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D-Conformal Curvature Tensor on $(LCS)_n$-Manifolds

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Abstract. This paper deals with the study of geometry of $(LCS)_n$-manifolds. We investigate some properties of D-conformally flat and D-conformally semi-symmetric curvature conditions on $(LCS)_n$-manifold. We classify $(LCS)_n$-manifolds, which satisfy the curvature conditions $B(\xi, Y)P = 0$ and $B(\xi, Y)S = 0$, where B is the D-conformal curvature tensor and S is the Ricci tensor of manifold.

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1. Introduction

In 2003, Shaikh [15] introduced the notion of Lorentzian concircular structure manifolds (briefly, $(LCS)_n$-manifolds) with an example, which generalizes the notion of LP-Sasakian manifolds introduced by Matsumoto [10] and also by Mihai and Rosca [11]. Then Shaikh and Baishya [16, 17] investigated the applications of $(LCS)_n$-manifolds to the general theory of relativity and cosmology. The $(LCS)_n$-manifolds are also studied by Atçeken et. al. [2, 3, 9], Hui [8], Narain and Yadav [12], Atçeken and Yıldırım [4] and many authors.

The concept of D-conformal curvature tensor was defined by Chuman [6]. Again Chuman [7] studied D-conformal vector fields in para-Sasakian manifolds. Adati [1] studied D-conformal para-Killing vector fields in special para-Sasakian manifolds. Shah [14] researched some curvature properties of D-conformal curvature tensor on LP-Sasakian manifolds. Recently, D-conformal curvature tensor were studied by many geometers [5, 18].

Motivated by the studies of the above authors, we have studied some curvature properties of D-conformal curvature tensors on $(LCS)_n$-manifolds.

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2. Preliminaries

An n-dimensional Lorentzian manifold M is a smooth connected paracompact Hausdorff manifold with a Lorentzian metric g, that is, M admits a smooth symmetric tensor field g of type $(0,2)$ such that for each point $p \in M$, the tensor $g_p : T_pM \times T_pM \to \mathbb{R}$ is non-degenerate inner product of signature $(-,+,...,+)$, where T_pM denotes the tangent vector space of M at p and $\mathbb{R}$ is the real number space. A non-zero vector $v \in T_pM$ is said to be timelike (resp., non-spacelike, null, spacelike) if satisfies $g_p(v,v) < 0$ (resp., $\leq 0, = 0, > 0$) [13]. The category to which a given vector falls is called its casual chatacter.

Definition 2.1. In a Lorentzian manifold (M,g), a vector field P defined by

$$g(X,P) = A(X)$$

for any $X \in \Gamma(TM)$ is said to be a concircular vector field if

$$(\nabla_X A)Y = \alpha\{g(X,Y) + \omega(X)A(Y)\}$$

for $Y \in \Gamma(TM)$, where α is a nonzero scalar function, A is a 1-form, ω is also closed 1-form, and ∇ denotes the Levi-Civita connection on M.

Let M bea Lorentzian manifold admitting a unit timelike concircular vector field ξ, called the characteristic vector field of the manifold. Then we have

$$g(\xi,\xi) = -1.$$

Since ξ is a unit vector field, there exists a nonzero 1-form η such that

$$g(X,\xi) = \eta(X).$$

The equation of the following form holds:

$$(\nabla_X \eta)Y = \alpha\{g(X,Y) + \eta(X)\eta(Y)\}, \alpha \neq 0 \tag{2.1}$$

for all $X, Y \in \Gamma(TM)$, where α is nonzero scalar function satisfying

$$\nabla_X \alpha = X(\alpha) = d\alpha(X) = \rho\eta(X),$$

ρ being a certain scalar function given by $\rho = -\xi(\alpha)$. Let us put

$$\nabla_X \xi = \alpha\phi x, \tag{2.2}$$

then from (2.1) and (2.2), we can derive

$$\phi X = X + \eta(X)\xi$$

which tell us that ϕ is symmetric $(1,1)$-tensor. Thus the Lorentzian manifold M together with the unit timelike concircular vector field ξ, ,ts associated 1-form η and $(1,1)$-type tensor field ϕ is said to be a Lorentzian concircular structure manifold.

