fuzzy soft subsemigroup of $(\bigcap_r)_{i\in\Lambda}(F_i, A_i)$.
(ii) $\tilde{\bigcap}^e_{i\in\Lambda}(f_i, E_i)$ is SP-fuzzy soft subsemigroup of $(\bigcap_e)_{i\in\Lambda}(F_i, A_i)$.
(iii) $\tilde{\bigcap}^r_{i\in\Lambda}(f_i, E_i)$ is SP-fuzzy soft subsemigroup of $(\bigcap_e)_{i\in\Lambda}(F_i, A_i)$.
(iv) $\prod_{i\in\Lambda}(f_i, E_i)$ is a SP-fuzzy soft subsemigroup of $\prod_{i\in\Lambda}(F_i, A_i)$.

Proof.

(i) *Let $\tilde{\bigcap}^r_{i\in\Lambda}(f_i, E_i) = (f, E)$ and $(\bigcap_r)_{i\in\Lambda}(F_i, A_i) = (F, A)$. Clearly, $E = \bigcap_{i\in\Lambda} E_i \subseteq \bigcap_{i\in\Lambda} A_i = A$. Let $a, b \in F(x)$ for any $x \in E$. $f(x)(ab) = (\bigwedge_{i\in\Lambda} f_i(x))(ab) = \bigwedge_{i\in\Lambda}(f_i(x)(ab)) \geq \bigwedge_{i\in\Lambda}(f_i(x)(a) \wedge f_i(x)(b)) = \bigwedge_{i\in\Lambda}(f_i(x)(a)) \wedge \bigwedge_{i\in\Lambda}(f_i(x)(b)) = f(x)(a) \wedge f(x)(b)$ for all $a, b \in F(x)$ since $F(x) = \bigcap_{i\in\Lambda} F_i(x)$. We obtain that $f(x)$ is fuzzy subsemigroup of $F(x)$ for all $x \in E$. Thus $\tilde{\bigcap}^r_{i\in\Lambda}(f_i, E_i)$ is a SP-fuzzy soft subsemigroup of $(\bigcap_r)_{i\in\Lambda}(F_i, A_i)$.*
(ii) *Let $\tilde{\bigcap}^e_{i\in\Lambda}(f_i, E_i) = (f, E)$ and $(\bigcap_e)_{i\in\Lambda}(F_i, A_i) = (F, A)$. Clearly, $E = \bigcup_{i\in\Lambda} E_i \subseteq \bigcup_{i\in\Lambda} A_i = A$. Let $i \in \Lambda$ be arbitrary and constant, and $J = \{j \in \Lambda \mid x \in E_j, i \neq j\}$, and $x \in E$. If $x \in E_i$, then there are two cases: $x \in E_i \setminus \bigcup_{i\neq j} E_j$ or $J \neq \emptyset$. If $x \in E_i \setminus \bigcup_{i\neq j} E_j$, then $F(x) = F_i(x)$ and $f(x) = f_i(x)$, and since $f(x)$ is a fuzzy subsemigroup of $F(x)$, then (f, E) is SP-fuzzy soft subsemigroup of (F, A). If $J \neq \emptyset$, then $F(x) \subseteq (\bigcap_{j\in J} F_j(x)) \cap F_i(x)$ and $f(x) = (\tilde{\bigwedge}_{j\in J} f_j(x)) \wedge f_i(x)$. Hence (f, E) is SP-fuzzy soft subsemigroup of (F, A) by similar way in (i). Let $x \notin E_i$. Then $F(x) \subseteq \bigcap_{j\in J} F_j(x)$ and $f(x) = \tilde{\bigwedge}_{j\in J} f_j(x)$. Hence (f, E) is SP-fuzzy soft subsemigroup of (F, A) by similar way in (i).*
(iii) *Let $x \in E$. Then $x \in \bigcap_{i\in\Lambda} A_i$, since $E = \bigcap_{i\in\Lambda} E_i \subseteq \bigcap_{i\in\Lambda} A_i \subseteq \bigcup_{i\in\Lambda} A_i = A$. Hence $f(x) = \bigwedge_{i\in\Lambda} f_i(x)$ and $F(x) = \bigcap_{i\in\Lambda} F_i(x)$. Since $\bigwedge_{i\in\Lambda} f_i(x)$ is a fuzzy subsemigroup of $\bigcap_{i\in\Lambda} F_i(x)$, (f, E) is a SP-fuzzy soft subsemigruop of (F, A).*

