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MATHEMATICS TEACHER TRAINEES' SKILLS AND OPINIONS ON SOLVING NON-ROUTINE MATHEMATICAL PROBLEMS

MATEMATİK ÖĞRETMENİ ADAYLARININ RUTİN OLMAYAN MATEMATİKSEL PROBLEMLERİ ÇÖZME BECERİLERİ VE BU KONUDAKİ DÜŞÜNCELERİ

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ABSTRACT
Problem solving is one of the most important subjects for Mathematics Educators. The subject of this study is problem solving and the non-routine mathematical problem solving competences and opinions on problem solving of mathematics teacher trainees. The study was carried out with 61 mathematics teacher trainees. The study group was given problem solving instruction for 4 hours a week throughout 8 weeks. Pre, post, and retention tests were conducted and participants' ideas on problem solving were determined. Statistical analysis of the study revealed that the instruction increased the trainees' success of problem solving at different levels and that simplifying the problem, looking for a pattern, reasoning, writing a diagram, making a systematic list, guessing and checking, and working backwards, respectively were affected the most. In addition to the separation of successful and unsuccessful participants, it was observed that the strategies of reasoning, working backwards, writing a diagram, making a table and simplifying the problem, respectively had a big impact. The analysis also confirmed that 80% of the problem solving success could be explained by the problem solving strategies. Teacher trainees stated that the study widened their perspectives, developed their self confidence, presented them with new ideas on how to study systematically, and, thanks to the study, they also recognized that there might be a mathematical order even in complex events.

Key words: Problem solving, non-routine problems, beliefs about problem solving, teacher training.

ÖZ
Bu çalışmanın amacı, matematik öğretmen adaylarının rutin olmayan matematiksel problemleri çözme becerilerini ve bu tür problemler ile bunları çözmede kullanılan stratejilere ilişkin düşüncelerini incelemektir. Matematik öğretmeni adayı olan ve 61 öğrenciden oluşan çalışma grubuna haftada 4 saat olmak üzere ve toplam 7 hafta süre ile problem çözme öğretimi dersleri verilmiştir; ön test, son test ve kalıcılık testi uygulanmıştır; öğrencilerin problem çözme konusundaki düşünceleri tespit edilmiştir. İstatistiksel analizler, stratejilerin öğretilmesinde yapılan öğretimin farklı düzeylerde etkili olduğunu ve sırayla problemi basitleştirme, örüntü arama, muhakeme etme, diyagram çizme, sistematik liste yapma, tahmin ve kontrol, geriye doğru çalışma stratejilerinin çok etkilendiğini ortaya koymuştur. Ayrıca, problem çözmede başarılı-başarısız ayırımı yapmada sırayla muhakeme etme, geriye doğru çalışma, diyagram çizme, tablo yapma ve problemi basitleştirme stratejilerinin güçlü etkiye sahip oldukları görülmüştür. Yapılan regresyon analizi, problem çözme stratejilerinin problem çözme başarısını %80 açıklayabildiğini ortaya koymuştur. Öğretmen adayları; çalışmanın problemlere bakış açılarını ve güven duygusunu geliştirdiğini, sistematik çalışmayı öğrettiğini, çalışma sayesinde karmaşık olayların içinde bile bir matematiksel düzen olduğunu fark ettiklerini belirtmişlerdir.

Anahtar kelimeler: Problem çözme, rutin olmayan problemler, çözüme yönelik inançlar, öğretmen eğitimi.

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INTRODUCTION

The aim of this study is to disclose mathematics teacher trainees' non-routine problem-solving skills, their thoughts on these kinds of problems, and the strategies used when solving them.

In educational reform studies of recent years in such countries as the US, England, Australia, Netherlands, Singapore and South Korea, there is a strong emphasis on acquisition of problem-solving skills, application of those skills on real problems in daily life, and developing positive attitudes towards mathematics (Verschaffel, De Corte, Lasure, Vaerenbergh, Bogaerts & Ratinckx, 1999; Cai, 2003). Teaching problem solving has gained more significance as recent studies on teaching mathematics determine giving mathematical disposition as the ultimate target of mathematics education. Mathematical disposition requires mastery in problem solving strategies and other skills (De Corte, 2004). For such reasons, teaching problem solving has become one of the important education issues in the last 25 years (Santos-Trigo, 1996; De Corte, 2004). On the other hand, studies on teaching mathematics carried out during the same period report that mathematics education at schools is not helpful enough to solve real life problems. Students do not think about the problems and produce solution strategies but just try to find the result with arithmetical calculations (Marschall, 1988; Fitzpatrick, 1994; De Corte, 2004; Nancarrow, 2004), negative attitudes to mathematics prevail, and even the students good at mathematics adopt such attitudes towards learning mathematics (Verschaffel et al., 1999; De Corte, 2004). Countries continuously perform program development studies to combat these problems and enable mathematics education to achieve up-to-date targets. The educationalists and researchers working on program development examine the programs of successful countries and make comparisons to find the content and practices that are congruent with the targets in question (Nancarrow, 2004). In the third international comparison performed by the "Trends in International Mathematics and Science Study" (TIMSS), Singapore and other far-eastern countries had high averages, attaching importance to open-ended discussions and strategy training in problem solving and organizing their programs with a primary focus on problem solving (Cai, 2003; Kaur, 2001), and this attracted the program developers' attention to those countries.

In recent years, teaching problem solving in the classroom has been performed based on Polya's four-stepped model (understanding the problem, choosing a suitable strategy, using it, and evaluating the solution). Even though the order of those steps has not changed in practice, it is true that they are worded differently and some steps, especially the solution evaluation step, are divided within themselves (see: Verschaffel et al., 1999). As a result of discussions on the expectations from problem solving teaching, the problem types studied have undergone some partial changes. Thinking that they would improve metacognitive strategies better, the focus has been on non-routine problems (Santos-Trigo, 1996). In addition to those changes in problem solving, some other changes have also occurred in how mathematics is conceived, and instead of a collection of abstract concepts and items of information to be learnt, it has been thought of as a group of problem solving activities that are based on modeling reality (De Corte, 2004). Parallel to all those developments, the aim of learning mathematics has been gaining a mathematical disposition rather than learning isolated concepts and skills.

The above-mentioned determinations bring up the question of how an effective problem solving instruction and learning environment could be planned and created (Verschaffel et al., 1999). Although there is still not an agreed way of organizing problem solving instruction (Santos-Trigo, 1998), studies have shown that situative learning, social constructivist learning environments, where students can express and share views on individual or collective works and form their own personal views as a result of the interactions, or benefiting from both at the same time, are more effective than the other methods (Verschaffel et al., 1999; Santos-Trigo, 1996). Dealing with the subject matter in context prompts social interaction and situative learning helps learning become an activity to which the equipment and cultural elements around contribute (Verschaffel et al., 1999). Santos-Trigo (1996) reported that the students in Schoenfeld's problem solving classes were working on mathematical issues as responsible members of a group doing mathematics.

