



## ON ACHROMATIC NUMBER OF CENTRAL GRAPH OF SOME GRAPHS

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**ABSTRACT.** The concept of coloring a graph will lead to the definition of a complete  $n$ -coloring of a graph  $G$  which results the achromatic number  $\psi(G)$  where the maximum number of colors required for the points of  $G$  in which every pair of colors appears on at least one pair of adjacent vertices. In this paper, we obtain the achromatic number for the Central graph of Ladder graph, Central graph of Dutch-Windmill graph, Central graph of Fan graph and Central graph of Flower graph is denoted as  $\psi[C(L_n)]$ ,  $\psi[C(D_3^{(n)})]$ ,  $\psi[C(F_{m,n})]$  and  $\psi[C(FL_n)]$  respectively.

### 1. INTRODUCTION AND PRELIMINARIES

Throughout this paper all graphs are finite, undirected and simple. A coloring of a graph  $G$  is a partitioning of the vertex set  $V$  into color classes. A proper  $k$ -coloring of a graph  $G$  is a function  $c : V(G) \rightarrow 1, 2, \dots, k$  such that  $c(u) \neq c(v)$  for all  $uv \in E(G)$ . Let  $c_i$  be the color class which is the subset of vertices of  $G$  that are assigned to each color  $i$ . The chromatic number  $\chi(G)$  is the minimum number of colors required in any proper coloring of  $G$ . An achromatic coloring [2] of a graph is a proper vertex coloring such that each pair of color classes is adjacent by at least one edge. The achromatic number was defined and studied by Harary, Hedetniemi and Prins [3]. They shown that, for every complete  $n$ -coloring  $\tau$  of a graph  $G$ , there exists a complete homomorphism  $\pi$  of  $G$  onto  $K_n$  and conversely. They considered the largest possible number of colors in an achromatic coloring is called the achromatic number and is denoted by  $\psi$ . The greatest number of colors used in a complete coloring of  $G$  is the achromatic number  $\psi(G)$  of  $G$ . It is clear that  $\chi(G) \leq \alpha(G) \leq \psi(G)$ .

Computing the achromatic number of a general graph was proved to be  $NP$  complete by Yannakakis and Gavril [13] in 1980. In 1976, Pavol Hell and Donald

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Received by the editors: February 05, 2018; Accepted: June 16, 2018.

2010 *Mathematics Subject Classification.* 05C15.

*Key words and phrases.* Achromatic coloring, achromatic number, central graph.

Submitted via International Conference on Current Scenario in Pure and Applied Mathematics [ICCSPAM 2018].

J. Miller [7] who found the achromatic number of Paths and Cycles. The achromatic number of disjoint union of graphs and the achromatic number of the categorical products of graphs are founded by Pavel Hell and Donald. J. Miller [7].

Hedetniemi conjectured that pseudoachromatic number and achromatic number are equal for all trees. But Keith. J. Edwards disproves by giving an infinite family of trees for which pseudoachromatic number strictly exceeds the achromatic number. The following definitions are considered for the result and discussion of this paper[5][6][8][9][10][11].

The central graph [12]  $C(G)$  of a graph  $G$  is obtained by introducing a new vertex on every edge of  $G$  and hence joining every pair of vertices of the given graph  $G$  which were non-adjacent in previous.

A Ladder graph  $L_n$  is a planar undirected graph with  $2n$  vertices and  $3n - 2$  edges where  $\Delta(L_n) = 3$  and  $\delta(L_n) = 2$ .

The Dutch-windmill graph  $D_3^{(n)}$ , also called a Friendship graph, is the graph obtained by taking  $n$  copies of the Cycle graph  $C_3$  with a vertex in common and therefore corresponds to the usual windmill graph  $W_m(n)$ .

The Fan graph denoted by  $F_{m,n}$  can be constructed by joining  $\overline{K_m} + P_n$ , where  $\overline{K_m}$  is the empty graph on  $m$  nodes and  $P_n$  is the Path graph on  $n$  nodes. Let  $(X, Y)$  be the bipartition of  $F_{m,n}$  with  $|X| = m$  and  $|Y| = n$ .