(iv) *Let $\tilde{\prod}_{i\in\Lambda}(f_i, E_i) = (f, E)$ and $\prod_{i\in\Lambda}(F_i, A_i) = (F, A)$. Clearly, $E = \prod_{i\in\Lambda} E_i \subseteq \prod_{i\in\Lambda} A_i = A$. Let $a, b \in F((x_i)_{i\in\Lambda})$ for any $(x_i)_{i\in\Lambda} \in E$. Then*

$$\begin{aligned}
f((x_i)_{i\in\Lambda})(a) \wedge f((x_i)_{i\in\Lambda})(b) &= \Big(\bigvee_{\substack{J\subseteq\Lambda\\ J\text{is finite}}} \big(\bigwedge_{i\in J} f_i(x_i)\big)\Big)(a) \wedge \Big(\bigvee_{\substack{J\subseteq\Lambda\\ J\text{is finite}}} \big(\bigwedge_{i\in J} f_i(x_i)\big)\Big)(b) \\
&= \bigvee_{\substack{J\subseteq\Lambda\\ J\text{is finite}}} \big(\bigwedge_{i\in J} f_i(x_i)(a)\big) \wedge \bigvee_{\substack{J\subseteq\Lambda\\ J\text{is finite}}} \big(\bigwedge_{i\in J} f_i(x_i)(b)\big) \\
&= \bigvee_{\substack{J\subseteq\Lambda\\ J\text{is finite}}} \Big(\bigwedge_{i\in J} \big(f_i(x_i)(a) \wedge f_i(x_i)(b)\big)\Big) \\
&\leq \bigvee_{\substack{J\subseteq\Lambda\\ J\text{is finite}}} \Big(\bigwedge_{i\in J} \big(f_i(x_i)(ab)\big)\Big) = \bigvee_{\substack{J\subseteq\Lambda\\ J\text{is finite}}} \Big(\bigwedge_{i\in J} f_i(x_i)\Big)(ab) \\
&= f((x_i)_{i\in\Lambda})(ab).
\end{aligned}$$

Thus $\tilde{\prod}_{i\in\Lambda}(f_i, E_i)$ is a SP-fuzzy soft subsemigroup of $\prod_{i\in\Lambda}(F_i, A_i)$ since $f((x_i)_{i\in\Lambda})$ is a fuzzy subsemigroup of $F((x_i)_{i\in\Lambda})$.

Theorem 3.11. Let (f_i, E_i) be SP-fuzzy soft left (right) ideal of (F_i, A_i) for all $i \in \Lambda$. Then

(i) $\tilde{\bigcap}^r_{i\in\Lambda}(f_i, E_i)$ is SP-fuzzy soft left (right) ideal of $(\bigcap_r)_{i\in\Lambda}(F_i, A_i)$.
(ii) $\tilde{\bigcap}^e_{i\in\Lambda}(f_i, E_i)$ is SP-fuzzy soft left (right) ideal of $(\bigcap_e)_{i\in\Lambda}(F_i, A_i)$.
(iii) $\tilde{\bigcap}^r_{i\in\Lambda}(f_i, E_i)$ is SP-fuzzy soft left (right) ideal of $(\bigcap_e)_{i\in\Lambda}(F_i, A_i)$.
(iv) $\tilde{\prod}_{i\in\Lambda}(f_i, E_i)$ is SP-fuzzy soft left (right) ideal of $\prod_{i\in\Lambda}(F_i, A_i)$.

Proof.

(i) *Let $\tilde{\bigcap}^r_{i\in\Lambda}(f_i, E_i) = (f, E)$ and $(\bigcap_r)_{i\in\Lambda}(F_i, A_i) = (F, A)$. $E \subseteq A$ by Theorem 3.10 (i). Let $a, b \in F(x)$ for any $x \in E$. $f(x)(ab) = (\bigwedge_{i\in\Lambda} f_i(x))(ab) = \bigwedge_{i\in\Lambda}(f_i(x)(ab)) \geq \bigwedge_{i\in\Lambda}(f_i(x)(b)) = (\bigwedge_{i\in\Lambda} f_i(x))(b) = f(x)(b)$ for all $a, b \in F(x)$ since $F(x) = \bigcap_{i\in\Lambda} F_i(x)$. We obtain that $f(x)$ is fuzzy left ideal of $F(x)$ for all $x \in E$. Thus $\tilde{\bigcap}^r_{i\in\Lambda}(f_i, E_i)$ is SP-fuzzy soft left ideal of $(\bigcap_r)_{i\in\Lambda}(F_i, A_i)$.*

