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# Theoretical and mathematical analysis of double-circuit solar station with thermo siphon circulation

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*Yazar(lar) (Author(s)):* Yedilkhan AMİRGALİYEV<sup>1</sup>, Murat KUNELBAYEV<sup>2</sup>, Aliya KALİZHANOVA<sup>3</sup>, Omirlan AUELBEKOV<sup>4</sup>, Nazbek KATAYEV<sup>5</sup>, Ainur KOZBAKOVA<sup>6</sup>

ORCID<sup>1</sup>: 0000-0002-6528-0619

ORCID<sup>2</sup>: 0000-0002-5648-4476

ORCID<sup>3</sup>: 0000-0002-5979-9756

ORCID<sup>4</sup>: 0000-0002-2903-9056

ORCID<sup>5</sup>: 0000-0003-0501-3719

ORCID<sup>6</sup>: 0000-0002-5213-4882

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# Termo Sifon Sirkülasyonu ile Çift Devreli Güneş İstasyonunun Teorik ve Matematiksel Analizi

*Araştırma Makalesi / Research Article*

**Yedilkhan AMIRGALIYEV<sup>1</sup>, Murat KUNELBAYEV<sup>2\*</sup>, Aliya KALIZHANOVA<sup>3</sup>, Omirlan AUERBEKOV<sup>4</sup>, Nazbek KATAYEV<sup>5</sup>, Ainur KOZBAKOVA<sup>6</sup>**

<sup>1,2,3,4,5,6</sup>Institute Information and Computational Technologies CS MES RK, Almaty, Kazakhstan

<sup>1,3,6</sup>Al-Farabi Kazakh National University Almaty, Kazakhstan

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## ÖZ

Bu makalede, bireysel yapıların matematiksel modelleri ve termo sifon sirkülasyonu ile bir çift devreli güneş kolektörü çalışma modları anlatılmaktadır. Bu görevi gerçekleştirmek için, panel ve ısı yalıtımı arasındaki ek bölümleri ortadan kaldırarak ısı transfer katsayısının arttığı termo sifon dolaşımı ile düz bir güneş kolektörünün yeni bir tasarımını düşündük. Güneş kolektörünün verimliliği, kondenser ve evaporatörün “spiral spiral” tipi bir ısı eşanjörü şeklinde yapıldığı ölçüm tankının ve tanktaki ısı pompasının varlığı nedeniyle elde edilir ve ısı eşanjörü boru hatları, ısı değişiminin alanını ve yoğunluğunu arttırmaya izin veren diğerinin üzerine yerleştirilir. Bu çalışmanın sonucu, göz önüne alınan çalışma modlarında düz güneş kolektörlerinin kararsız termal rejiminin teorik ve matematiksel bir analizidir. Analiz sonuçlarına dayanarak, bireysel yapısal unsurları optimize etmenin yanı sıra termal rejimi tahmin etmek ve düz güneş kolektörlerinin tasarımı ve çalışma modlarının seçimi için alternatif çözümler seçmek mümkündür.

**Anahtar Kelimeler:** Çift devreli güneş istasyonu, ısı pompası, güneş kolektörü, termo sifon sirkülasyonu.

## Theoretical And Mathematical Analysis Of Double-Circuit Solar Station With Thermo Siphon Circulation

### ABSTRACT

This article discusses the mathematical models of the individual structures and modes of operation of a double-circuit solar collector with thermo siphon circulation. To perform this task, we considered a new design of a flat solar collector with thermo siphon circulation, in which the heat transfer coefficient was increased by eliminating additional partitions between the panel and thermal insulation. The efficiency of the solar collector is achieved due to the presence of the metering tank and the heat pump in the tank, where the condenser and evaporator are made in the form of a “spiral in a spiral” type heat exchanger, and the heat exchanger pipelines are placed one above the other, which allows increasing the area and intensity of heat exchange. The result of this work is a theoretical and mathematical analysis of the unsteady thermal regime of flat solar collectors on the modes of operation under consideration. Based on the results of the analysis, it is possible to optimize individual structural elements, as well as to predict the thermal regime and select alternative solutions for the design of flat solar collectors and the choice of operating modes.

