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SITEM FOR THE CONFORMABLE SPACE-TIME FRACTIONAL COUPLED KD EQUATIONS

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Abstract

In the present paper, new analytical solutions for the space-time fractional coupled Konopelchenko-Dubrovsky (KD) equations are obtained by using the simplified $\tan(\frac{\phi(\xi)}{2})$- expansion method (SITEM). Here, fractional derivatives are described in conformable sense. The obtained traveling wave solutions are expressed by the trigonometric, hyperbolic, exponential and rational functions. Simulation of the obtained solutions are given at the end of the paper.

Keywords: Space-time fractional coupled KD equations, simplified $\tan(\frac{\phi(\xi)}{2})$-expansion method (SITEM), conformable derivative.

1. Introduction

In recent years, to model and describe phenomena in various fields of science such as plasma physics, nonlinear optics, nonlinear transmission lines, solid state physics, chemical kinematics, and biology, nonlinear partial differential equations have been used. The popularity of these equations is because of their capacity to model many real systems. Therefore, nonlinear equations have gained a very significant place in the current research. To solve the nonlinear partial differential equations, various methods have been developed (see, for example, [1, 2, 3, 4, 5, 6]).

KD equations were introduced by Konopelchenko and Dubrovsky [7]. These equations constitute applications in the ocean dynamics, fluid mechanics and plasma physics. To solve the coupled KD equations, various methods have been proposed such as the standard truncated Painlevé analysis, homotopy perturbation method, generalized F-expansion method, $(G'/G,1/G)$-expansion method, first integral method, extended Riccati equation rational method, Xu's stable-range method, tanh–sech method, cosh–sinh method and exponential functions method [8, 9, 10, 11, 12,13, 14, 15]. There is not much work on the fractional coupled KD equation. Fractional coupled KD equations have been solved by using sub equation method, Jacobi elliptic equation method and extended G'/G -expansion method in [16, 17, 18]. Here fractional derivatives are described in modified Riemann-Liouville sense and Caputo sense.

Recently, $\tan(\frac{\phi(\xi)}{2})$ -expansion method has been applied by many authors [19, 20, 21, 22, 23]. In [19], ITEM has been simplified and called simplified ITEM (SITEM). SITEM has been applied to Kundu-Eckhaus equation. To our knowledge, there is no other application of the SITEM in the literature. In this paper, we consider space-time fractional coupled KD equation. Here fractional derivatives are described in conformable sense. We obtain some traveling wave solutions such as trigonometric, hyperbolic, exponential and rational functions.

2. Description of the conformable fractional derivative and its properties

For a function $f:(0,\infty)\to R$, the conformable fractional derivative of f of order $0<\alpha<1$ is defined as (see, for example, [24])

$$T_t^{\alpha} f(t)=\lim_{\varepsilon\to 0}\frac{f(t+\varepsilon t^{1-\alpha})-f(t)}{\varepsilon}. \tag{1}$$

Some important properties of the the conformable fractional derivative are as follows:

$$T_t^{\alpha}(af+bg)(t) \;=aT_t^{\alpha}f(t)+bT_t^{\alpha}g(t)\;,\;\text{ for all }\; a,b\in R\;,$$
$$T_t^{\alpha}(t^{\mu}) \;=\; \mu t^{\mu-\alpha},$$
(2)
$$T_t^{\alpha}(f(g(t)) \;=t^{1-\alpha}g'(t)f'(g(t)).$$

3. Description of the simplified $\tan(\frac{\phi(\xi)}{2})$ -expansion method (SITEM) for solving conformable partial differential equations

Let us consider general nonlinear fractional partial differential equation of the type

$$P(u,T_t^{\alpha}u,T_x^{\beta}u,T_t^{\alpha}T_t^{\alpha}u,T_t^{\alpha}T_x^{\beta}u,T_x^{\beta}T_x^{\beta}u,...)=0,\; 0<\alpha\le 1\;,0<\beta\le 1\;, \tag{3}$$

where u is an unknown function and P is a polynomial of u and its partial fractional derivatives. Using the following transformation

$$u(x,t)=U(\xi),\; \xi=k\frac{t^{\alpha}}{\alpha}+m\frac{x^{\beta}}{\beta}, \tag{4}$$

where k and m are non zero arbitrary constants, Eq. (3) can be written as the following nonlinear ordinary differential equations

$$\phi(U,U',U'',U''',...)=0\,. \tag{5}$$

Suppose that traveling wave solution of Eq. (5) can be expressed as follows

$$U(\xi)=\sum_{k=0}^{m}A_k\left[p+\tan\left(\frac{\phi(\xi)}{2}\right)\right]^k+\sum_{k=1}^{m}B_k\left[p+\tan\left(\frac{\phi(\xi)}{2}\right)\right]^{-k}, \tag{6}$$

