Available online: January 4, 2019

Commun. Fac. Sci. Univ. Ank. Ser. A1 Math. Stat.
Volume 68, Number 1, Pages 1114–1122 (2019)
DOI: 10.31801/cfsuasmas.508205
ISSN 1303–5991 E-ISSN 2618-6470

http://communications.science.ankara.edu.tr/index.php?series=A1

COMMUNICATIONS
SERIES A1

OUTPUT STABILIZATION OF SEMILINEAR PARABOLIC SYSTEMS WITH BOUNDED FEEDBACK

ABDESSAMAD EL ALAMI, ALI BOUTOULOUT, AND RADOUANE YAFIA

Abstract. In this paper, we will study the output feedback stabilization of infinite-semilinear parabolic systems evolving on a spatial domain Ω and in a subregion ω of Ω (interior to Ω or on its boundary $\partial\Omega$). We consider the condition of admissibility and the decomposition methods technique of the state space via the spectral properties of the system. Then we apply this approach to a regional exponential stabilization problem using bounded feedback. Applications are presented.

1. Introduction

In this work, we study the stabilization of the system described by the equation:

$$\frac{dy(t)}{dt} = Ay(t) + Ny(t) + v(t)By(t), \quad y(0) = y_0, \tag{1}$$

where Ω is an open bounded subset of $\mathcal{R}^n$ with smooth boundary $\partial\Omega$. The state space $\mathcal{H}$ endowed with the inner product $\langle ., . \rangle$, and the corresponding norm $\|.\|$. A is the dynamic unbounded operator with domain $D(A) \subset \mathcal{H}$ and generates a semigroup of contractions $(S(t))_{t\geq 0}$ on $\mathcal{H}$, and N is a nonlinear operator such that $N(0) = 0$. B is a linear operator from $\mathcal{H}$ to $\mathcal{H}$. The valued function $t \mapsto v(t)$ represents the control. Let ω be an open and positive Lebesgue measurable subset of Ω, and $\Gamma \subset \partial\Omega$. The operator χ_Γ is define by

$$\begin{array}{rcl} \chi_\Gamma : \mathcal{H} & \longrightarrow & \mathcal{H}_\Gamma, \\ y & \longrightarrow & \chi_\Gamma y = y_{/\Gamma}, \end{array} \tag{2}$$

where χ_Γ^* the adjoint operator of χ_Γ and set $i_\Gamma = \chi_\Gamma^* \chi_\Gamma$.
Taking $\mathcal{H} = L^2(\partial\Omega)$ then $\mathcal{H}_\Gamma = L^2(\Gamma)$. We define the stability of the semilinear system (1) on Γ as:

Received by the editors: May 30, 2018, Accepted: August 30, 2018.
2010 *Mathematics Subject Classification.* 35K58, 35B35.
Key words and phrases. Output stabilzation, semilinear systems, bounded feedback.
Submitted via ICCDS2018.

©2019 Ankara University
Communications Faculty of Sciences University of Ankara-Series A1 Mathematics and Statistics

Definition 1.1. *The system (1) is $\Gamma-$ weakly (resp.$\Gamma-$ strongly, exponentially) stabilizable if there exists a feedback control $v(t) = f(y(t))$, $t \geq 0$, $f : \mathcal{H} \longrightarrow K := \mathcal{R}, \mathcal{C}$ such that the corresponding mild solution $y(t)$ of the system (1) satisfies the properties:*

(1)
(2) *For each initial state y_0 of the system (1) there exists a unique mild solution defined for all $t \in \mathcal{R}^+$ of the system (1).*
(3) *$\{0\}$ is an equilibrium state of the system (1).*
(4) *$\chi_\Gamma y(t) \rightarrow 0$, weakly (resp. strongly, exponentially), as $t \rightarrow +\infty$, for all $y_0 \in \mathcal{H}$.*