The recent reform studies are congruent with all the things mentioned above in terms of both content and methodology. The success of such efforts for change depends on teachers adopting the programs, and it is clear that the efforts will fail if teachers, as the ones to implement the programs, do not adopt them (Enochs, Smith & Huinker, 1999). De Mesquita & Drake (1994) demonstrated that there is a direct relationship between teachers' perception levels of a novelty and the success of that new thing. These determinations suggest that the teachers and teacher candidates who are supposed to have some responsibilities during a reform process should be examined in terms of their knowledge, skills, and beliefs about what the reform requires.

Revealing mathematics teacher trainees' knowledge, skills and opinions about non-routine problems and solution strategies, this study is expected to provide opportunities to comment on the future of the abovementioned reform movement. On the other hand, revealing to what extent social constructivist instruction increases problem solving success at university level, which strategies are learnt harder, and the role the strategies play in explaining the problem solving success, is expected to contribute to the studies on problem solving, which are scanty especially at university level and still decreasing in number (Nancarrow, 2004).

The research questions of this study are as follows:

1. What is the success rate of mathematics teacher trainees in solving non-routine problems? Does problem solving strategies instruction increase students' problem solving success? If it does, what are the roles of the strategies taught in it? (Which strategies are the leading indicators of problem solving skills?)

2. What are the trainees' opinions on the necessity of problem solving strategies instruction and the difficulty levels of the strategies?

The hypothesis of the study was that the teacher trainees were going to be just partly successful in solving non-routine problems and try to solve those by writing equations, which is a result of the traditional education they had had. In addition, the learning atmosphere and the instruction itself were supposed to improve the problem solving skills of the trainees, and there was supposed to be some significant differences between the results of the pretest, posttest and retention test. It was impossible to estimate which strategies were going to be more effective in discriminating between successful and unsuccessful students.

The meanings of the concepts of non-routine problem and problem solving strategies are as follows. *Non-routine Problem* is the problem about which the solving person does not know any ready-made methods. Unlike the routine problems, they require using the knowledge and skills in unusual ways that the solutions required. For that reason, they reveal the best way for the choice of the metacognitive activities in the solution process, planning how to use them and the process of metacognitive control (Nancarrow, 2004). *Problem Solving Strategy* is the original metacognitive activity the students engage in when trying to solve problems. The most frequent solution strategies are *Making Systematic List, Guess and Check, Drawing Diagram, Writing Equation, Looking for Pattern, Making Table, Reasoning and Simplifying the Problem.* Success in solving a problem is directly related to the choice of the appropriate strategy (Cai, 2003). Knowing the problem solving strategies does not guarantee solving the problems correctly, but it enables students to take right and systematic actions, and that increases the possibility of achieving the correct solutions (Verschaffel et al., 19999).

Related Research Studies

There have been many research studies discussing the problem solving teaching and its results.

Verschaffel et al (1999) wrote that the problem solving instruction given to the fourth and fifth grade students helped them solve mathematical application problems and that the students could learn problem solving strategies. Folmer (2000) wrote that in the fourth grade, the instruction on non-routine problems improved the students' use of cognitive strategies and their awareness of how they solved the problems. Pugale (2001) suggested that successful high school students could be distinguished from others in terms of the ways they focus on problems, organize data, make calculations and give meaning to the results. Krutetski reported that in the sixth grade, the students who succeed in solving non-routine problems are the ones who can analyze problems from different perspectives before solving, make syntheses, generalize the solution methods and benefit from solutions to similar problems (Niederer and Irwin, 2001). Pape and Wang (2003) had the result that successful secondary school students are those who are able to determine their goals, make plans, control their own behaviors, organize the places where they study and evaluate themselves with the help of others. De Hoys, Gray and Simpson (2004) analyzed the non-routine problem solving processes of two undergraduate students and reported that the more successful student was the one who focused on developing a method himself according to the qualities of the problem while the other sought for a method to work in the solution only. Nancarrow (2004) examined how the students solved the non-routine algebra problems in a lesson based on a problem solving method designed to sustain the students' heuristics to solve the problems and their creativity. The study, which was conducted with a control group, showed that there is a correlation between the success in solving a problem and the knowledge of the basic concepts and methods about the problem, and that the experimental instruction was useful for improving the students' cognitive strategies.

The Present Study

In program development studies that started in Singapore in 1997, students were encouraged to study in problem solving environments to improve their skills and it produced positive results (Kaur, 2001), which created the need to examine with a new perspective routine and non-routine problems, open-ended discussions and the solving strategies those discussions included.

The effects of problem solving training can be detected more clearly in the course of time (Cai, 2003). Therefore, instead of considering it a subject to be taught within a certain period of time, problem solving should be an issue with which teachers and students should consistently deal at schools. In the study, that fact was taken into account and in the lessons following the experimental period, related problems and contexts requiring the employment of some specific problem solving strategies were used.

This study is similar to the above-mentioned ones in terms of giving problem-solving instruction first and then evaluating the success in solving problems. However, it is different as it explains the roles of the instructed strategies in the success levels, being at university level (a level at which the researches on problem solving is relatively few), for being conducted with teacher trainees, and focusing on the teacher trainees' opinions about problem solving instruction.

METHOD

Participants

The study was conducted in the academic year 2005-2006 with 61 mathematics teacher trainees, of whom 31 were secondary school mathematics teacher trainees while 30 were being trained to be high school mathematics teachers. In the syllabuses of both groups, lessons on mathematics teaching were given within the framework of the course of special teaching methods that were run for two semesters in 4 hours per week.

The Introduction of the Experimental Study

As the rationale of the study was problem solving strategies instruction, what was done first was the review of the problem solving strategies in the literature. In different resources in the taxonomy of the strategies, it was analyzed what kind of differences there were between the strategies in terms of both how they are named and what is understood from the names used. Considering the level of the group the study was done with, nine particular strategies were decided to be taught.

The social constructivist model was taken as the basis when determining the teaching technique. As tested sample studies, it benefited from Verschafell et al.'s (1999) instruction employed in their experimental study on mathematical application problems and Shoenfeld's way of teaching in the problem solving instruction course offered at the university (Santos-Trigo 1998). Due to this approach, the lessons started with works done by all the students as whole, discussions in heterogeneous groups of two or three people followed them and after that, classroom discussions were opened up as the concluding activities. The main function of the instructor (researcher) in the works done in the classroom was just to help the students develop their creative approaches while solving the problems. Due to that, the instructor observed the works carried out in small groups, he did not answer the questions on problem analyses as long as he did not have to. Instead, he asked particular questions to make the students understand the problems and suggested to make discussions with peers to see their relations to some similar problems and situations they had already known. In case of hesitations and pauses, the students were asked such questions as "What is the difficulty you have faced?", "Would it work if you drew a figure about the situation?", "Are your thoughts on special values confirmed?" When they solved any problem correctly, they were induced to evaluate their solutions with questions such as "Can there be another way to solve it and can your solution be generalized?" During all these works, the students were free to have discussions with the instructor and the students in other groups. After completing any solution, they were asked what they had understood from that solution and what kind of strategies the groups used when dealing with the problem. The groups were demanded to defend their own strategies and tell about in what ways they

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would use that strategy in similar or more complicated problems. They were sometimes asked to solve the problems produced afterwards. What was achieved with those studies was that the students had the opportunities to know more about the strategies and their suitability and contributions in solving problems.

