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The Role of Quark-Gluon Plasma in the Early Universe

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Abstract — *In an era of the early Universe at a time estimated to be a millionth of a second after the Big Bang, the Universe was filled with quark-gluon plasma. In this plasma and due to the high temperature the strong coupling constant, that characterizes the magnitude of the strong force acting on quarks and gluons, becomes so small. As a consequence quarks and gluons inside this plasma can be considered as an ideal gas of gluons and massless quarks that weakly interact with each others. Thus, for this plasma, one can describe its characteristics by the equations of states that relate both energy density and pressure to its temperature. This has been done in several models in the literature with the recent information about the properties of the quark-gluon plasma provided by relativistic heavy-ion collision experiments and some astrophysical measurement. In this article we review three of these models namely the MIT bag model, Model 1 and Model 2. Moreover, we solve Einstein's field equations of the general relativity,that describe our universe, to show the time evolution of energy density, pressure and temperature in the early universe in these three models. This kind of a study is important as our present universe evolved from a universe filled with quark-gluon plasma.*

Keywords: Quark-Gluon Plasma, Early Universe.
Mathematics Subject Classification: 80C50, 30A40, 90C26.

1 Introduction

Protons, neutrons and other hadrons are composed of quarks. The gluons are the massless particles that carry the strong nuclear force between quarks, or, in the language of modern particle physics, gluons mediate the strong nuclear force between quarks. The widely accepted theory that can describe the strong nuclear force(strong interaction) is known as Quantum Chromodynamics (QCD)[1]. At large momentum transfer or equivalently short distances the QCD coupling constant α_s decreases and thus the quarks and gluons become asymptotically free [2, 3]. This is referred as asymptotic freedom where

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quarks and gluons interact weakly at short distances. Based on this phenomena in the mid-seventies, Quark-Gluon plasma (QGP) was proposed as a new state of nuclear matter [4, 5]. QGP can exist at high temperatures and densities when hadrons break down into their constituents quarks and gluons. On the other hand, it is believed that a millionth of a second after the Big Bang, the universe was filled with QGP. This plasma can be described by the thermodynamic quantities such as energy density, pressure and temperature which vary with time as Universe later cooled down. Studying the time variation of these thermodynamic quantities can be done through using cosmological models based on Einstein. These equations are the fundamental field equations of general relativity.

In particle physics we deal with so tiny numbers such as the masses of the leptons and quarks, the lengths of the radii of proton and neutron and the cross sections of particles interactions. For instances the radius of the proton is $\simeq 10^{-15}m$ and the cross sections are commonly measured in barns where $1b = 10^{-28}m^2$. Clearly units like meter and kilogram that we use in our daily life are not suitable to be used in particle physics and make even simple calculations difficult. As alternative units, the natural units are used. These units are suitable for the cases where the dimensions are so small. These units are based on taking $\hbar = 1$ and $c = 1$. Here $\hbar = h/2\pi$ with h is Planck constant and c is the speed of light in vacuum. In any system of units the relativistic formula of the total energy E of a particle with mass m and a momentum p is given as

$$E^2 = (pc)^2 + (mc^2)^2 \tag{1}$$

Clearly, all E, pc and mc^2 have units of energy. Since in natural units we take $c = 1$ we find that

$$E^2 = p^2 + m^2 \tag{2}$$

which means that the momentum p and the mass m in natural units will have units of energy. From De Broglie famous relation the wave length λ of a particle of momentum p is given by

$$\lambda = \frac{h}{p} = \frac{2\pi\hbar}{p} \tag{3}$$

and since in natural units we take $\hbar = 1$ we find that length has unit $p^{-1} = E^{-1}$. From speed of light $c = 1$ and unit of length is E^{-1} we can deduce that time has also unit E^{-1} as speed is equal to distance divided by time. So, we finally conclude that in natural units mass, length and time can be expressed in units of E, E^{-1} and E^{-1}, respectively.

In natural units energy can be measured in electron volts (eV) or its multiples such as giga-electron volts (GeV) where ($1GeV = 10^9 eV = 1.6 \times 10^{-10} J$). Thus we can express fundamental quantities such as mass, length and time in terms of GeV, GeV^{-1} and GeV^{-1}, respectively. Other derived physical quantities can be then expressed in terms of powers of GeV. Throughout this paper we will adopt natural units system and use GeV as a unit of energy.

This paper is organized as follows: In section 2 we review the equation of states of the QGP relevant to the study in this work. In section 3 we will review Einstein field equations of the general relativity. Based on this equations, we show the derivation of the differential equation that governs the time evolution of the energy density in the early universe when it was in a QGP phase. Finally in section 4 we present our results for the time evolution of the energy density, pressure and temperature in this era of the early Universe.

