http://www.newtheory.org ISSN: 2149-1402

Journal of New Theory

Received: 31.08.2018 *Year*: **2019**, *Number*: **26**, *Pages*: **64-72**
Published: 04.01.2019 *Original Article*

Weak Soft Binary Structures

Arif Mehmood Khattak[1,*] <mehdaniyal@gmail.com>
Zia Ullah Haq[2] <ktk8@gmail.com>
Muhammad Zamir Burqi[2] <zamirburqi@ust.edu.pk>
Saleem Abdullah[3] <saleemabdullah81@yahoo.com>

[1]*Department of Mathematics and Statistics, Riphah International University Islamabad, Pakistan*
[2]*Department of Mathematics, University of Science and Technology, Bannu, KPK, Pakistan*
[3]*Department of Mathematics, Abdul Wali Khan University, Mardan, Pakistan.*

Abstract – The main aim of this paper is to introduce a single structure which carries the subsets of X as well as the subsets of Y under the parameter E for studying the information about the ordered pair of soft subsets of X and Y. Such a structure is called a binary soft structure from X to Y. The purpose of this paper is to introduce certain binary soft weak axioms that are analogous to the axioms of topology.

Keywords – *Binary soft topology, binary soft weak open sets, binary soft weak closed sets, binary soft weak separation axioms and binary soft T_0 space with respect to coordinates.*

1. Introduction

The concept of soft sets was first introduced by Molodtsov [3] in 1999 as a general mathematical technique for dealing with uncertain substances. In [3,4] Molodtsov magnificently applied the soft theory in numerous ways, such as smoothness of functions, game theory, operations research, Riemann integration, Perron integration, probability, theory of measurement, and so on. Point soft set topology deals with a non-empty set X to gether with a collection τ of sub set X under some set of parameters satisfying certain conditions. Such a collection τ is called a soft topological structure on X.

In 2016 Acikgöz and Tas [1] introduced the notion of binary soft set theory on two master sets and studied some basic characteristics. In prolongation, Benchalli *et al*. [2] planned the idea of binary soft topology and linked fundamental properties which are defined over two master sets with appropriate parameters. Benchalli *et al.*, [6] threw his detailed discussion on binary soft topological Kalaichelvi and Malini [7] beautifully discussed.

[*] *Corresponding Author.*

Application of fuzzy soft sets to investment decision and also discussed some more results related to this particular field. Özgür and Taş [8] studied some more applications of fuzzy soft sets to investment decision making problem. Taş *et al.* [9] worked over an application of soft set and fuzzy soft set theories to stock management Alcantud *et al.* [10] carefully discussed valuation fuzzy soft sets: A Flexible fuzzy soft set-based decision-making procedure for the valuation of assets [11] Çağman and Enginoğlu attractively explored soft matrix theory and some very basic results related to it and its decision making.

In continuation, in the present paper binary soft topological structures known as soft weak structures with respect to first coordinate as well as with respect to second coordinate are defined. Moreover, some basic results related to these structures are also planted in this paper. The same structures are defined over soft points of binary soft topological structure and related results are also reflected here with respect to ordinary and soft points.

2. Preliminaries

Definition 2.1. [5]. Let X be an initial universe and let E be a set of parameters. Let $\mathrm{P(X)}$ denote the power set of X and let A be a non-empty subset of E. A pair $\mathrm{(F, A)}$ iscalled a soft set overX, where F is a mapping given by: $\mathrm{A} \rightarrow \mathrm{P(X)}$. In other words, a soft set over X is a parameterized family of subsets of the universe X. For $\varepsilon \in \mathrm{A}$, F ($\varepsilon$) may be considered as the set of ε-approximate elements of the soft set (F, A). Clearly, a soft set is not a set.

Let $\mathrm{U_1, U_2}$ be two initial universe sets and E be a set of parameters.

Let$\mathrm{P(U_1)}$, $\mathrm{P(U_2)}$ denote the power set of $\mathrm{U_1, U_2}$ respectively. Also, let$\mathrm{A, B, C} \subseteq$ E.

Definition 2.2. [1]. A pair (F, A) is said to be a binary soft set over $\mathrm{U_1, U_2}$ where F is defined as below:

F: $\mathrm{A} \rightarrow \mathrm{P(U_1)} \times \mathrm{P(U_2)}$, $\mathrm{F(e)} = \mathrm{(X, Y)}$ for each e $\in$ A such that$\mathrm{X} \subseteq \mathrm{U_1}$, $\mathrm{Y} \subseteq \mathrm{U_2}$

Definition 2.3. [1]. A binary soft set (F, A) over $\mathrm{U_1, U_2}$ is called a binary absolute soft set, denoted by$\tilde{\tilde{\mathrm{A}}}$ if F (e) = $\mathrm{(U_1, U_2)}$ for eache $\in$ A.

Definition 2.4. [1]. The intersection of two binary soft sets of (F, A) and (G, B) over the common $\mathrm{U_1, U_2}$ is the binary soft set (H, C), where $\mathrm{C} = \mathrm{A} \cap \mathrm{B}$ and for all $\mathrm{e} \in \mathrm{C}$

$$\mathrm{H(e)} = \begin{cases} (\mathrm{X_1, Y_1}) \text{ if } \mathrm{e} \in \mathrm{A - B} \\ (\mathrm{X_2, Y_2}) \text{ if } \mathrm{e} \in \mathrm{B - A} \\ (\mathrm{X_1 \cup X_2, Y_1 \cup Y_2}) \text{ if } \mathrm{e} \in \mathrm{A \cap B} \end{cases}$$

Such that $\mathrm{F(e)} = \mathrm{(X_1, Y_1)}$ for each e $\in$ A and $\mathrm{G(e)} = \mathrm{(X_2, Y_2)}$ for eache $\in$ B. We denote it $\mathrm{(F, A)} \ \tilde{\tilde{\cup}} \ \mathrm{(G, A)} = \mathrm{(H, C)}$.

Definition 2.5. [1]. The intersection of two binary soft sets (F, A) and (G, B) over a common $\mathrm{U_1, U_2}$ is the binary soft set (H, C), where $\mathrm{C} = \mathrm{A} \cap \mathrm{B}$ and $\mathrm{H(e)} = (\mathrm{X_1 \cap X_2, Y_1 \cap Y_2})$) for each

$e \in C$ such that $F(e) = (X_1, Y_1)$ for each $e \in A$ and $G(e) = (X_2, Y_2)$for each$e \in B$. We denote it as $(F, A) \,\tilde{\tilde{\cap}}\, (G, B) = (H, C)$.

Definition 2.6. [1]. [1] Let (F, A) and (G, B) be two binary soft sets over a common U_1, U_2. (F, A) is called a binary soft subset of (G, B) if

(i) $A \subseteq B$,
(ii) $X_1 \subseteq X_2$ and $Y_1 \subseteq Y_2$ Such that $F(e) = (X_1, Y_1)$, $G(e) = (X_2, Y_2)$ for each$e \in A$.

We denote it as $(F, A) \,\tilde{\tilde{\subseteq}}\, (G, B)$.

Definition 2.7. [1]. A binary soft set (F, A) over U_1, U_2 is called a binary null soft set, denoted by if $F(e) = (\varphi, \varphi)$ for each$e \in A$.

