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CONTOUR SURFACES IN THE (2+1)-DIMENSIONAL SINE-POISSON MODEL

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Original scientific paper

This paper apply the modified exp(-Omega(xi))-expansion function method to the (2+1)-dimensional Sine-Poisson equation. Many complex soliton solutions are successfully constructed. 2D and 3D figures along with contour surfaces by using several computational programs such as Mathematica and Matlab are plotted. Finally, at the end of manuscript, general conclusion about these novel findings which they are differ from existing results are given.

Keywords: *Complex function solution, Exponential function solution, Modified exp(-Omega(xi))-expansion function approach, Sine-Poisson equation.*

1 Introduction

Solving nonlinear evolution equations (NEEs) and seeking explicit and exact solutions of NEEs has become one of the most exciting and extremely active domains in nonlinear theory. Especially, solitons, as an important concept of analytical solutions of nonlinear partial differential equations fields, have attracted attention of scientist from all over the world during three last decades. During these years, it has been submitted many works to the literature related with the numerical solutions of NEEs together with analytical, exact, travelling, and approximate solutions. Moreover, in real world problems, Fuzzy approaches for risk analysis and evaluation in multi-criteria decision making problems. Furthermore, multi-criteria decision-making methods have been used in the selection of personnel in daily life problems and many others with mathematical aspects [1-27, 31-44].

The main aim of this paper, using modified exp(-Omega(xi))-expansion function method (MEFM) is to find new soliton solutions to the (2+1)-dimensional Sine-Poisson equation defined as [28]

$$u_{tt} - u_{xx} - u_{yy} + m^2 \sin(u) = 0, \qquad (1)$$

where m is real constant and non-zero.

2 General Facts of MEFM

Let's consider the partial differential equation:

$$P(u_x, u_t, u_{xx}, u_{tx}, \cdots) = 0, \qquad (2)$$

where, $u = u(x,t)$ is an unknown function, P is a polynomial in $u(x,t)$ and its derivative in which highest order derivatives and nonlinear terms are involved and the subscripts stand for the partial derivatives. The basic phases of method are expressed as follows:

Step 1: Let's consider the following travelling transformation defined by

$$u(x,y,t) = U(\xi),\ \xi = kx + ly - wt, \qquad (3)$$

By using Eq(3), we can convert Eq.(2) into nonlinear ordinary differential equation (NODE) defined by;

$$N(U, U', U'', \cdots) = 0. \qquad (4)$$

where N is a polynomial of U and its derivatives and the superscripts indicate the ordinary derivatives with respect to ξ.

Step 2: Suppose the travelling wave solution of Eq.(4) can be rewritten as following manner;

$$U(\xi) = \frac{\sum_{i=0}^{N} A_i \left[e^{-\Omega(\xi)} \right]^i}{\sum_{j=0}^{M} B_j \left[e^{-\Omega(\xi)} \right]^j}, \qquad (5)$$

where $A_i, B_j, (0 \le i \le N, 0 \le j \le M)$ are constants to be determined later, such that $A_N \neq 0, B_M \neq 0,$ and $\Omega = \Omega(\xi)$ verifies the following ordinary differential equation;

$$\Omega'(\xi) = \exp(-\Omega(\xi)) + \mu \exp(\Omega(\xi)) + \lambda. \qquad (6)$$

Eq. (6) has the following solution families [9,29,30]:

Family-1: When $\mu \neq 0,\ \lambda^2 - 4\mu > 0,$

$$\Omega(\xi) = \ln\left(\frac{-\sqrt{\lambda^2 - 4\mu}}{2\mu} \tanh\left(\frac{\sqrt{\lambda^2 - 4\mu}}{2}(\xi + E) \right) - \frac{\lambda}{2\mu} \right), \qquad (7)$$

Family-2: When $\mu \neq 0,\ \lambda^2 - 4\mu < 0,$

$$\Omega(\xi) = \ln\left(\frac{\sqrt{-\lambda^2 + 4\mu}}{2\mu} \tan\left(\frac{\sqrt{-\lambda^2 + 4\mu}}{2}(\xi + E) \right) - \frac{\lambda}{2\mu} \right), \qquad (8)$$

Family-3: When $\mu = 0,\ \lambda \neq 0,$ and $\lambda^2 - 4\mu > 0,$

$$\Omega(\xi) = -\ln\left(\frac{\lambda}{\exp(\lambda(\xi + E)) - 1} \right), \qquad (9)$$