$\phi(\xi)$ satisfies the following ordinary differential equation

$$\phi'(\xi)=a\sin(\phi(\xi))+b\cos(\phi(\xi))+c, \tag{7}$$

where a, b, c , $A_k(0\le k\le m)$ and $B_k(1\le k\le m)$ are constants to be determined. The solution of Eq. (7) is given as:

For $b=c,\ a=0$,

$$\tan(\frac{\phi}{2})=b\xi+c_1-p. \tag{8}$$

For $b=c,\ a\neq 0$,

$$\tan(\frac{\phi}{2})=c_1\exp(a\xi)-\frac{b}{a}. \tag{9}$$

For $b\neq c,\ \Delta=a^2+b^2-c^2>0$,

$$\tan(\frac{\phi}{2})=\frac{2}{b-c}\frac{c_1r_1\exp(r_1\xi)+c_2r_2\exp(r_2\xi)}{c_1\exp(r_1\xi)+c_2\exp(r_2\xi)}-p. \tag{10}$$

For $b\neq c,\ \Delta=a^2+b^2-c^2=0$,

$$\tan(\frac{\phi}{2})=\frac{a}{b-c}+\frac{2}{b-c}\frac{c_2}{c_1+c_2\xi}. \tag{11}$$

For $b\neq c,\ \Delta=a^2+b^2-c^2<0$,

$$\tan(\frac{\phi}{2})=\frac{a}{b-c}+\frac{\sqrt{-\Delta}}{b-c}\frac{-c_1\sin(\frac{\sqrt{-\Delta}}{2}\xi)+c_2\cos(\frac{\sqrt{-\Delta}}{2}\xi)}{c_1\cos(\frac{\sqrt{-\Delta}}{2}\xi)+c_2\sin(\frac{\sqrt{-\Delta}}{2}\xi)}, \tag{12}$$

where c_1 and c_2 are arbitrary constants, $r_1=(a+p(b-c)+\sqrt{\Delta})/2$ and $r_2=(a+p(b-c)-\sqrt{\Delta})/2$. Substituting Eq. (6) into Eq. (5) and by balancing the highest order derivatives and nonlinear terms appearing in Eq. (5), the value of m can be computed. Collecting the coefficients of $(p+\tan(\frac{\phi}{2}))^k$, $(p+\tan(\frac{\phi}{2}))^{-k}(k=0,1,2,...)$, we have system of algebraic equations. Solving the system with the aid of the Mathematica, the values of A_0 , A_k , $B_k(k=1,2,...,m)$, a , b , c and p are computed.

4. Application

Conformable space-time fractional Konopelchenko-Dubrovsky equation is given in the following from [18]

$$T_t^{\alpha}u+T_x^{\beta}T_x^{\beta}T_x^{\beta}u-6\lambda_2uT_x^{\beta}u+\frac{3}{2}\lambda_1^2u^2T_x^{\beta}u-3T_y^{\theta}v+3\lambda_1vT_x^{\beta}u=0, \tag{13}$$

$$T_y^{\theta}u=T_x^{\beta}v,\ 0<\alpha\le 1,\ 0<\beta\le 1. \tag{14}$$

Let us consider the following transformation

$$u(x,y,t)=U(\xi),\ v(x,y,t)=V(\xi),\ \xi=k\frac{t^{\alpha}}{\alpha}+m\frac{x^{\beta}}{\beta}+n\frac{y^{\theta}}{\theta}, \tag{15}$$

where k, m, n are constants. Substituting (15) into Eqs.(13)-(14), we obtain the following differential equations

$$kU'-m^3U'''-6\lambda_2 mUU'+\frac{3}{2}\lambda_1^2 mU^2U'-3nV'+3\lambda_1 mU'V=0, \tag{16}$$

$$n\,U'=m\,V'. \tag{17}$$

Integrating of Eqs.(16)-(17) with zero constant of integration and eliminating V, we have

$$(k-\frac{3n^2}{m})U+(3\lambda_1 n-6\lambda_2 m)\frac{U^2}{2}+\frac{\lambda_1^2 m}{2}U^3-m^3U''=0. \tag{18}$$