Definition 2.8. [1]. The difference of two binary soft sets (F, A) and (G, A) over the common U_1, U_2 is the binary soft set (H, A), where $H(e)\ (X_1 - X_2, Y_1 - Y_2)$ for each $e \in A$ such that$(F, A) = (X_1, Y_1)$ and $(G, A) = (X_2, Y_2)$.

Definition 2.9. [2]. Let $\tau_\triangle$ be the collection of binary soft sets over U_1, U_2 then $\tau_\triangle$ is said to be a binary soft topology on U_1, U_2 if

(i) $\tilde{\tilde{\varphi}}, \tilde{\tilde{X}} \in \tau_\triangle$
(ii) The union of any member of binary soft sets in $\tau_\triangle$ belongs to $\tau_\triangle$.
(iii) The intersection of any two binary soft sets in $\tau_\triangle$ belongs to $\tau_\triangle$.

Then $(U_1, U_2, \tau_\triangle, E)$ is called a binary soft topological space over U_1, U_2 .

3. Weak Soft Binary Separation Axioms

This section id devoted to binary soft set and related results. Moreover, binary soft weak separation axioms in binary soft topological spaces are reflected.

Definition 3.1. Let (F, A) be any binary soft sub set of a binary soft topological space (X, Y, τ, E) then (F, A) is called

1) Binary soft b-open set of (X, Y, τ, E) if $(F, A) \subseteq cl(int((F, A) \cup in(cl((F, A)$ and
2) Binary soft b-closed set of (X, Y, τ, E) if $(F, A) \supseteq cl(int(F, A))) \cap in(cl(F, A))))$

The set of all binary b-open soft sets is denoted by BSBO (U) and the set of all binary b-closed sets is denoted by BSBO (U).

Definition 3.2. A binary soft topological space $(\tilde{X}, \tilde{Y}, \mathcal{M}, A)$ is called a binary soft b-T_0 space if for any two binary soft points $(x_1, y_1), (x_2, y_2) \tilde{\mathcal{E}} (\tilde{X}, \tilde{Y})$ such that $x_1 > x_2$, $y_1 > y_2$ there exists binary soft b-open sets (F_1, A) and (F_2, A) which behaves as$(x_1, y_1) \,\tilde{\tilde{\in}}\, (F_1, A)$, $(x_2, y_2) \,\tilde{\tilde{\notin}}\, (F_1, A)$ or $(x_2, y_2) \,\tilde{\tilde{\in}}\, (F_2, A)$ and $(x_1, y_1) \,\tilde{\tilde{\notin}}\, (F_2, A)$.

Definition 3.3. A binary soft topological space $(\widetilde{X}, \widetilde{Y}, \mathcal{M}, A)$ is called a binary soft b- T_1 space if for any two binary soft points $(x_1, y_1), (x_2, y_2) \tilde{\mathcal{E}} (\widetilde{X}, \widetilde{Y})$ such that $x_1 > x_2$, $y_1 > y_2$ If there exists binary soft b-open sets (F_1, A) and (F_2, A) which behaves as$(x_1, y_1) \tilde{\tilde{\in}} (F_1, A)$ and $(x_2, y_2) \tilde{\tilde{\notin}} (F_1, A)$ and $(x_2, y_2) \tilde{\tilde{\in}} (F_2, A)$ and $(x_1, y_1) \tilde{\tilde{\notin}} (F_2, A)$.
Definition 3.4. Two binary soft b-open sets $((F, A), (G, A))$ and $(H, A), (I, A)$ are said to be disjoint if $\big((F, A) \sqcap (H, A), (G, A) \sqcap (I, A)\big) = (\Phi, \Phi)$. That is $(F, A) \sqcap (H, A) = (\Phi, \Phi)$ and $(G, A) \sqcap (I, A) = (\Phi, \Phi)$.

Definition 3.5. A binary soft topological space $(\widetilde{X}, \widetilde{Y}, \mathcal{M}, A)$ is called a binary soft b-T_2 space if for any two binary soft points $(x_1, y_1), (x_2, y_2) \tilde{\mathcal{E}} (\widetilde{X}, \widetilde{Y})$ such that $x_1 > x_2$, $y_1 > y_2$ If there exists binary soft b-open sets (F_1, A) and (F_2, A) which behaves as$(x_1, y_1) \tilde{\tilde{\in}} (F_1, A)$ and $(x_2, y_2) \tilde{\tilde{\in}} (F_2, A)$ and moreover $(F_1, A)$and(F_2, A) are disjoint that is $(F_1, A) \sqcap (F_2, A) = (\Phi, \Phi)$.

Definition 3.6. A binary soft topological space $(\widetilde{X}, \widetilde{Y}, \tau \times \sigma, A)$ is called a binary soft b-T_0 with respect to the first coordinate if for every pair of binary points $(x_1, \alpha), (y_1, \alpha)$ there exists $\big((F, A), (G, A)\big) \tilde{\mathcal{E}} \tau \times \sigma$ with$x_1 \tilde{\mathcal{E}} (F, A), y_1 \tilde{\tilde{\notin}} (F, A), \alpha \tilde{\mathcal{E}} (G, A)$.where b-open (F, A)`in τ and b-open (G, A)in σ.

Definition 3.7. A binary soft topological space $(\widetilde{X}, \widetilde{Y}, \tau \times \sigma, A)$ is called a binary soft b-T_0 with respect to the second coordinate if for every pair of binary points $(\beta, x_2), (\beta, y_2)$ there exists $\big((F, A), (G, A)\big) \tilde{\mathcal{E}} \tau \times \sigma$ with$\beta \tilde{\mathcal{E}} (F, A), x_2 \tilde{\mathcal{E}} (G, A)$,$y_2 \tilde{\tilde{\notin}} (G, A)$. where b- open (F, A)`in $\tau$ and b-`open (G, A)in σ.

Definition 3.8. A binary soft topological space $(\widetilde{X}, \widetilde{Y}, \mathcal{M}, A)$ is called a binary soft b- T_0 space if for any two binary soft points $\big(e_{\mathbb{G}_1}, e_{\mathbb{H}_1}\big), \big(e_{\mathbb{G}_2}, e_{\mathbb{H}_2}\big) \tilde{\mathcal{E}} (\widetilde{X_A}, \widetilde{Y_A})$ such that $e_{\mathbb{G}_1} > e_{\mathbb{G}_2}, e_{\mathbb{H}_1} > e_{\mathbb{H}_2}$there exists binary soft b-open sets (F_1, A) and (F_2, A) which behaves as$\big(e_{\mathbb{G}_1}, e_{\mathbb{H}_1}\big) \tilde{\tilde{\in}} (F_1, A)$, $\big(e_{\mathbb{G}_2}, e_{\mathbb{H}_2}\big) \tilde{\tilde{\notin}} (F_1, A)$ or $\big(e_{\mathbb{G}_2}, e_{\mathbb{H}_2}\big) \tilde{\tilde{\in}} (F_2, A)$ and $\big(e_{\mathbb{G}_1}, e_{\mathbb{H}_1}\big) \tilde{\tilde{\notin}} (F_2, A)$.