A Flower graph  $FL_n$  is the graph obtained from a Helm by joining each pendent vertex to the root vertex of the Helm.

In the following sections we discuss about achromatic number of central graph of some graphs.

## 2. ACHROMATIC NUMBER OF CENTRAL GRAPH OF LADDER GRAPH

**Theorem 2.1.** *For  $n \geq 2$ , the achromatic number for central graph of Ladder graph is  $2n$ . ie.,  $\psi[C(L_n)] = 2n, n \geq 2$ .*

*Proof.* Let  $L_n$  be the Ladder graph with  $2n$  vertices and  $3n - 2$  edges and the vertex set of Ladder graph is  $V(L_n) = \{v_i : 1 \leq i \leq n\} \cup \{u_i : 1 \leq i \leq n\}$ . Consider  $C(L_n)$ , by the definition of central graph, each edge of the graph is subdivided by a new vertex and here each edge  $(v_i, v_{i+1})$ ,  $(u_i, u_{i+1})$  and  $(v_j, u_j)$  are subdivided by the vertices  $x_i, y_i$  and  $w_j$  for  $1 \leq i \leq n - 1$  and  $1 \leq j \leq n$  respectively. The vertex set of  $C(L_n)$  is defined as follows:

$$V[C(L_n)] = \{v_i : 1 \leq i \leq n\} \cup \{u_i : 1 \leq i \leq n\} \cup \{x_i : 1 \leq i \leq n - 1\} \\ \cup \{y_i : 1 \leq i \leq n - 1\} \cup \{w_j : 1 \leq j \leq n\}.$$

Consider the color class  $C = \{c_1, c_2, \dots, c_{2n}\}$ .

Now assign a proper coloring to the vertices of  $C(L_n)$  to make the coloring as achromatic. To get the maximum number of pair of colors, we assign the colors as follows:

- For  $1 \leq i \leq n$ , assign the color  $c_i$  to the vertex  $v_i$ .

- For  $1 \leq i \leq n$ , assign the color  $c_{n+i}$  to the vertices  $u_n, u_{n-1}, \dots, u_{n-(n-1)}$  respectively.
- For  $1 \leq i \leq n - 1$ , assign the color  $c_{i+2}$  to the vertex  $x_i$ .
- For  $1 \leq i \leq n - 1$ , assign the color  $c_i$  to the vertex  $y_i$ .
- For  $w_j$ , assign the color  $c_{j+1}$  for  $j = 1, 2$  and for  $3 \leq j \leq n$  assign the color  $c_{2n-k}$  for  $1 \leq k \leq n - 2$ .

By the procedure of achromatic coloring, the above coloring pattern fails to receive some of the pairs  $(c_2, c_{2n-1})$  and  $(c_{2n-1}, c_{2n})$ . Thus to accommodate the pairs, color the vertex  $x_1$  by assigning the color  $c_{2n-1}$  and for the vertex  $y_2$  assign the color  $c_{2n}$ . Thus an easy check shows that any pair in the color class is adjacent by at least one edge and by the very construction it is the maximal color class  $C$ . Therefore  $\psi[C(L_n)] = |C| = 2n$ . Thus for any Ladder of length  $n$ ,  $\psi[C(L_n)] = 2n$ , for  $n \geq 2$ .  $\square$

### 3. ACHROMATIC NUMBER OF CENTRAL GRAPH OF DUTCH-WINDMILL GRAPH

**Theorem 3.1.** *For any Dutch-Windmill graph  $D_3^{(n)}$ , the achromatic number  $\psi[C(D_3^{(n)})] = 2n + 1$ , for every  $n \geq 2$ .*