**Keywords:** Double-circuit solar station, heat pump, solar collector, thermo siphon circulation.

### 1. INTRODUCTION

Renewable energy sources have the potential to meet current and future projected global energy needs without any environmental impact. Renewable energy sources such as solar, wind, hydropower, biogas and geothermal energy are potential candidates for a sustainable global energy demand. The best alternative to meet the growing demand for energy is solar energy. The conversion of solar radiation into heat is one of the simplest and most direct applications of this energy. To analyze the mathematical model of the developed double-circuit solar stations with thermo siphon circulation, we considered a number of existing models.

In the article there has been noted, that virtual prototyping of the solar collector helps predict collector's performance until operation [1]. The work herein admits, that in order to ground the solar collector project at physical state space it should be drove to a state of modelling to research collector's dimensions, which gives the air outlet low temperature.

Upon studying the design and operation models there is also taken into account the performance parameters. Duffie and Beckman in the work have constructed the solar collectors, which are applied to the models. In the current research there are several novelty aspects in the model-based approach [1].

With numerical models the restrictions of the approach thereof can be alleviated. Duffie and Beckman mention, that for the heat loss overall coefficient calculation to

\* Sorumlu yazar (Corresponding Author)  
e-posta : murat7508@yandex.kz

define the spatial average temperature of the absorbing plate is difficult. In the spatially distributed model as presented here, this is not a problem. Moreover, the solar collectors are often characterized by the heat which shall be defined experimentally.

Koyunsu in the work concerning the solar collector has noted that the special attention is paid to the heat increase from the solar power radiation factor and heat output [3]. Kicsiny has experimentally simulated the solar collector where the temperature output does not help design the solar collector's dimensions, particularly the solar collectors being operated under different conditions [4]. Gao et al., Alvarez et al. have given the information on the solar collector's parameters calculation [5,6].

Buzás et alii in their work on the dynamic model spatial distribution do not consider the collector as a combined reservoir [7]. In the works it is told about spatial distribution of the PV / water system, tunnel dryer, solar water heater. In the work thereof the authors strive to use the model-based approach to design and construct the solar collectors for low and high outlet temperature in the dryer of the adsorptive solar system to dry agricultural products and for regeneration there is used the adsorbent for the drying during the night [8,9,10,11].

Solar thermal energy is a renewable energy source which is widely used all over the world. From the widespread use of household solar water heaters (SWH) to complex solar farms for power generation, they have an extensive field of application. For the production of hot water in the domestic and commercial sector, SWH proved to be reliable and economically viable. It is shown in the paper, that the basic elements of SWH are collectors (flat plates or tubular pipes), connecting pipes, a water tank and auxiliary heating elements [12]. To avoid damage from freezing (boiling) and indirect heating of SWHs, a heat exchanger is used between the reservoir and the reservoir, or inside the tank. The tank can be installed either above the collector (thermosyphon system) or at a lower level (forced circulation). Prapas and others developed and investigated that the thermal characteristics of SWS thermosyphons are comparable with the thermal characteristics of forced circulation systems [13,14]. The area of the solar collector is about 120,000 m<sup>2</sup> [15]. On an industrial scale, the proportion of electrical energy provided by systems for the production of hot water from 40 to 80 ° C can be very significant [16]. For developments in water storage systems, solar collectors are usually separated, and using a forced circulation system installed at the desired temperature, is fixed at the collector outlet, which ensures maximum efficiency and economy [17]. The mathematical model includes the differential laws of conservation of mass, momentum, and energy. The drawback of this model is the absence of environmental influence. The actual results of the physical process and the mathematical modeling of the heat transfer process of the thermal conductivity of walls in a reservoir can lead to specific values for the calculation of a flat collector with a thermosyphon [19]. A multidimensional

mathematical model of the influence of the filling factor of a working fluid is presented in under the stationary regime of water heating in a thermosyphon. In the researchers developed a mathematical simulation of a thermosyphon with a solar collector based on equations for laminar flows of a compressible ideal gas [20].

