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Fuzzy Soft Locally Closed Sets in Fuzzy Soft Topological Space

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Abstract - The purpose of this paper is to introduce fuzzy soft locally closed and fuzzy soft b-locally closed sets and study their properties in fuzzy soft topological space. Further we define and study fuzzy soft LC-continuous and fuzzy soft b-LC-continuous functions.

Keywords *- Fuzzy soft locally closed sets, Fuzzy soft b-locally closed set, Fuzzy soft LC-continuous functions.*

1. Introduction

The notion of fuzzy sets for dealing with uncertainties was introduced by Zadeh [15]. Fuzzy topology was introduced by Chang [4]. To overcome difficulties in fuzzy set theory soft sets were introduced in 1999 [11]. The hybridisation of fuzzy set and soft set known as fuzzy soft set was introduced by Maji et.al. [10]. The notion of topological structure of Fuzzy soft sets was introduced by Tanay and Kandemir [13] and studied further by many authors [5,6,12,14]. The concept of fuzzy soft semi open set was introduced by Kandil et al. [8] whereas fuzzy soft pre-open and regular open sets was introduced by Hussain [7] and fuzzy soft b-open sets was introduced by Anil [1]. In this paper we introduce fuzzy soft locally closed and fuzzy soft b-locally closed sets and study their properties. Further we define fuzzy soft LC-continuous and fuzzy soft b-LC-continuous functions and study few of the properties.

2. Preliminaries

Definition2.1 [10] Let X be an initial universal set, I^X be set of all fuzzy sets on X and E be a set of parameters and let $A \subseteq E$. A pair (f, A) denoted by f_A is called fuzzy soft set over X, where f is a mapping given by $f : A \to I^X$ i.e. for each $a \in A$, $f(a) = f_a : X \to I$

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is a fuzzy set on X

Definition2.2 [12] Let τ be a collection of all fuzzy soft sets over a universe X with a fixed parameter set E then (X,τ,E) is called fuzzy soft topological space if i. $\tilde{0}_E,\ \tilde{1}_E \in \tau$ ii. Union of any members of τ is a member of τ, iii. Intersection of any two members of τ is a member of τ. Each member of τ is called fuzzy soft open set i.e. A fuzzy soft set f_A over X is fuzzy soft open if and only if $f_A \in \tau$. A fuzzy soft set f_A over X is called fuzzy soft closed set if the complement of f_A is fuzzy soft open set.

Definition2.3 [14] The fuzzy soft closure of f_A, denoted by $Fscl(f_A)$ is defined as $Fscl(f_A) = \bigcap \{h_D : h_D \textit{ is fuzzy soft closed set and } f_A \subseteq h_D\}$

Definition2.4 [14] The fuzzy soft interior of g_B denoted by $Fs\ int(g_B)$ is defined as $Fs\ int(g_B) = \bigcup \{h_D : h_D \textit{ is fuzzy soft open set and } h_D \subseteq g_B\}$

Definition2.5 [7] Fuzzy soft set f_A of a fuzzy soft topological space (X,τ,E) is called fuzzy soft pre-open set if $f_A \leq Fs\text{int}\, Fscl(f_A)$ and fuzzy soft pre-closed if $FsclFs\,\text{int}(f_A) \leq f_A$

Definition2.6 [7] Fuzzy soft set f_A of a fuzzy soft topological space (X,τ,E) is called fuzzy soft α-open set if $f_A \leq Fs\,\text{int}(Fscl(Fs\,\text{int}(f_A)))$

Definition 2.6 [1] A fuzzy soft set f_A in a fuzzy soft topological space (X,τ,E) is called fuzzy soft b-open set if $f_A \leq Fs\,\text{int}\, Fscl(f_A) \vee FsclFs\,\text{int}(f_A)$ and fuzzy soft b-closed set if $f_A \geq Fs\,\text{int}\, Fscl(f_A) \vee FsclFs\,\text{int}(f_A)$