*Proof.* Let  $D_3^{(n)}$  be the Dutch-Windmill graph obtained by taking  $n$ -copies of the Cycle  $C_3$  with vertices  $v_1^i, v_2^i, v_3^i, i = 1, 2, \dots, n$ , named in anti-clockwise direction in which the vertex  $v_1^i (1 \leq i \leq n)$  is the common vertex. Consider  $C(D_3^{(n)})$ . For each  $i (1 \leq i \leq n)$  with  $p = 1, q = 2, 3$  take  $v_{p,q}^i$  which represents the newly introduced vertex in the edge connecting  $(v_p^i, v_q^i)$  and subdividing the edges  $v_p^i$  and  $v_q^i$  for  $1 \leq i \leq n$  and  $p = 2$  and  $q = 3$  by introducing a new vertex  $v_{p,q}^i$ . Thus the vertex set of  $C(D_3^{(n)})$  is defined as follows:

$$V[C(D_3^{(n)})] = \{v_1^i : 1 \leq i \leq n\} \cup \{v_k^i : 1 \leq i \leq 2n \text{ and } k = 2, 3\} \\ \cup \{v_{p,q}^i : 1 \leq i \leq 2n \text{ and } p = 1, q = 2\} \cup \{v_{p,q}^i : 1 \leq i \leq 2n \text{ and } p = 1, q = 3\} \cup \{v_{p,q}^i : 1 \leq i \leq 2n \text{ and } p = 2, q = 3\}.$$

Define  $c : V[C(D_3^{(n)})] \longrightarrow \{c_1, c_2, \dots, c_{2n+1}\}$

The coloring procedure is defined as follows:

$$c[V(C(D_3^{(n)}))] = \begin{cases} c_1 & \text{to } v_1^i \text{ for } 1 \leq i \leq n \\ c_i & \text{to } v_k^i \text{ for } 1 \leq i \leq 2n \text{ and } k = 2, 3 \\ c_{2i} & \text{to } v_{p,q}^i \text{ for } 1 \leq i \leq n \text{ and } p = 1, q = 2 \\ c_{2i+1} & \text{to } v_{p,q}^i \text{ for } 1 \leq i \leq n \text{ and } p = 1, q = 3 \\ c_{2n+1} & \text{to } v_{p,q}^i \text{ for } 1 \leq i \leq n \text{ and } p = 2, q = 3 \end{cases}$$

It is trivial to see that it is a  $2n$ -regular graph so that we can assign  $2n + 1$  colors. To prove the above said coloring is achromatic, consider any pair  $(c_i, c_j)$ .

**Case 1**

If  $p = 1$  and  $q = 2, 3$  then the edge joining  $v_p^1$  and  $v_{p,q}^j$  will stand for the pairs  $(c_1, c_{j+i})$  and  $(c_1, c_{j+i+1})$  for  $1 \leq i, j \leq n$ .

**Case 2**

- If  $p = 2$  and  $q = 3$  then the edge joining  $v_p^j$  and  $v_{p,q}^j$  will stand for the pair  $(c_{2j-1}, c_{2n+1})$  for  $1 \leq j \leq n$ .
- If  $p = 3$  and  $q = 2$  then the edge joining  $v_p^j$  and  $v_{q,p}^j$  will stand for the pair  $(c_{2j}, c_{2n+1})$  for  $1 \leq j \leq n$ .

**Case 3**

- If  $p = 1$  and  $q = 2$  then the edge joining  $v_q^j$  and  $v_{p,q}^j$  will stand for the pair  $(c_{2j-1}, c_{2j})$  for  $1 \leq j \leq n$ .
- If  $p = 1$  and  $q = 3$  then the edge joining  $v_q^j$  and  $v_{p,q}^j$  will stand for the pair  $(c_{2j}, c_{2j+1})$  for  $1 \leq j \leq n$ .