A differentiable manifold M of dimension n is called (LCS)-manifold if it admits a $(1,1)$-type tensor field ϕ, a covariant vector field η and a Lorentzian metric g which satisfy

$$\eta(\xi) = g(\xi,\xi) = -1, \tag{2.3}$$

$$\phi^2 X = X + \eta(X)\xi,$$

$$g(X,\xi) = \eta(X)\xi,$$

$$\phi\xi = 0, \quad \eta \circ \phi = 0,$$

for all $X \in \Gamma(TM)$. Particulary, if we take $\alpha = 1$, then we can obtain the LP-Sasakian structure of Matsumoto [10].

Also, in an $(LCS)_n$-manifold M, the following relations are satisfied

$$\eta(R(X,Y)Z) = (\alpha^2 - \rho)[g(Y,Z)\eta(X) - g(X,Z)\eta(Y)],$$

$$R(\xi,X)Y = (\alpha^2 - \rho)[g(X,Y)\xi - \eta(Y)X], \tag{2.4}$$

$$R(X,Y)\xi = (\alpha^2-\rho)[\eta(Y)X-\eta(X)Y], \tag{2.5}$$

$$(\nabla_X\phi)Y = \alpha[g(X,Y)\xi + 2\eta(X)\eta(Y)\xi + \eta(Y)X],$$

$$S(X,\xi) = (n-1)(\alpha^2-\rho)\eta(X), \tag{2.6}$$

$$S(\phi X,\phi Y) = S(X,Y) + (n-1)(\alpha^2-\rho)\eta(X)\eta(Y)$$

for all X, Y, Z on M, where R and S denote the Riemannian curvature tensor and Ricci curvature, respectivelly, Q is also the Ricci operator given by $S(X,Y) = g(QX,Y)$.
In 1983, Chuman defined a tensor field B on a n-dimensional Riemannian manifold (M^n, g) $(n > 4)$ as

$$\begin{aligned} B(X,Y)Z &= R(X,Y)Z + \frac{1}{n-3}[S(X,Z)Y - S(Y,Z)X + g(X,Z)QY - g(Y,Z)QX \\ &+ S(Y,Z)\eta(X)\xi - S(X,Z)\eta(Y)\xi + \eta(Y)\eta(Z)QX - \eta(X)\eta(Z)QY] \\ &- \frac{K-2}{n-3}[g(X,Z)Y - g(Y,Z)X] \\ &+ \frac{K}{n-3}[g(X,Z)\eta(Y)\xi - g(Y,Z)\eta(X)\xi + \eta(X)\eta(Z)Y - \eta(Y)\eta(Z)X]. \end{aligned} \tag{2.7}$$

Such a tensor field B is known as D-conformal curvature tensor, where $K = \frac{r+2(n-1)}{n-2}$, R is Riemannian curvature tensor, Q is the Ricci operator, S is the Ricci tensor and r is the scalar curvature of M.
The projective curvature tensor P of n-dimensional Riemann manifold is defined by

$$P(X,Y)Z = R(X,Y)Z - \frac{1}{(n-1)}[S(Y,Z)X - S(X,Z)Y],$$

where S is the Ricci tensor of the manifold.
Also, in n-dimensional (LCS)-manifold M, the following is satisfied,

$$B(X,Y)\xi = \frac{2}{n-3}\Big[\frac{r+n}{n-2} - (\alpha^2-\rho)\Big]\Big[\eta(Y)X - \eta(X)Y\Big] + \frac{2}{n-3}\Big[\eta(X)QY - \eta(Y)QX\Big],$$