2 Equations of state of a Quark-Gluon Plasma

Characteristics of solids, fluids and mixtures of fluids can be described using equations of state. These equations are thermodynamic equations that relate mathematically two or more state functions associated with the matter. For instance, the state functions can be the pressure, temperature, volume or internal energy. In the following, we briefly show the derivation of the equations of state of the quark gluon plasma in the MIT bag model [6]. The derivation is based on the approximation that the hot quark-gluon plasma with energy scale ~ 200 MeV just contains massless u and d quarks with neglected interactions inside the plasma. The degrees of freedom for the gluons (quarks) denoted by N_g (N_q) that constitute the plasma can be calculated as follows:

$$N_g = 8(colour) \times 2(polarizations) = 16 \tag{4}$$

$$N_q = 3(colour) \times 2(flavour) \times 2(spin) = 12 \tag{5}$$

The next step is to derive the energy density corresponding to each degree of freedom for the quarks and the gluons. In the absence of interactions, gluons form perfect relativistic Bose gas. The energy density of this gas can be calculated knowing its temperature T as (D. A. Foga et al. 2010)

$$E_g = \int \frac{d^3k}{(2\pi)^3} \frac{k}{(e^{k/T} - 1)} = \frac{\pi^2 T^4}{30} \tag{6}$$

We move now to the energy densities of the quarks and antiquarks in the quark-gluon plasma. In general, there will be a slight excess of quarks over antiquarks in the QGP created from ordinary atomic nuclei (heavy ions). To account for this excess, one has to introduce a chemical potential μ. The chemical potential is defined as the energy required to add another quark to the plasma at a temperature equals zero. At this temperature, there is no antiquark and so the energy necessary to add it to the plasma is zero and thus one expects that its chemical potential is zero. However, this is not the case and the chemical potential of the antiquarks must be chosen to be $-\mu$. This can be explained as the additional antiquark has the possibility to annihilate one of the quarks that lies at the surface of the Fermi sea and release the energy μ. In terms of the chemical potential and temperature, the energy densities for a quark and an antiquark can be calculated from the relations [7]

$$\begin{aligned} E_q &= \int \frac{d^3k}{(2\pi)^3} \frac{k}{[e^{(k-\frac{1}{3}\mu)/T} + 1]} \\ E_{\bar{q}} &= \int \frac{d^3k}{(2\pi)^3} \frac{k}{[e^{(k+\frac{1}{3}\mu)/T} + 1]} \end{aligned} \tag{7}$$

Upon performing the integration, we get

$$E_q + E_{\bar{q}} = \frac{7\pi^2}{120}T^4 + \frac{\mu^2}{36}T^2 + \frac{\mu^4}{648\pi^2} \tag{8}$$

The total energy density of the QGP, denoted (ε) and the total pressure (p) can be expressed as

$$\begin{aligned} \varepsilon &= \frac{E_k}{V} + \mathcal{B} \\ p &= -\mathcal{B} + \frac{1}{3}\frac{E_k}{V} \end{aligned} \tag{9}$$

where $\mathcal{B}$ is known as the bag constant and E_k is the internal energy that arises from the kinetic energies of the quarks and gluons inside the bag and thus it is given by

$$E_k = (16E_g + 12(Eq + E_{\bar{q}}))V \tag{10}$$

The factors 16 and 12 appear in the previous equations account for the degrees of freedom of gluons, quarks and antiquarks. Recall that we treat the QGP as an ideal gas of negligible rest masses of quarks and thus the pressure in this case is just third the internal energy density which is the reason for the second term in the second line of Eq.(9). The first term in the second line of Eq.(9) accounts for the fact that the external pressure on the bag surface $\mathcal{B}$ should be balanced by internal pressure of equal magnitude in the absence of QGP to keep the bag stable. Finally, we get

$$\begin{aligned} \varepsilon &= \mathcal{B} + 16E_g + 12(Eq + E_{\bar{q}}) \\ p &= -\mathcal{B} + \frac{1}{3}\Big(16E_g + 12(Eq + E_{\bar{q}})\Big). \end{aligned} \tag{11}$$

In this paper, we are interested in studying the baryon-number symmetric quark-gluon plasma which corresponds to $\mu = 0$. Thus, setting $\mu = 0$ in Eq.(8) we finally obtain the energy density and pressure as

$$\begin{aligned} \varepsilon &= \frac{37\pi^2}{30}T^4 + \mathcal{B} \\ p &= \frac{37\pi^2}{90}T^4 - \mathcal{B}. \end{aligned} \tag{12}$$

Upon eliminating the temperature T from the above equations we get:

$$p(\varepsilon) = \frac{1}{3}(\varepsilon - 4\mathcal{B}) \tag{13}$$

In natural units, the length has dimension $energy^{-1}$ and thus ε will have dimension $(energy)^4$ and thus from Eq.(12) we find that T and p will have dimensions $energy$ and $(energy)^4$, respectively.

