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NC-Smarandache Curve and NW-Smarandache Curve According to Alternative Frame

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Abstract. In this paper, NC-Smarandache curve and NW-Smarandache curve are defined, according to Alternative frame. Then, some characters of this curves are calculated.

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1. Introduction

In differential geometry, special curves have an important role. One of these curves Smarandache curves. Smarandache curves was firstly defined by M. Turgut and S. Yılmaz in 2008 [5]. Let $\alpha = \alpha(s)$ be a regular unit speed curve in E^3. This curves Frenet frame and Alternative frame are $\{T, N, B\}$ and $\{N, C, W\}$, respectively. In there, N is normal vector, W is unit Darboux vector and $C = W \wedge N$ [1].

In this paper, we created the Smarandache curves according to the alternative frame of the unit speed curve. Firstly, we introduced Frenet frame, Alternative frame and its properties. After that we mentioned the relationship with Alternative frame and Frenet frame. Then we defined two curves. And we calculated curvature, torsion, Frenet frame and Alternative frame of this curves.

2. Preliminaries

Let $\alpha = \alpha(s)$ be a regular curve with unit speed. Then the Frenet apparatus of the curve (α) [4]

$$\begin{aligned} T(s) &= \alpha'(s), \quad N(s) = \frac{T'(s)}{\|T'(s)\|}, \quad B(s) = T(s) \wedge N(s) \\ \kappa(s) &= \|T'(s)\|, \quad \tau(s) = \frac{\det(\alpha'(s), \alpha''(s), \alpha'''(s))}{(\|\alpha'(t) \wedge \alpha''(t)\|)^2} \\ T' &= \kappa N, \quad N' = -\kappa T + \tau B, \quad B' = -\tau N \end{aligned} \tag{2.1}$$

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In Euclidean 3-space any regular curve $\alpha(s)$ depending on the Frenet vectors moves around the axis of Darboux vector and the Darboux vector and defining a unit vector field are given as [2]

$$W = \frac{\tau T + \kappa B}{\sqrt{\kappa^2 + \tau^2}}, \quad C = W \wedge N$$

So build another orthonormal moving frame along the curve $\alpha(s)$. This frame defined as Alternative frame and is represented by $\{N, C, W\}$. The derivative formulae of the Alternative frame is given by [3]

$$N' = \beta C, \quad C' = -\beta N + \gamma W, \quad W' = -\gamma C, \quad \beta = \sqrt{\kappa^2 + \tau^2}, \quad \gamma = \frac{\kappa^2}{\kappa^2 + \tau^2}\Big(\frac{\tau}{\kappa}\Big)'.$$

The relationship between the Frenet frame and Alternative frame is

$$C = \overline{\kappa} T + \overline{\tau} B, \quad W = \overline{\tau} T + \overline{\kappa} B \quad or \quad T = -\overline{\kappa} C + \overline{\tau} W, \quad B = \overline{\tau} C + \overline{\kappa} W$$

where

$$\overline{\kappa} = \frac{\kappa}{\beta}, \quad \overline{\tau} = \frac{\tau}{\beta}.$$

Principal normal vector N is common both frames.

3. Smarandache Curves of Alternative Frame

Definition 3.1. Let $\alpha(s)$ be a regular curve with unit speed in E^3 and $\{N, C, W\}$ is Alternative frame. Then α_{NC}-Smarandache curve can be identified as

$$\alpha_{NC} = \frac{1}{\sqrt{2}}(N + C). \tag{3.1}$$

Theorem 3.2. *Let $\alpha(s)$ be a regular curve with unit speed in E^3 and $\{N, C, W\}$ is Alternative frame. The Frenet frame of α_{NC}-Smarandache curve is $\{T_{NC}, N_{NC}, B_{NC}\}$.*

$$T_{NC} = \frac{-\beta N + \beta C + \gamma W}{\sqrt{2\beta^2 + \gamma^2}}$$

$$N_{NC} = \frac{\chi_1 N + \nu_1 C + \mu_1 W}{\sqrt{\chi_1^2 + \nu_1^2 + \mu_1^2}}$$