$$B(\xi,Y)Z = \frac{2}{n-3}\Big[\frac{r+n}{n-2} - (\alpha^2-\rho)\Big]\Big[g(Y,Z)\xi - \eta(Z)Y\Big] - \frac{2}{n-3}\Big[S(Y,Z)\xi - \eta(Z)QY\Big], \tag{2.8}$$

$$P(\xi,Y)Z = (\alpha^2-\rho)g(Y,Z)\xi - \frac{1}{n-1}S(Y,Z)\xi, \tag{2.9}$$

$$P(\xi,Y)\xi = P(X,Y)\xi = 0, \tag{2.10}$$

for all $X, Y, Z \in \Gamma(TM)$.

3. D-Conformally Flat $(LCS)_n$-Manifolds

Theorem 3.1. *If an n-dimensional $(LCS)_n$-manifold M is D-conformally flat, the M is an η-Einstein manifold.*

Proof. For an n-dimensional D-conformally flat LCS_n-manifold, we have, from (2.7),

$$\begin{aligned} R(X,Y)Z &= \frac{1}{n-3}\Big[S(Y,Z)X - S(X,Z)Y + g(Y,Z)QX - g(X,Z)QY \\ &+ S(X,Z)\eta(Y)\xi - S(Y,Z)\eta(X)\xi + \eta(X)\eta(Z)QY - \eta(Y)\eta(Z)QX\Big] \\ &+ \frac{K-2}{n-3}\Big[g(X,Z)Y - g(Y,Z)X\Big] + \frac{K}{n-3}\Big[g(Y,Z)\eta(X)\xi - g(X,Z)\eta(Y)\xi \\ &+ \eta(Y)\eta(Z)X - \eta(X)\eta(Z)Y\Big] \end{aligned} \tag{3.1}$$

for all $X, Y, Z \in \Gamma(TM)$. If we put $Z = \xi$ in(3.1) and also using (2.3), (2.5) and (2.6), we obtain

$$\begin{aligned} (\alpha^2-\rho)\Big[\eta(Y)X - \eta(X)Y\Big] &= \frac{(n-1)(\alpha^2-\rho)}{n-3}\Big[\eta(Y)X - \eta(X)Y\Big] + \frac{2}{n-3}\Big[\eta(Y)QX - \eta(X)QY\Big] \\ &+ \frac{2K-2}{n-3}\Big[\eta(X)Y - \eta(Y)X\Big]. \end{aligned} \tag{3.2}$$

Let $X = \xi$ be in (3.2), then also by using (2.3) we obtain

$$QY = \left[\frac{r+n}{n-2} - (\alpha^2 - \rho)\right]Y + \left[\frac{r+n}{n-2} - n(\alpha^2 - \rho)\right]\eta(Y)\xi.$$

Taking inner product on both sides of the last equation by $W \in \Gamma(TM)$, we obtain

$$S(Y,W) = \left[\frac{r+n}{n-2} - (\alpha^2 - \rho)\right]g(Y,W) + \left[\frac{r+n}{n-2} - n(\alpha^2 - \rho)\right]\eta(Y)\eta(W).$$

□

4. D-Conformally Semi-Symmetric $(LCS)_n$-Manifold

Theorem 4.1. *Let M be an $(LCS)_n$ manifold. Then M is D-conformally semi-symmetric if and only if the scalar curvature of manifold is*

$$r = \frac{(\alpha^2 - \rho)(n+1)(n+10)}{2}.$$

Proof. Suppose that n-dimensional an (LCS) manifold M be a D-conformal semi-symmetric. Then, we have

$$\begin{aligned}(R(X,Y))B(U,W,Z) &= R(X,Y)B(U,W)Z - B(R(X,Y)U,W)Z \\ &- B(U,R(X,Y)W)Z - B(U,W)R(X,Y)Z \\ &= 0 \end{aligned} \tag{4.1}$$

for all $X, Y, Z \in \Gamma(TM)$. In (4.1) choosing $X = \xi$, we obtain

$$\begin{aligned} 0 &= R(\xi,Y)B(U,W)Z - B(R(\xi,Y)U,W)Z \\ &- B(U,R(\xi,Y)W)Z - B(U,W)R(\xi,Y)Z. \end{aligned} \tag{4.2}$$