Let us suppose that the solution of Eq.(18) can be expressed in the form Eq.(6) for $p=0$.
Substituting Eq.(6) into Eq.(18) and then by balancing the highest order derivative term and nonlinear term in result equation, the value of m can be determined as 1. Therefore, (6) reduces to

$$U(\xi)=A_0+A_1\left[\tan\left(\frac{\phi(\xi)}{2}\right)\right]+B_1\left[\tan\left(\frac{\phi(\xi)}{2}\right)\right]^{-1}. \tag{19}$$

Substituting Eq.(19) into (18), collecting all the terms with the same power of $\tan(\frac{\phi}{2})$, we can obtain a set of algebraic equations for the unknowns A_0, A_1, B_1, k, m, n:

$$A_1^3\lambda_1^2m^2-A_1b^2m^4+2A_1bcm^4-A_1c^2m^4=0,$$

$$3A_0A_1^2\lambda_1^2m^2-6A_1^2\lambda_2m^2+3aA_1bm^4-3aA_1cm^4+3A_1^2\lambda_1mn=0,$$

$$-2a^2A_1m^4+3A_0^2A_1\lambda_1^2m^2+6A_0A_1\lambda_1mn-12\lambda_2A_0A_1m^2+3B_1A_1^2\lambda_1^2m^2$$
$$+A_1b^2m^4-A_1c^2m^4+2kA_1m-6A_1n^2=0,$$

$$2A_0km-6A_0^2\lambda_2m^2-6A_0n^2+A_0^3\lambda_1^2m^2-aA_1bm^4-aA_1cm^4+abB_1m^4$$
$$-aB_1cm^4-12A_1B_1\lambda_2m^2+3A_0^2\lambda_1mn+6A_0A_1B_1\lambda_1^2m^2+6A_1B_1\lambda_1mn=0,$$

$$-2a^2B_1m^4+3A_0^2B_1\lambda_1^2m^2+6A_0B_1\lambda_1mn-12\lambda_2A_0B_1m^2+b^2B_1m^4$$
$$+3A_1B_1^2\lambda_1^2m^2-B_1c^2m^4+2kB_1m-6B_1n^2=0,$$

$$3A_0B_1^2\lambda_1^2m^2-6B_1^2\lambda_2m^2-3abB_1m^4-3aB_1cm^4+3B_1^2\lambda_1mn=0,$$

$$-b^2B_1m^4-2bB_1cm^4+B_1^3\lambda_1^2m^2-B_1c^2m^4=0.$$

Solving the algebraic equations in the Mathematica, we obtain the following set of solutions:

Case 1: $A_0=\pm\frac{an}{2\lambda_2}$, $A_1=\mp\frac{(b-c)n}{2\lambda_2}$, $B_1=0$, $m=\frac{\lambda_1 n}{2\lambda_2}$,

$$k=\frac{96\lambda_2^4n-(a^2+b^2-c^2)\lambda_1^4n^3}{16\lambda_1\lambda_2^3}$$

For $b=c$ and $a\neq 0$,

$$U_{1,2}(\xi)=\pm\frac{an}{2\lambda_2}. \tag{20}$$

For $\Delta>0$ and $b\neq c$,

$$U_{3,4}(\xi)=\pm\frac{an}{2\lambda_2}\mp\frac{n}{\lambda_2}\Big[\frac{c_1\frac{a+\sqrt{\Delta}}{2}\exp(\frac{a+\sqrt{\Delta}}{2}\xi)+c_2\frac{a-\sqrt{\Delta}}{2}\exp(\frac{a-\sqrt{\Delta}}{2}\xi)}{c_1\exp(\frac{a+\sqrt{\Delta}}{2}\xi)+c_2\exp(\frac{a-\sqrt{\Delta}}{2}\xi)}\Big]. \tag{21}$$

For $\Delta=0$ and $b\neq c$,

$$U_{5,6}(\xi)=\pm\frac{an}{2\lambda_2}\mp\frac{n}{2\lambda_2}\Big[a+\frac{2c_2}{c_1+c_2\xi}\Big]. \tag{22}$$