In Refs.[8, 9] modification of the bag model has been proposed so that the resultant models can describe the lattice QCD data [10]. These modifications include a reduction in the Stephan-Boltzmann constant and an introduction of another temperature dependent term (linear or quadratic) in the pressure and also in the energy density and finally a bag constant term with negative sign. These modifications resulted in two viable simple models that can be regarded as variants of the MIT bag model. These two models can be referred as Model 1 and Model 2. In model 1, the pressure p_1 and energy density ε_1 as functions of temperature T, are given by:

$$p_1 = \frac{\sigma_1}{3} T^4 - AT - \mathcal{B}_1 \tag{14}$$

$$\varepsilon_1 = \sigma_1 T^4 + \mathcal{B}_1 \tag{15}$$

From these two equations we obtain the following relation:

$$p_1[\varepsilon_1(t)] = \frac{1}{3}\left[\varepsilon_1(t) - 4\mathcal{B}_1\right] - A\left[\frac{\varepsilon_1(t) - \mathcal{B}_1}{\sigma_1}\right]^{1/4} \tag{16}$$

with the parameter: $\sigma_1 = 4.73$, $A = 3.94\, T_c{}^3$ and $\mathcal{B}_1 = -2.37\, T_c{}^4$, where T_c is the critical temperature for the QGP. In Model 2 the pressure p_2 and energy density ε_2 as functions of temperature T, are given by:

$$p_2 = \frac{\sigma_2}{3} T^4 - CT^2 - \mathcal{B}_2 \;\text{ and }\; \varepsilon_2 = \sigma_2 T^4 - CT^2 + \mathcal{B}_2 \tag{17}$$

and so:

$$\begin{aligned} p_2[\varepsilon_2(t)] &= \frac{1}{3\sigma_2}\{[\varepsilon_2(t) - 4\mathcal{B}_2] \\ &- C[C + \sqrt{C^2 + 4\sigma_2[\varepsilon_2(t) - \mathcal{B}_2]}]\} \end{aligned} \tag{18}$$

where $\sigma_2 = 13.01$, $C = 6.06\, T_c{}^2$ and $\mathcal{B}_2 = -2.34\, T_c{}^4$ with $T_c = 0.175$ GeV is the critical temperature. In chapter 4 we will use the equations of states presented above in the MIT model, Model 1 and Model 2 to study the time evolution of the energy density, pressure and temperature in the early Universe.

3 A Relativistic cosmological model for the Universe

Studying the Universe as a whole, including its origin, nature and evolution is the subject of cosmology. Mathematical models used to describe the large-scale features of the universe are usually referred as cosmological models. In 1917 Einstein formulated such a model based on Einstein field equations which are the fundamental field equations of general relativity. These field equations can be obtained from the relation [11, 12]

$$R_{\mu\nu} - \frac{1}{2} R\, g_{\mu\nu} = -\kappa\, T_{\mu\nu} \tag{19}$$

The indices, μ and ν run from 0 to 3 and hence the previous equation represents a set of 16 equations. It should be noted that due to some symmetries, as we will discuss below in the next subsections, only 10 of these equations are independent. In Eq.(19) κ is a constant, $R_{\mu\nu}$ is the Ricci tensor which depends on the metric tensor and its derivatives, R is the Ricci scalar and $T_{\mu\nu}$ is the energy momentum tensor. The consistency of general relativity and Newtonian gravitation requires $\kappa = 8\pi G$. In the following we discuss the energy momentum and the Ricci tensors relevant to the field equations.

3.1 Energy-momentum tensor

The energy-momentum tensor, denoted by $T^{\mu\nu}$, with μ and ν run from 0 to 3, describes the distribution and flow of energy and momentum in a region of spacetime resulting from the presence and propagation of matter and radiation. This tensor is a symmetric tensor i.e. $T^{\mu\nu} = T^{\nu\mu}$ and has a rank 2. Thus, at any point in spacetime it has sixteen components but due to the symmetry only ten of the sixteen components are independent. These independent components are $T^{00}, T^{11}, T^{22}, T^{33}, T^{01}, T^{02}, T^{03}, T^{12}, T^{13}, T^{23}$. Each component of $T^{\mu\nu}$ can be measured in units of energy density. Thus, in the international system of units, SI, it is measured in Jm^{-3} units while in the natural units it is measured in GeV^4 units. In the following, we give description of each component of the energy-momentum tensor $T^{\mu\nu}$ [12]:

- The component T^{00} is the local energy density resulting from the existing masses and energies.
- The component $T_{0i} = T_{i0}$, for $i = 1, 2, 3$, is defined as the rate of flow of energy per unit area at an angle right to the i-direction, divided by the speed of light c. Another but equivalent definition, $T_{0i} = T_{i0}$ is the density of the i-component of momentum multiplied by c.
- The component $T_{ij} = T_{ji}$, for $i, j = 1, 2, 3$, is defined as the rate of flow of the i-component of the momentum per unit area at an angle right to the j-direction.