Besides, some related supplementary questions were prepared before the studies were given as homework and all the students were asked to solve them in their free time.

After administering the posttest, the students continued the course with the topics included in the syllabus. The retention test was given six weeks after the posttest. Thinking that the effects of problem solving instruction would be seen more clearly with time (Cai, 2003), problem solving was frequently recycled in the school environment and it was not dealt as a topic to be taught within a certain period of time. In accordance with that approach, no time was set aside in that six weeks period for teaching only non-routine problems and problem solving strategies, but there was always an emphasis on using the strategies and finding the correlations for the suitable patterns encountered during the course of the instruction. As examples to it, such questions as "Why is the subgroup number of the n element group 2^n ?", "In how many different ways can a natural number be written as the sum of two or three natural numbers?", "How many diagonals does a polygon with an n side have?" could be cited.

Data Collection Instruments

The instruments used to collect data for the study were two kinds. The first one of them was *the problem solving tests* (Appendix 1). The other was *a likert type scale* consisting of 10 questions, by which the students could determine to what extent they had found the problem solving strategies difficult, and *an open-ended question* aimed at revealing what they had been thinking about problem solving.

At the beginning of the study, it was planned to ask the same questions in pretest, posttest and retention test all but it was not done thinking that new problems and their solutions would arouse interest.

In order to assure the content validity of the problem solving tests, the problem solving units and the problems themselves in the university, course books were analyzed and then each problem solving test was composed of 10 questions, of which each one of them was intended to disclose the students' knowledge and skills about 1 strategy at least. To make the questions parallel to each other equal, all the questions were discussed within a group of three specialists. Furthermore, the questions selected were asked to forty 4th grade students who had taken the mathematics-teaching course at undergraduate level, then it was analyzed whether the texts of the problems were comprehensible and what strategies the problem solutions required. The required corrections were made. The questions in the posttest and retention test were not included in the studies during the instruction and thus, the students encountered them just when they took the tests in question.

In the likert type scale as a part of the second instrument, the teacher trainees were given the strategy names in the first column and in the second column, some sample problems which they had encountered in the tests and some other ones requiring the use of that particular strategy. They were asked to take those problems and the instruction process into account and then choose one of the options of *hard, moderately hard, middle level, moderately easy, easy*. The aim of using that instrument was to disclose the way in which the trainees perceived the levels of learning the strategies.

The open-ended question that formed the other part of the second instrument was *"Would you recommend that this instruction you have been given on non-routine problem solving strategies be given to the students who will be in your position next year? Please support your answer giving reasons"*. It was aimed with this question to learn indirectly the students' thoughts on the issue.

Implementation

The study was carried out within the framework of the "Special Teaching Methods" course, which had been included in the syllabuses of both groups and was given for two terms in 4 hours per week.

The problem solving education continued 8 weeks (the lessons were planned in two different days with two hours each week. The students were given the pretest in the first session. There were two class hours to answer the 10 questions in the test.) All the students got interested in the questions and used two class hours at least to answer them.

The second session of the first week and the second week were allocated for the introduction of the problem solving process. The points dealt during that phase were problems, problem solving, the place of problem solving in mathematics, classification of problems (the differences between oral-real, routine-non-routine ones). The problem solving process was introduced as based on Polya's four-stepped model. The problem selected to zero on the meanings of the concepts in those steps and which explained student behaviors was "drawing in a given triangle a square whose two corners are to be on one side of the triangle while the two other corners are to be on the other sides of the triangle". The groups were given enough time for the solutions and then a classroom discussion started. The phases of the solution were introduced and discussed in detail considering the solution of that problem.

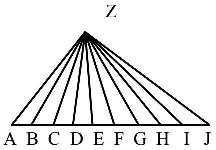
During the hours in the following weeks, two strategies were introduced per week. When a strategy was to be studied, a problem requiring the use of that particular strategy was given without giving any information about it in advance. During the classroom discussion phase, the solution process was associated with the four steps of problem solving. After completing the solution, the students were asked to suggest names for the strategy used. It was discussed to what extent the names were suitable and what the names were in the literature was decided upon.

After completing the strategy training in the way mentioned above, the problems the students had experienced difficulties when solving and the ones they demanded to study afterwards were studied throughout two weeks without focusing on any particular strategy. Finishing the instruction, the posttest was administered, and 6 weeks after that, the retention test was given.

Evaluation of the Results

The questions in the test measuring the problem solving success had a value of 10 points each. Thus, the scores given according to the success in solving the problems varied between 0 and 100. Each answer was categorized under the titles of *correct, little mistakes stemming from calculation errors or inattentiveness, insufficient answers despite understanding the problem and taking the right action, wrong answers* and *no answer*. Two specialists evaluated the answer sheets independently and the scores given were compared to each other. In case of different scores, the sheets were reexamined to determine scores in common. As the main aim of the study was not to improve the calculation skills but to choose among the strategies and problem solving, the answers containing unimportant calculation errors were accepted as correct. In order to reply such a question as “what is the contribution of the problem solving strategies in this success?”, the scorers determined to what extent the students used those strategies in their solutions. The answers of the students slightly varied among each other. Situations of using one instead of another were seen especially between the strategies of *guess and check* and *writing an equation*, and between *working backwards* and *writing an equation*. To find the frequency of any strategy use, each solution in the test papers was examined. When deciding a strategy was used, that strategy was given one point. The following examples could clarify how the strategy points were generated: To the question of “How many triangles do ten dots on a line and one outside produce?”, some students answered:

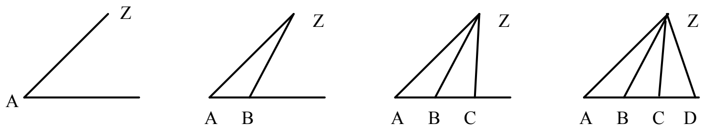
(i) drawing the figure below or similar ones and said;
“as the Z point is found in every triangle, we can ignore it and in that case 45 triangles come out with



9 triangles taking the A point as the corner
8 taking the B point as the corner
……………………………………………
……………………………………………
1 taking the I point as the corner.

Such students were accepted as having used *drawing a diagram* and *looking for a pattern*.

(ii) Some students thought in such a way as the following: *"Let's determine the dot number on the line not as 10 but 1, 2, 3, 4,… and analyze the solutions.*



0 triangle 1 triangle 3 triangle (2+1) 6 triangle (3+2+1)

We can have a solution analyzing the solutions above in a chart:

Number of dots	*Number of triangles*
1	*0*
2	*1*
3	*3*
4	*6*
…	…

"It can be seen in the chart that the numbers of the triangles go up in the way of 1, 2, 3, 4… So when the number of the dots is 10, the number of the triangles becomes 45." Such students were accepted as having used "looking for a pattern", "drawing a diagram", and *"simplifying the problem (benefiting similar problems)*.