1.1. **Motivation.** The following system gives the motivation behind our study

$$\begin{cases} \dfrac{dy(x_1,x_2,t)}{dt} = \Delta y(x_1,x_2,t) + \dfrac{y^2(x_1,x_2,t)}{1+y^2(x_1,x_2,t)} + u(t), \quad on\ ]0,2[^2 \times]0,+\infty[, \\ y(0) = y_0 \in Z, \end{cases} \tag{3}$$

where $u(t) = e^{(\frac{1}{2}-x_2)t}(\frac{1}{2} - x_2 - t^2 - \frac{e^{(\frac{1}{2}-x_2)t}}{1+e^{2(\frac{1}{2}-x_2)t}})$.
and the state space $Z := \{y \in \mathcal{H}^1(\Omega)/y = 0\ on\ \Gamma_0 = \{0\} \times [0,1]\}$ (Z is a Hilbert space), that is a closed subspace of $\mathcal{H}^1(\Omega)$ endowed with its natural inner product.

Proposition 1.2. *The system (3) is not stable on any subregion $\omega \subset \Omega$, but it is stable on $\Gamma \subset \partial\omega$, where $\Gamma = \{0\} \times [0,2]$.*

Proof : The system (3) is not stable on any subregion $\omega \subset \Omega$, indeed the solution of systems (3) is $y(t) = e^{(\frac{1}{2}-x_2)t}$ which does not tend to zero as $t \longrightarrow +\infty$ on any subregion $\omega \subset \Omega$, but for $\Gamma = \{0\} \times [0,2]$, we have

$$\|\chi_\Gamma y(t)\|_{L^2(\Gamma)} \leq e^{-t}\|y_0\|_{L^2(\Gamma)}\,\forall t > 0.$$

Definition 1.3. *(see [12]) Let us consider $(S(t))_{t\geq 0}$ linear semigroup, we say that Γ is called an admissible subregion for $(S(t))_{t\geq 0}$, if:*

-
- *The mapping $\chi_\Gamma : D(A) \mapsto D(\chi_\Gamma)$ is continuous, when $D(A)$ is equipped with the graph norm $\|.\|_A$,*
- *there is some $\gamma > 0$ such that*

$$\int_0^\infty \|\chi_\Gamma S(t)y_0\|^2_{L^2(\Gamma)}dt \leq \gamma\|y_0\|^2, forall y_0 \in D(A). \tag{4}$$

Remark 1.4.

(1)
(2) *The first condition of admissiblity implies,in particular, that $D(A) \subset D(\chi_\Gamma)$.*

(3) *It follows from 4 that the mapping* $y_0 \mapsto \chi_{\Gamma} S(\cdot) y_0$ *has a continuous extension from* $D(A)$ *to* $L^2(0,\infty; L^2(\Gamma))$ *for every* $t > 0$.

(4) *If* $(S(t))_{t\geq 0}$ *is exponentially stable, then the notion of finite-time admissibility and infinite-time admissibility are equivalent.*

We present some results showing how to obtain the boundary stabilization. Much work has been done in this direction, both for linear and bilinear problems, see for instance about:
In [3], the quadratic control

$$v(t) = -\langle y(t), i_{\Gamma} B y(t) \rangle, \tag{5}$$

was proposed to study the feedback stabilization of (1) with $N = 0$, and a weak stabilization result was established under the condition

$$\langle BS(t)y, S(t)y \rangle = 0, \ \forall t \geq 0 \Rightarrow \chi_{\Gamma} y = 0 \,. \tag{6}$$

In [6], it has been proved that under (6), the same quadratic control(5) ensures the strong stabilization for a class of semilinear systems.
Recently, the regional exponential stabilization problem of distributed semilinear systems has been resolved (see [2]). Then it has been proved that under the assumption:

$$\int_0^T |\langle i_\omega BS(t)y, S(t)y \rangle| dt \geq \delta \|\chi_\omega y\|^2, \ \forall \ y \in H, \ \forall T, \delta > 0, \tag{7}$$

the feedback defined by
$v_j(t) = \frac{-B^* i_\omega y(t)}{R_j(i_\omega y(t))} (j = 1, 2)$,
where $R_1(i_\omega y) = 1 + \|B^* i_\omega y\|_U$,
and, $R_2(i_\omega y) = sup(1, \|B^* i_\omega y\|_U)$, guarantees the
regional exponential stabilization. The bilinear finite-dimensional case has been treated in ([2]).