The quark-gluon plasma is usually treated as an ideal fluid and thus, its corresponding energy-momentum tensor $T^{\mu\nu}$ is given as

$$T_{\mu\nu} = (\varepsilon + p)\, u_\mu u_\nu - p\, g_{\mu\nu} \tag{20}$$

where u_μ, ε and p are the velocity four-vector, energy density and the pressure density of the fluid respectively. The form of $T_{\mu\nu}$, given in the previous equation, can be further reduced upon restricting ourselves to using locally inertial frames with Cartesian coordinates. In this case, the metric $g_{\mu\nu}$ can be represented simply by the Minkowski metric, $\eta_{\mu\nu} = diag.(1, -1, -1, -1)$. In the fluid, we expect that thermal effects can lead to flows of energy and momentum. If we adopt the point of view of an observer in the instantaneous rest frame of the fluid we find that those flows will not contribute to the flow of energy. Because in that frame $u_\mu = (1, 0, 0, 0)$ and thus from Eq.(20) we get $T_{0i} = T_{i0} = 0$ for $i = 1, 2, 3$. Moreover, due to the lack of interactions between the particles in the fluid, we see from Eq.(20) that $T_{ij} = 0$ for $i \neq j$. As a result, we are left with only non-zero components T_{00} and T_{ii} for $i = 1, 2, 3$. The component T_{00} receives contributions from the random thermal motion of the particles inside the fluid and from Eq.(20) is given by $T_{00} = \varepsilon$. On the other hand, due to the thermal motion of the particles inside the fluid, the momentum will be transferred with equal magnitude per unit area per unit time in all directions and from Eq.(20) we have $T_{ii} = p$. Collecting all components together, we can write [12]

$$T_{\mu\nu} = \begin{pmatrix} \varepsilon & 0 & 0 & 0 \\ 0 & p & 0 & 0 \\ 0 & 0 & p & 0 \\ 0 & 0 & 0 & p \end{pmatrix} \tag{21}$$

It should be noted that the energy-momentum conservation is ensured by

$$\partial_\mu T^{\mu\nu} = 0 \tag{22}$$

3.2 Ricci tensor

The possibility that space is flat or curved can be determined through studying the Riemann curvature tensor or shortly the Riemann tensor. This tensor is fundamental to study curved spaces. A space is flat when Riemann tensor vanishes at all points in that space. Contracting the first and last indices on the Riemann tensor $R^\gamma_{\alpha\beta\gamma}$ results in Ricci tensor $R_{\alpha\beta}$ as

$$R_{\alpha\beta} \equiv \sum_\gamma R^\gamma_{\alpha\beta\gamma} \tag{23}$$

The Ricci scalar R, known also as the curvature scalar, is obtained through further contracting of the indices on the Ricci tensor as

$$R \equiv \sum_{\alpha,\beta} g^{\alpha\beta} R_{\alpha\beta} \tag{24}$$

Ricci tensor is symmetric i.e. $R_{\alpha\beta} = R_{\beta\alpha}$. This property can be seen from the definition of the Riemann tensor. This tensor describes the curvature of the space and has rank 4. Thus, it has four indices and can be denoted as $R^\ell ijk$. In four-dimensional spacetime each of these indices can take four values, so it has $4^4 = 256$ components. However, due to some symmetries of the tensor with respect to interchanging its indices, there are just 20 independent components. In three dimensions there are 6 independent components, and in two dimensions only one. In an n-dimensional Riemannian space the Riemann tensor $R^\ell ijk$ is defined as

$$R^\ell ijk = \frac{\partial \Gamma^\ell ik}{\partial x^j} - \frac{\Gamma^\ell ij}{\partial x^k} + \sum_m \Gamma^m ik\, \Gamma^\ell mj - \sum_m \Gamma^m ij\, \Gamma^\ell mk \tag{25}$$

where the quantities $\Gamma^i jk$, for $i, j, k = 1, 2, ..., n$, are known as connection coefficients. There numbers are n^3, however due to the symmetry, they are not all independent. These connection coefficients are important for differentiation in curved space. They can be defined from the differentiation of the basis vectors $\hat{e}$ of the space through [12]

$$\frac{\partial \hat{e}_j}{\partial x^k} = \sum_i \Gamma^i jk\, \hat{e}_i \tag{26}$$

The above equation shows that the connection coefficient $\Gamma^i jk$ simply denotes the component of the rate of the change of the basis vector $\hat{e}_j$ with respect to changes in the coordinate x^k in the direction of basis vector $\hat{e}_i$. The connection coefficient has a direct relation to the metric tensor $g_{\mu\nu}$ via

$$\Gamma^i jk = \frac{1}{2}\sum_{\ell} g^{i\ell}\Big(\frac{\partial g_{\ell k}}{\partial x^j} + \frac{\partial g_{j\ell}}{\partial x^k} - \frac{\partial g_{jk}}{\partial x^\ell}\Big) \tag{27}$$

where the dual metric tensor g^{ij} is the matrix inverse of g_{ij} satisfying

$$\sum_k g^{ik} g_{kj} = \delta^i_j \tag{28}$$

with $\delta^i_j = 1$ for $i = j$ and $\delta^i_j = 0$ for $i \neq j$.