Definition 3.9. A binary soft topological space $(\widetilde{X}, \widetilde{Y}, \mathcal{M}, A)$ is called a binary soft b-T_1 space if for any two binary soft points $\big(e_{\mathbb{G}_1}, e_{\mathbb{H}_1}\big), \big(e_{\mathbb{G}_2}, e_{\mathbb{H}_2}\big) \tilde{\mathcal{E}} (\widetilde{X_A}, \widetilde{Y_A})$ such that $e_{\mathbb{G}_1} > e_{\mathbb{G}_2}, e_{\mathbb{H}_1} > e_{\mathbb{H}_2}$ If there exists binary soft b-open sets (F_1, A) and (F_2, A) which behaves as $\big(e_{\mathbb{G}_1}, e_{\mathbb{H}_1}\big) \tilde{\tilde{\in}} (F_1, A)$ *and* $\big(e_{\mathbb{G}_2}, e_{\mathbb{H}_2}\big) \tilde{\tilde{\notin}} (F_1, A)$ and $\big(e_{\mathbb{G}_2}, e_{\mathbb{H}_2}\big) \tilde{\tilde{\in}} (F_2, A)$ and $\big(e_{\mathbb{G}_1}, e_{\mathbb{H}_1}\big) \tilde{\tilde{\notin}} (F_2, A)$.

Definition 3.10. A binary soft topological space $(\tilde{X}, \tilde{Y}, \mathcal{M}, A)$ is called a binary soft b- T_2 space if for any two binary soft points $\big(e_{\mathbb{G}_1}, e_{\mathbb{H}_1}\big), \big(e_{\mathbb{G}_2}, e_{\mathbb{H}_2}\big) \tilde{\mathcal{E}} (\widetilde{X_A}, \widetilde{Y_A})$ such that $e_{\mathbb{G}_1} > e_{\mathbb{G}_2}, e_{\mathbb{H}_1} > e_{\mathbb{H}_2}$ If there exists binary soft b-open sets (F_1, A) and (F_2, A) which behaves as $\big(e_{\mathbb{G}_1}, e_{\mathbb{H}_1}\big) \tilde{\tilde{\in}} (F_1, A)$ and $\big(e_{\mathbb{G}_2}, e_{\mathbb{H}_2}\big) \tilde{\tilde{\in}} (F_2, A)$ and moreover $(F_1, A) and (F_2, A)$ are disjoint.

Definition 3.11. A binary soft topological space $(\tilde{X}, \tilde{Y}, \tau \times \sigma, A)$ is called a binary soft b- T_0 with respect to the first coordinate if for every pair of binary points $(e_{\mathbb{G}_1}, \alpha), (e_{\mathbb{H}_1}, \alpha)$ there exists $\big((F, A), (G, A)\big) \tilde{\mathcal{E}} \tau \times \sigma$ with $e_{\mathbb{G}_1} \tilde{\mathcal{E}} (F, A), e_{\mathbb{H}_1} \tilde{\tilde{\notin}} (F, A), \alpha \tilde{\mathcal{E}} (G, A)$ where b-open (F, A)`in τ and b-open (G, A)in σ.

Definition 3.12. A binary soft topological space $(\tilde{X}, \tilde{Y}, \tau \times \sigma, A)$ is called a binary soft b- T_0 with respect to the second coordinate if for every pair of binary points $(\beta, e_{\mathbb{G}_2}), (\beta, e_{\mathbb{H}_2})$ there exists $\big((F,A),(G,A)\big)\tilde{\mathcal{E}}\tau \times \sigma$ with$\beta\tilde{\mathcal{E}}(F,A), e_{\mathbb{G}_2}\tilde{\mathcal{E}}(G,A)$,$e_{\mathbb{H}_2} \tilde{\notin} (G,A)$. where b- open (F,A)`in τ and b-open (G,A)in σ.

4. Soft Binary Structures with Respect to Ordinary Points

Theorem 4.1. If the binary soft topological space $(\tilde{X}, \tilde{Y}, \rho \times \sigma, A)$ is a binary soft b- T_0, then $(\tilde{X}, \rho, A)$ and $(\tilde{Y}, \sigma, A)$ are soft b- T_0.

Proof. We suppose $(\tilde{X}, \tilde{Y}, \rho \times \sigma, A)$ is a binary soft b- T_0. Suppose $x_1, x_2\tilde{\mathcal{E}}\tilde{X}$ and $y_1, y_2\tilde{\mathcal{E}}\tilde{Y}$ with such that $x_1 > x_2$, $y_1 > y_2$. Since $(\tilde{X}, \tilde{Y}, \rho \times \sigma, A)$ is a binary soft b- T_0, accordingly there binary soft b-open set $((F,A),(G,A))$ such that

$$(x_1, y_1)\tilde{\mathcal{E}}((F,A),(G,A))\ ;\ (x_2, y_2)\tilde{\mathcal{E}}(F^C,A),(G^C,A)$$

or

$$(x_1, y_1)\tilde{\mathcal{E}}((F^C,A),(G^C,A))\ ;\ (x_2, y_2)\tilde{\mathcal{E}}((F,A),(G,A))$$

This implies that either $x_1\tilde{\mathcal{E}}(F,A); x_2\tilde{\mathcal{E}}(F^C,A); y_1\tilde{\mathcal{E}}(G,A); y_2\tilde{\mathcal{E}}(G^C,A)$; or $x_1\tilde{\mathcal{E}}(F^C,A); y_1\tilde{\mathcal{E}}(G^C,A)$; $y_2\tilde{\mathcal{E}}(G,A)$. This implies either $x_1\tilde{\mathcal{E}}(F,A)\ ; x_2\tilde{\mathcal{E}}(F^C,A)$or $x_1\tilde{\mathcal{E}}(F^C,A)$; $x_1\tilde{\mathcal{E}}(F,A)$ and either $y_1\tilde{\mathcal{E}}(G,A)$; $y_2\tilde{\mathcal{E}}(G^C,A)ory_1\tilde{\mathcal{E}}(G^C,A)$; $y_2\tilde{\mathcal{E}}(G,A)$. Since $\big((F,A),(G,A)\big)\tilde{\mathcal{E}}\rho \times \sigma$, We have b-open $(F,A)\tilde{\mathcal{E}}\rho$ and b-open$(F,A)\tilde{\mathcal{E}}\ \sigma$. this proves that $(\tilde{X}, \rho, A)$ and $(\tilde{Y}, \sigma, A)$ are soft b-T_0.

Theorem 4.2. A binary soft topological space $(\tilde{X}, \tilde{Y}, \tau \times \sigma, A)$ is binary soft b-T_0 space with respect to first and the second coordinates, then $(\tilde{X}, \tilde{Y}, \tau \times \sigma, A)$is binary soft b-$T_0$ space.

Proof. Let $(\tilde{X}, \tilde{Y}, \tau \times \sigma, A)$ is binary soft b-T_0 space with respect to first and the second coordinates. Let $(x_1, y_1), (x_2, y_2)\tilde{\mathcal{E}}X \times Y$ with $x_1 > x_2$, $y_1 > y_2$. Take $\alpha\tilde{\mathcal{E}}Y$ and $\beta\tilde{\mathcal{E}}X$.Then $(x_1, \alpha), (x_2, \alpha)\tilde{\mathcal{E}}X \times Y$.Since $(\tilde{X}, \tilde{Y}, \tau \times \sigma, A)$ is a binary soft b-T_0space with respect to the first coordinate, by using definition, there exists b-open sets (F,A) such that (G,A) $\big((F,A),(G,A)\big)\tilde{\mathcal{E}}\tau \times \sigma$ with $x_1\tilde{\mathcal{E}}(F,A)$, $x_2 \tilde{\notin} (F,A)$, $\alpha\tilde{\mathcal{E}}(G,A)$. Since$(\beta, y_1), (\beta, y_2)\tilde{\mathcal{E}}X \times Y$, by using the arguments and using definition there exists $\big((H,A),(K,A)\big)\tilde{\mathcal{E}}\tau \times \sigma$ with $y_1\tilde{\mathcal{E}}(K,A), y_1 \tilde{\notin} (K,A), \widetilde{\beta\mathcal{E}}(H,A)$. Therefore, $(x_1, y_1)\ \tilde{\mathcal{E}}\big((F,A),(K,A)\big)$ and $(x_2, y_2)\ \tilde{\mathcal{E}}((F^C,A),(K^C,A))$. Hence $(\tilde{X}, \tilde{Y}, \tau \times \sigma, A)$ is called a binary soft b- T_0.