2. Stabilization results

In this paper we need some assumptions:

- A_1- Assume that $i_{\Gamma} A$ is dissipative and:

$$\|\chi_{\Gamma} S(t) y(t)\|_{L^2(\Gamma)} \leq \|\chi_{\Gamma} y(t)\|_{L^2(\Gamma)},$$

- A_2- The nonlinear operator N is dissipative on ω,

$$(i.e, \langle i_{\Gamma} N y, y \rangle_{L^2(\Gamma)} \leq 0, \forall y \in H),$$

  such that:

$$\|\chi_{\Gamma} N y(t)\|_{L^2(\Gamma)} \leq \|\chi_{\Gamma} y(t)\|_{L^2(\Gamma)},$$

- A_3- A generates a C_0-semigroup $(S(t))_{t\geq 0}$ on Hilbert H, and $N(\cdot) : H \to H$ is a locally Lipschitz, that is, there exists a positive constant K such that

$$\|N(y) - N(z)\|_{L^2(\Omega)} \leq K \|y - z\|_{L^2(\Omega)}, \forall (y, z) \in H,$$

and we can show that

$$K(N) = \sup_{y \neq z} \frac{\|N(y) - N(z)\|_{L^2(\Omega)}}{\|y - z\|_{L^2(\Omega)}}.$$

- Assume that the operator B satisfies:

$$\int_0^T |\langle i_\Gamma BS(t)y, S(t)y\rangle|_U dt \geq \delta \|\chi_\Gamma y\|^2_{H_\omega},\ \forall y \in H,\ (T, \delta > 0 \tag{8}$$

with $\delta \leq \delta(B) = \inf\limits_{\|y_0\|=1} |\langle i_\Gamma BS(t)y, S(t)y\rangle|_{L^2(0,T,\mathcal{H})}$.

- A $\Gamma-$weak stabilization result was obtained under the weak observability condition:

$$\langle BS(t)y, S(t)y\rangle = 0,\ \forall t \geq 0 \Longrightarrow \chi_\Gamma y = 0. \tag{9}$$

2.1. **Regional Boundary exponential stabilization.** The first result concerns the Regional Boundary exponential stability of the system (1) using properties of the spectrum $\sigma(A)$ of A. For this we will assume that the operator A satisfies the following condition:
(H) : A is self-adjoint with compact resolvent, so that A possesses a sequence $(\lambda_n)_{n\geq 1}$of real eigenvalues, which can be numbered in decreasing order in such away that $\lambda \to -\infty$. Moreover, there are at most finitely many nonnegative eigenvalues $\varsigma = \{\lambda_1, \lambda_2, ..., \lambda_N\}$(which can be empty) of A, each with finite-dimensional eigenspace. The eigenvectors $(\varphi_{n_j}), 1 \leq j \leq m_n$ associated with λ_n (m_n is the multiplicity of λ_n) compose a complete system in $\mathcal{H}$ (see [3]).

Proposition 2.1. *let* Γ *is an admissible subregion for* $(S(t))_{t\geq 0}$ *Then the following properties are equivalents:*

(1)
(2) *The system (1) is Regionally exponentially stable On* Γ *by the control*

$$v(t) = -\frac{\rho\langle i_\Gamma By(t), y(t)\rangle}{1 + |\langle i_\Gamma By(t), y(t)\rangle|},\ \rho,\ t > 0. \tag{10}$$

(3) $\forall \lambda \in \sigma(A), Re(\lambda) \geq 0 \Longrightarrow \forall \phi \in N(A - \lambda I), \chi_\Gamma \varphi = 0.$

Proof: First we show that (1) $\Longrightarrow$ (2) note that from the assumption (H), we have $S(t)y_0 \in D(A), \forall y_0 \in \mathcal{H}$ and $t > 0$. we have

$$S(t)y_0 = \sum_{n=1}^{+\infty} \exp(\lambda_n t) \sum_{j=1}^{m_n} \langle y_0, \varphi_{n_j}\rangle . \varphi_{n_j},$$