If we put $U = Z = \xi$ in (4.2) and also using (2.4), we obtain

$$\begin{aligned} 0 &= \frac{2}{n-3}\left[\frac{r+n}{n-2} - (2n-1)(\alpha^2-\rho)\right]R(\xi,Y)\left[(\alpha^2-\rho)\eta(Y)\xi + (\alpha^2-\rho)Y\right] \\ &+ \frac{2}{n-3}\left[\frac{r+n}{n-2} - (\alpha^2-\rho)\right]\left[(\alpha^2-\rho)g(Y,W)\xi - (\alpha^2-\rho)\eta(W)Y\right] \\ &+ \frac{2}{n-3}\left[(\alpha^2-\rho)g(Y,QW)\xi - (\alpha^2-\rho)\eta(QW)Y\right] \\ &- (\alpha^2-\rho)\eta(Y)\left[\frac{2}{n-3}\left[\frac{r+n}{n-2} - (2n-1)(\alpha^2-\rho)\right]\eta(W)\xi\right. \\ &+ \left.\frac{2}{n-3}\left[\frac{r+n}{n-2} - (\alpha^2-\rho)\right]W + \frac{2}{n-3}QW\right] \\ &- (\alpha^2-\rho)\left[\frac{2}{n-3}\left[\frac{r+n}{n-2} - (\alpha^2-\rho)\right]\eta(W)Y - \frac{2}{n-3}\left[\frac{r+n}{n-2} - (\alpha^2-\rho)\right]\eta(Y)W\right. \\ &+ \left.\frac{2}{n-3}\eta(Y)QW - \frac{2}{n-3}\eta(W)QY\right] \\ &+ (\alpha^2-\rho)\eta(W)\left[\frac{2}{n-3}\left[\frac{r+n}{n-2} - (2n-1)(\alpha^2-\rho)\right]\eta(Y)\xi\right. \\ &+ \left.\frac{2}{n-3}\left[\frac{r+n}{n-2} - (\alpha^2-\rho)\right]Y + \frac{2}{n-3}QY\right] \\ &- (\alpha^2-\rho)\eta(Y)\left[\frac{2}{n-3}\left[\frac{r+n}{n-2} - (2n-1)(\alpha^2-\rho)\right]\eta(W)\xi\right. \\ &+ \left.\frac{2}{n-3}\left[\frac{r+n}{n-2} - (\alpha^2-\rho)\right]W + \frac{2}{n-3}QW\right] \\ &- (\alpha^2-\rho)\left[\frac{2}{n-3}\left[\frac{r+n}{n-2} - (\alpha^2-\rho)\right]g(Y,W)\xi - \frac{2}{n-3}S(Y,W)\xi\right. \\ &- \left.\frac{2}{n-3}\left[\frac{r+n}{n-2} - (\alpha^2-\rho)\right]\eta(Y)W + \frac{2}{n-3}\eta(Y)QW\right]. \end{aligned}$$

By direct calculations, we have

$$\begin{aligned} 0 &= (\alpha^2-\rho)\Big[\frac{2}{n-3}\Big[\frac{r+n}{n-2}-(2n-1)(\alpha^2-\rho)\Big]\eta(W)Y \\ &+ \frac{2}{n-3}\Big[2S(Y,W)\xi-\eta(QW)Y-4\eta(Y)QW\Big] \\ &- \frac{2}{n-3}\Big[\frac{r+n}{n-2}-(\alpha^2-\rho)\Big]\eta(W)Y+2\eta(W)QY\Big]. \end{aligned}$$