We turn now to discuss the metric tensor $g_{\mu\nu}$ required to the calculations of the connection coefficients necessary for the evaluation of both Riemann and Ricci tensors. Recall that the line element ds^2, the infinitesimal generalization of the spacetime separation of two points, in Minkowski spacetime is given as

$$ds^2 = dt^2 - dx^2 - dy^2 - dz^2 \tag{29}$$

If we denote the points (t, x, y, z) in Minkowski spacetime as $x^\mu = (x^0, x^1, x^2, x^3)$ we can express the previous equation in terms of the Minkowski metric tensor $\eta_{\mu\nu} = diag.(1, -1, -1, -1)$ as

$$ds^2 = \sum_{\mu,\nu=0}^{3} \eta_{\mu\nu}\, dx^\mu\, dx^\nu \tag{30}$$

For later use, we show the expression of ds^2 in spherical coordinate system

$$ds^2 = dt^2 - dr^2 - r^2\, d\theta^2 - r^2\, sin^2\theta\, d\phi^2 \tag{31}$$

where the points in Minkowski spacetime are given as $x^\mu = (x^0, x^1, x^2, x^3) = (t, r, \theta, \phi)$. The coefficients of the Minkowski spacetime metric tensor $\eta_{\mu\nu}$ are constants showing that the Minkowski spacetime of the special relativity is flat. In the curved spacetime, Riemannian space, of general relativity the metric tensor coefficients are functions of the coordinates. Thus, to calculate the line element ds^2 in curved spacetime, we need to replace the metric tensor $\eta_{\mu\nu}$ by a general metric tensor $g_{\mu\nu}$ that has coefficients as functions of the coordinates. So we have

$$ds^2 = \sum_{\mu,\nu=0}^{3} g_{\mu\nu}\, dx^\mu\, dx^\nu \tag{32}$$

Relativistic cosmological models are based on three principals. These principals are the applicability of general relativity to the Universe as a whole, the cosmological principle and Weyl's postulate. The first principal is based on the assumption that all matter and radiation in the universe exist in four-dimensional spacetime that can be described by an appropriate metric tensor $g_{\mu\nu}$. On the other hand, the cosmological principle tells that on an enough large scale and at any time the universe is homogeneous and isotropic. The fact that our universe is not static but expanding is based on observational evidence published by Edwin Hubble in 1929. Following studies have shown that the large-scale motion, sometimes referred as the Hubble flow, is isotropic and thus it can be characterized by a single rate of expansion at any time. This rate of expansion is currently increasing with time indicating that the expansion of the universe is accelerating as confirmed by many recent observations. Observers who move with the Hubble flow are referred as fundamental observers. Only these observers who can find that the Universe around them is isotropic. Weyl's postulate states that in cosmic spacetime there exists a set of privileged fundamental observers whose world-lines form a smooth bundle of time-like geodesics. These geodesics never meet at any event, apart perhaps from an initial singularity in the past and/or a final singularity in the future.

Howard Robertson and Arthur Walker had shown that all cosmological relativistic models that are homogeneous and isotropic can be described by a single spacetime metric. Currently, this metric is referred as the Robertson-Walker metric and can be obtained from ds^2 given in terms of co-moving coordinates r, θ and ϕ as [13, 14, 15, 16]

$$ds^2 = dt^2 - a^2(t)\Big[\frac{dr^2}{1-Kr^2} + r^2(d\theta^2 + sin^2\theta d\phi^2)\Big] \tag{33}$$

The constant K describes the geometry of the spatial section of space time, with closed, flat and open universes corresponding to $K = +1, 0, -1$, respectively. a(t) is the scale factor that gives the information about distance ratios at different times. When a(t) increases with time, the fundamental observers become more widely separated with time. As a consequence, the galaxies containing those fundamental observers get further apart indicating that the universe is expanding. Contrarily, when a(t) decreases with time, the fundamental observers and their associated galaxies come close to each others, and the universe may be said to be contracting. Comparing Eqs.(31,33) we can read off the the Robertson-Walker metric tensor $g_{\mu\nu}$

$$g_{\mu\nu} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & \frac{a^2(t)}{1-Kr^2} & 0 & 0 \\ 0 & 0 & a^2(t) & 0 \\ 0 & 0 & 0 & a^2(t) \end{pmatrix} \tag{34}$$

In this metric, t represents the cosmic time that can be related to the proper time measured by any fundamental observer. The rate of the change of the proper distance d_p, the distance between two fundamental observers or their galaxies, with respect to cosmic time define the so-called proper radial velocity v_p. In terms of v_p and d_p the Hubble parameter $H(t)$ is given as

$$H(t) = \frac{v_p}{d_p} \tag{35}$$

$H(t)$ can be defined from the scale factor $a(t)$ as follows

$$H \equiv H(t) = \frac{1}{a(t)}\frac{d\,a(t)}{dt} \equiv \frac{\dot{a}}{a} \tag{36}$$

Positive values of v_p indicate that fundamental observers or their galaxies are moving away from each others while negative values of v_p indicating that they are coming toward each others.