(ii) *Let $\tilde{\bigcap}^e_{i\in\Lambda}(f_i, E_i) = (f, E)$ and $(\bigcap_e)_{i\in\Lambda}(F_i, A_i) = (F, A)$. Clearly, $E \subseteq A$ by Theorem 3.10 (ii). Let $i \in \Lambda$ be arbitrary and constant, and $J = \{j \in \Lambda \mid x \in E_j, i \neq j\}$, and $x \in E$. If $x \in E_i$, then there are two cases: $x \in E_i \setminus \bigcup_{i\neq j} E_j$ or $J \neq \emptyset$. If $x \in E_i \setminus \bigcup_{i\neq j} E_j$, then $F(x) = F_i(x)$ and $f(x) = f_i(x)$, and since $f(x)$ is fuzzy left ideal of $F(x)$, then (f, E) is SP-fuzzy soft left ideal of (F, A). If $J \neq \emptyset$, then $F(x) \subseteq (\bigcap_{j\in J} F_j(x)) \cap F_i(x)$ and $f(x) = (\tilde{\bigwedge}_{j\in J} f_j(x)) \wedge f_i(x)$. Hence (f, E) is SP-fuzzy soft left ideal of (F, A) by similar way in (i). Let $x \notin E_i$. Then $F(x) \subseteq \bigcap_{j\in J} F_j(x)$ and $f(x) = \tilde{\bigwedge}_{j\in J} f_j(x)$. Hence (f, E) is SP-fuzzy soft left ideal of (F, A) by similar way in (i).*

(iii) *Let $x \in E$. Then $x \in \bigcap_{i\in\Lambda} A_i$, since $E \subseteq A$ by Theorem 3.10 (iii). Hence $f(x) = \bigwedge_{i\in\Lambda} f_i(x)$ and $F(x) = \bigcap_{i\in\Lambda} F_i(x)$. Since $\bigwedge_{i\in\Lambda} f_i(x)$ is a left ideal of $\bigcap_{i\in\Lambda} F_i(x)$, (f, E) is a SP-fuzzy soft left ideal of (F, A).*

(iv) *Let $\tilde{\prod}_{i\in\Lambda}(f_i, E_i) = (f, E)$ and $\prod_{i\in\Lambda}(F_i, A_i) = (F, A)$. Clearly, $E \subseteq A$ by Theorem 3.10 (iv). Let $a, b \in F((x_i)_{i\in\Lambda})$ for any $(x_i)_{i\in\Lambda} \in E$. Then*

$$\begin{aligned}
f((x_i)_{i\in\Lambda})(b) &= \Big(\bigvee_{\substack{J\subseteq\Lambda\\ J\text{is finite}}} \big(\bigwedge_{i\in J} f_i(x_i)\big)\Big)(b) = \bigvee_{\substack{J\subseteq\Lambda\\ J\text{is finite}}} \big(\bigwedge_{i\in J} f_i(x_i)(b)\big) \\
&\leq \bigvee_{\substack{J\subseteq\Lambda\\ J\text{is finite}}} \Big(\bigwedge_{i\in J} \big(f_i(x_i)(ab)\big)\Big) = \bigvee_{\substack{J\subseteq\Lambda\\ J\text{is finite}}} \Big(\bigwedge_{i\in J} f_i(x_i)\Big)(ab) = f((x_i)_{i\in\Lambda})(ab).
\end{aligned}$$

Thus $\tilde{\prod}_{i\in\Lambda}(f_i, E_i)$ is a SP-fuzzy soft left ideal of $\prod_{i\in\Lambda}(F_i, A_i)$ since $f((x_i)_{i\in\Lambda})$ is a fuzzy left ideal of $F((x_i)_{i\in\Lambda})$. The proof is similar for SP-fuzzy soft right ideals.

Theorem 3.12. Let (f_i, E_i) be generalized SP-fuzzy soft bi-ideal of (F_i, A_i) for all $i \in \Lambda$. Then

(i) $\tilde{\bigcap}^r_{i\in\Lambda}(f_i, E_i)$ is generalized SP-fuzzy soft bi-ideal of $(\bigcap_r)_{i\in\Lambda}(F_i, A_i)$.
(ii) $\tilde{\bigcap}^e_{i\in\Lambda}(f_i, E_i)$ is generalized SP-fuzzy soft bi-ideal of $(\bigcap_e)_{i\in\Lambda}(F_i, A_i)$.

(iii) $\tilde{\bigcap}^{r}_{i\in\Lambda}(f_i, E_i)$ is generalized SP-fuzzy soft bi-ideal of $(\bigcap_e)_{i\in\Lambda}(F_i, A_i)$.
(iv) $\tilde{\prod}_{i\in\Lambda}(f_i, E_i)$ is a generalized SP-fuzzy soft bi-ideal of $(\prod)_{i\in\Lambda}(F_i, A_i)$.

Proof.