(iii*)*The ones who gave such answers as *"Lines make a triangle with any point different from the two points on itself, then the true answer is as many as 10 points' combinations with 2 and the number of the triangles is C(10/2)=10!/8!.2! =45."* were considered to have used formulas they had already known and not the strategies used in this study, and the strategies they followed were noted as *other.*

In order for a strategy to be given +1 point, the questions such as the contribution of that strategy to the solution, and whether or not it was used appropriately were analyzed and a correct and complete solution was not sought. For the reliability levels of the strategy points, the Cronbach alpha coefficients regarding the pretest, posttest and retention test were calculated *0.53, 0.75* and *0.68* respectively.

To understand if there were any significant differences between the problem solving tests scores and students' strategy scores in those tests, variance analysis was used. In order to understand which strategies had stronger effects in discriminating between successful and unsuccessful students, discriminant analysis was used. In order to reveal how functional the strategies were implemented in explaining the problem solving success, multiple regression analysis was used.

The students' responses to the likert type questionnaire items on how difficult learning the strategies was found were scored with 1, 2, 3, 4, and 5. No numerical grades were awarded to the open-ended question on the necessity of such an instruction as the one given in this study, but the points the students had focused on were determined and classified.

FINDINGS

The primary aim of the present study was to investigate the mathematics teacher trainees' knowledge and skills on problem solving strategies, the effects of a related instruction on those skills and knowledge, the roles of problem solving strategies in explaining the success in solving problems. The secondary aim was to find out the students' opinions on the difficulty levels of the strategies and the necessity of such an instruction as the one they received. The findings could be cited as follows:

The Findings about the Strategies

The aim of the first problem solving test was to determine to what extent the students had known and used the strategies before being taught. The posttest was targeted on finding out how and how much the instruction provided affected the perception of the strategies. The retention test was performed with the aim of assessing how much the students could remember about what they had learned. The statistics about the problem solving tests results of the groups can be seen in Table 1:

Table 1: Problem Solving Tests Results of Study Groups

Tests / Groups (n)	Pre-test		Post-test		Retention Test	
	Scale of scores	**$\bar{x}$ (ss)**	**Scale of scores**	**$\bar{x}$ (ss)**	**Scale of scores**	**$\bar{x}$ (ss)**
Elem. School Math. Teacher Trainees (31)	15-60	29.58 (10.56)	42-100	73.51 (11.61)	53-98	77.12 (11.99)
High School Math. Teacher Trainees (30)	19-63	39.16 (11.07)	48-95	72.87 (13.90)	23-98	75.83 (7.49)
General (61)	15-63	33.75 (11.87)				

The average success rates of the subgroups of the study, which were composed of elementary and high school mathematics teacher trainees, were compared to the t test, and a significant difference for the pretest was found (t_{59}= -3.46, $p<0.001$). However, no significant difference was detected between the posttest and retention test averages of the same groups (t_{59}= 0.19, $p<0.84$) t_{59}= 0.46, $p<0.65$). The comparative averages in the pretest, posttest and retention test are given in Figure 1.

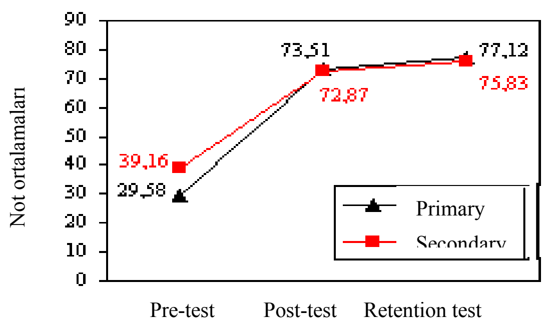


Figure 1. Pretest, Posttest and Retention Test Averages of Study Groups

What the groups in this study had in common was that the members of both were mathematics teacher trainees. The effects of the instruction on the whole group (n=61) were compared to the results attained from the three problem solving tests ($F(2.59)=230.23$, $p<0.001$) and significant differences were found between the pretest and posttest, and the pretest and retention test, which is in accordance with what had been expected**.** Below, explanations are given about how and how much each strategy was learnt and used. Though all the questions had been devised in a way that would prompt the use of the strategies taught, it was impossible to predict precisely what strategies the students were to follow. As no explanations or clues about any strategy use were given in the test papers and during the administration of the tests, the numbers of the strategy uses could be considered the students' original frequencies of behavior. *Writing an equation* and *drawing a diagram* were the most frequent strategies in the pretest for both the groups, and there were no students using the strategies of *making table, looking for pattern* and *working backwards*. The average frequencies of the problem solving strategy uses are given in Table 2:

Table 2. Frequency of Strategy Use

	Average Use number			**Number of the using students**			**Difficulty level**
Problem Solving Strategies	Pretest (1) x (ss)	Posttest (2) x(ss)	Retention Test (3) x(ss)	F	p	Significant Difference	$\bar{x}$ (ss)
Making Systematic List	0.28 (0.45)	1.05 (0.46)	0.96 (0.47)	40.017	.000	1-2, 1-3,	4.33 (3.90)
Guess and Check	0.20 (0.40)	1.15 (0.83)	1.07 (0.53)	38.463	.000	1-2, 1-3,	4.59 (0.84)
Drawing Diagram	0.92 (0.76)	2.49 (0.99)	2.36 (0.86)	54.882	.000	1-2, 1-3,	3.44 (1.12)
Looking for Pattern	0.00 (0.00)	0.96 (0.86)	1.18 (0.64)	59.556	.000	1-2, 1-3,	3.78 (1.19)
Writing Equation	1.48 (0.81)	0.90 (0.98)	0.89 (0.71)	9.765	.002	1-2, 1-3,	4.56 (0.80)
Working Backwards	0.15 (0.36)	0.72 (0.45)	0.64 (0.48)	33.054	.000	1-2, 1-3,	3.70 (1.29)
Making Table	0.03 (0.18)	0.25 (0.47)	0.46 (0.50)	16.190	.011 .000	1-2, 1-3,	1.63 (0.74)
Reasoning	0.07 (0.25)	1.15 (0.77)	0.57 (0.50)	55.707	.000	1-2, 1-3, 2-3,	2.74 (1.09)
Simplifying the Problem	0.00 (0.00)	0.90 (0.72)	1.30 (0.74)	67.043	.000 .011	1-2, 1-3, 2-3	1.85 (0.77)
Other	0.07 (0.25)	0.10 (0.35)	0.41 (0.63)	11.276	.001 .002	1-3, 2-3	-

* $p<0.001$

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The instruction provided in this study brought about the strongest effects on the use of the strategies of simplifying the problem, looking for pattern, reasoning and drawing diagram (See: Table 2). The other detectable effects could be observed on the use of making systematic list, guess and check and working backwards. Affected modestly but significantly yet, the strategy of making tables followed them. In figure 2, changes in the use of each strategy are demonstrated in graphics.