In this article, researchers consider a theoretical and mathematical analysis of a dual-circuit solar installation with a thermo siphon circulation. For the theoretical and mathematical analysis, the dual-circuit solar plant was divided into four blocks: the first block, consisting of a thermo syphon metering tank and a flat solar collector; the second unit is a flat solar collector and a heat pump evaporator; a third unit, a condenser and a storage tank, and a fourth unit with a heat pump. In the first block, we obtained the equations of the thermo siphon flow between the thermo syphon dosing tank and the flat solar collector. In the second block, the analysis showed that when a dual-circuit solar system is operating with a heat pump and solar collector capacitor, the direction of heat flow between the solar collector and the environment changes. In the third block, we analyzed the mode of constant heat consumption from the storage tank, when hot water was supplied and when it was heated. In the fourth block, the analysis showed that a new solution for the development of a heat pump contributes to an increase in the thermal characteristics of a dual-circuit solar installation.

## 2. METHOD OF RESEARCH

Operation functional diagram of the thermal pump with the solar collectors is shown in the Figure 1 [21]. One of the main purposes of the system is to provide heating and hot water supplying. One of the levels or the first cause of the system development is to use primarily for the purpose thereof the solar power atmospheric air heat.

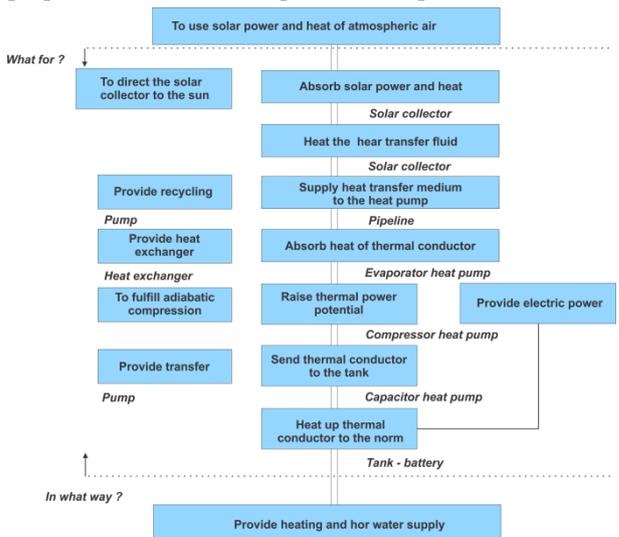


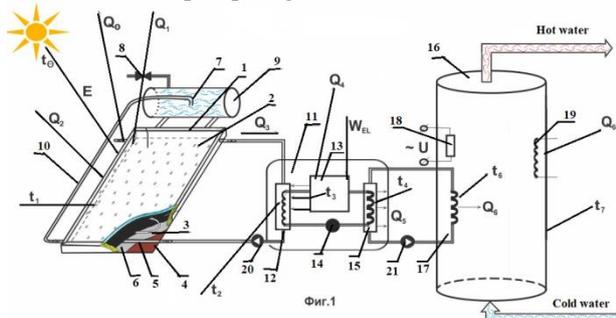
Figure 1. Solar installation fast model

To implement the complex of subsequent principal functions, located on the chart between the upper and

lower dashed line is one of the system's aim achievement. The first function is to absorb, by means of the solar collector, the solar power and atmospheric air heat and warm them up with the power of the heat conductor flowing through it. Further to supply the heat conductor to the thermal pump evaporator; absorb the warm of the heat conductor with a cooling medium (freon) boiling in the evaporator (recirculation process); concurrently to fulfill the cooling agent adiabatic contraction with a thermal pump compressor and transfer it into the liquid state; transfer the liquid and warmed cooling agent to the condenser; release the cooling agent heat from the condenser with a heat conductor; direct the heat conductor to the accumulator tank, return the heat conductor back to the condenser for reheating (recirculation process); accumulate thermal power in the tank; bring the water temperature in the accumulator tank to the required technological value (electrical heating).