**Case 4**

- If  $p = q = 3$  then the edge joining  $v_p^j$  and  $v_q^{j+1}$  will stand for the pair  $(c_{2j}, c_{2j+2})$  for  $1 \leq j \leq n - 1$ .
- If  $p = 3$  and  $q = 2$  then the edge joining  $v_p^j$  and  $v_q^{j+2}$  will stand for the pair  $(c_{2j}, c_{2j+3})$  for  $1 \leq j \leq n - 2$ .
- If  $p = q = 3$  then the edge joining  $v_p^j$  and  $v_q^{j+2}$  will stand for the pair  $(c_{2j}, c_{2j+4})$  for  $1 \leq j \leq n - 2$ .
- If  $p = q = 2$  then the edge joining  $v_p^{j+1}$  and  $v_q^{j+2}$  will stand for the pair  $(c_{2j+1}, c_{2j+3})$  for  $1 \leq j \leq n - 2$ .
- If  $p = 2$  and  $q = 3$  then the edge joining  $v_p^{j+1}$  and  $v_q^{j+2}$  will stand for the pair  $(c_{2j+1}, c_{2j+4})$  for  $1 \leq j \leq n - 2$ .
- If  $p = q = 2$  then the edge joining  $v_p^{j+1}$  and  $v_q^{j+3}$  will stand for the pair  $(c_{2j+1}, c_{2j+5})$  for  $1 \leq j \leq n - 3$ .

Hence any pair in the color class is adjacent by at least one edge and by the construction it is the maximal color class  $C$ . Therefore  $\psi[C(D_3^{(n)})] = 2n + 1$ , for every  $n \geq 2$ .  $\square$

## 4. ACHROMATIC NUMBER OF CENTRAL GRAPH OF FAN GRAPH

**Theorem 4.1.** For any Fan graph  $F_{1,n}$ , the achromatic number  $\psi[C(F_{1,n})] = n + 1$ .

*Proof.* Let  $F_{m,n}$  be the Fan graph defined as the graph join of  $\overline{K_m} + P_n$ , where  $\overline{K_m}$  is the empty graph on  $m$  nodes and  $P_n$  is the Path graph on  $n$  nodes. Let  $(X, Y)$  be the bipartition of  $F_{m,n}$  with  $|X| = m$  and  $|Y| = n$ . Let  $X = \{v\}$  and  $Y = \{v_1, v_2, \dots, v_n\}$ .

By the definition of central graph, each vertex  $vv_i$  for  $1 \leq i \leq n$  is subdivided by the newly introduced vertex  $v'_i$  for  $1 \leq i \leq n$  and  $u'_i$  is another newly introduced vertex between  $v_i v_{i+1}$  for  $(1 \leq i \leq n - 1)$ . The vertex set of  $C(F_{1,n})$  is defined as,

$$V[C(F_{1,n})] = \{v\} \cup \{v_i : 1 \leq i \leq n\} \cup \{v'_i : 1 \leq i \leq n\} \cup \{u'_i : 1 \leq i \leq n - 1\}.$$

Define  $c : V[C(F_{1,n})] \longrightarrow \{c_1, c_2, \dots, c_{n+1}\}$  The coloring procedure is defined as follows:

$$c[V(C(F_{1,n}))] = \begin{cases} c_i & \text{to } v_i & \text{for } 1 \leq i \leq n \\ c_{n+1} & \text{to } u'_i & \text{for } 1 \leq i \leq n - 1 \\ c_{i+1} & \text{to } v'_i & \text{for } 1 \leq i \leq n - 1 \\ c_2 & \text{to } v'_n \end{cases}$$

To get the maximum number of colors, assign the color  $c_{n+2}$  to the root vertex  $v$ . If we color in the above pattern some pair of colors  $(c_1, c_{n+2})$  and  $(c_{n+1}, c_{n+2})$  are missing. Thus to accommodate all the missing pairs, color the vertex  $v$  by  $c_{n+1}$ . Thus the coloring of  $c_{n+2}$  is not possible for this graph. Hence by the coloring procedure and under observation, the above said coloring is achromatic and it is maximum. Therefore  $\psi[C(F_{1,n})] = n + 1$ .  $\square$

**Theorem 4.2.** For  $m = 2$  and  $n \geq 2$ , the achromatic number for central graph of Fan graph is  $m + n + 1$ . i.e.,  $\psi[C(F_{m,n})] = m + n + 1$ , for  $m = 2$  and  $n \geq 2$ .