(i) *Let $\tilde{\bigcap}^{r}_{i\in\Lambda}(f_i, E_i) = (f, E)$ and $(\bigcap_r)_{i\in\Lambda}(F_i, A_i) = (F, A)$. Clearly, $E \subseteq A$ by Theorem 3.10(i). Let $a, b, c \in F(x)$ for any $x \in E$. $f(x)(acb) = (\bigwedge_{i\in\Lambda} f_i(x))(acb) = \bigwedge_{i\in\Lambda}(f_i(x)(acb)) \geq \bigwedge_{i\in\Lambda}(f_i(x)(a) \wedge f_i(x)(b)) = \bigwedge_{i\in\Lambda}(f_i(x)(a)) \wedge \bigwedge_{i\in\Lambda}(f_i(x)(b)) = f(x)(a) \wedge f(x)(b)$ for all $a, b, c \in F(x)$ since $F(x) = \bigcap_{i\in\Lambda} F_i(x)$. We obtain that $f(x)$ is generalized fuzzy bi-ideal of $F(x)$ for all $x \in E$. Thus $\tilde{\bigcap}^{r}_{i\in\Lambda}(f_i, E_i)$ is a generalized SP-fuzzy soft bi-ideal of $(\bigcap_r)_{i\in\Lambda}(F_i, A_i)$.*

(ii) *Let $\tilde{\bigcap}^{e}_{i\in\Lambda}(f_i, E_i) = (f, E)$ and $(\bigcap_e)_{i\in\Lambda}(F_i, A_i) = (F, A)$. Clearly, $E \subseteq A$ by Theorem 3.10 (ii). Let $i \in \Lambda$ be arbitrary and constant, and $J = \{j \in \Lambda \mid x \in E_j, i \neq j\}$, and $x \in E$. If $x \in E_i$, then there are two cases: $x \in E_i \setminus \bigcup_{i\neq j} E_j$ or $J \neq \emptyset$. If $x \in E_i \setminus \bigcup_{i\neq j} E_j$, then $F(x) = F_i(x)$ and $f(x) = f_i(x)$, and since $f(x)$ is generalized fuzzy bi-ideal of $F(x)$, then (f, E) is generalized SP-fuzzy soft bi-ideal of (F, A). If $J \neq \emptyset$, then $F(x) \subseteq (\bigcap_{j\in J} F_j(x)) \cap F_i(x)$ and $f(x) = (\tilde{\bigwedge}_{j\in J} f_j(x)) \wedge f_i(x)$. Hence (f, E) is generalized SP-fuzzy soft bi-ideal of (F, A) by similar way in (i). Let $x \notin E_i$. Then $F(x) \subseteq \bigcap_{j\in J} F_j(x)$ and $f(x) = \tilde{\bigwedge}_{j\in J} f_j(x)$. Hence (f, E) is generalized SP-fuzzy soft bi-ideal of (F, A) by similar way in (i).*

(iii) *Let $x \in E$. Then $x \in \bigcap_{i\in\Lambda} A_i$, since $E \subseteq A$ by Theorem 3.10 (iii). Hence $f(x) = \bigwedge_{i\in\Lambda} f_i(x)$ and $F(x) = \bigcap_{i\in\Lambda} F_i(x)$. Since $\bigwedge_{i\in\Lambda} f_i(x)$ is a generalized bi-ideal of $\bigcap_{i\in\Lambda} F_i(x)$, (f, E) is a generalized SP-fuzzy soft bi-ideal of (F, A).*

(iv) *Let $\tilde{\prod}_{i\in\Lambda}(f_i, E_i) = (f, E)$ and $\prod_{i\in\Lambda}(F_i, A_i) = (F, A)$. Clearly, $E \subseteq A$ by Theorem 3.10 (iv). Let $a, b \in F((x_i)_{i\in\Lambda})$ for any $(x_i)_{i\in\Lambda} \in E$. Then*

$$
\begin{aligned}
f((x_i)_{i\in\Lambda})(b) &= \Big(\bigvee_{\substack{J\subseteq\Lambda \\ J \text{ is finite}}} \big(\bigwedge_{i\in J} f_i(x_i)\big)\Big)(b) = \bigvee_{\substack{J\subseteq\Lambda \\ J \text{ is finite}}} \big(\bigwedge_{i\in J} f_i(x_i)(b)\big) \\
&\leq \bigvee_{\substack{J\subseteq\Lambda \\ J \text{ is finite}}} \Big(\bigwedge_{i\in J} (f_i(x_i)(ab))\Big) = \bigvee_{\substack{J\subseteq\Lambda \\ J \text{ is finite}}} \Big(\bigwedge_{i\in J} f_i(x_i)\Big)(ab) = f((x_i)_{i\in\Lambda})(ab).
\end{aligned}
$$

Thus $\tilde{\prod}_{i\in\Lambda}(f_i, E_i)$ is a generalized SP-fuzzy soft bi-ideal of $\prod_{i\in\Lambda}(F_i, A_i)$ since $f((x_i)_{i\in\Lambda})$ is a generalized bi-ideal of $F((x_i)_{i\in\Lambda})$.