### 2.1 Substantiation of constructive-technological scheme

With account of the demand to raise the installation operational performance there has been elaborated the principle diagram of the double-circuit solar installation with a thermal pump (Figure 2).



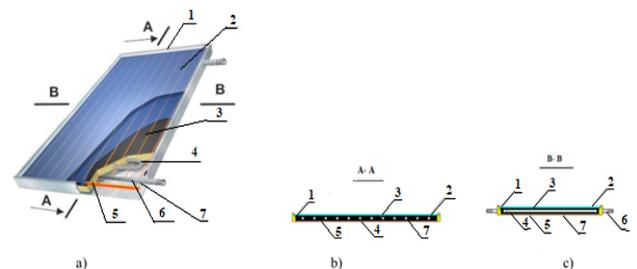
**Figure 2.** Principle diagram of the double-circuit solar installation with a thermal pump

The work of the proposed installation is as follows. Solar energy  $E$  with a temperature  $t_0$  is absorbed by the solar collector 1, with a temperature  $t_1$ , heating the flow of solar energy passes through a translucent insulating glass unit 2. The heat received from the solar flow heats the fluid in the coils 3, which is removed from the collector, and its place comes cold water from the pipeline with a valve for cold water 8, and from the siphon of the dispenser tank 7 there is a constant term siphon circulation through the circulation pipe 10. Then the liquid enters the heat pump 11, which consists of condenser evaporator 12 with temperature  $t_2$ , in which the heat exchanger is made in the form of a spiral, absorbing heat carrier heat, lowers its temperature below atmospheric air temperature ( $Q_2$ ) by means of a throttling valve 14, thereby contributing to additional heat absorption from atmospheric air. The diagram also shows solar radiation reflected from the translucent coating ( $Q_0$ ) and the surface of the absorbing panel ( $Q_1$ ). In the heat pump energy is transferred to the coolant with a relatively low temperature, to the coolant of the heat

exchanger of the condenser 15 in the form of a spiral with a higher temperature  $t_2$ , which increases the area, as well as the intensity of heat transfer. For the implementation of such a cycle, a compressor 13 with a temperature  $t_3$  is used with an electric drive 17. Further, through a heat exchanger of a condenser 15 with a temperature  $t_4$ , heat from the heat pump ( $Q_5$ ) is transferred to the tank of the heat exchanger  $Q_6$  with temperature  $t_6$  of the heating system 18. Since the installation has two circuits, it is equipped with automatic circulation pumps 19 and 20 for the circulation of fluid between the solar collector and the evaporator, the condenser and the storage tank. The water temperature is brought to the required technological level and is supplied to the consumer for the purpose of hot water supply and heating

### 2.2 Nonstationarity of thermal processes in the collector

Nonstationarity of thermal processes in the collector takes place due to the daily changes of the solar radiation intensity, atmospheric air temperature, radiation balance and other climatic factors. At the stage of designing the flat-plate solar collectors in order to reach the efficiency it is necessary to conduct the design analysis of solar hot water supply systems, as well, to decrease the thermal losses into the atmosphere, which is possible merely at joint consideration of optical and thermal properties of the collector, thermal accumulator and paths of hydrodynamics. Below there are principal schemes of the solar water heaters with heat absorbers from the steel stamp-welded heater and demountable tube-on-sheet heat receiver.