Theorem 4.3. A binary soft topological space $(\tilde{X}, \tau, A)$ and $(\tilde{Y}, \sigma, A)$ are soft b-T_1spaces if and only if the binary soft topological space $(\tilde{X}, \tilde{Y}, \tau \times \sigma, A)$ is soft binary b-T_1.

Proof. Suppose $(\tilde{X}, \tau, A)$ and $(\tilde{X}, \sigma, A)$ are soft b-T_1 spaces. Let $(x_1, y_1), (x_2, y_2)\tilde{\mathcal{E}}X \times Y$ with $x_1 > x_2$, $y_1 > y_2$. since $(\tilde{X}, \tau, A)$ is soft b-T_1 space, there exists soft b-open sets such that $(F,A),(G,A)\tilde{\mathcal{E}}\tau$, $x_1\mathcal{E}(F,A)$ and $x_2\mathcal{E}(G,A)$ such that $x_1 \tilde{\notin} (G,A)$ and $x_2 \tilde{\notin} (F,A)$. Also, since $(\tilde{Y}, \sigma, A)$ is soft b-T_1space, there exists soft b-open sets such that $(H,A),(I,A)\tilde{\mathcal{E}}\sigma$, $y_1\mathcal{E}(H,A)$ and $y_2\mathcal{E}(I,A)$ such that $y_1 \tilde{\notin} (I,A)$ and $y_2 \tilde{\notin} (H,A)$.thus $(x_1, y_1)\mathcal{E}\big((F,A),(H,A)\big)$and $(x_2, y_2)\mathcal{E}\big((G,A),(I,A)\big)$ with $(x_1, y_1)\mathcal{E}\big((G^C,A),(I^C,A)\big)$ and

$(x_1, y_1)\mathcal{E}\big((F^C, A), (H^C, A)\big)$.This implies that $(\tilde{X}, \tilde{Y}, \tau \times \sigma, A)$ is soft binary b-T_1. Conversely assume that $(\tilde{X}, \tilde{Y}, \tau \times \sigma, A)$ is soft binary b-T_1. Let $x_1, x_2\mathcal{E}X$ and $y_1, y_2\mathcal{E}Y$ such that $x_1 > x_2$, $y_1 > y_2$. Therefore $(x_1, y_1), (x_2, y_2)\tilde{\mathcal{E}}X \times Y$. Since $(\tilde{X}, \tilde{Y}, \tau \times \sigma, A)$ is soft binary b-T_1, there exists b-open sets $(F, A), (G, A)$ and b-open sets $(H, A), (I, A)\mathcal{E}(\tau \times \sigma)$, $(x_1, y_1)\tilde{\mathcal{E}}((F, A), (G, A))$ and $(x_1, y_1)\tilde{\mathcal{E}}((H, A), (I, A))$ such that $(x_1, y_1)\mathcal{E}(H^C, \widetilde{A), (I^C}, A)$ and $(x_2, y_2)\tilde{\mathcal{E}}\big((F^C, A), (G^C, A)\big)$. Therefore, $x_1\mathcal{E}(F, A), x_2\mathcal{E}(H, A)$ and $x_1\mathcal{E}(H^C, A)$ and $x_2\mathcal{E}(F^C, A)$ and $, y_1\mathcal{E}(G^C, A)$ and $y_1\mathcal{E}(I, A)$ and $, y_1\mathcal{E}(I^C, A)$ and $y_2\mathcal{E}(G^C, A)$. Since $(F, A), (G, A)\tilde{\mathcal{E}}\tau \times \sigma$, We have$(F, A), (H, A)\mathcal{E}\tau$ and $(G, A), (I, A)\mathcal{E}\sigma$. This proves that $(\tilde{X}, \tau, A)$ and $(\tilde{X}, \sigma, A)$ are soft b-T_1spaces.

Theorem 4.4. A binary soft topological space $(\tilde{X}, \tilde{Y}, \mathcal{M}, A)$ is binary soft b-T_1 space if and only if every binary soft point $\wp(X) \times \wp(Y)$ is binary soft b-closed.

Proof. Suppose that $(\tilde{X}, \tilde{Y}, \mathcal{M}, A)$ is binary soft b-T_1 space. Let $(x, y)\tilde{\mathcal{E}}X \times Y$. Let $(\{x\}, \{y\})\tilde{\mathcal{E}}\wp(X) \times \wp(Y)$.We shall show that $(\{x\}, \{y\})$ is binary soft b-closed.it is sufficient to show that $(X\backslash\{x\}, Y\backslash\{y\})$ is binary soft b-open. Let $(a, b)\mathrm{E}(X\backslash\{x\}, Y\backslash\{y\})$. This implies that $a\tilde{\mathcal{E}}X\backslash\{x\}$ and $b\tilde{\mathcal{E}}Y\backslash\{y\}$. hence $a \neq x \; and \; b \neq y$ .That is, (a, b) and $(x, y)$are distinct binary soft points of $X \times Y$. Since $(\tilde{X}, \tilde{Y}, \mathcal{M}, A)$ is binary soft b-T_1 space, there exists binary soft b-open sets $((F, A), (G, A))$ and $(H, A), (I, A)$ such that $(a, b)\mathrm{E}((F, A), (G, A))$ and $(x, y)\mathrm{E}((H, A), (I, A))$ such that $(a, b)\mathrm{E}((H^C, A), (I^C, A))$ and $(x, y)\mathrm{E}((F^C, A), (G^C, A))$. Therefore, $\big((F, A), (G, A)\big) \subseteq (\{x\}^c, \{y\}^c)$. Hence$(\{x\}^c, \{y\}^c)$ is a soft neighbourhood of (a, b).This implies that $(\{x\}, \{y\})$is binary soft b- closed. Conversely, suppose that $(\{x\}, \{y\})$is binary soft b-closed for every $(x, y)\mathrm{E}X \times Y$.Suppose $(x_1, y_1), (x_2, y_2)\tilde{\mathcal{E}}X \times Y$ with $x_1 > x_2$, $y_1 > y_2$.Therefore, $(x_2, y_2)\mathcal{E}(\{x_1\widetilde{\}^c, \{y_1}\}^c)$ and $\mathcal{E}$` $(\{x_1\}^c, \{y_1\}^c)$is binary soft b-open. Also $(x_1, y_1)\mathcal{E}(\{x_2\widetilde{\}^c, \{y_2}\}^c)$ and $\mathcal{E}(\{x_1\widetilde{\}^c, \{y_1}\}^c)$ is binary soft b-open set. Also $(x_1, y_1)\mathcal{E}(\{x_2\widetilde{\}^c, \{y_2}\}^c)$ and $\mathcal{E}(\{x_2\widetilde{\}^c, \{y_2}\}^c)$ is binary soft b-open set. This shows that $(\tilde{X}, \tilde{Y}, \mathcal{M}, A)$ is binary soft b-T_1 space.