3.3 Energy density evolution equation

Using the metric tensor $g_{\mu\nu}$ given in Eq.(34) one can proceed to calculate the connection coefficients using Eq.(27) and then evaluate Riemann tensor in Eq.(25). This will allow to calculate both Ricci tensor and Ricci scalar with the help of Eqs.(23,24). The result is [14]

$$R_{00} = \frac{3\ddot{a}}{a}, \tag{37}$$

$$R^i_j = \Big(\frac{\ddot{a}}{a} + \frac{2\dot{a}^2}{a^2} + \frac{2K}{a^2}\Big)\delta^i_j, \tag{38}$$

$$R = 6\Big(\frac{\ddot{a}}{a} + \frac{\dot{a}^2}{a^2} + \frac{K}{a^2}\Big), \tag{39}$$

here $\ddot{a}$ means a derivative with respect to t. After substituting Ricci tensor components given in Eqs.(37,38), Ricci scalar given in Eq.(39) and the energy-momentum tensor given in Eq.(21) in the Einstein field equations given in Eq.(19) we obtain the following two differential equations

$$H^2 = \Big(\frac{\dot{a}}{a}\Big)^2 = \frac{8\pi G\varepsilon}{3} - \frac{K}{a^2}, \tag{40}$$

$$\dot{H} = -4\pi G(p+\varepsilon) + \frac{K}{a^2}, \tag{41}$$

The energy momentum tensor is conserved by virtue of the Bianchi identities, prompting the continuity equation[49]

$$\dot{\varepsilon} + 3H(\varepsilon + p) = 0 \tag{42}$$

Recent measurements showed that the Universe is flat and hence K = 0 and thus Eq.(40) and Eq.(42) result in

$$-\frac{d\varepsilon}{3\sqrt{\varepsilon}\,(\varepsilon+p)} = \sqrt{\frac{8\pi G}{3}}dt \tag{43}$$

which allows us to find the time evolution of the energy density ε once we know the pressure p as a function of ε.

4 Results and discussion

In this chapter, we present our results for the time evolution of the total energy density (ε), pressure (p) and temperature (T) in the early universe in the MIT model, Model 1 and Model 2. To do this, we substitute the $p(\varepsilon)$, $p_1(\varepsilon_1)$ and $p_2(\varepsilon_2)$ given in Eq.(13), Eq.(16) and Eq.(18) in Eq.(43) and solve this differential equation numerically to obtain the time evolution of ε, ε_1 and ε_2 which represent the time variation of the total energy density in the MIT model, Model 1 and Model 2, respectively. Next step is to use the two equations of states of ε to calculate the time evolution of the temperature T and the pressure p. In our analysis, we use the following initial conditions [17]:

$$\varepsilon_i(t_i) = 10^7\,\mathrm{MeV/fm^3} \quad \text{at} \quad t_i = 10^{-9}\ s \tag{44}$$

and run the evolution from the time of the electroweak phase transition, $t_i = 10^{-9}\ s$, to the time of the QCD phase transition, $t_f = 10^{-4}\ s$.

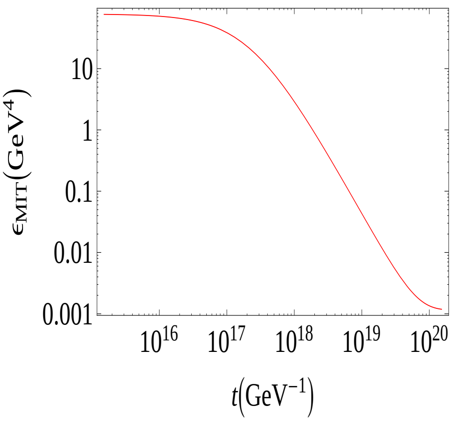

Figure 1: Time evolution of the energy density in MIT bag model.

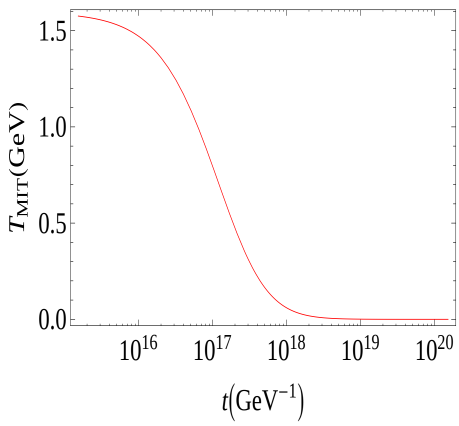

Figure 2: Time evolution of the temperature in MIT bag model.