Then

$$\|\chi_\Gamma S(t)y_0\|^2_{L^2(\Gamma)} = \sum_{n=1}^{+\infty} \exp((\lambda_n + \lambda_m)t) \sum_{j=1}^{m_n} \sum_{k=1}^{m_m} \langle y_0, \varphi_{n_j}\rangle \langle y_0, \varphi_{m_k}\rangle \langle \varphi_{n_j}, \varphi_{m_k}\rangle_{L^2(\Gamma)},$$

taking $y_0 = \varphi_{n_0 j_0}$ we obtained

$$\|\chi_\Gamma S(t) y_0\|^2_{L^2(\Gamma)} = \exp((2\lambda_{n_0})t)\|\chi_\Gamma \varphi_{n_0 j_0}\|^2_{L^2(\Gamma)}$$

Furthermore

$$\lim_{t\to+\infty} \|\chi_\Gamma S(t) y_0\|_{L^2(\Gamma)} = 0 \Longleftrightarrow \lambda_{n_0} < 0, or : \|\varphi_{n_0 j_0}\|_{L^2(\Gamma)} = 0,$$

if $\|\varphi_{n_j}\|_{L^2(\Gamma)} \neq 0$, we deduce that

$$\lim_{t\to+\infty} \|\chi_\Gamma S(t) y_0\|_{L^2(\Gamma)} = 0 \Longleftrightarrow \lambda_{n_j} < 0,$$

using the variation of constant formula with y_0 as the initial state, we get:

$$y(t) = S(t-s)y_0 + \int_0^t v(s)S(t-s)By(s) + S(t-s)Ny(s)ds, \forall t \in [0,T], \tag{11}$$

which give

$$\|\chi_\Gamma S(t)y_0\|_{L^2(\Gamma)} \leq (1 + (\rho\|B\| + L_{\|y_0\|})T)\|\chi_\Gamma y(s)\|_{L^2(\Gamma)}, \forall : 0 < s < t.$$

Or

$$\exp((2\lambda_{n_0})t)\|\chi_\Gamma \varphi_{n_0 j_0}\|^2_{L^2(\Gamma)} = \|\chi_\Gamma S(t)y_0\|^2_{L^2(\Gamma)} = \|\chi_\Gamma S(t-s)S(s)y_0\|^2_{L^2(\Gamma)}.$$

Then if there exist $\lambda \in \sigma(A)$, and $\varphi_{n_0 j_0} \in N(A-\lambda)$ such that $Re(\lambda) \geq 0$ and $\chi_\Gamma \varphi_{n_0 j_0} \neq 0$, which give

$$\exp((2\lambda_{n_0})t)\|\chi_\Gamma \varphi_{n_0 j_0}\|^2_{L^2(\Gamma)} \leq (1 + (\rho\|B\| + L_{\|y_0\|})T)\|\chi_\Gamma y(s)\|_{L^2(\Gamma)}, \forall : 0 < s < t.$$

and we deduce that the system (1) is not regionally stable on Γ. Suppose that (2), and taking $y_0 = y_{01} + y_{02} \in \mathcal{H}$ where

$$y_{01} = \sum_{n=1}^{N}\sum_{j=1}^{m_n} \langle y_0, \varphi_{n_j}\rangle.\varphi_{n_j},$$

and

$$y_{02} = \sum_{n=N+1}^{+\infty}\sum_{j=1}^{m_n} \langle y_0, \varphi_{n_j}\rangle.\varphi_{n_j},$$

From (2), we have $\chi_\Gamma S(t)y_0 = \chi_\Gamma S(t)y_{02}$, and $\|S(t)y_{02}\| \leq \exp((2\lambda_{N+1})t)\|y_{02}\|$. It follows that $S(t)y_{02} \longrightarrow 0$ exponentially as $t \longrightarrow +\infty$. If $y_{02} \in D(A)$, then $S(t)y_{02} \in D(A), : \forall t \geq 0$ and $AS(t)y_{02} = S(t)Ay_{02} \longrightarrow 0$ exponentially as $t \longrightarrow +\infty$, which give $\|S(t)y_{02}\|_A = (\|S(t)y_{02}\| + \|S(t)Ay_{02}\|)^{\frac{1}{2}} \longrightarrow 0$ exponentially as $t \longrightarrow +\infty$. Using the continuity of $\chi_\Gamma : (D(A), \|.\|_A) \mapsto L^2(\Gamma)$, we deduce $\chi_\Gamma S(t)y_{02} \longrightarrow 0$ exponentially as $t \longrightarrow +\infty$, or

$$(1 - (\rho\|B\| + L_{\|y_0\|})T)\|\chi_\Gamma y(t)\|_{L^2(\Gamma)} \leq \|\chi_\Gamma S(t)y_0\|_{L^2(\Gamma)}, \forall : t > 0.$$