For $\Delta<0$ and $b\neq c$,

$$U_{7,8}(\xi)=\pm\frac{an}{2\lambda_2}\mp\frac{n}{2\lambda_2}\Big[a+\sqrt{-\Delta}\frac{-c_1\sin(\frac{\sqrt{-\Delta}}{2}\xi)+c_2\cos(\frac{\sqrt{-\Delta}}{2}\xi)}{c_1\cos(\frac{\sqrt{-\Delta}}{2}\xi)+c_2\sin(\frac{\sqrt{-\Delta}}{2}\xi)})\Big]. \tag{23}$$

Case 2: $A_0=\pm\frac{an}{2\lambda_2}$, $A_0=\pm\frac{an}{2\lambda_2}$ A_1=0, $B_1=\pm\frac{(b+c)n}{2\lambda_2}$, $m=\frac{\lambda_1 n}{2\lambda_2}$,

$$k=\frac{96\lambda_2^4 n-(a^2+b^2-c^2)\lambda_1^4 n^3}{16\lambda_1\lambda_2^3}$$

For $b=c$ and $a=0$,

$$U_{9,10}(\xi)=\pm\frac{bn}{\lambda_2}\big[b\xi+c_1\big]^{-1}. \tag{24}$$

For $b=c$ and $a\neq 0$,

$$U_{11,12}(\xi)=\pm\frac{an}{2\lambda_2}\pm\frac{bn}{\lambda_2}\Big[c_1\exp(a\xi)-\frac{b}{a}\Big]^{-1}. \tag{25}$$

For $\Delta>0$ and $b\neq c$,

$$U_{13,14}(\xi)=\pm\frac{an}{2\lambda_2}\pm\frac{(b+c)n}{4\lambda_2}\Big[\frac{1}{b-c}\frac{c_1\frac{a+\sqrt{\Delta}}{2}\exp(\frac{a+\sqrt{\Delta}}{2}\xi)+c_2\frac{a-\sqrt{\Delta}}{2}\exp(\frac{a-\sqrt{\Delta}}{2}\xi)}{c_1\exp(\frac{a+\sqrt{\Delta}}{2}\xi)+c_2\exp(\frac{a-\sqrt{\Delta}}{2}\xi)}\Big]. \tag{26}$$

For $\Delta=0$ and $b\neq c$,

$$U_{15,16}(\xi)=\pm\frac{an}{2\lambda_2}\pm\frac{(b+c)n}{2\lambda_2}\Big[\frac{a}{b-c}+\frac{2}{b-c}\frac{c_2}{c_1+c_2\xi}\Big]^{-1}. \tag{27}$$

For $\Delta<0$ and $b\neq c$,

$$U_{17,18}(\xi)=\pm\frac{an}{2\lambda_2}\pm\frac{(b+c)n}{2\lambda_2}\Big[\frac{a}{b-c}+\frac{\sqrt{-\Delta}}{b-c}\frac{-c_1\sin(\frac{\sqrt{-\Delta}}{2}\xi)+c_2\cos(\frac{\sqrt{-\Delta}}{2}\xi)}{c_1\cos(\frac{\sqrt{-\Delta}}{2}\xi)+c_2\sin(\frac{\sqrt{-\Delta}}{2}\xi)}\Big]^{-1}. \tag{28}$$

Here $\xi=(\frac{96\lambda_2^4 n-(a^2+b^2-c^2)\lambda_1^4 n^3}{16\lambda_1\lambda_2^3}\frac{t^\alpha}{\alpha}+\frac{\lambda_1 n}{2\lambda_2}\frac{x^\beta}{\beta}+n\frac{x^\theta}{\theta})$.

Figs.1 and 2 show 3D and 2D plots of the king wave solution $u_4(x,0.25,t)$ and $u_4(x,0.25,1)$ in (21) for $\alpha=0.75,\ \beta=1,\ \theta=0.5$, $\lambda_1=0.25,\ \lambda_2=0.2$, $n=1$, $a=3,\ b=2,\ c=1,\ c_1=2,\ c_2=1$, respectively.

Figs.3 and 4 show 3D and 2D plots of the periodic wave solution $u_8(x,1,t)$ and $u_8(x,1,1)$ in Eq.(23) for $\alpha=0.5, \beta=1, \theta=0.5$, $\lambda_1=0.25, \lambda_2=0.2$, $n=-1$, $a=0.1, b=0.2, c=0.5, c_1=2, c_2=1$, respectively.