In Fig.1, Fig.2 and Fig.3 we show our results for the time evolution of the energy density, temperature and pressure, respectively, in the MIT bag model. Recall that, in natural units system energy density has a unit GeV^4 and the time has a GeV^{-1}. In natural units system, also the temperature has a unit GeV and pressure has a unit GeV^4.

In Fig.4, Fig.5 and Fig.6 we present our results for the time evolution of the energy density, temperature and pressure, respectively, in Model 1. Regarding Model 2 we present our results for the time evolution of the energy density, temperature and pressure in Fig.7, Fig.8 and Fig.9.

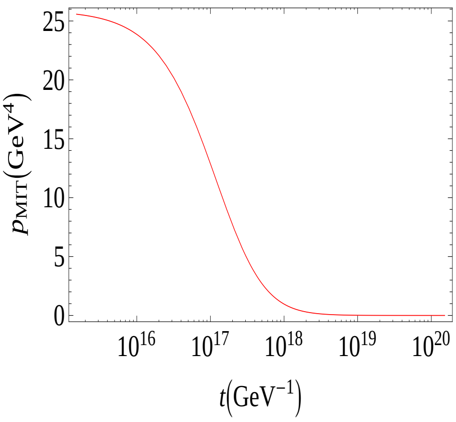

Figure 3: Time evolution of the pressure in MIT bag model.

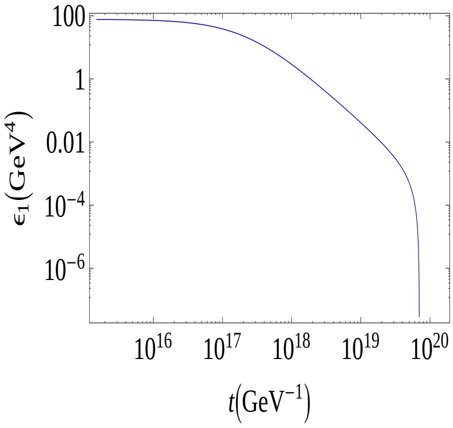

Figure 4: Time evolution of the energy density in Model 1.

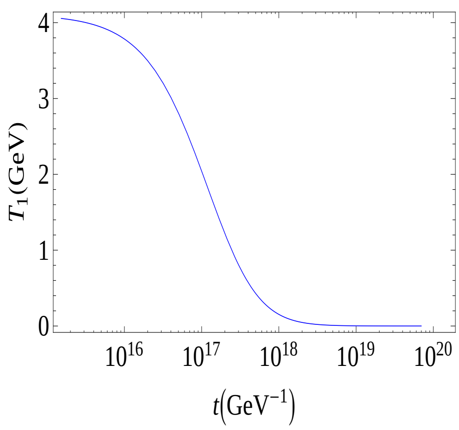

Figure 5: Time evolution of the temperature in Model 1.

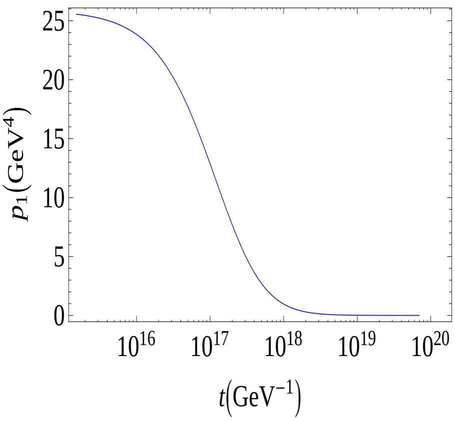

Figure 6: Time evolution of the pressure in Model 1.

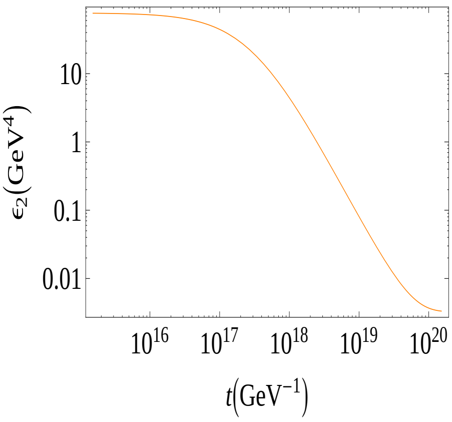

Figure 7: Time evolution of the energy density in Model 2.

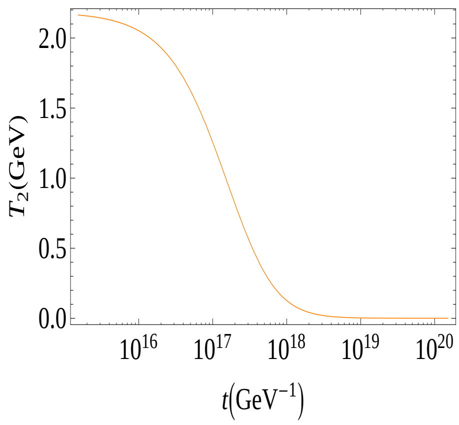

Figure 8: Time evolution of the temperature in Model 2.