Family-4: When $\mu \neq 0,\ \lambda \neq 0,$ and $\lambda^2 - 4\mu = 0,$

$$\Omega(\xi)=\ln\left(-\frac{2\lambda(\xi+E)+4}{\lambda^2(\xi+E)}\right), \tag{10}$$

Family-5: When $\mu=0,\ \lambda=0,$ and $\lambda^2-4\mu=0,$

$$\Omega(\xi)=\ln(\xi+E), \tag{11}$$

being $A_0, A_1, A_2, \cdots A_N, B_0, B_1, B_2, \cdots B_M, E, \lambda, \mu$ are constants to be determined later. The positive integer N and M can be determined by considering the homogeneous balance between the highest order derivatives and the nonlinear terms occurring in Eq. (8).

Step 3: Setting Eq.(5) and Eq.(6) into Eq. (4), we get a polynomial of $\exp(-\Omega(\xi))$. We equate all the coefficients of same power of $\exp(-\Omega(\xi))$ to zero. This procedure yields a system of equations whichever can be solved to find $A_0, A_1, A_2, \cdots A_N$, $B_0, B_1, B_2, \cdots B_M, E, \lambda, \mu$ with the aid of computer programs. Substituting the values of $A_0, A_1, A_2, \cdots A_N, B_0, B_1, B_2, \cdots B_M, E, \lambda, \mu$ in Eq. (9), the general solutions of Eq. (4) complete the determination of the solution of Eq. (2).

3 Application of MEFM

This section apply the MEFM to the Eq.(1) for obtaining some new complex soliton solutions.

Example-1 When we apply Eq.(3) to the Eq.(1), we obtain the following nonlinear ordinary differential equation;

$$\left(w^2-k^2-l^2\right)U''+m^2\sin(U)=0. \tag{12}$$

In Eq.(12), when we use $\sin(U)=\left(e^{iU}-e^{-iU}\right)/2i$, we can find the following equality;

$$2\left(w^2-k^2-l^2\right)ie^{iU}U''+m^2e^{2iU}-m^2=0. \tag{13}$$

If we use $V=e^{iU}$, we reach the following equality;

$$\begin{aligned}&2\left(w^2-k^2-l^2\right)V''V-2\left(w^2-k^2-l^2\right)\left(V'\right)^2\\&+m^2V^3-m^2V=0.\end{aligned} \tag{14}$$

Balance principle between $V''V$ and V^3;

$$N=M+2. \tag{15}$$

By using this relationship, we can attain some new soliton solutions for Eq.(1) as follows:

Case 1: By getting $M=1$ and $N=3$, after then, we can write follows;

$$V=\frac{A_0+A_1\,e^{-\Omega}+A_2\,e^{-2\Omega}+A_3\,e^{-3\Omega}}{B_0+B_1\,e^{-\Omega}}, \tag{16}$$

$$\begin{aligned}V'&=\frac{\left[A_1\,e^{-\Omega}(-\Omega')+A_2\,e^{-2\Omega}(-2\Omega')+A_3\,e^{-3\Omega}(-3\Omega')\right]\left[B_0+B_1\,e^{-\Omega}\right]}{\left[B_0+B_1\,e^{-\Omega}\right]^2}\\&\quad-\frac{\left[A_0+A_1\,e^{-\Omega}+A_2\,e^{-2\Omega}+A_3\,e^{-3\Omega}\right]\left[B_1\,e^{-\Omega}(-\Omega')\right]}{\left[B_0+B_1\,e^{-\Omega}\right]^2}=\frac{\Upsilon}{\Psi},\end{aligned} \tag{17}$$

$$V''=\frac{\Upsilon'\Psi-\Upsilon\Psi'}{\Psi^2},$$

$\vdots,$

where $A_3\neq 0$ and $B_1\neq 0$. Substituting Eqs.(16,17) in Eq.(14), we get an equation including $\exp(-\Omega(\xi))$ and it has various powers. Therefore, we have a system of algebraic equations from the coefficients of polynomial of $\exp(-\Omega(\xi))$. Solving this system of equations, it yields us the following coefficients;