Figs.5 and 6 show 3D and 2D plots of the solitary wave solution $a=0.1, b=0.2, c=0.5, c_1=2, c_2=1$ $u_{12}(x,1,t)$ and $u_{12}(x,1,1)$ in Eq. (25) for $\alpha=0.5, \beta=1, \theta=0.5$, $\lambda_1=0.25, \lambda_2=0.2$, $n=-2$, $a=3, b=1, c=1, c_1=1, c_2=2$, respectively.

Figs.7 and 8 show 3D and 2D plots of the periodic wave solution $u_{18}(x,1,t)$ and $u_{18}(x,1,1)$ in Eq.(28) for $\alpha=0.5, \beta=1, \theta=0.5$, $\lambda_1=0.25, \lambda_2=0.2$, $n=-2$, $a=0.1, b=0.2, c=0.5, c_1=2, c_2=1$, respectively.

5. Conclusion

In this paper, the conformable space-time fractional coupled KD equations have been solved by using the simplified $\tan(\frac{\phi(\xi)}{2})$ -expansion method (SITEM) and new exact traveling wave solutions containing hyperbolic, trigonometric, exponential and rational functions have been obtained. Note that SITEM has been applied to the Kundu-Eckhaus equation only for the parameter $p=0$ in [19]. In the literature, fractional coupled KD equations with modified Riemann-Liouville and Caputo fractional derivatives have been investigated. In our work, SITEM has been applied to space-time fractional coupled KD equations with conformable fractional derivative.

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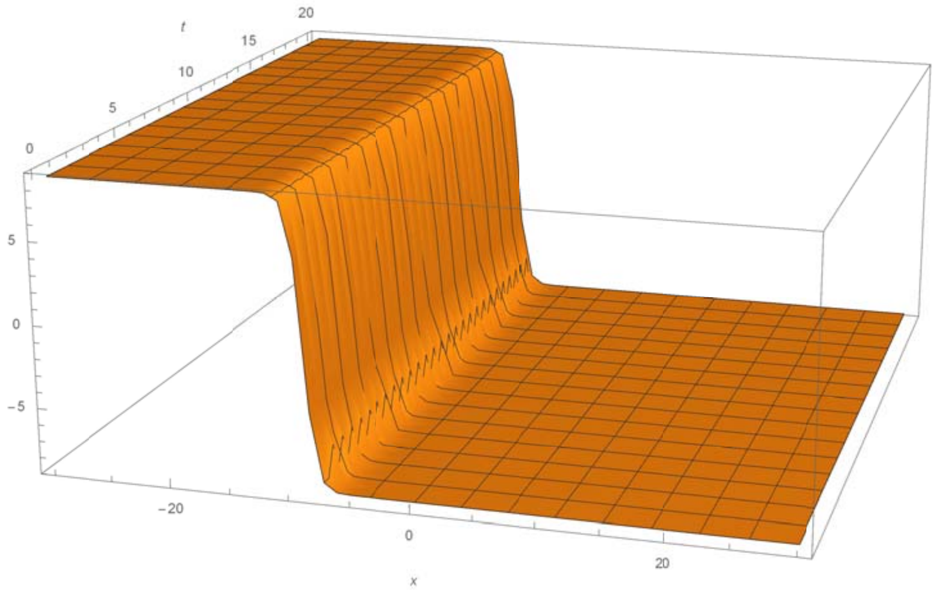

Figure 1: 3D plot of the obtained traveling wave solution $u_4(x, 0.25, t)$ of Eq.(21).