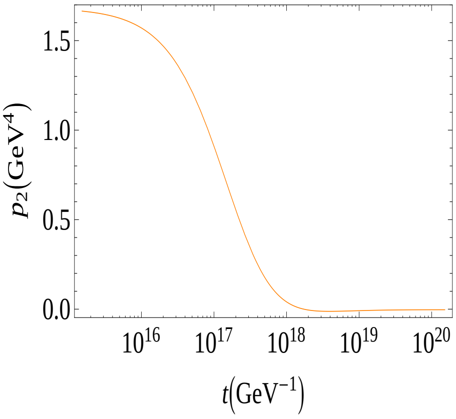

Figure 9: Time evolution of the the pressure in Model 2.

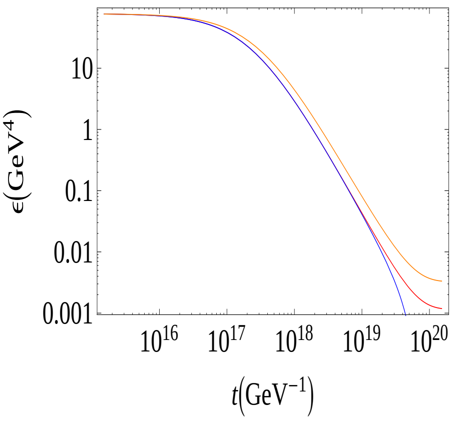

Figure 10: Time evolution of the energy density in MIT bag model (red), Model 1(blue) and Model 2 (orange).

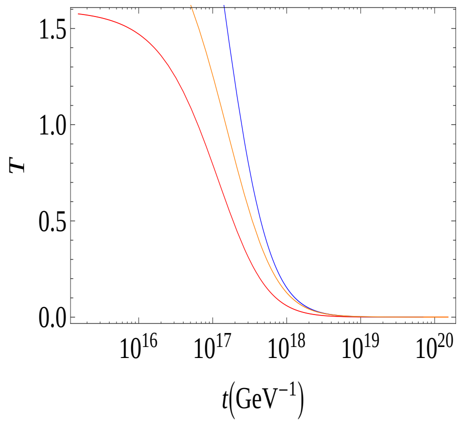

Figure 11: Time evolution of the Temperature in MIT bag model (red), Model 1(blue) and Model 2 (orange).

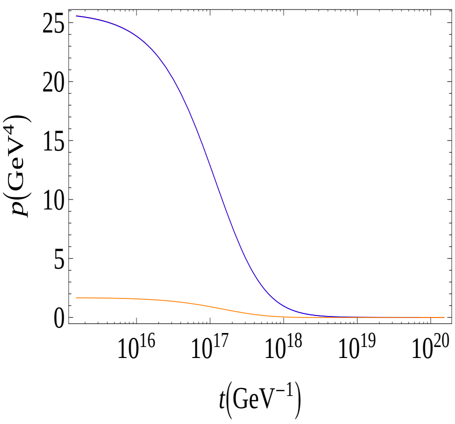

Figure 12: Time evolution of the pressure in MIT bag model (red), Model 1(blue) and Model 2 (orange).

Models 1 and 2 can be thought as new versions of the MIT bag model that are combatable with recent lattice QCD data. As shown in Fig.10 time evolution of the energy density in Model 1 and Model 2 have similar behaviors to time evolution of the energy density in MIT bag model. We note also from the same figure that at most of the time the prediction of time evolution of the energy density in Model 1 coincide with the MIT bag model prediction for the time evolution of the energy density. However, at the end of the chosen time interval, model 1 predicts smaller energy density than that in MIT bag model. Regarding Model 2, and from the same figure, we see that at the beginning of the time interval the prediction of time evolution of the energy density in Model 2 coincide with the MIT bag model prediction. However, after short time, models 2 predicts larger energy density than that in MIT bag model at a given time.

Turning now to the the time evolution of the temperature in the three model, we note from Fig.11 we see that starting from the beginning of the time interval the prediction of the time evolution of the Temperature in Model 1 and Model 2 differ than that in the MIT bag model prediction and at the end of the time interval the three models coincide in their predictions.

We proceed now to consider the time evolution of the pressure. We see that from Fig.12 the prediction of Model 1 coincides with that in the MIT bag model. Moreover, both models predict larger pressure than that predicted in Model 2 at the beginning of the time interval. Starting from the middle of the time interval the difference in the predictions of the time evolution of the the pressure is small in the three models and with increasing time the predictions coincide in the three models.

Finally we discuss the difference between the study considered in this work and a similar study in Ref.[17]. First, in this work, we showed the time evolution of the pressure in the three models that was not discussed in Ref.[17]. Second we showed our results for the time evolution of the energy density in the three models in natural units system which is different than corresponding results shown in Ref.[17] where they have used different system of units.

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