Case 1.1: When

$$A_0=-\frac{1}{12}\lambda(-4A_1+\lambda A_2),\ B_0=\frac{\lambda(-4A_1+\lambda A_2)}{4(A_1-\lambda A_2)}B_1,$$

$$A_3=\frac{4(-A_1+\lambda A_2)}{3\lambda^2},\ \mu=\frac{1}{4}\lambda^2\left(1+\frac{3B_1}{-A_1+\lambda A_2}\right),$$

$$m=\frac{i\sqrt{3}\sqrt{-k^2-l^2+w^2}\,\lambda\sqrt{B_1}}{\sqrt{-A_1+\lambda A_2}},$$

we find the following soliton solutions under the *Family-1* condition as

$$u_1=-i\ln\left[-\frac{\left(3\sqrt{B_1}+\sqrt{3}\sqrt{A_1-\lambda A_2}\tanh\left(\frac{\sqrt{3}\lambda\sqrt{B_1}}{2\sqrt{A_1-\lambda A_2}}(E+kx+ly-wt)\right)\right)^2}{3\left(\sqrt{A_1-\lambda A_2}+\sqrt{3}\sqrt{B_1}\tanh\left(\frac{\sqrt{3}\lambda\sqrt{B_1}}{2\sqrt{A_1-\lambda A_2}}(E+kx+ly-wt)\right)\right)^2}\right]. \tag{18}$$

For better understanding of physical meaning of Eq.(18), 2D and 3D figures along with contour graphs may be observed in Figures (1) and (2) for suitable values of parameters as follows;

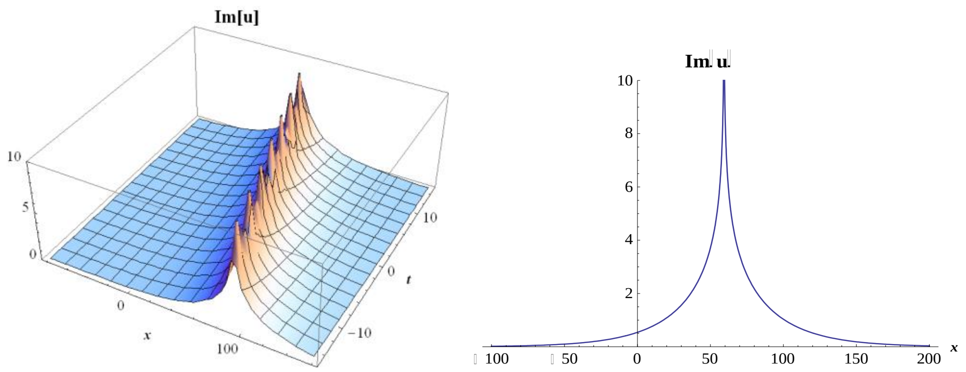

Figure 1. The 2D and 3D surfaces of Eq.(18)

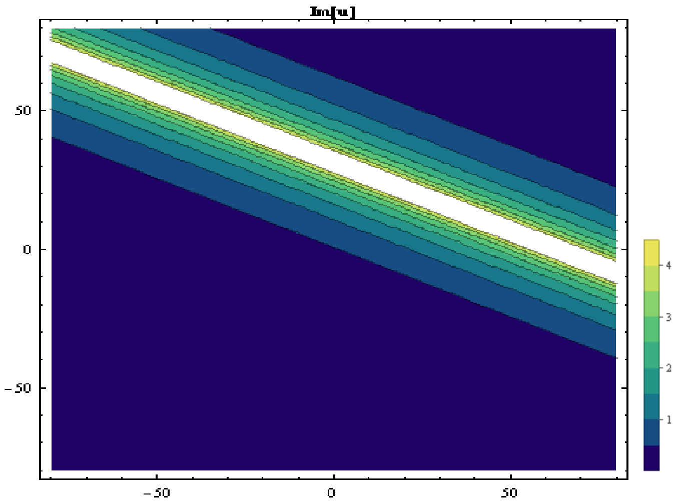

Figure 2. The contour surfaces of Eq.(18)