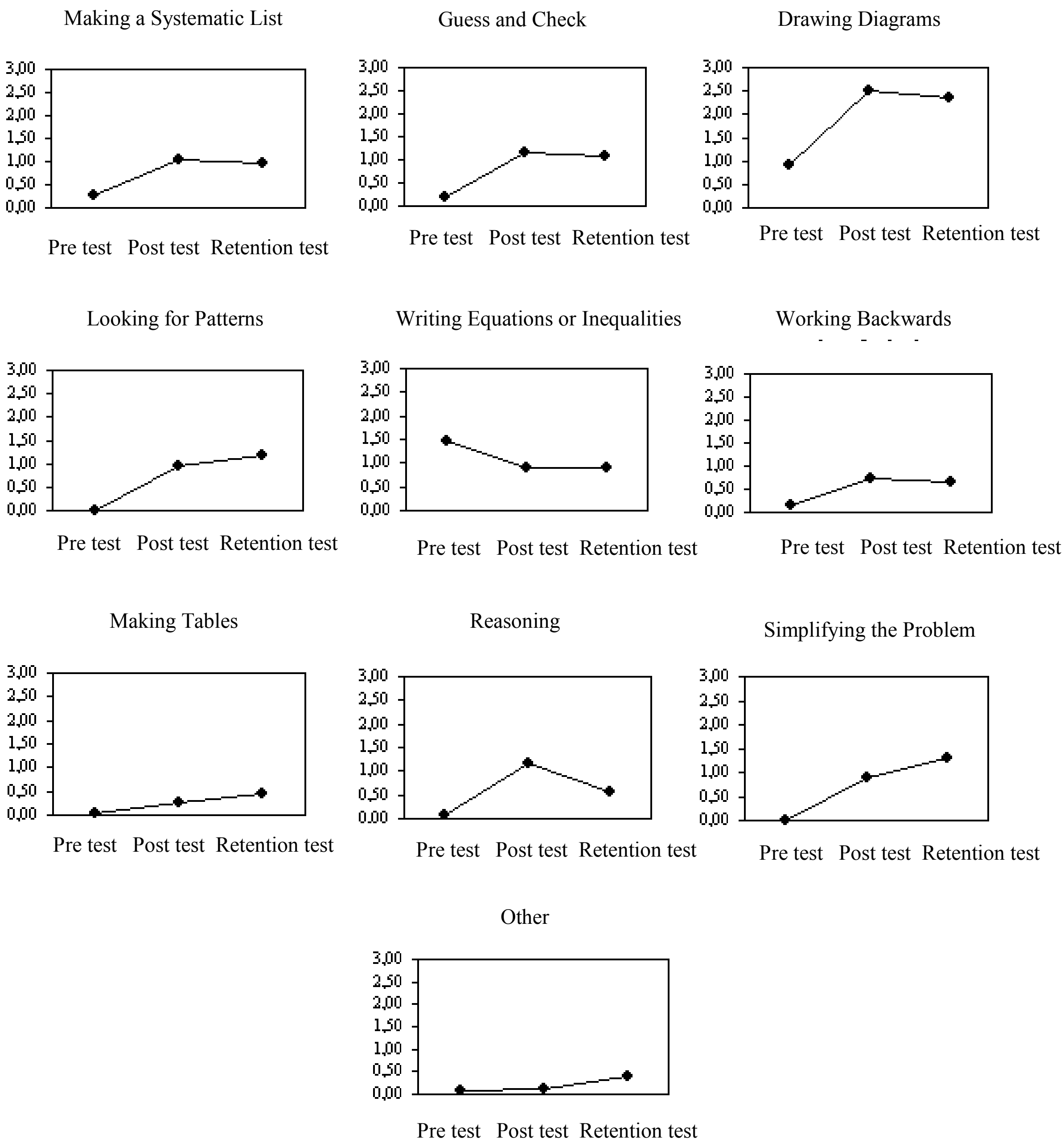

Figure 2. Pretest, Posttest and Retention Test Averages of Problem Solving Strategies for Study Group

The changes observed in the use of the strategies can be summarized up as follows:

Making a Systematic List: In the pretest, the students hardly ever used this strategy when trying to solve the *apple basket* problem, which had been included to give the students the opportunity to use the strategy. However, there was an important increase in the number of the use of the strategy in the posttest and all students employed it to solve the *twenty's disruption* problem, which had been considered parallel to that *apple basket* problem. Despite following the strategy, there were also students who could not achieve the correct solutions.

Guess and Check: At the beginning of the study, this strategy was being used very rarely. In the pretest, the students tended to write equations when solving the problems that could be solved with this strategy. That changed after the instruction, and it was observed that the students used *guess and check* to solve even some problems which were expected to be solved through equations.

Drawing Diagrams: This strategy is one of those used most frequently in both the pretest and the posttest. The students generally seemed intent on using it, but there were ones who could not have the solutions despite drawing the required diagrams as a result of not understanding the problems wholly or making calculation errors.

Looking for Patterns: This is a strategy in which we try to have some final results by seeking relations between the data in hand and the results attained during the solving processes. It works to have the final general result examining the results attained. Achieving a result through specific situations could be a working method, but some specific results might be in conformity with more than one pattern and that might sometimes make solutions lead to wrong results (Santos-Trigo, 1996). All the same, such a method is suitable in terms of the secondary and high school levels. While there was no attempt to use the strategy of looking for patterns in the pretests, an important increase occurred in its use after the instruction.

Writing Equations or Inequalities (Using Unknowns): As the strategy of writing equations is the most frequently pursued one in the traditional education system, the students were using it more often than any other strategy, but it decreased much in frequency after the instruction. The main reason of that decrease is that the students who had followed the strategy in the pretest solved the similar problems in the posttest using the ones they learned. Despite that decrease, the strategy was used successfully and when really needed.

Working Backwards: No students employed the strategy of working backwards in the pretest, but there was an important increase in the frequency of its use after the instruction. In solving the problems for which this strategy

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could be used efficiently, the strategy of writing equations works and there were still some students preferring it in the posttest.

Making Tables: The strategy of making tables has not only very simple uses but also some quite complicated ones. Especially, determining the general solutions to the problems about the situations based on two variables requires the organization of the ordered specific solutions in tables. While only two students could solve the *diagonal of the rectangle* problem in the pretest, the instruction had its effects on the use of this strategy, and 13 students in the posttest got the positive result in the solution of the *billiard ball* problem using the strategy of making tables.

Reasoning: Reasoning is a strategy that is based on forming and testing out hypotheses about the solutions, revising and retrying them according to the results obtained. Almost all problem solutions involve a process of reasoning, but in this study, the problems requiring forming and testing hypotheses were accepted in the scope of reasoning. The problem of *the color of the handkerchief* in the pretest and that of *bridge* in the posttest were closely related with this strategy. At the beginning of the instruction, only 4 students attempted to use it, but the number went up to 74 after the instruction.