Theorem 4.5. A binary soft topological space $(\tilde{X}, \tau, A)$ and $(\tilde{Y}, \sigma, A)$ are soft b-T_2 spaces if and only if the binary soft topological space $(\tilde{X}, \tilde{Y}, \tau \times \sigma, A)$ is soft binary b-T_2.

Proof. Suppose $(\tilde{X}, \tau, A)$ and $(\tilde{X}, \sigma, A)$ are soft b-T_2 spaces. Let $(x_1, y_1), (x_2, y_2)\tilde{\mathcal{E}}X \times Y$ with $x_1 > x_2$, $y_1 > y_2$. Since $(\tilde{X}, \tau, A)$ is soft b-T_2 space, there exists soft b-open sets such that $(F, A), (G, A)\tilde{\mathcal{E}}\tau$, $x_1\mathcal{E}(F, A)$ and $x_2\mathcal{E}(G, A)$ such that $x_1 \tilde{\notin} (G, A)$ and $x_2 \tilde{\notin} (F, A)$. Also, since $(\tilde{Y}, \sigma, A)$ is soft b-T_2space, there exists distoint soft b-open sets such that $(H, A), (I, A)\tilde{\mathcal{E}}\sigma, y_1\mathcal{E}(H, A)$ and $y_2\mathcal{E}(I, A)$ such that $y_1 \tilde{\notin} (I, A)$ and $y_2 \tilde{\notin} (H, A)$. Thus $(x_1, y_1)\mathcal{E}\big((F, A), (H, A)\big)$ and $(x_2, y_2)\mathcal{E}\big((G, A), (I, A)\big)$ with $(x_1, y_1)\mathcal{E}\big((G^C, A), (I^C, A)\big)$ and $(x_1, y_1)\mathcal{E}\big((F^C, A), (H^C, A)\big)$. Since $(F, A) \; and \; (G, A)$ are disjoint, $(F, A) \sqcap (H, A) = (\Phi, \Phi)$. Also, since $(H, A) \sqcap (I, A) = (\Phi, \Phi)$. Thus $\big((F, A) \sqcap (H, A), (G, A) \sqcap (I, A)\big) = (\Phi, \Phi)$. This implies that we have this implies that $(\tilde{X}, \tilde{Y}, \tau \times \sigma, A)$ is soft binary b-T_2. Conversely assume that $(\tilde{X}, \tilde{Y}, \tau \times \sigma, A)$ is soft binary b-T_2. Let $x_1, x_2\mathcal{E}X$ and $y_1, y_2\mathcal{E}Y$ such that $x_1 > x_2$, $y_1 > y_2$. Therefore $(x_1, y_1), (x_1, y_1)\tilde{\mathcal{E}}X \times Y$. Since $(\tilde{X}, \tilde{Y}, \tau \times \sigma, A)$ is soft binary b-T_2, there exists binary soft b-open sets $(F, A), (G, A)$ and there exists binary soft b-open sets $(H, A), (I, A)\mathcal{E}(\tau \times \sigma)$, $(x_1, y_1)\tilde{\mathcal{E}}((F, A), (G, A))$ and $(x_2, y_2)\tilde{\mathcal{E}}((\mathrm{H}, A), (I, A))$ such that $(x_1, y_1)\mathcal{E}(H^C, \widetilde{A), (I^C}, A)$ and $(x_2, y_2)\tilde{\mathcal{E}}\big((F^C, A), (G^C, A)\big)$. Therefore, $x_1\mathcal{E}(F, A), x_2\mathcal{E}(H, A)$ and $x_1\mathcal{E}(H^C, A)$ and $x_2\mathcal{E}(F^C, A)$ and $y_1\mathcal{E}(G^C, A)$ and $y_2\mathcal{E}(I, A)$ and $, y_1\mathcal{E}(I^C, A)$ and $y_2\mathcal{E}(G^C, A)$. Since

$(F,A),(G,A)\tilde{\mathcal{E}}\tau\times\sigma$, We have$(F,A),(H,A)\mathcal{E}\tau$ and $(G,A),(I,A)\mathcal{E}\sigma$. This proves that $(\tilde{X},\tau,A)$ and $(\tilde{X},\sigma,A)$ are soft b-T_2spaces.

5. Soft Binary Structures with respect to Soft Points

Theorem 5.1. If the binary soft topological space $(\tilde{X},\tilde{Y},\rho\times\sigma,A)$ is a binary soft b-T_0, then $(\tilde{X},\rho,A)$ and $(\tilde{Y},\sigma,A)$ are soft b- T_0.

Proof. We suppose $(\widetilde{X_A},\tilde{\tilde{Y}}_A,\rho\times\sigma,A)$ is a binary soft b-T_0. Suppose $e_{\mathbb{G}_1},e_{\mathbb{G}_2}\tilde{\mathcal{E}}\widetilde{X_A}$and $e_{\mathbb{H}_1},e_{\mathbb{H}_2}\tilde{\mathcal{E}}\widetilde{Y_A}$ with such that $e_{\mathbb{G}_1}>e_{\mathbb{G}_2},e_{\mathbb{H}_1}>e_{\mathbb{H}_2}$.Since $(\widetilde{X_A},\tilde{\tilde{Y}}_A,\rho\times\sigma,A)$ is a binary soft b-T_0,accordingly there binary soft b-open set $((F,A),(G,A))$ such that

$$(e_{\mathbb{G}_1},e_{\mathbb{H}_1})\tilde{\mathcal{E}}((F,A),(G,A))\,;(e_{\mathbb{G}_2},e_{\mathbb{H}_2})\tilde{\mathcal{E}}(F^C,A),(G^C,A)$$

or

$$(e_{\mathbb{G}_1},e_{\mathbb{H}_1})\tilde{\mathcal{E}}((F^C,A),(G^C,A));(e_{\mathbb{G}_2},e_{\mathbb{H}_2})\tilde{\mathcal{E}}((F,A),(G,A))$$

This implies that either $e_{\mathbb{G}_1}\tilde{\mathcal{E}}(F,A);e_{\mathbb{G}_2}\tilde{\mathcal{E}}(F^C,A);e_{\mathbb{H}_1}\tilde{\mathcal{E}}(G,A);e_{\mathbb{H}_2}\tilde{\mathcal{E}}(G^C,A)$ or $e_{\mathbb{G}_1}\tilde{\mathcal{E}}(F^C,A);e_{\mathbb{H}_1}\tilde{\mathcal{E}}(G^C,A);\ e_{\mathbb{H}_2}\tilde{\mathcal{E}}(G,A)$. This implies either $e_{\mathbb{G}_1}\tilde{\mathcal{E}}(F,A)\,;e_{\mathbb{G}_2}\tilde{\mathcal{E}}(F^C,A)$ or $e_{\mathbb{G}_1}\tilde{\mathcal{E}}(F^C,A);\ x_1\tilde{\mathcal{E}}(F,A)$and either $e_{\mathbb{H}_1}\tilde{\mathcal{E}}(G,A);\ e_{\mathbb{H}_2}\tilde{\mathcal{E}}(G^C,A)ore_{\mathbb{H}_1}\tilde{\mathcal{E}}(G^C,A);\ e_{\mathbb{H}_2}\tilde{\mathcal{E}}(G,A)$. Since $\big((F,A),(G,A)\big)\tilde{\mathcal{E}}\rho\times\sigma$, We have b-open $(F,A)\tilde{\mathcal{E}}\rho$ and b-open$(F,A)\widetilde{\mathcal{E}}\,\sigma$. this proves that $(\tilde{X},\rho,A)$ and $(\tilde{Y},\sigma,A)$ are soft b- T_0.