Then if $0 < \rho < \frac{1-\|B\|L}{\|B\|T}$ we have $\chi_\Gamma y(t) \longrightarrow 0$ exponentially as $t \longrightarrow +\infty$, 2.2. **Regional Boundary Strong stabilization.** The next result concerns the regional Boundary strong stabilization of (1) on Γ.

Theorem 2.2. *Let us consider the following assumptions:*

(1)
(2) *A generate a linear C_0-contraction semigroup $(S(t))_{t\geq 0}$ on H such that (8) holds.*
(3) *The nonlinear operator N satisfies A_2.*

Then the system(1) is $\Gamma-$ strongly stabilizable by the feedback

$$v(t) = -\frac{\rho\langle i_\Gamma By(t), y(t)\rangle}{1+|\langle i_\Gamma By(t), y(t)\rangle|},\ \rho,\ t>0. \tag{12}$$

Proof : Let $y(t)$ denote the corresponding solution of (1). For $t \geq 0$ we define the function

$$\tau \longrightarrow z(\tau) := \int_t^\tau v(s)S(\tau-s)By(s) + S(\tau-s)Ny(s)ds$$

Applying the variation of constant formula with $y(t)$ as the initial state, we get

$$y(\tau) = S(\tau-t)y(t) + z(\tau), \forall\tau \in [t, t+T], \tag{13}$$

we obtain
$\|\chi_\Gamma y(\tau)\| \leq \|\chi_\Gamma S(\tau-t)y(t)\| + \|\chi_\Gamma z(\tau)\|$ Then we have

$$\|\chi_\Gamma y(\tau)\| \leq \|\chi_\Gamma y(t)\| + \|\chi_\Gamma z(\tau)\|.$$

Furthermore:

$$\|\chi_\Gamma y(\tau)\| \leq \|\chi_\Gamma y(t)\| + (\rho\|B\| + K_{\|y_0\|})\int_t^\tau \|\chi_\Gamma y(s)\|ds,$$

where $\|B\|$ is the norm of B. The Gronwall inequality then yields

$$\|\chi_\Gamma y(\tau)\| \leq \|\chi_\Gamma y(t)\| e^{(\rho\|B\|+k_{\|y_0\|})T}, \forall\tau \in [t, t+T], \tag{14}$$

we have the relation $\langle i_\Gamma BS(\tau-t)y(t), S(\tau-t)y(t)\rangle = \langle i_\Gamma B(y(\tau)-z(\tau)), S(\tau-t)y(t)\rangle$
$= -\langle i_\Gamma Bz(\tau), S(\tau-t)y(t)\rangle - \langle i_\Gamma By(\tau), z(\tau)\rangle$
$+ \langle i_\Gamma By(\tau), y(\tau)\rangle$.

It follows that:
$|\langle i_\Gamma BS(\tau-t)y(t), S(\tau-t)y(t)\rangle|$
$\leq \|B\|\|\chi_\Gamma y(t)\|\|\chi_\Gamma z(\tau)\|$
$+ \|B\|\|\chi_\Gamma z(\tau)\|\|\chi_\Gamma y(\tau)\| + |\langle i_\Gamma By(\tau), y(\tau)\rangle|$.
Therefore

$$|\langle i_\Gamma BS(\tau-t)y(t), S(\tau-t)y(t)\rangle| \leq \|B\|\|\chi_\Gamma z(\tau))\|(\|\chi_\Gamma y(t)\| + \|\chi_\Gamma y(\tau)\|) + |\langle i_\Gamma By(\tau), y(\tau)\rangle|. ,$$

Using (14) we deduce that:
$|\langle i_\Gamma BS(\tau-t)y(t), S(\tau-t)y(t)\rangle|$
$\leq (\rho\|B\|^2 + B\|k_{\|y_0\|})T\|\chi_\Gamma y(t)\|^2$
$(1+e^{(\rho\|B\|+K_{\|y_0\|})T})e^{(\rho\|B\|+k_{\|y_0\|})T} + |\langle i_\Gamma By(\tau), y(\tau)\rangle|$.