**Figure 3.** a) flat solar collector; b) section A-A of a flat solar collector; c) section B-B of a flat solar collector

The main aim is to develop and master the solar stations production economically feasible in Kazakhstan conditions using conceptually new solar collectors. To reach the set aim it is offered to implement a new approach to the solar collectors design applying modern materials, by means of which substantially cut (2-3 times) the solar installation cost. Subject matter and novelty of the method being offered consist in the fact that alternatively to the known design principle the collector thereof has a transparent double glass pane 2 with double glass and reduced pressure, as well a perimetritic frame 1. Wood frame bottom 7 is made of the 8 mm width plywood and heat insulating film 5 with foil is glued upon. In the gap between the glass pane and frame bottom there is laid a flexible thin-walled stainless gofferred tube 4 with a diameter of  $\square\square 16$  mm in the form

of a spiral. Tube's ends are attached to inlet and outlet protruding tubes 6.

Differential equations system represents a mathematical model of the flat-plate collectors thermal regime, the solution of which gives a possibility to define the water flow speed upon thermo-siphon circulation, as well as the solar installation's performance efficiency for the given weather conditions. Change of weather and flow conditions upon water circulation in the system through the tank and solar collector at constant flow rate using the pump, changes the conditions, i.e. the liquid temperature at the outlet upon subsequent flowing is continuously changing. The weather function and the system geometry as well as the natural circulation flow conditions in the flat-plate solar collectors, caused by thermo siphon impact are difficult to define. Being offered thermal model has been modified in such a way that there is accounted the system capacity and solar collector's efficiency coefficient. To solve the thermal regime of flat-plate solar collectors and their main parameters, as solar radiation and ambient-temperature, which are included as factors, shall be taken Fourier series expansion. This approach makes the method applicable for the cloudy weather as well, which is proved by experimental researches. However, the system's operating conditions have been maintained on grounds of the analysis simplicity. To execute the constructive researches there has been elaborated the program for computer-aided calculations. The system's researched principal parameters: circulation tubes diameter and equipment placement, tank configuration, ratio of the solar collector length to the width, of the tank height to the diameter. Proceeding from the nature of the occurring phenomena of thermo physical activity in the solar collector there have been derived following assumptions: tank temperature to the liquid temperature in the tank, the tank is broken down into  $n$  sections with the uniform liquid temperature in each; the liquid temperature distribution along the receiving tank length is linear; the thermal processes, taking place between the separate areas of the model are characterized as average values of heat output factors within each area.

Taking into account the foregoing in the system's thermal model we can distinguish the areas:  $m$  – solar power receiving units;  $n$  – tank sections;  $k$  – conduits;  $i$  – collector's components. Let's formulate the energy conservation law for every thermal model area (fig. 4).

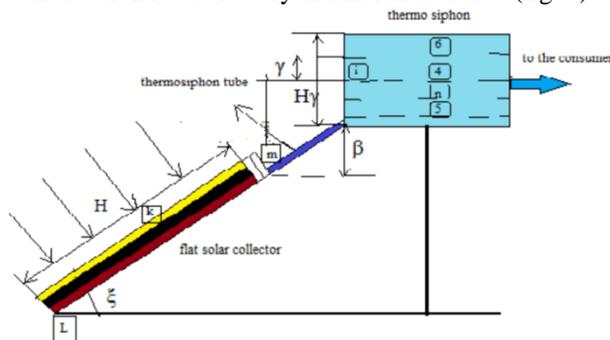


Figure 4. Model of the solar collector with a thermo siphon

Area  $m$ . The thermal flow absorbed from the sun by the receiver is spent for enthalpy changing  $C_n \frac{dt_c}{d\theta} + C_T \frac{dt_f}{d\theta} + G_0 C_p (t_3 - t_5)$  of the receiver and liquid in it, heat output to the elements  $\sum_m U_p (t_p - t_a)$  and the environment  $U_c (t_m - t_a)$ .