$$B_{NC} = \frac{(\beta\mu_1 - \gamma\nu_1)N + (\beta\mu_1 + \gamma\chi_1)C + (-\beta\nu_1 - \beta\chi_1)W}{\sqrt{(\chi_1^2 + \nu_1^2 + \mu_1^2)(2\beta^2 + \gamma^2)}}$$

where

$$\chi_1 = -\beta^2(2\beta^2 + \gamma^2) - \gamma(\gamma\beta' - \beta\gamma'), \quad \nu_1 = -\beta^2(2\beta^2 + 3\gamma^2) - \gamma(\gamma^3 - \gamma\beta' + \beta\gamma'), \quad \mu_1 = \beta\gamma(2\beta^2 + \gamma^2) - 2\beta(\gamma\beta' - \beta\gamma').$$

Proof. If we take the derivative of the equation (3.1),

$$T_{NC}\frac{ds_{NC}}{ds} = \frac{-\beta N + \beta C + \gamma W}{\sqrt{2}}, \qquad \frac{ds_{NC}}{ds} = \sqrt{\frac{2\beta^2 + \gamma^2}{2}}. \tag{3.2}$$

From equations (3.2) tangent vector of α_{NC} curve is

$$T_{NC} = \frac{-\beta N + \beta C + \gamma W}{\sqrt{2\beta^2 + \gamma^2}}. \tag{3.3}$$

If we take the derivative of the equation (3.3), we can write

$$T'_{NC} = \frac{\sqrt{2}}{(2\beta^2 + \gamma^2)^2}(\chi_1 N + \nu_1 C + \mu_1 W) \tag{3.4}$$

where the coefficients are,

$$\chi_1 = -\beta^2(2\beta^2 + \gamma^2) - \gamma(\gamma\beta' - \beta\gamma'), \quad \nu_1 = -\beta^2(2\beta^2 + 3\gamma^2) - \gamma(\gamma^3 - \gamma\beta' + \beta\gamma'), \quad \mu_1 = \beta\gamma(2\beta^2 + \gamma^2) - 2\beta(\gamma\beta' - \beta\gamma').$$

If we take norm of equation (3.4), we can write

$$\|T'_{NC}\| = \frac{\sqrt{2}\sqrt{\chi_1^2+\nu_1^2+\mu_1^2}}{(2\beta^2+\gamma^2)^2}. \tag{3.5}$$

From equations (2.1), (3.4) and (3.5) principal normal vector of α_{NC} is

$$N_{NC} = \frac{\chi_1 N + \nu_1 C + \mu_1 W}{\sqrt{\chi_1^2+\nu_1^2+\mu_1^2}}. \tag{3.6}$$

Binormal vector of α_{NC} is

$$\begin{aligned} B_{NC} &= T_{NC} \wedge N_{NC} \\ B_{NC} &= \frac{(\beta\mu_1 - \gamma\nu_1)N + (\beta\mu_1 + \gamma\chi_1)C + (-\beta\nu_1 - \beta\chi_1)W}{\sqrt{(\chi_1^2+\nu_1^2+\mu_1^2)(2\beta^2+\gamma^2)}}. \end{aligned}$$

□

Theorem 3.3. *Let $\alpha(s)$ be a regular curve with unit speed in E^3. The curvature and torsion according to α_{NC}-Smarandache curve of Alternative Frame are, respectively,*

$$\begin{aligned} \kappa_{NC} &= \frac{\sqrt{2}\sqrt{\chi_1^2+\nu_1^2+\mu_1^2}}{(2\beta^2+\gamma^2)^2}, \\ \tau_{NC} &= \sqrt{2}\frac{(\beta^2+\gamma^2-\beta')(\beta\bar{\mu}_1+\gamma\bar{\chi}_1)+\beta(\beta\gamma+\gamma')(\bar{\nu}_1-\bar{\chi}_1)+(\beta^2+\beta')(\beta\bar{\mu}_1-\gamma\bar{\nu}_1)}{\left(2\gamma\beta^2+\gamma^3+\beta\gamma'-\beta'\gamma\right)^2+(\beta\gamma'-\beta'\gamma)^2+(2\beta^2-\beta\gamma^2)^2} \end{aligned}$$

where

$$\bar{\chi}_1 = \beta^3+\beta(2\gamma^{-}3\beta')-\beta'', \quad \bar{\nu}_1 = -\beta^3-\beta(\gamma^2+3\beta')-3\gamma\gamma'+\beta'', \quad \bar{\mu}_1 = -\beta^2\gamma-\gamma^3+2\gamma\beta'+\beta\gamma'+\gamma''.$$