By integrating this inequality over $[t, t+T]$ we obtain the estimate

$$\int_0^T |\langle i_\Gamma BS(\tau-t)y(t), S(\tau-t)y(t)\rangle| d\tau$$
$$\leq (\rho\|B\|^2 + \|B\|k_{\|y_0\|})T^2 e^{(\rho\|B\|+k_{\|y_0\|})T}\|\chi_\Gamma y(t)\|^2$$
$$+ (\rho\|B\|^2\| + B\|k_{\|y_0\|})T^2 (e^{(\rho\|B\|+k_{\|y_0\|})T})^2\|\chi_\Gamma y(t)\|^2$$
$$+ \int_t^{t+T} |\langle i_\Gamma By(\tau), y(\tau)\rangle| d\tau.$$

We have

$$\int_t^{t+T} |\langle i_\Gamma By(\tau), y(\tau)\rangle| d\tau \longrightarrow 0, as: t \longrightarrow +\infty.$$

Taking $\rho > K > 0$ such that:

$$(\rho\|B\|^2 + \|B\|k_{\|y_0\|})T^2(1 + e^{(\rho\|B\|+K_{\|y_0\|})T})e^{(\rho\|B\|+k_{\|y_0\|})T} < \delta,$$

and using (8) we deduce that $\|\chi_\Gamma y(t)\| \longrightarrow 0, as: t \longrightarrow +\infty$.

Remark 2.3.

(1) *Note that the feedback (12) is a bounded function in time and is uniformly bounded with respect to initial states,*

$$|v(t)| \leq \rho\|B\|, \quad \forall t \geq 0,\ \forall y_0 \in H.$$

(2) *Since $\|y(t)\|$ decreases, then we have $\exists t_0 \geq 0$; $y(t_0) = 0 \Leftrightarrow y(t) = 0,\ \forall t \geq t_0$. In this case we have $v(t) = 0,\ \forall t \geq t_0$.*

3. Applications

3.1. **Example.** Let us consider the following semilinear heat equation, and $\Omega =]0,1[^2$:

$$\begin{cases} \dfrac{\partial y(x_1,x_2,t)}{\partial t} = Ay(x_1,x_2,,t) + Ny(x_1,x_2,t) + v(t)By(x_1,x_2,t), & in \quad Q, \\ y(0) = y_0, \dfrac{\partial y(.,t)}{\partial \nu} = 0, & on \quad \partial Q, \end{cases} \tag{15}$$

where $y(t)$ is the temperature profile at time t, and $Q = \Omega \times]0, +\infty[$. We suppose that the system is controlled via the flow of a liquid $v(t)$. Here we take the state space $H = L^2(0,1)$ and the operator A is defined by $Ay = \dfrac{\partial^2 y}{\partial x_1^2} + \dfrac{\partial^2 y}{\partial x_2^2}, \forall y \in \mathcal{D}(A)$ with

$$\mathcal{D}(A) = \{y \in H^2(0,1) |\ \frac{\partial y(0,t)}{\partial \nu} = 0, on: \partial Q\}.$$