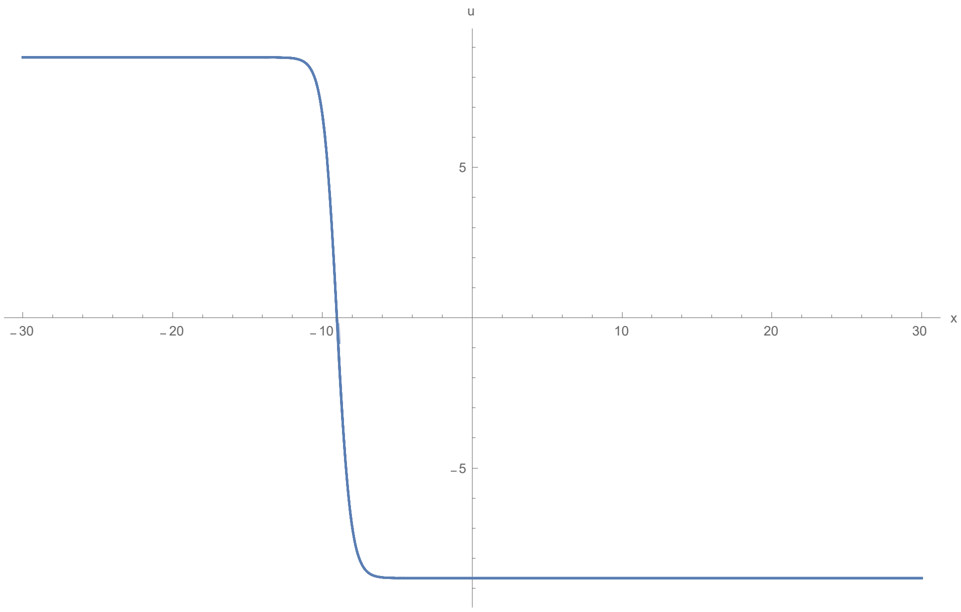

Figure 2: 2D plot of the obtained traveling wave solution $u_4(x, 0.25, 1)$ of Eq.(21).

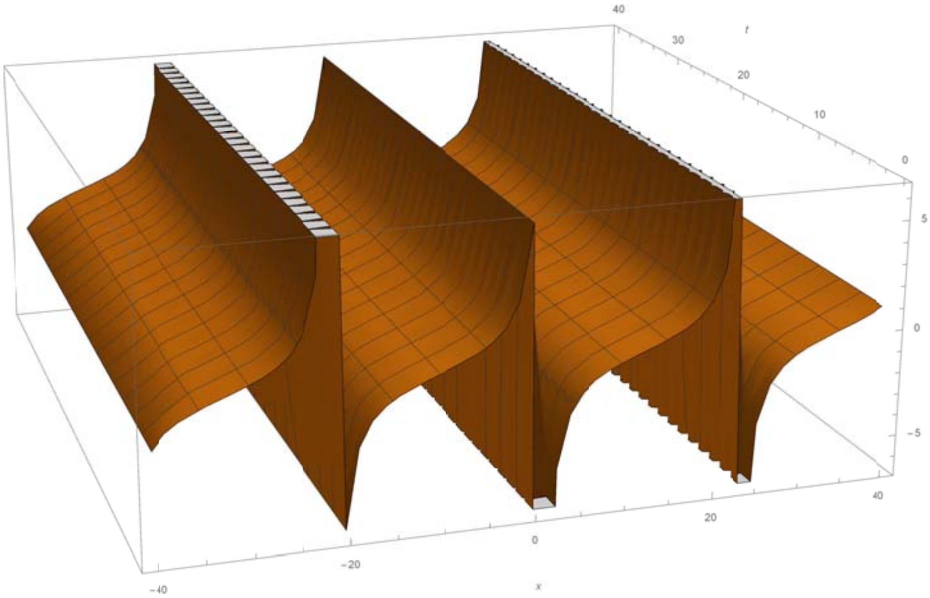

Figure 3: 3D plot of the obtained traveling wave solution $u_8(x,1,t)$ of Eq.(23)

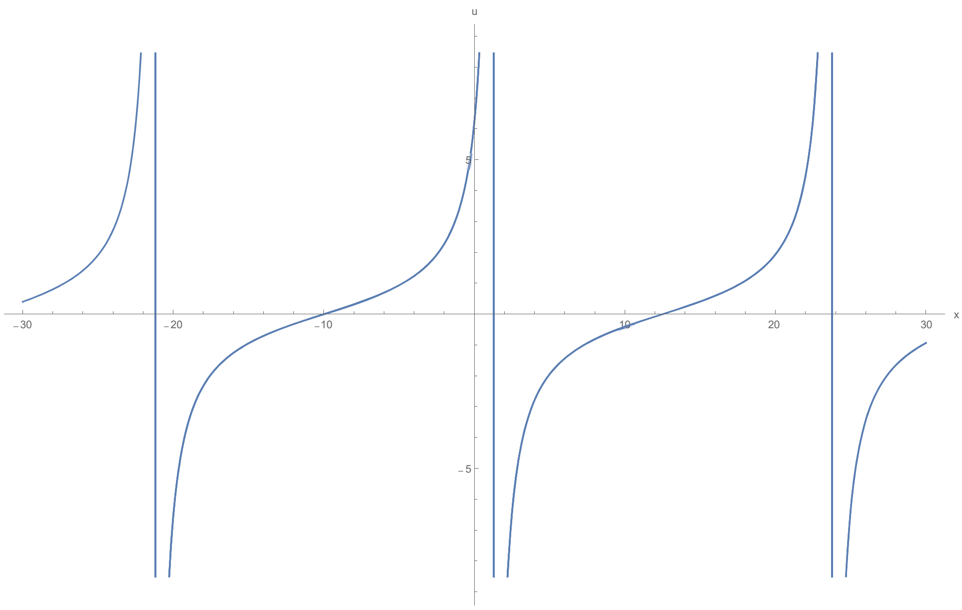

Figure 4: 2D plot of the obtained traveling wave solution $u_8(x,1,1)$ of Eq.(23).