Proof. From equations (2.1) and (3.5) the curvature according to α_{NC}-Smarandache curve κ_{NC} is

$$\kappa_{NC} = \frac{\sqrt{2}\sqrt{\chi_1^2+\nu_1^2+\mu_1^2}}{(2\beta^2+\gamma^2)^2}.$$

If we take second and third differential of equation (3.1) are, respectively

$$\begin{aligned} \alpha''_{NC} &= \frac{-(\beta^2+\beta')N+(\beta'-\beta^2-\gamma^2)C+(\beta\gamma+\gamma')W}{\sqrt{2}}, \\ \alpha'''_{NC} &= \frac{\bar{\chi}_1 N+\bar{\nu}_1 C+\bar{\mu}_1 W}{\sqrt{2}}. \end{aligned} \tag{3.7}$$

From equations (2.1) and (3.7) the torsion according to α_{NC}-Smarandache curve τ_{NC} is

$$\tau_{NC} = \sqrt{2}\frac{(\beta^2+\gamma^2-\beta')(\beta\bar{\mu}_1+\gamma\bar{\chi}_1)+\beta(\beta\gamma+\gamma')(\bar{\nu}_1-\bar{\chi}_1)+(\beta^2+\beta')(\beta\bar{\mu}_1-\gamma\bar{\nu}_1)}{\left(2\gamma\beta^2+\gamma^3+\beta\gamma'-\beta'\gamma\right)^2+(\beta\gamma'-\beta'\gamma)^2+(2\beta^2-\beta\gamma^2)^2}.$$

□

Theorem 3.4. *Let $\alpha(s)$ be a regular curve with unit speed in E^3 and $\{N, C, W\}$ is Alternative frame. The Alternative frame of α_{NC}-Smarandache curve is $\{N_{NC}, C_{NC}, W_{NC}\}$.*

$$\begin{aligned}
N_{NC} &= \frac{\chi_1 N + \nu_1 C + \mu_1 W}{\sqrt{\chi_1^2 + \nu_1^2 + \mu_1^2}}, \\
C_{NC} &= \frac{\mu_1(\chi_1 c - \mu_1 a) - \nu_1(\chi_1 b - \nu_1 a)}{(\chi_1^2 + \nu_1^2 + \mu_1^2)^{\frac{5}{2}}} N + \frac{\mu_1(\nu_1 c - \mu_1 b) - \chi_1(\chi_1 b - \nu_1 a)}{(\chi_1^2 + \nu_1^2 + \mu_1^2)^{\frac{5}{2}}} C + \frac{\nu_1(\nu_1 c - \mu_1 b) - \chi_1(\chi_1 c - \mu_1 a)}{(\chi_1^2 + \nu_1^2 + \mu_1^2)^{\frac{5}{2}}} W, \\
W_{NC} &= \frac{\nu_1 c - \mu_1 b}{(\chi_1^2 + \nu_1^2 + \mu_1^2)^2} N + \frac{\chi_1 c - \mu_1 a}{(\chi_1^2 + \nu_1^2 + \mu_1^2)^2} C + \frac{\chi_1 b - \nu_1 a}{(\chi_1^2 + \nu_1^2 + \mu_1^2)^2} W
\end{aligned}$$

where

$$\begin{aligned}
a &= (\chi_1' - \nu_1\beta)(\chi_1^2 + \nu_1^2 + \mu_1^2) - \chi_1(\chi_1 + \nu_1 + \mu_1)', \quad b = (\chi_1\beta + \nu_1' - \gamma\mu_1)(\chi_1^2 + \nu_1^2 + \mu_1^2) - \nu_1(\chi_1 + \nu_1 + \mu_1)', \\
c &= (\gamma\nu_1 + \mu_1')(\chi_1^2 + \nu_1^2 + \mu_1^2) - \mu_1(\chi_1 + \nu_1 + \mu_1)'
\end{aligned}$$