Case1.2. If

$$A_0=-\frac{1}{12}\lambda\left(-4A_1+\lambda A_2\right),B_0=\frac{\lambda\left(-4A_1+\lambda A_2\right)}{4\left(A_1-\lambda A_2\right)}B_1,$$

$$A_3=\frac{4\left(-A_1+\lambda A_2\right)}{3\lambda^2},\mu=\frac{1}{4}\lambda^2\left(1+\frac{3B_1}{-A_1+\lambda A_2}\right),$$

$$m=\frac{i\sqrt{3}\sqrt{-k^2-l^2+w^2}\lambda\sqrt{B_1}}{\sqrt{-A_1+\lambda A_2}},$$

we find the following another new soliton solutions under the *Family-2* condition as

$$u_2=i\ln(3)-i\ln\left[\frac{\left(3\sqrt{B_1}+\sqrt{-3A_1+3\lambda A_2}\tan\left(\frac{\sqrt{3}\lambda\sqrt{B_1}}{2\sqrt{-A_1+\lambda A_2}}(E+kx+ly-wt)\right)\right)^2}{\left(\sqrt{-A_1+\lambda A_2}-\sqrt{3B_1}\tan\left(\frac{\sqrt{3}\lambda\sqrt{B_1}}{2\sqrt{-A_1+\lambda A_2}}(E+kx+ly-wt)\right)\right)^2}\right].\tag{19}$$

For Eq.(19), 2D and 3D figures along with contour graphs may be observed in Figures (3) and (4) for suitable values of parameters as follows;

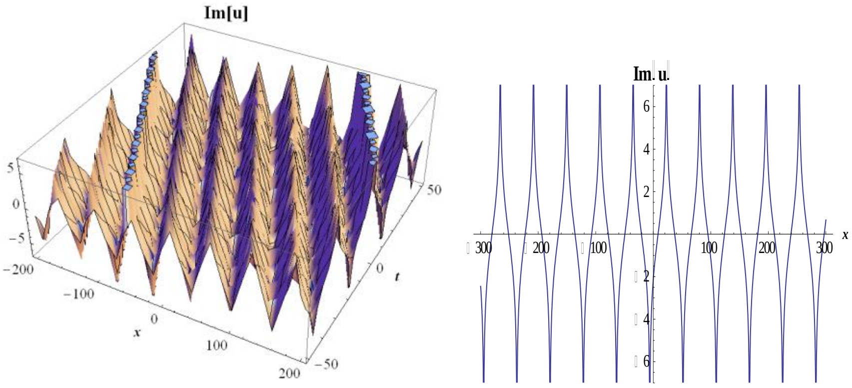

Figure 3. The 2D and 3D surfaces of Eq.(19)

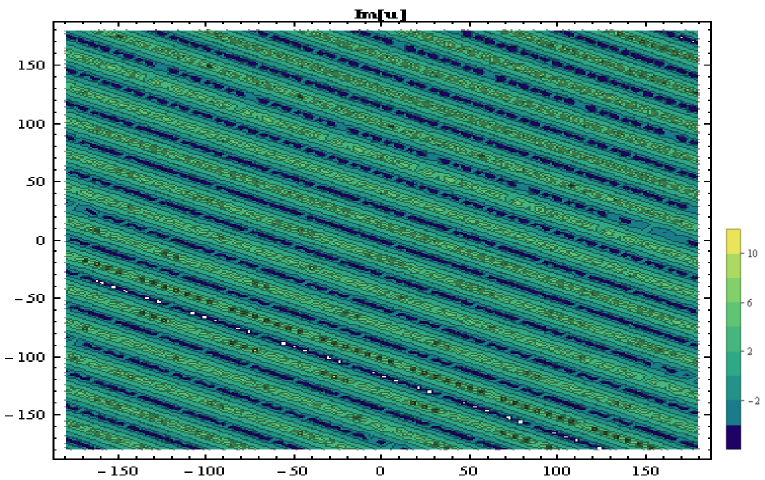

Figure 4. The contour surfaces of Eq.(19)