Now, taking inner product on both sides of the last equation by $\xi \in \Gamma(TM)$, we obtain

$$\begin{aligned} \frac{4}{n-3}S(Y,W) &= \Big[\frac{2}{n-3}\Big[\frac{r+n}{n-2}-(2n-1)(\alpha^2-\rho)\Big] \\ &- \frac{10(n-1)(\alpha^2-\rho)}{(n-3)}+2(n-1)(\alpha^2-\rho) \\ &- \frac{2}{n-3}\Big[\frac{r+n}{n-2}-(\alpha^2-\rho)\Big]\Big]\eta(W)\eta(Y), \end{aligned} \tag{4.3}$$

for $\{e_1, e_2, ..., e_{n-1}, \xi\}$ orthonormal basis of M, from (4.3), we obtain

$$r = \frac{(\alpha^2-\rho)(n+1)(n+10)}{2},$$

which proves our assertion. The converse is obvious. □

Theorem 4.2. *Let M be n-dimensional (LCS)-manifold. Then, $B(\xi, Y)P = 0$ if and only if there is between A and B the following relation*

$$A.r + B.n - \frac{2}{n-3}\| Q \|^2 = 0.$$

Here, A and B are as follows,

$$A = \Big[\frac{2}{n-3}\Big[\frac{r+n}{n-2}-(\alpha^2-\rho)\Big]-\frac{2(n-1)(\alpha^2-\rho)}{(n-3)}\Big],$$

$$B = \frac{2}{n-3}\Big[\frac{r+n}{n-2}-(\alpha^2-\rho)\Big](n-1)(\alpha^2-\rho).$$

Proof. Assume that $B(\xi, Y)P = 0$. Then we have,

$$\begin{aligned} 0 &= B(\xi,Y)P(U,W)Z - P(B(\xi,Y)U,W)Z \\ &- P(U,B(\xi,Y)W)Z - P(U,W)B(\xi,Y)Z \end{aligned} \tag{4.4}$$

for all $U, Y, W, Z \in \Gamma(TM)$. Now, using (2.8) and (2.9) in (4.4) and choosing $U = \xi$, we obtain

$$\begin{aligned} 0 &= \Big[(\alpha^2-\rho)g(W,Z)-\frac{1}{n-1}S(W,Z)\Big]B(\xi,Y)\xi \\ &- \frac{2}{n-3}\Big[\frac{r+n}{n-2}-(2n-1)(\alpha^2-\rho)\Big]\eta(Y)P(\xi,W)Z \\ &- \frac{2}{n-3}\Big[\frac{r+n}{n-2}-(\alpha^2-\rho)\Big]\Big[P(Y,W)Z-\eta(W)P(\xi,Y)Z-\eta(Z)P(\xi,W)Y\Big] \\ &- \frac{2}{n-3}\Big[P(QY,W)Z+\eta(W)P(\xi,QY)Z+\eta(Z)P(\xi,W)QY\Big]. \end{aligned} \tag{4.5}$$

In (4.5), putting $Z = \xi$, we have

$$\begin{aligned} 0 &= \frac{2}{n-3}(\alpha^2-\rho)g(W,QY)\xi - \frac{2}{(n-1)(n-3)}S(W,QY)\xi \\ &- \frac{2}{n-3}\Big[\frac{r+n}{n-2}-(\alpha^2-\rho)\Big]\Big[(\alpha^2-\rho)g(Y,W)\xi-\frac{1}{n-1}S(Y,W)\xi\Big]. \end{aligned}$$

Taking inner product on both sides of the last equation by $\xi \in \Gamma(TM)$ and by direct calculations, we obtain

$$\begin{aligned} 0 &= \left[\frac{2}{n-3}\left[\frac{r+n}{n-2} - (\alpha^2 - \rho)\right] - \frac{2(n-1)(\alpha^2 - \rho)}{(n-3)}\right] S(Y,W) \\ &- \frac{2}{n-3}\left[\frac{r+n}{n-2} - (\alpha^2 - \rho)\right](n-1)(\alpha^2 - \rho) g(Y,W) - \frac{2}{n-3} S(QY,W). \end{aligned} \tag{4.6}$$