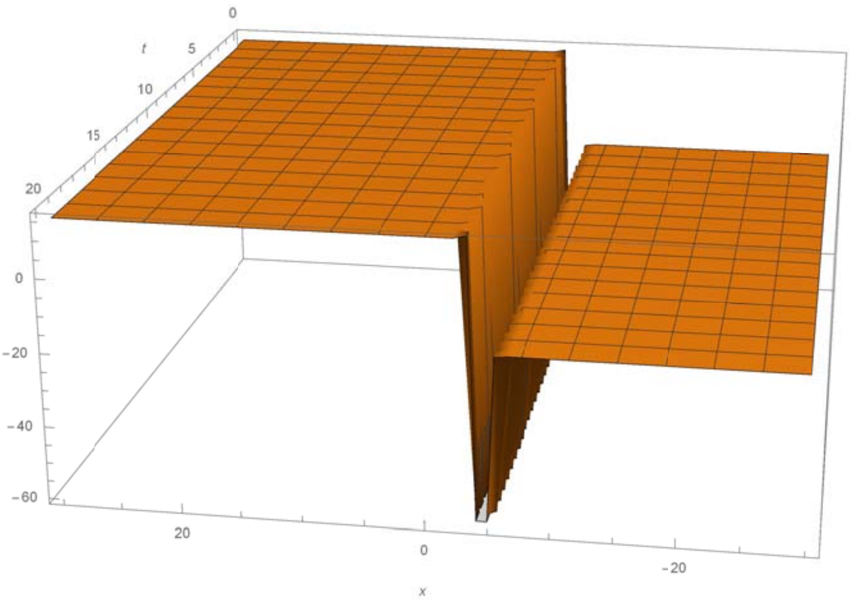

Figure 5: 3D plot of the obtained traveling wave solution $u_{12}\left(x,1,t\right)$ of Eq.(25).

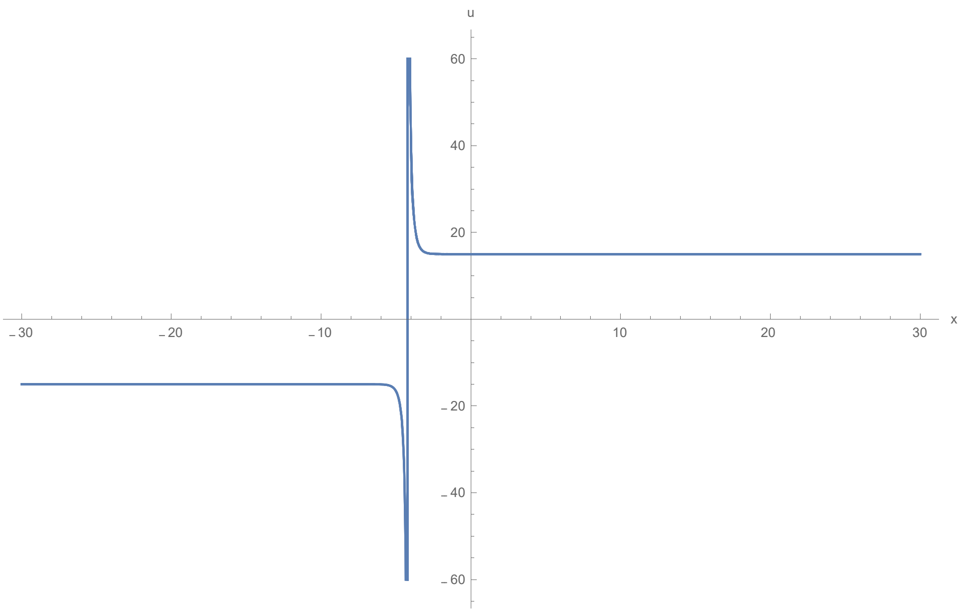

Figure 6: 2D plot of the obtained traveling wave solution $u\left(x,1,1\right)$ of Eq.(25).