Simplifying the Problem: The strategy of simplifying the problem (benefiting from similar simple problems), which is generally used for the problems containing numbers with many digits and complicated relations, was one of the strategies on which the instruction had major effects. No students used it in the pretest, but it was seen in the posttest that 55 students employed it in many problems. They used it mostly together with the strategies of looking for a pattern and drawing diagrams.

In order to see which ones were more functional in the differences between the successful and less successful students, bottom and top segments of 27% were determined according to the scores the students had got in the posttest. The group at the bottom consisting of 27% of the students was composed of those who had got 65 or lower, and the group at the top consisting of another 27% of the students involved those having got 80 or higher than that. According to the results of the discriminant analysis (considering the low values of Wilks Λ and high value of F) of the strategy use frequency points (nine parameters) of the students in those two groups, the strategies that contributed the most to the differences were *reasoning, working backwards, drawing diagrams, making tables* and *simplifying the problem* respectively. A significant difference of 0.05 was observed in the use of each one of those strategies. Moreover, there was a difference of 0.05 again regarding the use of the ways of solution that had been named *other* (Table 3).

Table 3: ^ and F Values of Bottom and Top Groups in terms of Success

Name of the Strategy	Wilks' Lambda	F
Making Systematic Lists	0.93	2.35
Guess and check	0.99	0.14
Drawing Diagrams	0.73	11.86*
Looking for Patterns	0.92	2.78
Writing Equations	0.98	0.53
Working Backwards	0.56	25.47*
Making Tables	0.83	6.43*
Reasoning	0.37	54.63*
Simplifying the Problem	0.88	4.41*
Other	0.88	4.25*

* p<0.05

The correlations of these strategies with the discriminant function were higher than the correlations of the other strategies. The correlations of these strategies with the discriminant function were observed to have been higher the other strategies in the same order. With the help of the discriminant function, a classification with an accuracy rate of 100% was achieved.

Multiple regression analysis was employed to disclose and discuss how functional the problem solving strategies were in the problem solving success. The results of the regression analysis are given in Table 4.

Table 4: Regression Analysis Results Related to Discussion of Problem Solving Success

Strategies	B	Standard Deviation	β	T	P	Dual Correlation	Partial Correlation
Constant	33.71	4.48		6.98	.00		
Making Systematic Lists	6.64	1.99	0.24	3.33	.00	0.23	0.43
Guess and check	3.34	1.62	0.22	2.06	.04	0.08	0.28
Drawing Diagrams	2.38	0.95	0.19	2.50	.02	0.45	0.33
Looking for Patterns	2.45	1.19	0.16	2.06	.04	0.28	0.28
Writing Equations	2.39	1.43	0.18	1.67	.10	-0.08	0.23
Working Backwards	8.01	2.00	0.29	4.00	.00	0.52	0.49
Making Tables	4.44	2.05	0.16	2.17	.03	0.36	0.29
Reasoning	7.45	1.29	0.45	5.74	.00	0.72	0.63
Simplifying the Problem	2.71	1.27	0.15	2.14	.04	0.27	0.29
Other	3.15	2.82	0.09	1.11	.27	0.33	0.16

After analyzing the dual and partial correlations between the problem solving success and problem solving strategies, it can be observed that the highest three correlation coefficients belong to the strategies of reasoning (0,73), working backwards (0,52) and drawing diagrams (0,45). Writing equations is at a negative and low level (-0,08) and other strategies are at a positive but low level (between 0,08 and 0,36).

There is a high level and significant relationship between the problem solving strategies and problem solving success (R=0,89, R2=0,80, p=0,01). The problem solving strategies involved in this study as the independent variables explain almost 80% of the problem solving success.

According to the standardized regression coefficients, the relative importance order of the problem solving strategies in terms of their effects on the problem solving success is as follows: reasoning, working backwards, making a systematic list, guessing and checking, drawing a diagram, writing an equation, looking for patterns, making tables, benefiting from similar problems, and others. After analyzing the t-test results about the regression coefficients, it can be said that all the strategies, except the ones that are defined as “others” and could not be considered within the nine strategies examined in this study, were significant. In other words, they had a decisive role in explaining the problem solving success.

According to the regression analyses results, the regression equation related to the success in problem solving is as follows: “problem solving =39,771 +6,64 making a systematic list +3,34 guessing and checking +2,38 drawing a diagram +2,4 looking for pattern +2,39 writing equations +8,01 working backwards +4,44 drawing a table +7,46 reasoning +2,71 simplifying the problem 3,15 other”.

Findings about Opinions on Problem Solving Strategies and Non-Routine Problems

With a likert type scale consisting of 10 items, it was investigated what the teacher trainees were thinking about the difficulty levels of the problem solving strategies.

Giving 1, 2, 3, 4 or 5 points to the responses made to the items in the scale, average scores out of 5 were determined for the difficulty levels. Table 2 gives the difficulty levels that the students determined for the strategies and according to that, they found the strategies of making tables, reasoning and simplifying the problem difficult while thinking that guess and check, writing equations and making systematic lists were easier.

All the students answered the open-ended question asked to measure their attitudes, which was *“Would you recommend that the students to be in your position the following year should be given this instruction you have been given on the strategies of solving non-routine problems? Support your answers giving reasons”* and they stated that the instruction should be provided. 59 students recommended that such a course be with this extent and two thought that it is unnecessary. 10 out of those 59 students emphasized that the course should be also offered to the students in the programs of social sciences.

In the students' answers to the open-ended questions, the thoughts expressed and the ways of expressing their thoughts differed from each other. The researches made a summary of those opinions and shared it with the two groups both. During those free talks in groups, the reasons the students gave why the instruction had been useful and why it had to be provided for the students of the following year were as follows:

- It presents new points of view,
- It teaches how to think correctly,
- It helps to get rid of memorizing everything and focusing only on a particular solution,
- It improves self-confidence and helps undertake the initiatives to make decisions,
- It helps to get rid of being dependant on formulas,
- It shows that patterns and correlations could be derived easily,
- It teaches how to study systematically,
- It makes easier to realize that all the events seeming complicated or not do have a mathematical order and it is not so hard to explain that order after having some clues,
- It teaches how to use the strategies and solve problems,
- It enables one to start to think that problems have more than only one way of solution,
- It enables one to analyze solutions in a more detailed way. Thus, it shows that solutions should be evaluated in terms of correctness and generality.