Here, for $\{e_1, e_2, ..., e_{n-1}, \xi\}$ orthonormal basis of M, from (4.6), we can result

$$\left[\frac{2}{n-3}\left[\frac{r+n}{n-2} - (\alpha^2 - \rho)\right] - \frac{2(n-1)(\alpha^2 - \rho)}{(n-3)}\right].r + \frac{2}{n-3}\left[\frac{r+n}{n-2} - (\alpha^2 - \rho)\right](n-1)(\alpha^2 - \rho).n - \frac{2}{n-3} \| Q \|^2 = 0,$$

which implies

$$A.r + B.n - \frac{2}{n-3} \| Q \|^2 = 0.$$

This proves our assertion. The converse is obvious. □

Theorem 4.3. *Let M be n-dimensional (LCS)-manifold. Then, $B(\xi, Y)S = 0$ if and only if there is between A and B the following relation*

$$A.r + B.n - \frac{2}{n-3} \| Q \|^2 = 0.$$

Here, A and B are as follows,

$$A = \left[\frac{2}{n-3}\left[\frac{r+n}{n-2} - (\alpha^2 - \rho)\right] - \frac{2(n-1)(\alpha^2 - \rho)}{(n-3)}\right],$$

$$B = \frac{2}{n-3}\left[\frac{r+n}{n-2} - (\alpha^2 - \rho)\right](n-1)(\alpha^2 - \rho).$$

Proof. Suppose that $B(\xi, Y)S = 0$. Then we have

$$-S(B(\xi, Y)U, W) - S(U, B(\xi, Y)W) = 0. \tag{4.7}$$

Using (2.8) in (4.7), we obtain

$$\begin{aligned} 0 &= \frac{2}{n-3}\left[\frac{r+n}{n-2} - (\alpha^2 - \rho)\right]\Big[g(Y,U)S(\xi,W) - \eta(U)S(Y,W) \\ &+ g(Y,W)S(U,\xi) - \eta(W)S(U,Y)\Big] + \frac{2}{n-3}\Big[\eta(U)S(QY,W) - S(U,Y)S(\xi,W) \\ &+ \eta(W)S(U,QY) - S(Y,W)S(U,\xi)\Big]. \end{aligned}$$

Again, choosing $U = \xi$ and by using (2.3) and (2.6), we obtain

$$\begin{aligned} 0 &= \left[\frac{2}{n-3}\left[\frac{r+n}{n-2} - (\alpha^2 - \rho)\right] + \frac{2(n-1)(\alpha^2 - \rho)}{n-3}\right] S(Y,W) \\ &- \frac{2}{n-3}\left[\frac{r+n}{n-2} - (\alpha^2 - \rho)\right](n-1)(\alpha^2 - \rho) g(Y,W) - \frac{2}{n-3} S(QY,W). \end{aligned} \tag{4.8}$$

Here, for $\{e_1, e_2, ..., e_{n-1}, \xi\}$ orthonormal basis of M, from (4.8), we can result

$$\left[\frac{2}{n-3}\left[\frac{r+n}{n-2} - (\alpha^2 - \rho)\right] - \frac{2(n-1)(\alpha^2 - \rho)}{(n-3)}\right].r + \frac{2}{n-3}\left[\frac{r+n}{n-2} - (\alpha^2 - \rho)\right](n-1)(\alpha^2 - \rho).n - \frac{2}{n-3} \| Q \|^2 = 0,$$

that is

$$A.r + B.n - \frac{2}{n-3} \| Q \|^2 = 0.$$

The proof is completed. The converse is obvious. □

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