Theorem 3.13. Let (f_i, E_i) be SP-fuzzy soft quasi ideal of (F_i, A_i) for all $i \in \Lambda$. Then

(i) $\tilde{\bigcap}^{r}_{i\in\Lambda}(f_i, E_i)$ is SP-fuzzy soft quasi ideal of $(\bigcap_r)_{i\in\Lambda}(F_i, A_i)$.
(ii) $\tilde{\bigcap}^{e}_{i\in\Lambda}(f_i, E_i)$ is SP-fuzzy soft quasi ideal of $(\bigcap_e)_{i\in\Lambda}(F_i, A_i)$.
(iii) $\tilde{\bigcap}^{r}_{i\in\Lambda}(f_i, E_i)$ is SP-fuzzy soft quasi ideal of $(\bigcap_e)_{i\in\Lambda}(F_i, A_i)$.
(iv) $\tilde{\prod}_{i\in\Lambda}(f_i, E_i)$ is SP-fuzzy soft quasi ideal of $(\prod)_{i\in\Lambda}(F_i, A_i)$.

Proof.

(i) *Let $\tilde{\bigcap}^{r}_{i\in\Lambda}(f_i, E_i) = (f, E)$ and $(\bigcap_r)_{i\in\Lambda}(F_i, A_i) = (F, A)$. Clearly, $E \subseteq A$ by Theorem 3.10(i). Since (f_i, E_i) is SP-fuzzy soft quasi ideal of (F_i, A_i) for any $i \in \Lambda$, then $(f_i(x) \circ 1) \wedge (1 \circ f_i(x)) \leq f_i(x)$ for all $x \in E$. Let $a \in F(x)$ for any $x \in E$. Hence $(f(x) \circ 1)(a) = ((\bigwedge_{i\in\Lambda} f_i(x)) \circ 1)(a) = \bigvee_{a=kt}((\bigwedge_{i\in\Lambda} f_i(x))(k) \wedge 1(t)) = \bigvee_{a=kt}(\bigwedge_{i\in\Lambda}(f_i(x)(k)) \wedge 1(t)) \leq \bigwedge_{i\in\Lambda} \bigvee_{a=kt}(f_i(x)(k) \wedge 1(t)) = \bigwedge_{i\in\Lambda}((f_i(x) \circ 1)(a)) = (\bigwedge_{i\in\Lambda}(f_i(x) \circ 1))(a)$. Thus $\bigwedge_{i\in\Lambda}(f_i(x) \circ 1) \geqslant f(x) \circ 1$. Analogously $\bigwedge_{i\in\Lambda}(1 \circ f_i(x)) \geqslant 1 \circ f(x)$. Since $f_i(x) \geqslant (f_i(x) \circ 1) \wedge (1 \circ f_i(x)) \geqslant \bigwedge_{i\in\Lambda}(f_i(x) \circ 1) \wedge \bigwedge_{i\in\Lambda}(1 \circ f_i(x)) \geqslant (f(x) \circ 1) \wedge (1 \circ f(x))$, then we have $f(x) = \bigwedge_{i\in\Lambda} f_i(x) \geqslant (f(x) \circ 1) \wedge (1 \circ f(x))$. We obtain that $f(x)$ is fuzzy quasi ideal of $F(x)$ for all $x \in E$. Thus $\tilde{\bigcap}^{r}_{i\in\Lambda}(f_i, E_i)$ is a SP-fuzzy soft quasi ideal of $(\bigcap_r)_{i\in\Lambda}(F_i, A_i)$.*

(ii) *Let $\tilde{\bigcap}^{e}_{i\in\Lambda}(f_i, E_i) = (f, E)$ and $(\bigcap_e)_{i\in\Lambda}(F_i, A_i) = (F, A)$. Clearly, $E \subseteq A$ by Theorem 3.10 (ii). Let $i \in \Lambda$ be arbitrary and constant, and $J = \{j \in \Lambda \mid x \in E_j, i \neq j\}$, and $x \in E$. If $x \in E_i$, then there are two cases: $x \in E_i \setminus \bigcup_{i\neq j} E_j$ or $J \neq \emptyset$. If $x \in E_i \setminus \bigcup_{i\neq j} E_j$, then $F(x) = F_i(x)$ and $f(x) = f_i(x)$, and since $f(x)$ is fuzzy quasi ideal of $F(x)$, then (f, E) is SP-fuzzy soft quasi ideal of (F, A). If $J \neq \emptyset$, then $F(x) \subseteq (\bigcap_{j\in J} F_j(x)) \cap F_i(x)$ and $f(x) = (\tilde{\bigwedge}_{j\in J} f_j(x)) \wedge f_i(x)$. Hence (f, E) is SP-fuzzy soft quasi ideal of (F, A) by similar way in (i). Let $x \notin E_i$. Then $F(x) \subseteq \bigcap_{j\in J} F_j(x)$ and $f(x) = \tilde{\bigwedge}_{j\in J} f_j(x)$. Hence (f, E) is SP-fuzzy soft quasi ideal of (F, A) by similar way in (i).*