Definition 2.7 [1] Let f_A be a fuzzy soft set in a fuzzy soft topological space (X,τ,E) then fuzzy soft b-closure of f_A and fuzzy soft b-interior of f_A are defined as

(i) $fsb-cl\,(f_A) = \bigcap \{g_B : g_B \textit{ is a } fsb-closed\ set\ \&\ g_B \geq f_A\}$

(ii) $fsb-\text{int}\,(f_A) = \bigcup \{h_c : h_c \textit{ is a } fsb-open\,set\ \&\ h_C \leq f_A\}$

3. Soft Locally Closed Sets

Definition 3.1. A fuzzy soft set (F, E) is called fuzzy soft locally closed set in a fuzzy soft topological space (X,τ,E) if (F, E) = (G, E)∩(H,E) where (G, E) is fuzzy soft open and (H, E) is fuzzy soft closed in X.

The family of all fuzzy soft locally closed sets of a fuzzy soft topological space (X,τ,E) is denoted by FSLCS (X,τ,E).

Theorem 3.2. In a fuzzy soft topological space (X,τ,E), every fuzzy soft open set is fuzzy soft locally closed.

Proof. Let (F, E) be fuzzy soft open in X, then (F, E) is fuzzy soft locally closed in X, since (F, E)=(F, E) ∩ $\tilde{1}$.

Theorem 3.3. Let (X, τ, E) be a fuzzy soft topological space. If (F_1, E) and (F_2, E) are two fuzzy soft locally closed sets in X then (F_1, E) ∩ (F_2, E) is a fuzzy soft locally closed set in X.

Proof. Let (F_1, E) = (G_1, E) ∩ (H_1, E) and (F_2, E) = (G_2, E) ∩ (H_2, E) where (G_1, E) and (G_2, E) are fuzzy soft open and (H_1, E) and (H_2, E) are fuzzy soft closed in X. Then (F_1, E) ∩ (F_2, E) = ((G_1, E) ∩ (H_1, E)) ∩((G_2, E) ∩ (H_2, E)) = ((G_1, E) ∩ (G_2, E)) ∩((H_1, E) ∩ (H_2, E)), where (G_1, E) ∩ (G_2, E) is fuzzy soft open and (H_1, E) ∩ (H_2, E) is fuzzy soft closed and hence (F_1, E) ∩ (F_2, E) is a fuzzy soft locally closed set in X.

Theorem 3.4. Let (X, τ, E) be a fuzzy soft topological space. Then (F, E) is fuzzy soft locally closed if and only if (F, E) = (G, E)∩Fs-cl(F, E) for some fuzzy soft open set (G, E).

Proof. Let (F, E) be fuzzy soft locally closed set in X. Hence (F, E) = (G, E)∩(H, E) where (G, E) is fuzzy soft open and (H, E) is fuzzy soft closed in X. Then Fs-cl(F,E) = Fs-cl((G, E)∩(H, E)) $\subset$ Fs-cl(G, E)∩ Fs-cl(H, E) = Fs-cl(G, E)∩ (H, E). We have Fs-cl(F,E) $\subset$ (H, E) and hence (F, E) $\subset$ (G, E)∩Fs-cl(F, E) $\subset$ (G, E)∩(H, E) = (F, E). Therefore (F, E) = (G, E)∩Fs-cl(F, E).

Conversely, if (F, E) = (G, E)∩Fs-cl(F, E) for some fuzzy soft open set (G, E) then (F, E) is fuzzy soft locally closed since Fs-cl(F, E) is fuzzy soft closed in X.

Definition 3.5. Let (F, E) and (G, E) be any two fuzzy soft sets. Then (F, E) and (G, E) are said to be separated if (F, E)∩Fs-cl(G, E) = (G, E)∩Fs-cl(F, E) = $\tilde{0}$.