Theorem 5.2. A binary soft topological space $(\tilde{X},\tilde{Y},\tau\times\sigma,A)$ is binary soft b-T_0 space with respect to first and the second coordinates, then $(\tilde{X},\tilde{Y},\tau\times\sigma,A)$is binary soft b-$T_0$ space.

Proof. Let $(\tilde{X},\tilde{Y},\tau\times\sigma,A)$ is binary soft b-T_0 space with respect to first and the second coordinates. Let $(e_{\mathbb{G}_1},e_{\mathbb{H}_1}),(e_{\mathbb{G}_2},e_{\mathbb{H}_2})\tilde{\mathcal{E}}X\times Y$ with $e_{\mathbb{G}_1}>e_{\mathbb{G}_2}\,,e_{\mathbb{H}_1}>e_{\mathbb{H}_2}$. Take $\alpha\tilde{\mathcal{E}}Y$ and $\beta\tilde{\mathcal{E}}X$.Then $(e_{\mathbb{G}_1},\alpha),(e_{\mathbb{G}_2},\alpha)\tilde{\mathcal{E}}X\times Y$.Since $(\tilde{X},\tilde{Y},\tau\times\sigma,A)$ is a binary soft b-T_0space with respect to the first coordinate, by using definition, there exists b-open sets $\big((F,A),(G,A)\big)\tilde{\mathcal{E}}\tau\times\sigma$ with $e_{\mathbb{G}_1}\tilde{\mathcal{E}}(F,A),\ e_{\mathbb{G}_2}\ \widetilde{\notin}\ (F,A),\ \alpha\tilde{\mathcal{E}}(G,A)$. Since$\big(\beta,e_{\mathbb{H}_1}\big),\big(\beta,e_{\mathbb{H}_2}\big)\tilde{\mathcal{E}}X\times Y$, by using the arguments and using definition there exists b-open sets $\big((H,A),(K,A)\big)\tilde{\mathcal{E}}\tau\times\sigma$ with $e_{\mathbb{H}_1}\tilde{\mathcal{E}}(K,A),e_{\mathbb{H}_1}\ \widetilde{\notin}\ (K,A),\widetilde{\beta\mathcal{E}}(H,A)$. Therefore, $(e_{\mathbb{G}_1},e_{\mathbb{H}_1})\ \tilde{\mathcal{E}}\big((F,A),(K,A)\big)$ and $(e_{\mathbb{G}_2},e_{\mathbb{H}_2})\ \tilde{\mathcal{E}}((F^C,A),(K^C,A))$. Hence $(\tilde{X},\tilde{Y},\tau\times\sigma,A)$ is called a binary soft b-T_0.

Theorem 5.3. A binary soft topological space $(\tilde{X},\tau,A)$ and $(\tilde{Y},\sigma,A)$ are soft b-T_1spaces if and only if the binary soft topological space $(\tilde{X},\tilde{Y},\tau\times\sigma,A)$ is soft binary b-T_1.

Proof. Suppose $(\tilde{X},\tau,A)$ and $(\tilde{X},\sigma,A)$ are soft b-T_1spaces. Let $(e_{\mathbb{G}_1},e_{\mathbb{H}_1})\,,(e_{\mathbb{G}_2},e_{\mathbb{H}_2})\ \tilde{\mathcal{E}}X\times Y$ with $e_{\mathbb{G}_1}>e_{\mathbb{G}_2}\,,e_{\mathbb{H}_1}>e_{\mathbb{H}_2}$. since $(\tilde{X},\tau,A)$ is soft b-T_1space, there exists soft b-open sets such that $(F,A),(G,A)\tilde{\mathcal{E}}\tau$, $e_{\mathbb{G}_1}\mathcal{E}(F,A)$ and $e_{\mathbb{G}_2}\mathcal{E}(G,A)$ such that $e_{\mathbb{G}_1}\ \widetilde{\notin}\ (G,A)$ and $e_{\mathbb{G}_2}\ \widetilde{\notin}\ (F,A)$. Also, since $(\tilde{Y},\sigma,A)$ is soft b-T_1space, there exists soft b-open sets such that $(H,A),(I,A)\tilde{\mathcal{E}}\sigma$, $e_{\mathbb{H}_1}\mathcal{E}(H,A)$ and $e_{\mathbb{H}_2}\mathcal{E}(I,A)$ such that $e_{\mathbb{H}_1}\ \widetilde{\notin}\ (I,A)$ and $e_{\mathbb{H}_2}\ \widetilde{\notin}\ (H,A)$.thus $(e_{\mathbb{G}_1},e_{\mathbb{H}_1})\mathcal{E}\big((F,A),(H,A)\big)$and $(e_{\mathbb{G}_2},e_{\mathbb{H}_2})\mathcal{E}\big((G,A),(I,A)\big)$ with $(e_{\mathbb{G}_1},e_{\mathbb{H}_1})\mathcal{E}\big((G^C,A),(I^C,A)\big)$

and $(e_{\mathbb{G}_1}, e_{\mathbb{H}_1})\mathcal{E}\big((F^C, A), (H^C, A)\big)$.This implies that $(\tilde{X}, \tilde{Y}, \tau \times \sigma, A)$ is soft binary b-T_1. Conversely assume that $(\tilde{X}, \tilde{Y}, \tau \times \sigma, A)$ is soft binary b-T_1. Let $e_{\mathbb{G}_1}, e_{\mathbb{G}_2}\mathcal{E}X$ and $e_{\mathbb{H}_1}, e_{\mathbb{H}_2}\mathcal{E}Y$ such that $e_{\mathbb{G}_1} > e_{\mathbb{G}_2}$, $e_{\mathbb{H}_1} > e_{\mathbb{H}_2}$. Therefore $(e_{\mathbb{G}_1}, e_{\mathbb{H}_1}), (e_{\mathbb{G}_2}, e_{\mathbb{H}_2})\tilde{\mathcal{E}}X \times Y$. Since $(\tilde{X}, \tilde{Y}, \tau \times \sigma, A)$ is soft binary b-T_1, there exists b-open sets $(F, A), (G, A)$ and b-open sets $(H, A), (I, A)\mathcal{E}(\tau \times \sigma)$, $(e_{\mathbb{G}_1}, e_{\mathbb{H}_1})\tilde{\mathcal{E}}((F, A), (G, A))$ and $(e_{\mathbb{G}_2}, e_{\mathbb{H}_2})\tilde{\mathcal{E}}((H, A), (I, A))$ such that $(e_{\mathbb{G}_1}, e_{\mathbb{H}_1})\mathcal{E}(H^C, \widetilde{A), (}I^C, A)$ and $(e_{\mathbb{G}_2}, e_{\mathbb{H}_2})\tilde{\mathcal{E}}\big((F^C, A), (G^C, A)\big)$. Therefore, $e_{\mathbb{G}_1}\mathcal{E}(F, A), e_{\mathbb{G}_2}\mathcal{E}(H, A)$ and $e_{\mathbb{G}_1}\mathcal{E}(H^C, A)$ and $e_{\mathbb{G}_2}\mathcal{E}(F^C, A)$ and , $y_1\mathcal{E}(G^C, A)$ and $e_{\mathbb{H}_1}\mathcal{E}(I, A)$ and , $e_{\mathbb{H}_1}\mathcal{E}(I^C, A)$ and , $e_{\mathbb{H}_2}\mathcal{E}(G^C, A)$. Since $(F, A), (G, A)\tilde{\mathcal{E}}\tau \times \sigma$, We have$(F, A), (H, A)\mathcal{E}\tau$ and $(G, A), (I, A)\mathcal{E}\sigma$. This proves that $(\tilde{X}, \tau, A)$ and $(\tilde{X}, \sigma, A)$ are soft b-T_1spaces.