All the students indicated that the instruction had some effects on them and wrote that it caused the below mentioned changes in the ways they thought and behaved:

- They began to solve the problems they once considered impossible to solve, and the classroom atmosphere throughout the instruction had its effects on that change,
- They thought that students would like mathematics more with such ways of teaching,
- When becoming teachers, they would never teach rules and patterns based on memorization but try to make their students infer them with the help of such discussions as the ones they themselves had,
- They saw that during the group discussions, opinions expressed by one or more students were made complete by others' thoughts, that the students felt the need to prove to each other the rationales of their thoughts and that they felt more assured of the results attained when compared to the ones they attained individually,
- They began to believe that students need such learning environments to be aware of the mathematical order of the events happening around

them, and improving that awareness is a primary responsibility of teachers.

DISCUSSION and CONCLUSION

The subject of this study in a general sense is the solution of non-routine problems. In a narrower sense, it is to what extent problem solving strategy training increases problem solving success, which strategies are learnt more easily, which ones are functional in choosing between successful and unsuccessful students, how important knowledge about strategies is to explain success in problem solving and what teacher trainees think about that.

This study examined examples of strategy uses that are suitable for university level and, being different from other studies, it was aimed to determine what difficulties are experienced when learning about the strategies and what functions and roles the strategies have regarding success in problem solving. The length of time (8 weeks) used for the instruction was longer than those in other studies were and it was sufficient. Besides, the problem solving instruction in the study was done in a related way to other topics in mathematics teaching, and problem solving was revised when studying the topics that followed the posttest and the classroom environment tested throughout the study was kept alive. Studying with a larger sample group than the one in the study (n=61) was preferable, but it was not possible as there was no more than 61 students registered for the program. The former studies showed that problem-solving strategies can be learnt (Verschaffel et al., 1999) and this study confirmed that.

The students were interested in almost all of the strategies and participated willingly in the instructional and practical processes. The expected positively significant differentiation occurred in the success in problem solving (Table 2). These findings are in congruity with Higgins's (1997), Folmer's (2000) findings that studying with non-routine problems improves thinking skills and the awareness of the ways one thinks in, and Verschaffel et al.'s (1999) and De Corte's (2004) findings that social constructivist learning environment increases the correct solutions rate in non-routine problems. That the increase is such a high one ($\bar{x}_{pretest}$=33.75, $\bar{x}_{posttest}$=72.93, $\bar{x}_{kalıcılık}$=76.56) could be commented that the social constructivist learning environment and the content of the instruction wiped out such negative attitudes and beliefs students have as "a mathematical problem has only one way of solution and one correct answer, ordinary students can never solve an unusual problem correctly" (Verschaffel et al., 1999). That seems to be confirmed by the students' evaluations and views of the problem solving strategies, ways of perceiving non-routine problems and learning environments.

Considering the fact that the instruction took 8 weeks and the retention test was administered 6 weeks after the posttest, it can be claimed that the increase in success was not a coincidental one or a result of the unusual teaching technique and content.

The fact that the sample group of the study was a small one (n=61) could make the generalization of the results somewhat hard, however some clear signs could be detected that teaching problem solving strategies and studying non-routine problems would improve the problem solving skills and change students' views of problems and problem solving. The considerable increase in the attempts to solve problems and the retention test average higher than that of the pretest can be seen as the indicators suggesting that the students were using the problem solving strategies when really needed and that they made them a part of their educational lives. The teacher trainees' appreciation of the mathematical order in every event, their expressions of commitment that they would get their own students to infer patterns and formulas and not spoon-feed them show that they now have more will *to do mathematics* and more self-confidence to achieve that. This result is also an indicator of the fact that the instruction of problem solving strategies provides trainees with mathematical disposition (De Corte, 2004).

The students were able to learn all the 9 strategies studied in this study, but they differed significantly from each other when using those strategies in problem solving (Table 2). Success rates might be affected by some factors such as the difficulty levels of problems, which are determined by some other factors such as having not encountered similar problems before and not knowing about the concepts in problems. In order to cope with such problems, a special effort was made at the beginning of the study, which is mentioned in the "method" section. Besides, it is another fact that some strategies (drawing diagrams, writing equations etc.) might be employed more frequently as they are convenient to use in many problems. Considering all these, it could be suggested that the differences between the results of the pretest, posttest and retention test in the study are more important than the frequencies of strategy uses.

In this study, the strategies whose uses were affected mostly by the instruction were simplifying the problem, looking for patterns, reasoning, drawing diagrams, making a systematic list, guess and check, and working backwards. This is in conformity with Altun (2006) where the strategies affected by the instruction were reported to be looking for patterns, guessing and checking, making a systematic list, making a table and working backwards. However, it is different from the findings of the study of Verschaffel et al (1999) at elementary education level, which states that drawing diagrams and looking for patterns are hard to use.

Simplifying the problem is a rather effective strategy in solving the university-level problems with complicated relations in, and thus the difference in its use could be attributed to the difference between the levels in the two studies. It was expected that the difficulty levels the students would declare fully accord with the difficulty levels to be inferred from the problem solving tests, but there came out a partial accordance between the two in the end. This condition indicates that there is still a need to conduct studies on the subject with larger groups.

When the averages in the pretest, posttest and retention test are compared, it is observed that the changes in the uses of *making systematic lists, guess and check, looking for patterns, simplifying the problem* and *drawing diagrams* are similar to each other. The students had some presuppositions about those strategies coming from traditional education. The instruction transformed what was known about the strategies into formal knowledge and caused significant differences in their uses when solving problems. The changes in the uses of *working backwards* and *making tables* were similar as well and there was no considerable increase in their frequencies of use in the posttest, but no students in the pretest had employed them. In the situations where the strategy of working backwards could be used, the ones who wrote equations were still forming the majority. The problems about making tables were ones that required making of tables based on two variables and making tables came out the most difficult strategy to learn. For instance, in the solution of the *billiard ball* problem, the student saw that if the number of the points in were augmented, the area would get bigger as much as the half rate of that augmentation, but they had difficulties in determining the general solution. That situation could be explained with the help of the fact that the solutions based on two variables are more complicated than the solutions that are based on a single variable.

Reasoning was the strategy in whose use the least difference was detected between the pretest and posttest. Reasoning has its part in the solution of every problem but in this study, it was sought in the solutions of the problems requiring forming, testing and revising hypotheses to have the results. The fact that the difference was the least in its use can be interpreted that developing reasoning skills needs longer periods of instructional studies

The use of writing equations or inequalities (using unknowns) became less and less frequent, which is the opposite of the cases in other strategies. The students tended to solve many problems in the pretest writing equations or using formulas of permutation, combination etc. However, in the posttest; they replaced all such attempts with the strategies newly learnt. It was observed that the students left seeking formulas, which seemed almost a habit of theirs. That change confirms and supports the students' expressions that strategy training could prevent one from trying to memorize everything. Although employing the strategy of writing equations was not much frequent in the posttest, it seemed to become frequent again in the retention test, which suggests that the tendency towards the behaviors before the instruction increased during the time after when the instruction had ended (6 weeks).