Theorem 3.6. Let (X, τ, E) be a fuzzy soft topological space and (F_1, E) and (F_2, E) are two fuzzy soft locally closed in X. If (F_1, E) and (F_2, E) are separated in X then (F_1, E)⋃ (F_2, E) is a fuzzy soft locally closed in X.

Proof. Since (F_1, E) and (F_2, E) are two fuzzy soft locally closed in X, we have (F_1, E) = (G_1, E)∩Fs-cl(F_1, E) and (F_2, E) = (G_2, E)∩Fs-cl(F_2, E), where (G_1, E) and (G_2, E) are fuzzy soft open in X. Since (F_1, E) and (F_2, E) are separated, we have (F_1, E)∩Fs-cl(F_2, E) = (F_2, E)∩Fs-cl(F_1, E) = $\tilde{0}$ and which implies (F_1, E)⋃(F_2, E) =(G_1, E) ⋃(G_2, E) ∩ Fs-cl((F_1, E) ⋃(F_2, E)). Hence (F_1, E)⋃(F_2, E) is fuzzy soft locally closed set in X.

Theorem 3.7. Let (X, τ, E) be a fuzzy soft topological space. For a fuzzy soft set (F, E) following are equivalent

(i) (F, E) is fuzzy soft open in X
(ii) (F, E) is fuzzy soft α-open and fuzzy soft locally closed
(iii) (F, E) is fuzzy soft pre-open and fuzzy soft locally closed
(iv) (F, E) is fuzzy soft b-open and fuzzy soft locally closed

Proof. (i) implies(ii), (ii) implies (iii) and (iii) implies (iv) are obvious

(iv) Implies (i): Let (F, E) be fuzzy soft b-open and fuzzy soft locally closed set in X. We have (F, E)$\subset$Fs-int(Fs-cl(F, E)) $\cup$ Fs-cl(Fs-int(F, E)) and (F, E) = (G, E)$\cap$Fs-cl(F, E) where (G, E) is fuzzy soft open. Then (F, E)$\subset$(G, E)$\cap$(Fs-int(Fs-cl(F, E)) $\cup$ Fs-cl(Fs-int(F, E)))= ((G, E)$\cap$Fs-int(Fs-cl(F, E))) $\cup$ ((G, E)$\cap$Fs-cl(Fs-int(F, E))) = Fs-int((G, E)$\cap$Fs-cl(F, E)) $\cup$ Fs-int(F, E) = Fs-int(F, E) $\cup$ Fs-int(F, E) = Fs-int(F, E). Hence (F, E) is fuzzy soft open in X.

Definition 3.8. A fuzzy soft set (F, E) is called fuzzy soft b-locally closed set in a fuzzy soft topological space (X,τ,E) if (F, E) = (G, E)$\cap$(H, E) where (G, E) is fuzzy soft b-open and (H, E) is fuzzy soft b-closed in X.

The family of all fuzzy soft b-locally closed sets of a fuzzy soft topological space (X,τ,E) is denoted by FSBLCS(X,τ,E).

Remark 3.9. It is obvious that every fuzzy soft b-closed set is fuzzy soft b-locally closed set.

Remark 3.10. Every fuzzy soft locally closed set is fuzzy soft b-locally closed set but converse need not be true.

Example 3.11. Let X = {a, b, c}, E = {e_1}, $\tau=\{\tilde{1},\tilde{0},(F_1,E),(F_2,E)\}$ where $(F_1,E)=\left\{\left\{\frac{1}{a},\frac{0}{b},\frac{0}{c}\right\}\right\}$ and $(F_2,E)=\left\{\left\{\frac{1}{a},\frac{0}{b},\frac{1}{c}\right\}\right\}$. Clearly the set $(F,E)=\left\{\left\{\frac{1}{a},\frac{1}{b},\frac{0}{c}\right\}\right\}$ is fuzzy soft b-locally closed set but not fuzzy soft locally closed.