Area  $n$ . Thermal flow  $G_0 C (t_3 - t_5)$ , received by the liquid, consumed for changing its enthalpy  $C_\delta \frac{dt_f}{d\theta} + C_p \frac{dt_n}{d\theta}$  thermal output to the environment  $U_T (t_n - t_a)$ . According to the energy conservation law it is possible to write down for every thermal model's area the following:

$$G_0 C_p (t_3 - t_5) = \eta_n [E(\tau\alpha) - U_c (t_m - t_a)] - C_n \frac{dt_c}{d\theta} - C_T \frac{dt_f}{d\theta} - U_p (t_p - t_a) \quad (1)$$

$$G_0 C_p (t_3 - t_5) = U_m (t_n - t_a) + C_\delta \frac{dt_\tau}{d\theta} - C_p \frac{dt_n}{d\theta} \quad (2)$$

In compliance with the accepted assumptions we might assume, that the solar collector's temperature and that of the tank equal to the corresponding water temperature. Therefore, taking into account, that  $t_c = t$ ,  $t_0 = t_n$ ,  $t_f = t_p$  the equations take the form:

$$G_0 C_p (t_3 - t_5) = \eta_n F_k [E(\tau\alpha) - \eta_c U_c (t_m - t_a)] + U_p (t_p - t_a) - C_n \frac{dt_m}{d\theta} - C_T \frac{dt_p}{d\theta} \quad (3)$$

$$G_0 C_p (t_3 - t_5) = (C_p + C_\delta) \frac{dt_n}{d\theta} + U_T (t_n - t_a) \quad (4)$$

Based on experimental researches, the water average temperature in the accumulator tank is the same as in the solar collector within a day, which is true as well for the tubes temperature if to consider the average temperature of both tubes. Thereby, accepting that  $t_m = t_n = t_p$  and having excluded  $G_0$  from the equations (3) and (4), we obtain the equation for the system's average temperature ( $t_n$ ):

$$W \frac{dt_n}{d\theta} + U (t_n - t_a) = \eta_n F_k E(\tau\alpha) \quad (5)$$

$$W = C_n + C_T + C_\delta + C_p \quad (6)$$

$$U = \eta_n F_k U_c + U_T + U_p \quad (7)$$

However, even this simplification, which allows calculating the system's average temperature ( $t_n$ ) through solving the equation (5), is the key reason of the fact that the analysis thereof cannot predict the system's temperature and water reverse flow at night. We assume, that the system at night is disconnected ( $E=0$ ). Therefore, the water average temperature in the solar collector and that of the water in the accumulator tank at night is specified separately by solving the number of equations:

$$W_1 \frac{dt_m}{d\theta} + U_1(t_m - t_a) = 0 \tag{8}$$

$$W_2 \frac{dt_n}{d\theta} + U_2(t_n - t_a) = 0 \tag{9}$$

$$W_1 = W - W_2, W_2 = C_\delta + C_{\delta\delta} \tag{10}$$

$$U_1 = U - U_2, U_2 = U_m \tag{11}$$

Equations (5), (8) and (9) are not uniform differential equations with constant coefficients and might be easily solved by means of integrating factor. Applying the values of Fourier's series to  $E$  and  $t_a$ , obtained using the Fourier analysis, we will have:

$$E = A_0 + \sum_{n=1}^N (A_n \cos n\omega\theta + B_n \sin m\omega\theta) \equiv \tag{12}$$

$$\equiv A_0 + \sum_{n=1}^N R[(A_n - jB_n)e^{jn\omega\theta}]$$

$$t_n = C_0 + \sum_{n=1}^N (\cos n\omega\theta + D_n \sin m\omega\theta) \equiv \tag{13}$$

$$\equiv C_0 + \sum_{n=1}^N R[(C_n - jD_n e^{jn\omega\theta})] j = V - I\omega = 2\pi/24]$$