Theorem 5.4. A binary soft topological space $(\tilde{X}, \tilde{Y}, \mathcal{M}, A)$ is binary soft b-T_1 space if and only if every binary soft point $\wp(X) \times \wp(Y)$ is binary soft b-closed.

Proof. Suppose that $(\tilde{X}, \tilde{Y}, \mathcal{M}, A)$ is binary soft b-T_1 space. Let $(x, y)\tilde{\mathcal{E}}X \times Y$. Let $(\{x\}, \{e_{\mathbb{H}}\})\tilde{\mathcal{E}}\wp(X) \times \wp(Y)$.We shall show that $(\{e_{\mathbb{G}}\}, \{e_{\mathbb{H}}\})$ is binary soft b-closed. It is sufficient to show that $(X\backslash\{e_{\mathbb{G}}\}, Y\backslash\{e_{\mathbb{H}}\})$ is binary soft b-open. Let $(a, b)\mathrm{E}(X\backslash\{e_{\mathbb{G}}\}, Y\backslash\{e_{\mathbb{H}}\})$. This implies that $a\tilde{\mathcal{E}}X\backslash\{e_{\mathbb{G}}\}$ and $b\tilde{\mathcal{E}}Y\backslash\{e_{\mathbb{H}}\}$. Hence $a \neq e_{\mathbb{G}}\ and\ b \neq e_{\mathbb{H}}$. That is, (a, b) and $(e_{\mathbb{G}}, e_{\mathbb{H}})$ are distinct binary soft points of $X \times Y$. Since $(\tilde{X}, \tilde{Y}, \mathcal{M}, A)$ is binary soft b- T_1 space, there exists binary soft b-open sets $((F, A), (G, A))$ and $(H, A), (I, A)$ such that $(a, b)\mathrm{E}((F, A), (G, A))$ and $(x, y)\mathrm{E}((H, A), (I, A))$ such that $(a, b)\mathrm{E}((H^C, A), (I^C, A))$ and $(e_{\mathbb{G}}, e_{\mathbb{H}})\mathrm{E}((F^C, A), (G^C, A))$. Therefore, $\big((F, A), (G, A)\big) \subseteq (\{e_{\mathbb{G}}\}^c, \{e_{\mathbb{H}}\}^c)$. Hence$(\{e_{\mathbb{G}}\}^c, \{e_{\mathbb{H}}\}^c)$ is a soft neighbourhood of (a, b). This implies that $(\{e_{\mathbb{G}}\}, \{e_{\mathbb{H}}\})$is binary soft b-closed. Conversely, suppose that $(\{e_{\mathbb{G}}\}, \{e_{\mathbb{H}}\})$ is binary soft b-closed for every $(e_{\mathbb{G}}, e_{\mathbb{H}})\mathrm{E}X \times Y$. Suppose $(e_{\mathbb{G}_1}, e_{\mathbb{H}_1})$, $(e_{\mathbb{G}_2}, e_{\mathbb{H}_2})\tilde{\mathcal{E}}X \times Y$with$e_{\mathbb{G}_1} > e_{\mathbb{G}_2}$, $e_{\mathbb{H}_1} > e_{\mathbb{H}_2}$ Therefore, $(e_{\mathbb{G}_2}, e_{\mathbb{H}_2})\mathcal{E}(\{e_{\mathbb{G}_1}\widetilde{\}^c, \{}e_{\mathbb{H}_1}\}^c)$ and $(\{e_{\mathbb{G}_1}\}^c, \{e_{\mathbb{H}_1}\}^c)$ is binary soft b-open. Also $(e_{\mathbb{G}_1}, e_{\mathbb{H}_1})\mathcal{E}(\{e_{\mathbb{G}_2}\widetilde{\}^c, \{}e_{\mathbb{H}_2}\}^c)$ and $\mathcal{E}(\{e_{\mathbb{G}_1}\widetilde{\}^c, \{}e_{\mathbb{H}_1}\}^c)$ is binary soft b-open set. Also $(e_{\mathbb{G}_1}, e_{\mathbb{H}_1})\mathcal{E}(\{e_{\mathbb{G}_2}\widetilde{\}^c, \{}e_{\mathbb{H}_2}\}^c)$ and $\mathcal{E}(\{e_{\mathbb{G}_2}\widetilde{\}^c, \{}e_{\mathbb{H}_2}\}^c)$ is binary soft b-open set. This shows that $(\tilde{X}, \tilde{Y}, \mathcal{M}, A)$ is binary soft b-T_1 space.

Theorem 5.5. A binary soft topological space $(\tilde{X}, \tau, A)$ and $(\tilde{Y}, \sigma, A)$ are soft b-T_2spaces if and only if the binary soft topological space $(\tilde{X}, \tilde{Y}, \tau \times \sigma, A)$ is soft binary b-T_2.