*Proof.* Let  $(X, Y)$  be the bipartition of  $F_{m,n}$  with  $|X| = m$  and  $|Y| = n$ . Let  $X = \{u_1, u_2\}$  and  $Y = \{v_1, v_2, \dots, v_n\}$ . In  $C(F_{m,n})$  let  $w_j$  be the newly introduced vertex in the edge connecting  $u_1$  and  $v_i, u_2$  and  $v_i$ , for  $1 \leq i \leq n$  and  $1 \leq j \leq 2n$ . Here  $v_i$  and  $v_{i+1}$  is subdivided by the vertex  $w'_j$  for  $1 \leq i, j \leq n - 1$  and the vertex set of  $C(F_{m,n})$  is,

$$V[C(F_{m,n})] = \{u_i : 1 \leq i \leq m\} \cup \{v_i : 1 \leq i \leq n\} \\ \cup \{w_j : 1 \leq j \leq 2n\} \cup \{w'_j : 1 \leq j \leq n - 1\}.$$

Consider in  $C(F_{m,n})$ , we see that for  $1 \leq i \leq n$ ,  $v_i$  is not adjacent with  $v_{i+1}, u_1$  and  $u_2$ . To make the coloring as achromatic, assign a proper coloring to all the vertices  $v_i$ 's and  $u_i$ 's.

Consider the color class  $C = \{c_1, c_2, \dots, c_{m+n+1}\}$ . Now assign a proper coloring to the vertices of  $C(F_{m,n})$  to make the coloring as achromatic. To get the maximum number of pair of colors, we assign the colors as follows:

- For  $1 \leq i \leq n$ , assign the color  $c_i$  to  $v_i$ .
- Assign the color  $c_{n+2}$  to  $u_1$  and  $c_{n+1}$  to  $u_2$ .
- For  $j = n + 2k$  where  $0 \leq k \leq 2$  but  $k \neq 1$ , assign the color  $c_{n+3}$  to  $w_j$ .
- For  $j = n + 1$ , assign the color  $c_{n+2}$  to  $w_j$ .
- For  $j = n + 2$ , assign the color  $c_1$  to  $w_j$ .
- For  $j = n + 3$ , assign the color  $c_n$  to  $w_j$ .
- For  $j = n + i$  ( $i = 5, 6, \dots$ ) when  $n \geq 5$  assign the color  $c_{i-1}$  to  $w_j$ .
- For  $1 \leq j \leq n - 1$  but  $j \neq 2$ , assign the color  $c_{n+3}$  to  $w'_j$  and for  $w'_2$  assign the color  $c_{n+1}$ .

If we increase one more color to the vertices of  $w'_j$  for  $1 \leq j \leq n - 1$ , some pair of colors  $(c_i, c_{n+3})$  for  $1 \leq i \leq n$  and  $(c_i, c_{n+1})$  for  $i = 2, 3$  will be missed. This contradicts the definition of achromatic coloring. So we assign the existing color

$c_{n+3}$  to all the vertices of  $w'_j$  for  $1 \leq j \leq n - 1$  but  $j \neq 2$  for producing an achromatic coloring. This proves the coloring is achromatic and hence it is maximal. Therefore  $\psi[C(F_{m,n})] = m + n + 1$ , for  $m = 2$  and  $n \geq 2$ .  $\square$

5. ACHROMATIC NUMBER OF CENTRAL GRAPH OF FLOWER GRAPH

**Theorem 5.1.** For  $n \geq 4$  the achromatic number for  $C(FL_n)$  is  $2n - 1$  i.e.,  $\psi[C(FL_n)] = 2n - 1, n \geq 4$ .

*Proof.* Let  $FL_n$  be the Flower graph with  $2n + 1$  vertices and  $4n$  edges and the vertex set of Flower graph is,

$$V(FL_n) = \{v\} \cup \{v_i : 1 \leq i \leq n - 1\} \cup \{u_i : 1 \leq i \leq n - 1\}.$$

Consider in central graph of  $FL_n$ , each edge  $(v_i, v_{i+1}), (v_i, u_i)$  and  $(v, v_i), (v, u_i)$  are subdivided by newly introduced vertices  $v'_i, u'_i$  and  $w_j$  for  $1 \leq i \leq n - 1$  and  $1 \leq j \leq n + 2$  respectively.