Proof. From equation (3.6) principal normal vector of α_{NC} is

$$N_{NC} = \frac{\chi_1 N + \nu_1 C + \mu_1 W}{\sqrt{\chi_1^2 + \nu_1^2 + \mu_1^2}}.$$

If we take derivative of equation (3.6), we can write

$$N'_{NC} = \frac{a}{(\chi_1^2 + \nu_1^2 + \mu_1^2)^{\frac{3}{2}}} N + \frac{b}{(\chi_1^2 + \nu_1^2 + \mu_1^2)^{\frac{3}{2}}} C + \frac{e}{(\chi_1^2 + \nu_1^2 + \mu_1^2)^{\frac{3}{2}}} W. \tag{3.8}$$

where the coefficients are

$$\begin{aligned}
a &= (\chi_1' - \nu_1\beta)(\chi_1^2 + \nu_1^2 + \mu_1^2) - \chi_1(\chi_1 + \nu_1 + \mu_1)', \quad b = (\chi_1\beta + \nu_1' - \gamma\mu_1)(\chi_1^2 + \nu_1^2 + \mu_1^2) - \nu_1(\chi_1 + \nu_1 + \mu_1)', \\
c &= (\gamma\nu_1 + \mu_1')(\chi_1^2 + \nu_1^2 + \mu_1^2) - \mu_1(\chi_1 + \nu_1 + \mu_1)'.
\end{aligned}$$

From equations (3.6) and (3.8) unit darboux vector of α_{NC} is

$$W_{NC} = \frac{\nu_1 c - \mu_1 b}{(\chi_1^2 + \nu_1^2 + \mu_1^2)^2} N + \frac{\chi_1 c - \mu_1 a}{(\chi_1^2 + \nu_1^2 + \mu_1^2)^2} C + \frac{\chi_1 b - \nu_1 a}{(\chi_1^2 + \nu_1^2 + \mu_1^2)^2} W. \tag{3.9}$$

From equations (3.6) and (3.9) unit vector C_{NC} is

$$\begin{aligned}
C_{NC} &= W_{NC} \wedge N_{NC} \\
C_{NC} &= \frac{\mu_1(\chi_1 c - \mu_1 a) - \nu_1(\chi_1 b - \nu_1 a)}{(\chi_1^2 + \nu_1^2 + \mu_1^2)^{\frac{5}{2}}} N + \frac{\mu_1(\nu_1 c - \mu_1 b) - \chi_1(\chi_1 b - \nu_1 a)}{(\chi_1^2 + \nu_1^2 + \mu_1^2)^{\frac{5}{2}}} C + \frac{\nu_1(\nu_1 c - \mu_1 b) - \chi_1(\chi_1 c - \mu_1 a)}{(\chi_1^2 + \nu_1^2 + \mu_1^2)^{\frac{5}{2}}} W.
\end{aligned}$$

□

Definition 3.5. Let $\alpha(s)$ be a regular curve with unit speed in E^3 and $\{N, C, W\}$ is Alternative frame. Then α_{NW}-Smarandache curve can be identified as

$$\alpha_{NW} = \frac{1}{\sqrt{2}}(N + W). \tag{3.10}$$

Theorem 3.6. *Let $\alpha(s)$ be a regular curve with unit speed in E^3 and $\{N, C, W\}$ is Alternative frame. The Frenet frame of α_{NW}-Smarandache curve is $\{T_{NW}, N_{NW}, B_{NW}\}$.*

$$T_{NW} = C, \quad N_{NW} = \frac{-\beta N + \gamma W}{\sqrt{\beta^2 + \gamma^2}}, \quad B_{NW} = \frac{\gamma N + \beta W}{\sqrt{\beta^2 + \gamma^2}}.$$