(iii) *Let $x \in E$. Then $x \in \bigcap_{i\in\Lambda} A_i$, since $E \subseteq A$ by Theorem 3.10 (iii). Hence $f(x) = \bigwedge_{i\in\Lambda} f_i(x)$ and $F(x) = \bigcap_{i\in\Lambda} F_i(x)$. Since $\bigwedge_{i\in\Lambda} f_i(x)$ is a fuzzy quasi ideal of $\bigcap_{i\in\Lambda} F_i(x)$, (f, E) is a SP-fuzzy soft quasi ideal of (F, A).*

(iv) *Let $\tilde{\prod}_{i\in\Lambda}(f_i, E_i) = (f, E)$ and $\prod_{i\in\Lambda}(F_i, A_i) = (F, A)$. Clearly, $E \subseteq A$ by Theorem 3.10 (iv). Let $a \in F((x_i)_{i\in\Lambda})$ for any $(x_i)_{i\in\Lambda} \in E$. Then*

$$
\begin{aligned}
((f((x_i)_{i\in\Lambda}) \circ 1) \wedge (1 \circ f((x_i)_{i\in\Lambda})))(a) &= (f((x_i)_{i\in\Lambda}) \circ 1)(a) \wedge (1 \circ f((x_i)_{i\in\Lambda}))(a) \\
&= (\bigvee_{a=kt} f((x_i)_{i\in\Lambda}))(k) \wedge 1(t)) \wedge (\bigvee_{a=mn} 1(m) \wedge f((x_i)_{i\in\Lambda}))(n)) \\
&= \bigvee_{a=kt} \Big((\bigvee_{\substack{T\subseteq\Lambda \\ T \text{ is finite}}} \bigwedge_{i\in T} f_i(x_i))(k) \wedge 1(t) \Big) \wedge \bigvee_{a=mn} \Big(1(m) \wedge (\bigvee_{\substack{J\subseteq\Lambda \\ J \text{is finite}}} \bigwedge_{i\in J} f_i(x_i))(n) \Big) \\
&= \bigvee_{\substack{T\subseteq\Lambda \\ T \text{ is finite}}} \Big(\bigwedge_{i\in T} (\bigvee_{a=kt} f_i(x_i)(k) \wedge 1(t)) \Big) \wedge \bigvee_{\substack{J\subseteq\Lambda \\ J \text{is finite}}} \Big(\bigwedge_{i\in J} (\bigvee_{a=mn} f_i(x_i)(m) \wedge 1(n)) \Big) \\
&= \bigvee_{\substack{T\subseteq\Lambda \\ T \text{ is finite}}} \Big(\bigwedge_{i\in T} (f_i(x_i) \circ 1)(a) \Big) \wedge \bigvee_{\substack{J\subseteq\Lambda \\ J \text{is finite}}} \Big(\bigwedge_{i\in J} (1 \circ f_i(x_i))(a) \Big) \\
&= \bigvee_{\substack{T\subseteq\Lambda \\ T \text{ is finite}}} \Big(\bigwedge_{i\in T} (f_i(x_i) \circ 1 \wedge 1 \circ f_i(x_i))(a) \Big) \\
&\leq \bigvee_{\substack{T\subseteq\Lambda \\ T \text{ is finite}}} \Big(\bigwedge_{i\in T} f_i(x_i)(a) \Big) \\
&\leq \Big(\bigvee_{\substack{T\subseteq\Lambda \\ T \text{ is finite}}} (\bigwedge_{i\in T} f_i(x_i)) \Big)(a) \\
&= f((x_i)_{i\in\Lambda})(a).
\end{aligned}
$$

We obtain that (f, E) is fuzzy quasi ideal of (F, A) for all $x \in E$. Thus (f, E) is SP-fuzzy soft quasi ideal of (F, A).