The vertex set of  $C(FL_n)$  is defined as follows:

$$V[C(FL_n)] = \{v\} \cup \{v_i : 1 \leq i \leq n - 1\} \cup \{u_i : 1 \leq i \leq n - 1\} \\ \cup \{v'_i : 1 \leq i \leq n - 1\} \cup \{u'_i : 1 \leq i \leq n - 1\} \cup \{w_j : 1 \leq j \leq n + 2\}.$$

Define  $c : V[C(FL_n)] \rightarrow \{c_1, c_2, \dots, c_{2n-1}\}$ .

Assign the following  $2n - 1$  colors to these vertices in a proper coloring manner to make the coloring as achromatic.

The coloring procedure is defined as follows:

$$c[V(C(FL_n))] = \begin{cases} c_1 & \text{to } v \\ c_{2i-1} & \text{to } v_{i-1} \text{ for } 2 \leq i \leq n \\ c_{2i} & \text{to } u_i \text{ for } 1 \leq i \leq n - 1 \\ c_{j+1} & \text{to } w_j \text{ for } 1 \leq j \leq n + 2 \\ c_{2i+3} & \text{to } u'_i \text{ for } 1 \leq i \leq n - 2 \\ c_3 & \text{to } u_{n-1} \\ c_{2i+2} & \text{to } v'_i \text{ for } 1 \leq i \leq n - 2 \\ c_{2n-3} & \text{to } v'_{n-1} \end{cases}$$

To prove the above said coloring is achromatic, consider any pair  $(c_i, c_j)$ .

**Case 1**

If  $j = i + 1$ , then the edge joining the vertices  $v$  and  $w_{j-1}$  will stand for the pair  $(c_1, c_j)$ , for  $1 \leq i \leq 2n - 1$ .

**Case 2**

If  $j = 2i - 1$ , then the edge joining the vertices  $w_j$  and  $v_i$ , will stand for the pair  $(c_{2i}, c_{j+2})$ , for  $1 \leq i \leq n - 1$ .

**Case 3**

For  $1 \leq i \leq n - 2$ , the edge joining the vertices  $v_i$  and  $u'_i$  will stand for the pair  $(c_{2i+1}, c_{2i+3})$ .

**Sub case 3.1**

For  $i = n - 1$ , then the edge joining the vertices  $u'_i$  and  $v_i$  will stand for the only pair  $(c_3, c_{2i+1})$ .

**Case 4**

- If  $i \neq 1$  and  $i \neq n$ , then the edge joining the vertices  $u_1$  and  $u_i$  will stand for the pair  $(c_2, c_{2i})$ .
- If  $i \neq 1$  and  $i \neq n$ , then the edge joining the vertices  $u_1$  and  $v_i$  will stand for the pair  $(c_2, c_{2i+1})$ .

**Case 5**

If  $i \neq 1$  and  $i \neq n$ , then the edge joining the vertices  $v_1$  and  $u_i$  will stand for the pair  $(c_3, c_{2i})$ .

**Case 6**

If  $i \neq 1$  and  $i \neq n$ , then the edge joining the vertices  $(u_2, u_{i+1})$  and  $(u_2, v_{i+1})$  will stand for the pair  $(c_4, c_{2i+2})$ , and  $(c_5, c_{2i+3})$ .

**Case 7**

If  $i \neq 1$  and  $i \neq n$ , then the edge joining the vertices  $(v_2, u_{i+1})$  and  $(v_2, v_{i+2})$  will stand for the pair  $(c_5, c_{2i+2})$ , and  $(c_5, c_{2i+3})$ .

Thus any pair in the color class is adjacent by at least one edge and by the construction it is the maximal color class  $C$ . Therefore  $\psi[C(FL_n)] = |C| = 2n - 1$ ,  $n \geq 4$ . □

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