Proof. Suppose $(\tilde{X}, \tau, A)$ and $(\tilde{X}, \sigma, A)$ are soft b-T_2spaces. Let $(e_{\mathbb{G}_1}, e_{\mathbb{H}_1}), (e_{\mathbb{G}_2}, e_{\mathbb{H}_2})\tilde{\mathcal{E}}X \times Y$ with$e_{\mathbb{G}_1} > e_{\mathbb{G}_2}$, $e_{\mathbb{H}_1} > e_{\mathbb{H}_2}$. Since $(\tilde{X}, \tau, A)$ is soft b-T_2space, there exists soft b-open sets such that $(F, A), (G, A)\tilde{\mathcal{E}}\tau$, $e_{\mathbb{G}_1}\mathcal{E}(F, A)$ and $e_{\mathbb{G}_2}\mathcal{E}(G, A)$ such that $e_{\mathbb{G}_1} \tilde{\notin} (G, A)$ and $e_{\mathbb{G}_2} \tilde{\notin} (F, A)$. Also, since $(\tilde{Y}, \sigma, A)$ is soft b-T_2 space, there exists disjoint soft b-open sets such that$(H, A), (I, A)\tilde{\mathcal{E}}\sigma$, $e_{\mathbb{H}_1}\mathcal{E}(H, A)$ and $e_{\mathbb{H}_2}\mathcal{E}(I, A)$ such that $e_{\mathbb{H}_1} \tilde{\notin} (I, A)$ and $e_{\mathbb{H}_2} \tilde{\notin} (H, A)$.thus $(e_{\mathbb{G}_1}, e_{\mathbb{H}_1})\mathcal{E}\big((F, A), (H, A)\big)$and $(e_{\mathbb{G}_2}, e_{\mathbb{H}_2})\mathcal{E}\big((G, A), (I, A)\big)$ with $(e_{\mathbb{G}_1}, e_{\mathbb{H}_1})\mathcal{E}\big((G^C, A), (I^C, A)\big)$ and $(e_{\mathbb{G}_1}, e_{\mathbb{H}_1})\mathcal{E}\big((F^C, A), (H^C, A)\big)$.Snce$(F, A)\ and\ (G, A)$are disjoint, $(F, A) \sqcap (H, A) = (\Phi, \Phi)$. Also, since$(H, A) \sqcap (I, A) = (\Phi, \Phi)$. Thus$\big((F, A) \sqcap (H, A), (G, A) \sqcap (I, A)\big) = (\Phi, \Phi)$. This implies that we have this implies that $(\tilde{X}, \tilde{Y}, \tau \times \sigma, A)$ is soft binary b-T_2. Conversely assume that $(\tilde{X}, \tilde{Y}, \tau \times \sigma, A)$ is soft binary b-T_2. Let $e_{\mathbb{G}_1}, e_{\mathbb{G}_2}\mathcal{E}X$ and $e_{\mathbb{H}_1}, e_{\mathbb{H}_2}\mathcal{E}Y$ such that $e_{\mathbb{G}_1} > e_{\mathbb{G}_2}$, $e_{\mathbb{H}_1} > e_{\mathbb{H}_2}$Therefore $(e_{\mathbb{G}_1}, e_{\mathbb{H}_1}), (e_{\mathbb{G}_2}, e_{\mathbb{H}_2})\tilde{\mathcal{E}}X \times Y$. Since $(\tilde{X}, \tilde{Y}, \tau \times \sigma, A)$ is soft binary b-T_2, there exists binary soft b-open sets $(F, A), (G,\mathrm{A})$ and there exists binary soft b-open sets $(\mathrm{H},\mathrm{A}),(\mathrm{I},\mathrm{A})\mathcal{E}(\tau\times\sigma)$, $(e_{\mathbb{G}_1},e_{\mathbb{H}_1})\tilde{\mathcal{E}}((\mathrm{F},\mathrm{A}),(\mathrm{G},\mathrm{A}))$ and $(e_{\mathbb{G}_2},e_{\mathbb{H}_2})\tilde{\mathcal{E}}((\mathrm{H},\mathrm{A}),(\mathrm{I},\mathrm{A}))$ such that $(e_{\mathbb{G}_1},e_{\mathbb{H}_1})\mathcal{E}\widetilde{(\mathrm{H}^{\mathrm{C}},\mathrm{A}),}(\mathrm{I}^{\mathrm{C}},\mathrm{A})$ and $(e_{\mathbb{G}_2},e_{\mathbb{H}_2})\tilde{\mathcal{E}}\left((\mathrm{F}^{\mathrm{C}},\mathrm{A}),(\mathrm{G}^{\mathrm{C}},\mathrm{A})\right)$. Therefore, $e_{\mathbb{G}_1}\mathcal{E}(\mathrm{F},\mathrm{A}),e_{\mathbb{G}_2}\mathcal{E}(\mathrm{H},\mathrm{A})$ and $e_{\mathbb{G}_1}\mathcal{E}(\mathrm{H}^{\mathrm{C}},\mathrm{A})$ and $e_{\mathbb{G}_2}\mathcal{E}(\mathrm{F}^{\mathrm{C}},\mathrm{A})$ and , $e_{\mathbb{H}_1}\mathcal{E}(\mathrm{G}^{\mathrm{C}},\mathrm{A})$ and $e_{\mathbb{H}_2}\mathcal{E}(\mathrm{I},\mathrm{A})$ and , $e_{\mathbb{H}_1}\mathcal{E}(\mathrm{I}^{\mathrm{C}},\mathrm{A})$ and , $e_{\mathbb{H}_2}\mathcal{E}(\mathrm{G}^{\mathrm{C}},\mathrm{A})$. Since $(\mathrm{F},\mathrm{A}),(\mathrm{G},\mathrm{A})\tilde{\mathcal{E}}\tau\times\sigma$, We have$(\mathrm{F},\mathrm{A}),(\mathrm{H},\mathrm{A})\mathcal{E}\tau$ and $(\mathrm{G},\mathrm{A}),(\mathrm{I},\mathrm{A})\mathcal{E}\sigma$. This proves that $(\tilde{\mathrm{X}},\tau,\mathrm{A})$ and $(\tilde{\mathrm{X}},\sigma,\mathrm{A})$ are soft b-T_2spaces.

6. Conclusion

The soft separation axioms namely b-T_o, b-T_1 and b-T_2 are extended to binary soft b-T_o, b-T_1 and b-T_2 structures. b-$\mathrm{T}_o$space with respect to first and second co-ordinates are beautifully reflected.

Acknowledgements

The author highly appreciated the referees for their valuable comments and suggestions for improving and bringing the paper in this meaningful form.

References

[1] A. Acıkgöz, N. Taş, Binary Soft Set Theory, Eur. J. of Pure and Appl. Math. 9/4 (2016) 452- 463.
[2] S.S. Benchalli. G.Patil et al, On Binary Soft Separation Axioms in Binary Soft Topological Spaces, Global journal and Applied Mathematics 13/9 (2017) 5393-5412.
[3] D. Molodtsov, Soft Set Theory First Results. Comput. Math. Appl. **37** (1999) 19-31.
[4] D. Molodtsov, V. Y. Leonov and D. V. Kovkov, Soft sets technique and its Application, Nechetkie Sistemy Myagkie Vychisleniya 1/1 (2006) 8-39.
[5] M. Shabir, M. Naz, On Some New Operations in Soft Set Theory, Computers and Math. With Appl. 57 (2011) 1786-1799.
[6] S. S. Benchalli, G. Patil, A. S. Dodamani and J. P. Kumar, On Binary Soft Topological Spaces, Int. Journal of Applied Mathematics 30/6 (2017) 437-453.
[7] Dr. A. Kalaichelvi and P.H. Malini, Application of Fuzzy Soft Sets to Investment Decision Making Problem, International Journal of Mathematical Sciences and Applications 1/3 (2011) 1583-1586.
[8] N. Y. Özgür and N. Taş, A Note on "Application of Fuzzy Soft Sets to Investment Decision Making Problem", Journal of New Theory, 7 (2015) 1-10.
[9] N. Taş, N. Y. Özgür and P. Demir, An Application of Soft Set and Fuzzy Soft Set Theories to Stock Management, Süleyman Demirel University Journal of Natural and Applied Sciences 21/2 (2017) 791-196.
[10] J. C. R. Alcantud, S. C. Rambaud and M. J. M. Torrecillas, Valuation Fuzzy Soft Sets: A Flexible Fuzzy Soft Set Based Decision Making Procedure for the Valuation of Assets, Symmetry 9/11 (2017) 253; doi:10.3390/sym 9110253.
[11] N. Çağman, S. Enginoğlu, Soft Matrix Theory and its Decision Making, Computers and Mathematics with Applications 59 (2010) 3308-3314.