Case1.3. If

$$A_0 = B_0\left(-1+\frac{\mu A_3}{B_1}\right), A_1 = \mu A_3 - \frac{1}{B_1}2\sqrt{A_3}B_0\sqrt{\mu A_3 - B_1} - B_1,$$

$$\lambda = -\frac{2\sqrt{\mu A_3 - B_1}}{\sqrt{A_3}},\ A_2 = -2\sqrt{A_3}\sqrt{\mu A_3 - B_1} + \frac{B_0 A_3}{B_1},$$

$$l = \frac{\sqrt{m^2 A_3 + 4\left(-k^2+w^2\right)B_1}}{2\sqrt{B_1}},$$

we obtain the following another new complex singular soliton solutions under the Family-1 condition as

$$u_3 = -i\ln\left[\frac{\mu A_3}{B_1} - 1 - \frac{4\mu\sqrt{A_3}\sqrt{\mu A_3 - B_1}}{\kappa - \varpi B_1\tanh\left(\frac{1}{2}\varpi\left(E+kx-tw+yl\right)\right)} + \frac{4A_3\mu^2}{B_1}\left[\frac{2\sqrt{\mu A_3 - B_1}}{\sqrt{A_3}} - \varpi\tanh\left(\frac{1}{2}\varpi\left(E+kx-tw+yl\right)\right)\right]^{-2}\right], \tag{20}$$

$$\varpi = \sqrt{-4\mu + \frac{4\left(\mu A_3 - B_1\right)}{A_3}}, \kappa = \frac{2\sqrt{\mu A_3 - B_1}}{\sqrt{A_3}}B_1,$$

For Eq.(20), 2D and 3D figures along with contour graphs may be observed in Figures (5) and (6) for suitable values of parameters as follows;

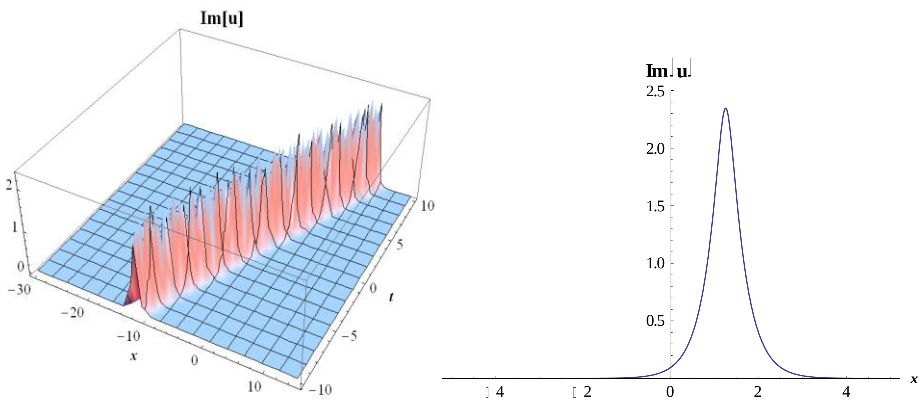

Figure 5. The 2D and 3D surfaces of Eq.(20)

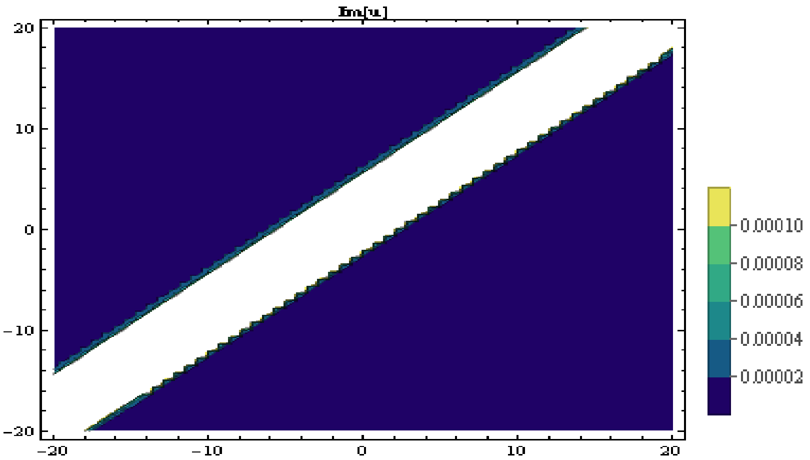

Figure 6. The contour surfaces of Eq.(20)

4 Conclusion

In this paper, we have applied MEFM to the Eq.(1). Then, we found some new complex soliton solutions such as hyperbolic and trigonometric functions. With the help of several computational computer programming, we have also plotted two and three-surfaces of results along with contour surfaces. All Figures (1-6), it can be observed that these surfaces have symbolized the physical properties of model considered in this paper. Comparising with papers presented in literature [28], it can be viewed that these soliton solutions are entirely new constructed by using MEFM. To the best of our knowledge, the application of MEFM to the Eq.(1) has not been submitted before.

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