The domain of A gives the homogeneous Neumann boundary. Here we take the state space $H = U = L^2(0,1)$. The spectrum of A is given by the simple eigenvalues $\lambda_{kj} = -\pi^2(j+k)^2,\ j,k \in \mathcal{N}^*$ and eigenfunctions $\varphi_{(k,j)}(x_1,x_2) = e_k(x_1)e_j(x_2)$, where $e_k(x_1) = \sqrt{2}\cos(k\pi x_1)$ if $k \neq 0$ and $e_0(x_1) = 1$. The operator of control B, is defined by : $By = \langle y, \varphi_{(1,0)}\rangle.\varphi_{(1,0)}$, B is linear operator, and we have

$$< By, y >= |\langle y, \varphi_{(1,0)}\rangle|^2,$$

and

$$Ny = -\frac{\langle y, \varphi_{(1,1)}\rangle.\varphi_{(1,1)}}{1 + |\langle y, \varphi_{(1,1)}\rangle|}$$

which is nonlinear operator, dissipative and locally Lipschitz. Let us show that the subregion $\Gamma = \{\frac{1}{2}\} \times [0, 1]$. From the Green formula and using the Schwartz inequality, we have

$$\int_0^{+\infty} \|\chi_\Gamma y(s)\|^2 ds < +\infty,$$

and thus Γ is admissible for $S(t)$ Then the systems (15) is Γ-exponentially stabilizable by the control:

$$v(t) = -\frac{2\pi^2|\langle i_\Gamma y(t), \varphi_{(1,0)}\rangle|^2}{1 + |\langle i_\Gamma y(t), \varphi_{(1,0)}\rangle|^2}, : t > 0.$$

Conclusion. In this work we have considered the problem of regional boundary exponential stabilization with output of a constrained parabolic semilinear system under the condition of admissibility for semigroup linear. Also the question of the regional boundary strong stabilizing controls is discussed.

References

[1] Tsouli, A., Boutoulout, A. and El Alami, A., Constrained Feedback Stabilization for Bilinear Parabolic Systems, *Intelligent Control and Automation*, 6, (2015), 103-115.

[2] El Harraki, I., El Alami, A., Boutoulout, A. and Serhani, M., Regional stabilization for semilinear parabolic systems, *IMA Journal of Mathematical Control and Information*, (2016), 2015-197.

[3] Zerrik, EL. and Ouzara, M., Output stabilization for infinite-dimentionel bilinear systems, *Int. J. Appl. Math. Comput. Sci.*, Vol. 15, No. 2, (2005), 187-195.

[4] Ball, J. and Slemrod, M., Feedback stabilization of distributed semilinear control systems, *Appl. Math. Opt.*, 5, (1979), 169-179.

[5] Kato, T., Perturbation theory for linear operators, New York., Springer, 1980.

[6] Berrahmoune, L., Stabilization and decay estimate for distributed bilinear systems, *Systems and Control Letters*, 36 (1999), 167-171.

[7] Berrahmoune, L., Stabilization of bilinear control systems in Hilbert space with nonquadratic feedback, *Rend. Circ. Mat. Palermo*, 58 (2009), 275-282.

[8] Ouzahra, M., Stabilization of infinite-dimensional bilinear systems using a quadratic feedback control, *International Journal of Control*, 82 (2009), 1657-1664.

[9] Ouzahra, M., Exponential and weak stabilization of constrained bilinear systems, S*IAM J. Control Optim.*, 48, Issue 6 (2010), 3962-3974.

[10] Pazy, A., Semi-groups of linear operators and applications to partial differential equations, Springer Verlag, New York, 1983.

[11] Quinn, J. P., Stabilization of bilinear systems by quadratic feedback control, *J. Math. Anal. Appl.,* 75 (1980), 66-80.

[12] Weiss,G., Admissibility of unbounded control operators. *SIAM J Control Optim.*, 27 (1989), 527-545.

Current address: Abdessamad El Alami (Corresponding author): Laboratory of MACS, Department of Mathematics, Moulay Ismail University, Faculty of Sciences Meknes, Morocco.

E-mail address: elalamiabdessamad@gmail.com

ORCID Address: http://orcid.org/0000-0002-5251-6047

Current address: Laboratory of MACS, Department of Mathematics, Moulay Ismail University, Faculty of Sciences Meknes, Morocco.

E-mail address: boutouloutali@yahoo.fr

ORCID Address: http://orcid.org/0000-0003-2545-1142

Current address: Ibn Zohr University, CST Campus Universitaire Ait Melloul Agadir, Morocco.

E-mail address: r.yafia@uiz.ac.ma

ORCID Address: http://orcid.org/0000-0002-9824-9036