Proof. If we take derivative of the equation (3.10)

$$T_{NW}\frac{ds_{NW}}{ds} = \frac{(\beta-\gamma)C}{\sqrt{2}}, \qquad \frac{ds_{NW}}{ds} = \sqrt{\frac{(\beta-\gamma)^2}{2}}. \tag{3.11}$$

From equations (3.11) tangent vector of α_{NW}-Smarandache curve is

$$T_{NW} = C \tag{3.12}$$

If we take the derivative of the equation (3.12), we can write ,

$$T'_{NW} = \frac{\sqrt{2}}{\beta-\gamma}(-\beta N+\gamma W). \tag{3.13}$$

If we take norm of equation (3.13), we can write

$$\|T'_{NW}\| = \frac{\sqrt{2(\beta^2+\gamma^2)}}{\beta-\gamma}. \tag{3.14}$$

From equations (2.1), (3.13) and (3.14) principal normal vector of α_{NW} is

$$N_{NW} = \frac{-\beta N+\gamma W}{\sqrt{\beta^2+\gamma^2}}. \tag{3.15}$$

Binormal vector of α_{NW} is

$$B_{NW} = T_{NW}\wedge N_{NW} = \frac{\gamma N+\beta W}{\sqrt{\beta^2+\gamma^2}}.$$

□

Theorem 3.7. *Let $\alpha(s)$ be a regular curve with unit speed in E^3 and $\{N, C, W\}$ is Alternative frame. The curvature and torsion according to α_{NW}-Smarandache curve of Alternative Frame are, respectively,*

$$\kappa_{NW} = \frac{\sqrt{2(\beta^2+\gamma^2)}}{\beta-\gamma},$$
$$\tau_{NW} = \sqrt{2}\frac{(\beta^3\bar{\mu_2}-2\beta^2\gamma\bar{\mu_2}+\beta\gamma^2\bar{\mu_2}+\beta^2\gamma\bar{\chi_2}+\gamma^3\bar{\chi_2}-2\beta\gamma^2\bar{\chi_2})}{\left(\gamma(\beta-\gamma)^2\right)^2+\left(\beta(\beta-\gamma)^2\right)^2}$$

where

$$\bar{\chi_2} = -3\beta\beta'+2\beta\gamma'+\beta'\gamma, \quad \bar{\nu_2} = \beta^3+\gamma\beta^2-\beta\gamma^2+\gamma^3+\beta''-\gamma'', \quad \bar{\mu_2} = \beta\gamma'+2\gamma'\beta-3\gamma\gamma'.$$

Proof. From equations (2.1) and (3.14) the curvature according to α_{NW}-Smarandache curve κ_{NW} is

$$\kappa_{NW} = \frac{\sqrt{2(\beta^2+\gamma^2)}}{\beta-\gamma}.$$

If we take second and third differential of equation (3.10) are, respectively

$$\alpha''_{NW} = \frac{(-\beta^2+\gamma\beta)N+(\beta'-\gamma')C+(\beta\gamma-\gamma^2)W}{\sqrt{2}}, \tag{3.16}$$

$$\alpha'''_{NW} = \frac{\bar{\chi_2}N+\bar{\nu_2}C+\bar{\mu_2}W}{\sqrt{2}}. \tag{3.17}$$

From equations (2.1), (3.16) and (3.17) the torsion according to α_{NW}-Smarandache curve τ_{NW} is

$$\tau_{NW} = \sqrt{2}\frac{(\beta^3\bar{\mu_2}-2\beta^2\gamma\bar{\mu_2}+\beta\gamma^2\bar{\mu_2}+\beta^2\gamma\bar{\chi_2}+\gamma^3\bar{\chi_2}-2\beta\gamma^2\bar{\chi_2})}{\left(\gamma(\beta-\gamma)^2\right)^2+\left(\beta(\beta-\gamma)^2\right)^2}.$$

where the coefficients are

$$\bar{\chi_2} = -3\beta\beta'+2\beta\gamma'+\beta'\gamma, \quad \bar{\nu_2} = \beta^3+\gamma\beta^2-\beta\gamma^2+\gamma^3+\beta''-\gamma'', \quad \bar{\mu_2} = \beta\gamma'+2\gamma'\beta-3\gamma\gamma'.$$