To classify the students as the successful and unsuccessful ones, it was already obvious that the determining factor would be the strategies that the successful students would be able to use and the unsuccessful ones would not. Moreover, it was quite expectable that the strategies that could be used by all or none would not have a function in distinguishing between the students. The strategies of *reasoning, working backwards, drawing diagrams* and *making tables* served the major functions in the study. This result is in harmony with Altun (2006) where the most effective strategies in discrimination were found to be writing equations, reasoning and drawing diagrams. Achievement of 100% accuracy in classification according to the discriminant analysis results shows that the strategies have a dominant and decisive role in determining where the success is.

The R^2 value found through the multiple regression analysis shows that knowledge of the strategies explains 80% of problem solving success. This reveals the importance of strategy training for success in problem solving.

Among the independent variables (problem solving strategies) that contribute to the dependant variable (problem solving success), the strategies of working backwards, reasoning and making systematic lists were more effective and functional. In terms of success in problem solving, this fact proves the importance of those strategies and the mental processes those strategies require.

The 20% part, which is far from the 80% variation explained by the problem solving strategies, can be explained through the other variables that were not involved in the model.

Instead of the open-ended question aiming to disclose the students' views on the instruction, it could have been used a test which would be objectively scored and measure the views, beliefs and attitudes of the students. In this study, rather than measuring the attitudes or beliefs, the researcher aimed to verify the observations made during the instruction, and that is the reason why an open-ended question was preferred. That question was intended to help the researcher have a general inventory of the students' views on the issue.

Having new points of view and the opportunities to get rid of memorizing everything and using formulas everywhere were among reasons that the students gave when explaining why a problem solving strategies course is necessary, and they remind a dimension of the traditional education system which is complained about really much and also give clues to change it. *Learning to think correctly and studying systematically, realizing that patterns and relations are easy to infer* were some other reasons that could be considered signs showing that the students prefer a process-oriented education rather than a product-oriented one.

Although it is known that firstly discussing the problems and concepts in heterogeneous groups of 3 or 4 people has positive effects on learning (Santos-Trigo, 1996; Verschaffel et al., 1999; de Corte, 2004), the literature still cannot recommend a best learning environment for students (Verschaffel et al., 1999). Both the cognitive and perceptive results gained in this study on teaching problem solving suggest that the learning environment and content helped the students notice what mathematics and problem-solving really means, increased their motivation to become teachers, changed their points of view and improved their skills to create similar atmospheres when they teach themselves. It is also understood that such teaching activities have a major role in motivating students to do mathematics and participate willingly in mathematical discussions and they could be taken into account when planning and organizing teaching processes. Another thing suggested by the findings is that students would adopt and support the educational reform studies congruous with the content of the present study. Conducting this study or other studies whose theoretical frameworks are similar to the one in this study with larger groups, and organizing mathematics curricula based on them could produce positive results in terms of both the specific subject in question and mathematics education in general.

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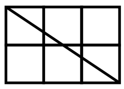
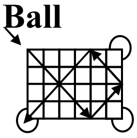
Appendix 1. Pre-test Questions

Pre-Test Problems	
Apple Basket	16 apples are to be put in 4 baskets. Keeping different numbers of apples in the baskets in each time, in how many different ways can you put the apples?
Marble	Ali and Veli are playing a game. If Ali wins, he gets 5 marbles from Veli; if Veli wins, he gets 7 marbles from Ali. At the end of 24 games, they have the same number of marbles. How many games have they won each?
Dancer	A dancer takes 5 steps forward and then 2 steps back on a straight line. Now he is 20 steps ahead of the point he started dancing Find the number of the steps he has made.
Cake	You are to cut a circle-shaped cake. You are allowed to use the knife 9 times. In how many pieces can you cut the cake? You do not have to cut the pieces in equal sizes.
Water Consumption	If you consume water up to 20 m^3, you pay a particular price per ton. If you use more than 20 tones, you pay a different price for each extra ton. A family using 25 tons is to pay 16 Liras, while another one using 35 tons is to pay 20 Liras. How much is a family to pay if they use only 8 tons?
Passengers	1 out of every 3 passengers gets off the bus at every bus stop. After leaving the third bus stop, 8 passengers are left in the bus. How many were there at the beginning?
Diagonal of the Rectangle 	Draw one of the diagonals of a rectangle drawn on a squared paper. In the figure below, the diagonal of a 2x3 rectangle passes over 4 squares, and the diagonal of a 3x5 rectangle passes over 7 squares. For any of the rectangles (ex: 7 x 11 or 17 x 20 rectangles), can you find out how many squares the diagonal passes through?
Color of the Handkerchief	Three people enter a competition. They stand in a queue. There are 2 white and 3 black handkerchiefs. Three of them are randomly chosen and fastened on the competitors' backs, but they cannot see anything. Then, each one is allowed to see the handkerchief on the back of the competitor in front of him, but they still cannot see anything back. They are told that the one who can tell the color on his own back will win the prize. The last one in the queue says, "I can't" and so does the one in between. The man in the front says, "I can" and guesses the color correctly. What is that color?
Chess Board	The chessboard is an 8x8 big square that consists of 64 small squares. How many squares are there on the board?

Appendix 2. Post-test Questions

Post-Test Problems	
Twenty's Disruption	What kind of 20 are there provided that sum of 8 odd numbers?
Scores in the Exam	Ayşe's team wins 302 points in an exam giving 43 correct answers to 50 questions. The correct answers are worth either 5 or 8 points. How many 5 points worth answers have the team given?
Designer	A designer prepares an ornament for a wall sticking one square on another.
Rectangles	There are ten points on a straight line and a point apart from that line. How many triangles can be drawn with the points?
Parachute	The parachute jumper falls 120 meters down in a second when his parachute is not open and falls 35 meters down per second when it is open. One day, he jumps from an altitude of 1785 meters and lands on the ground in 17 seconds After how many seconds did he open the parachute?
Treasure Hunters	Four treasure hunters find a basket of golden coins. They decide to share it equally, but one of them wakes up when the others are asleep and gets half of the coins counting "1 for me, 1 one for the others". One coin is left after it. He takes some of the coins and buries that one coin, leaves the others in the basket. Later, the other three do the same one after the other. When they all wake up, they see that only 5 coins are left and they share them. All of them keep that one coin in a secret place. How many coins were there in the basket before sharing?
Billiard Ball 	A rectangle billiard table has just 3 at the corners as shown in the figure. A ball from the corner that hasn't got a hole, has been pushed with 45^{o} angle. The ball reflects with 45^{o} angle from every side when it hit. How many rebounds will the ball make until it go into a hole? Find a rule or formula for different sized tables.
Bridge	Four warriors have to cross a narrow bridge at night. They have a torch and 17 minutes time. Some of them are wounded. So now, the first warrior has the strength to cross in 10 minutes, the second has the strength to do it in 5 minutes, the third has the strength to walk over the bridge in 2 minutes and the last is still able to cross the bridge in one minute. The bridge is wide enough for only two people to cross at one time with a torch. In what sequence do they have to cross the bridge to be on the other side altogether in 17 minutes?
Line Segment	There are 10 points on a straight line. How many line segment aroused with the points?