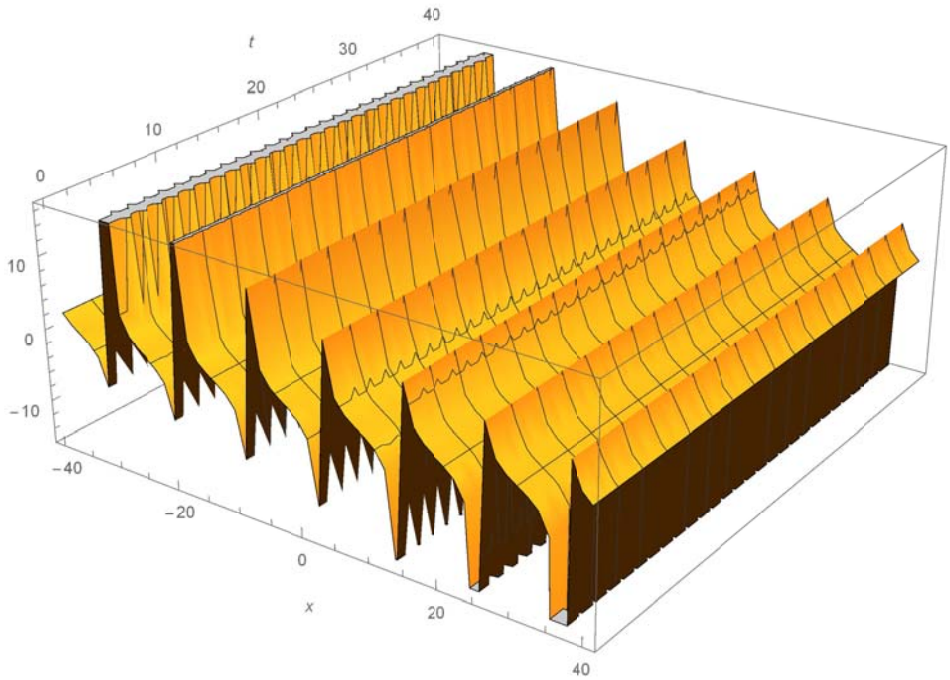

Figure 7: 3D plot of the obtained traveling wave solution $u_{18}(x,1,t)$ of Eq.(28).

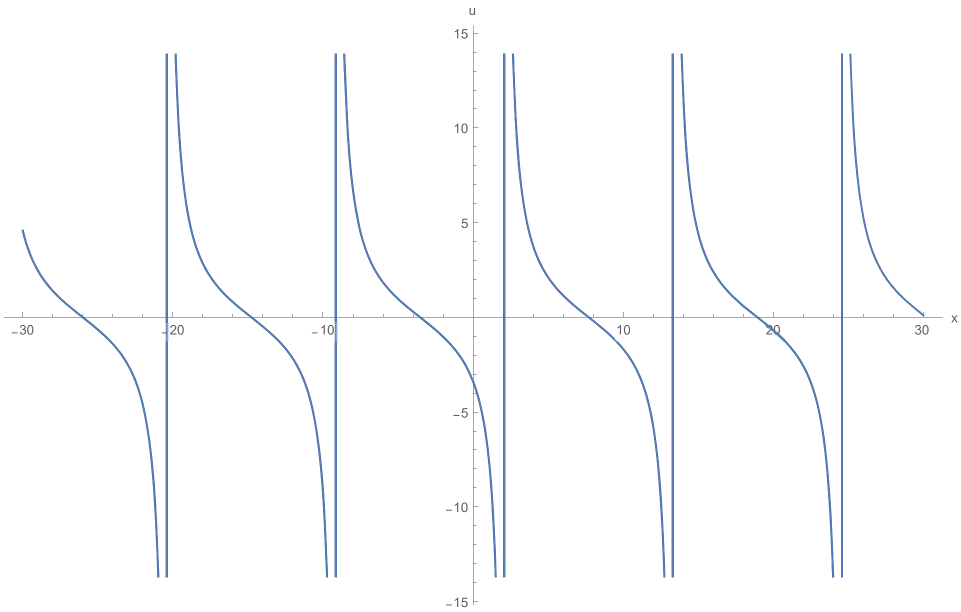

Figure 8: 2D plot of the obtained traveling wave solution $u_{18}(x,1,1)$ of Eq.(28).