□

Theorem 3.8. *Let $\alpha(s)$ be a regular curve with unit speed in E^3 and $\{N, C, W\}$ is Alternative frame. The Alternative frame of α_{NW}-Smarandache curve is $\{N_{NW}, C_{NW}, W_{NW}\}$.*

$$\begin{aligned} N_{NW} &= \frac{-\beta N + \gamma W}{\sqrt{\beta^2 + \gamma^2}}, \\ C_{NW} &= \frac{\beta\gamma(\beta^2 + \gamma^2)(\gamma - \gamma')}{(\beta^2 + \gamma^2)^{\frac{5}{2}}} N + \frac{(\beta^2 + \gamma^2)^2}{(\beta^2 + \gamma^2)^{\frac{5}{2}}} C + \frac{\beta^2(\beta^2 + \gamma^2)(\gamma - \gamma')}{(\beta^2 + \gamma^2)^{\frac{5}{2}}} W, \\ W_{NW} &= \frac{\gamma(\beta^2 + \gamma^2)}{(\beta^2 + \gamma^2)^2} N + \frac{\beta(\beta^2 + \gamma^2)(\gamma - \gamma')}{(\beta^2 + \gamma^2)^2} C + \frac{\beta(\beta^2 + \gamma^2)}{(\beta^2 + \gamma^2)^2} W. \end{aligned}$$

Proof. From equation (3.15) principal normal vector of α_{NW} is

$$N_{NW} = \frac{-\beta N + \gamma W}{\sqrt{\beta^2 + \gamma^2}}.$$

If we take derivative of equation (3.15), we can write

$$N'_{NW} = \frac{-\beta'\gamma^2 - \beta\gamma\gamma'}{(\beta^2 + \gamma^2)^{\frac{3}{2}}} N - \frac{\beta^4 + \gamma^4 + 2\beta^2 + \gamma^2}{(\beta^2 + \gamma^2)^{\frac{3}{2}}} C + \frac{\beta^2\gamma' - \gamma\beta\beta' + 2\gamma'\gamma^2}{(\beta^2 + \gamma^2)^{\frac{3}{2}}} W. \tag{3.18}$$

From equations (3.15) and (3.18) Darboux vector of α_{NW} is

$$\begin{aligned} W_{NW} &= \frac{-\gamma^3(\gamma^2 + 1) - \beta^2\gamma(\beta^2 + 2)}{(\beta^2 + \gamma^2)^2} N + \frac{-\beta'\gamma(\gamma^2 - \beta^2) - \beta\gamma'(\beta^2 + \gamma^2)}{(\beta^2 + \gamma^2)^2} C \\ &\quad + \frac{\gamma^2\beta(\gamma^2 + 1) + \beta^3\gamma(\beta^2 + 2)}{(\beta^2 + \gamma^2)^2} W. \end{aligned} \tag{3.19}$$

From equations (3.15) and (3.19) unit vector C_{NW} is

$$\begin{aligned} C_{NW} &= W_{NW} \wedge N_{NW} \\ C_{NW} &= \frac{-\beta'\gamma^2(\gamma^2 - \beta^2) - \beta\gamma\gamma'(\beta^2 + \gamma^2)}{(\beta^2 + \gamma^2)^{\frac{5}{2}}} N + \frac{-\gamma^4 - \gamma^2 - \beta^4\gamma - 2\beta^2\gamma}{(\beta^2 + \gamma^2)^{\frac{3}{2}}} C \\ &\quad + \frac{\beta\beta'\gamma(\gamma^2 - \beta^2) - \beta^2\gamma'(\beta^2 + \gamma^2)}{(\beta^2 + \gamma^2)^{\frac{5}{2}}} W. \end{aligned}$$

□

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