Theorem 3.12. Let (X,τ,E) be a fuzzy soft topological space. Then (F, E) is fuzzy soft b-locally closed if and only if (F, E) = (G, E)$\cap$Fsb-cl(F, E) for some fuzzy soft open set (G, E).

Proof. Let (F, E) be fuzzy soft b-locally closed set in X. Hence (F, E) = (G, E)$\cap$(H, E) where (G, E) is fuzzy soft b-open and (H, E) is fuzzy soft b-closed in X. Then Fsb-cl(F,E) $\subset$ (H, E) and hence (F, E) = (F, E)$\cap$Fsb-cl(F, E) = (G, E)$\cap$(H, E)$\cap$ Fsb-cl(F, E) = (G, E)$\cap$Fsb-cl(F, E).

Conversely, if (F, E) = (G, E)$\cap$Fsb-cl(F, E) for some fuzzy soft b- open set (G, E) and since Fsb-cl(F, E) is fuzzy soft closed, hence (F, E) is fuzzy soft b-locally closed in X.

Definition 3.13. Let (X,τ,E) and (Y,σ,K) be fuzzy soft topological spaces and $f:X\to Y$ be a function. Then f is called a

(i) fuzzy soft locally continuous (LC-continuous) if for each open set (G, K) in Y, $f^{-1}(G,K)$ is a fuzzy soft locally closed set in X.

(ii) fuzzy soft b-locally continuous (b-LC-continuous) if for each open set (G, K) in Y, $f^{-1}(G,K)$ is a fuzzy soft b-locally closed set in X.

(iii) fuzzy soft locally irresolute (LC-irresolute) if for each fuzzy soft locally closed set (G, K) in Y, $f^{-1}(G,K)$ is a fuzzy soft locally closed set in X.

(iv) fuzzy soft b-locally irresolute (b-LC-irresolute) if for each fuzzy soft b-locally closed set (G, K) in Y, $f^{-1}(G,K)$ is a fuzzy soft b-locally closed set in X.

Theorem 3.14. Every fuzzy soft LC- continuous function is fuzzy soft b-LC- continuous.

Proof. Let $f:(X,\tau,E)\to(Y,\sigma,K)$ be fuzzy soft LC- continuous function. Then for any fuzzy soft open set (G, K) in Y, $f^{-1}(G,K)$ is fuzzy soft locally closed in X. We have $f^{-1}(G,K)$ = (G_1, E)∩(H_1, E) where (G_1, E) is fuzzy soft open and (H_1, E) is fuzzy soft closed in X. Since every fuzzy soft open (closed) set is fuzzy soft b-open (b-closed) set. Therefore $f:(X,\tau,E)\to(Y,\sigma,K)$ is b-LC- continuous. Converse of this theorem need not be true as seen from the following example.

Example 3.15. Let X = Y={a, b, c}, E = K = {e_1}, $\tau=\left\{\tilde{1},\tilde{0},(F_1,E),(F_2,E)\right\}$ and $\sigma=\left\{\tilde{1},\tilde{0},(G,E)\right\}$ where $(F_1,E)=\left\{\left\{\frac{1}{a},\frac{0}{b},\frac{0}{c}\right\}\right\}$, $(F_2,E)=\left\{\left\{\frac{1}{a},\frac{0}{b},\frac{1}{c}\right\}\right\}$ and $(G,E)=\left\{\left\{\frac{1}{a},\frac{1}{b},\frac{0}{c}\right\}\right\}$.

Consider an identity function $f:X\to Y$, Clearly $f^{-1}(G,E)=\left\{\left\{\frac{1}{a},\frac{1}{b},\frac{0}{c}\right\}\right\}$ is fuzzy soft b-locally closed set but not fuzzy soft locally closed.

4. Conclusions

In this paper the concept of fuzzy soft locally closed set and fuzzy soft b-locally closed set is introduced in fuzzy soft topological space. Also fuzzy soft LC-continuous and fuzzy soft b-LC-continuous functions were defined in fuzzy soft topological space.

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