Theorem 3.14. Let (f_i, E_i) be SP-fuzzy soft interior ideal of (F_i, A_i) for all $i \in \Lambda$. Then

(i) $\tilde{\bigcap}^r_{i\in\Lambda}(f_i, E_i)$ is SP-fuzzy soft interior ideal of $(\bigcap_r)_{i\in\Lambda}(F_i, A_i)$.
(ii) $\tilde{\bigcap}^e_{i\in\Lambda}(f_i, E_i)$ is SP-fuzzy soft interior ideal of $(\bigcap_e)_{i\in\Lambda}(F_i, A_i)$.
(iii) $\tilde{\bigcap}^r_{i\in\Lambda}(f_i, E_i)$ is SP-fuzzy soft interior ideal of $(\bigcap_e)_{i\in\Lambda}(F_i, A_i)$.
(iv) $\tilde{\prod}_{i\in\Lambda}(f_i, E_i)$ is a SP-fuzzy soft interior ideal of $(\prod)_{i\in\Lambda}(F_i, A_i)$.

Proof.

(i) *Let $\tilde{\bigcap}^r_{i\in\Lambda}(f_i, E_i) = (f, E)$ and $(\bigcap_r)_{i\in\Lambda}(F_i, A_i) = (F, A)$. Clearly, $E \subseteq A$ by Theorem 3.10(i). Let $a, b, c \in F(x)$ for any $x \in E$. $f(x)((ac)b) = (\bigwedge_{i\in\Lambda} f_i(x))((ac)b) = \bigwedge_{i\in\Lambda}(f_i(x)((ac)b)) \geq \bigwedge_{i\in\Lambda}(f_i(x)(c)) = (\bigwedge_{i\in\Lambda} f_i(x))(c) = f(x)(c)$ for all $a, b, c \in F(x)$ since $F(x) = \bigcap_{i\in\Lambda} F_i(x)$. We obtain that $f(x)$ is fuzzy interior ideal of $F(x)$ for all $x \in E$. Thus $\tilde{\bigcap}^r_{i\in\Lambda}(f_i, E_i)$ is a SP-fuzzy soft interior ideal of $(\bigcap_r)_{i\in\Lambda}(F_i, A_i)$*

(ii) *Let $\tilde{\bigcap}^e_{i\in\Lambda}(f_i, E_i) = (f, E)$ and $(\bigcap_e)_{i\in\Lambda}(F_i, A_i) = (F, A)$. Clearly, $E \subseteq A$ by Theorem 3.10 (ii). Let $i \in \Lambda$ be arbitrary and constant, and $J = \{j \in \Lambda \mid x \in E_j, i \neq j\}$, and $x \in E$. If $x \in E_i$, then there are two cases: $x \in E_i \setminus \bigcup_{i\neq j} E_j$ or $J \neq \emptyset$. If $x \in E_i \setminus \bigcup_{i\neq j} E_j$, then $F(x) = F_i(x)$ and $f(x) = f_i(x)$, and since $f(x)$ is fuzzy interior ideal of $F(x)$, then (f, E) is SP-fuzzy soft interior ideal of (F, A). If $J \neq \emptyset$, then $F(x) \subseteq (\bigcap_{j\in J} F_j(x)) \cap F_i(x)$ and $f(x) = (\tilde{\bigwedge}_{j\in J} f_j(x)) \wedge f_i(x)$. Hence (f, E) is SP-fuzzy soft interior ideal of (F, A) by similar way in (i). Let $x \notin E_i$. Then $F(x) \subseteq \bigcap_{j\in J} F_j(x)$ and $f(x) = \tilde{\bigwedge}_{j\in J} f_j(x)$. Hence (f, E) is SP-fuzzy soft interior ideal of (F, A) by similar way in (i).*

(iii) *Let $x \in E$. Then $x \in \bigcap_{i\in\Lambda} A_i$, since $E \subseteq A$ by Theorem 3.10 (iii). Hence $f(x) = \bigwedge_{i\in\Lambda} f_i(x)$ and $F(x) = \bigcap_{i\in\Lambda} F_i(x)$. Since $\bigwedge_{i\in\Lambda} f_i(x)$ is a fuzzy interior ideal of $\bigcap_{i\in\Lambda} F_i(x)$, (f, E) is a SP-fuzzy soft interior ideal of (F, A).*

(iv) *Let* $\tilde{\prod}_{i\in\Lambda}(f_i, E_i) = (f, E)$ *and* $\prod_{i\in\Lambda}(F_i, A_i) = (F, A)$. *Clearly,* $E \subseteq A$ *by Theorem 3.10 (iv). Let* $a, b \in F((x_i)_{i\in\Lambda})$ *for any* $(x_i)_{i\in\Lambda} \in E$. *Then*

$$\begin{aligned} f((x_i)_{i\in\Lambda})((ac)b) &= \Big(\bigvee_{\substack{J\subseteq\Lambda\\ J \text{ is finite}}} \big(\bigwedge_{i\in J} f_i(x_i)\big)\Big)((ac)b) = \bigvee_{\substack{J\subseteq\Lambda\\ J \text{ is finite}}} \big(\bigwedge_{i\in J} f_i(x_i)\big)((ac)b) \\ &= \bigvee_{\substack{J\subseteq\Lambda\\ J \text{ is finite}}} \bigwedge_{i\in J} (f_i(x_i)((ac)b)) \leq \bigvee_{\substack{J\subseteq\Lambda\\ J \text{ is finite}}} \bigwedge_{i\in J} (f_i(x_i)(c)) \\ &= \bigvee_{\substack{J\subseteq\Lambda\\ J \text{ is finite}}} \Big(\bigwedge_{i\in J} f_i(x_i)\Big)(c) = f((x_i)_{i\in\Lambda})(c). \end{aligned}$$

Thus $\tilde{\prod}_{i\in\Lambda}(f_i, E_i)$ *is a* SP*-fuzzy soft interior ideal of* $\prod_{i\in\Lambda}(F_i, A_i)$ *since* $f((x_i)_{i\in\Lambda})$ *is a fuzzy interior ideal of* $F((x_i)_{i\in\Lambda})$.

Corollary 3.15. Let (f_i, E_i) be SP-fuzzy soft subsemigroup of (F, A) for all $i \in \Lambda$. Then

(i) $\tilde{\bigcap}^r_{i\in\Lambda}(f_i, E_i)$ is SP-fuzzy soft-subsemigroup of (F, A).
(ii) $\tilde{\bigcap}^e_{i\in\Lambda}(f_i, E_i)$ is SP-fuzzy soft-subsemigroup of (F, A).

Proof. It is straightforward from Theorem 3.10.

Corollary 3.16. Let (f_i, E_i) be SP-fuzzy soft left (right) ideal of (F, A) for all $i \in \Lambda$. Then

(i) $\tilde{\bigcap}^r_{i\in\Lambda}(f_i, E_i)$ is SP-fuzzy soft left (right) ideal of (F, A).
(ii) $\tilde{\bigcap}^e_{i\in\Lambda}(f_i, E_i)$ is SP-fuzzy soft left (right) ideal of (F, A).

Proof. It is straightforward from Theorem 3.11.

Corollary 3.17. Let (f_i, E_i) be (generalized) SP-fuzzy soft bi-ideal of (F, A) for all $i \in \Lambda$. Then

(i) $\tilde{\bigcap}^r_{i\in\Lambda}(f_i, E_i)$ is (generalized) SP-fuzzy soft bi-ideal of (F, A).
(ii) $\tilde{\bigcap}^e_{i\in\Lambda}(f_i, E_i)$ is (generalized) SP-fuzzy soft bi-ideal of (F, A).

Proof. It is straightforward from Theorem 3.12.

Corollary 3.18. Let (f_i, E_i) be SP-fuzzy quasi ideal of (F, A) for all $i \in \Lambda$. Then

(i) $\tilde{\bigcap}^r_{i\in\Lambda}(f_i, E_i)$ is SP-fuzzy soft quasi ideal of (F, A).
(ii) $\tilde{\bigcap}^e_{i\in\Lambda}(f_i, E_i)$ is SP-fuzzy soft quasi ideal of (F, A).

Proof. It is straightforward from Theorem 3.13.

Corollary 3.19. Let (f_i, E_i) be SP-fuzzy soft interior ideal of (F, A) for all $i \in \Lambda$. Then

(i) $\tilde{\bigcap}^r_{i\in\Lambda}(f_i, E_i)$ is SP-fuzzy soft interior ideal of (F, A).
(ii) $\tilde{\bigcap}^e_{i\in\Lambda}(f_i, E_i)$ is SP-fuzzy soft interior ideal of (F, A).

Proof. It is straightforward from Theorem 3.14.

4. Conclusions

The theory of soft sets has been very effective in many studies and its extensions on algebra have being interest since its introduction in 1999. This paper proposes UP-fuzzy soft sets of a soft set as a new concept. We introduce the notion of SP-fuzzy soft subsemigroups and SP-fuzzy soft left (right, bi-, quasi, interior) ideals of a soft semigroup. We investigate some basic properties of them. It can be easily seen that any fuzzy soft subsemigroup of S is a SP-fuzzy soft subsemigroup of a soft semigroup of S, where soft semigroup of S has a parameter set including the parameter set of the fuzzy soft subsemigroup of S. Our future work on this topic will focus on considering the results for prime, semiprime and quasi-prime ideals of a semigroup. According to us it is interesting to observation the results for other algebraic structures such as groups, rings, modules.

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