Solving the equation (5) is following

$$t_n = \frac{E_0}{x} [1 - e^{x(\theta-\theta_0)}] + t_0 e^{-x(\theta-\theta_0)} + \sum_{n=1}^N \left[ \frac{(xE_n - n\omega F_n)(\cos n\omega\theta - e^{-x(\theta-\theta_0)})}{(x^2 + n^2\omega^2)} + \frac{(n\omega E_n - xF_n)(\sin n\omega\theta - e^{-x(\theta-\theta_0)} \sin n\omega v_0)}{(x^2 + n^2\omega^2)} \right] \tag{14}$$

Where,

-  $t_0$  is the system's initial temperature corresponding to the period of the system's filling with fresh water,

- and  $X = \frac{U}{w}, E_n = xA_n + YC_n, n = 1, \dots, N$

$$Y = \frac{\eta_n F_n (\tau\alpha)}{w}; F_n = XB_n + YD; n = 1, \dots, N \tag{15}$$

To solve the equations (8) and (9) there were used corresponding values of  $U$  and  $W$  to calculate  $X$ ; the value  $Y$  is accepted as equal to the zero in case there is no sunshine. It follows that the equation (14) shows an average temperature in the system within all hours, when there were taken the appropriate constant values and there was no reverse thermo siphon flow (at night).

### 3. Results. Theoretical and mathematical analysis of double-circuit solar installation with thermo siphon circulation

To carry out the theoretical and mathematical analysis the installation has been broken down into four units backed

by thermal hydraulic circuits: 1<sup>st</sup>, formed by a thermo siphon and solar collector; 2<sup>nd</sup>, solar collector and heat conductor evaporator; 3rd, formed by the heat conductor condenser and accumulator tank, and the 4<sup>th</sup>, in the centre, formed by the heat conductor itself.

Inasmuch that the circuits form the sequencing circuit of units, the heat production maximum might be achieved at heat production capacity of every separate circuit, and inside the circuit – at maximum heat production capacity of every separate element.

#### 3.1 Theoretical analysis of the 1<sup>st</sup> circuit

The motions by means of natural convections formed in the result of water heating in the solar collector, produced by thermo siphon influence to define the liquid flow rate shall be considered at any moment, and the density in different points of 1-6 flow circulation. It is assumed that density distribution in the solar collector and tank is linear and power losses in the tube are insufficient comparing to heating in the collector. Following the description and supposing the quadratic dependence density-temperature:

$$D_w = At^2 + Bt + C \tag{16}$$

It may be shown, that thermal head, formed by solar heating, is defined through the equation:

$$h = \frac{(t_5 - t_3)}{2} [2At_n + B] f(s) \tag{17}$$

Where  $f(s)$  - system function depending on the lot of parameters: solar collector geometry, tubes location water height in the tank ( $H_t$ ), solar collector height and slope. Let's write down those dependencies

$$f(s) = l_0 \sin \varepsilon + 2\beta(H_t - \gamma) \tag{18}$$

$$t_n = \frac{(t_5 + t_3)}{2} \tag{19}$$

excluding  $(t_5 - t_3)$  from (16) and (19), we obtain

$$h_1 = \frac{[(W_{\omega t} + W_s)] \frac{dt_n}{d\theta} + U_T(t_n - t_a)}{2G_0 C_p} [2At_n + B] + f(s) \tag{20}$$

the thermo siphon flow rate will be such, that at any moment the thermal pressure head is balanced by means of frictional head loss ( $h_e$ ) in the circulation flow. Based on Darcy's equation the value of functional head loss might be represented as follows:  $h_f = \frac{fLU^2}{2gd_p}$ , where  $f$  equals to 0,035 for ordinary flows in the systems, which are probably the luminary flows. Applying the speed ( $U$ ) to the function of mass flow rate ( $G_0$ ) we will obtain:

$h_f = \frac{DG_0^2}{dp^2} l$  where D is the known invariable: 1 –effective length of circulation flow cycle with an internal diameter dp, depending on the collector configuration and tube location. For the collector having parallel tubes between water collector (collectors) and the tube location, it is as follows

$$l = \left\{ \frac{Sl_T}{l_k} \left( \frac{dp}{dc} \right)^5 + l_k \left( \frac{dp}{dh} \right)^5 \right\} + \{l_T + H_t - \gamma + \beta\chi(1 - \cos e\epsilon)\} + l_c + L_0 \tag{21}$$

having solved (20) and (21), we obtain the equation of thermo siphon flow:

$$G_0^3 = - \left( \frac{dp^5}{2lDC_p} \right) \left\{ W_{\alpha\chi} + C_\delta \frac{dt_n}{d\theta} + U_T(t_n + t_a) \right\} \{2At_n + B\} f(s) \tag{22}$$

the equation (22) for  $G_0$  is three-dimensional. Determining the values  $t_n$  and  $dt_n / d\theta$  the equation (14), through corresponding substitutions might be defined as  $G_0$ . With knowledge of  $G_0$  and  $t_n$  applying the equations (14) and (19) we can also define the temperatures  $t_3$  and  $t_5$  at the tank inlet and outlet. Proceeding from the above we can define all system’s parameters as well as performance efficiency of the solar collector and the overall system.

### 3.2 Theoretical analysis of the 2<sup>nd</sup> circuit

To solve the theoretical analysis of the 2<sup>nd</sup> circuit let’s formulate processes total of the 2<sup>nd</sup> circuit, which will be expressed by the thermal balance equation for the 2<sup>nd</sup> circuit:

$$(E - Q_0 - Q_1 + 2Q_2) \cdot S - Q_3 = C_1 \cdot \frac{dt_1}{d\tau} \tag{23}$$

where:

$$\int_0^\tau d\tau = C_1 \cdot \int_{t_1=t_0}^{t_1} \frac{dt_1}{[E - k_0E - k_1 \cdot E \cdot (1 - k_0) + U_L \cdot (t_0 - t_1)] \cdot S - W_{EL} \cdot (a - 1) + W_{EL}b \cdot (t_4 - t_1)} \tag{26}$$

$$t_1 = - \frac{[W_{EL}b \cdot (t_4 - t_1)] \exp \left[ - \frac{(W_{EL}b + U_L) \cdot \tau}{C_1} \right]}{(W_{EL}b + U_L S)} \tag{27}$$

$$+ \frac{[E(1 - k_0 - k_1 + k_1 \cdot k_0) \cdot S - W_{EL} \cdot (a - 1)] \left\{ 1 - \exp \left[ - \frac{(W_{EL}b + U_L) \cdot \tau}{C_1} \right] \right\} + W_{EL}b \cdot t_4 + U_L S \cdot t_0}{W_{EL}b + U_L S} \tag{28}$$

$$t_1 = \frac{[E - k_0E - k_1 \cdot E \cdot (1 - k_0) - U_L \cdot t_0] \cdot S - W_{EL} \cdot (a + b \cdot t_4 - 1)}{W_{EL}b + U_L S}$$

–  $C_1$  – total heat capacity of the 2<sup>nd</sup> circuit (includes the heat conductor’s heat capacity, material), kkal /degree;  
 –  $t_1$  – heat conductor current temperature; t- time, hour.  
 From (23) it follows, that the heat capacity criterion may

be the value of the derivative, that is the bigger, it is the higher than the thermal capacity:

$$\frac{dt_1}{d\tau} \Rightarrow max \tag{24}$$

Based on (23) there has been drawn up an expanded equation of the thermal balance:

$$[E - k_0E - k_1 \cdot E \cdot (1 - k_0) + U_L \cdot (t_0 + t_1)] \cdot S - W_{EL} \cdot [a - b \cdot (t_4 - t_1) - 1] = C_1 \frac{dt_1}{d\tau} \tag{25}$$

Executed the solution of the equation (10):

There is obtained the dependence of the current temperature of the 2<sup>nd</sup> circuit heat conductor on the concomitant factors total:

Conducted checking of (27). Obtained at  $t_1=t_0$ , that meets the condition. At we obtain the formula for the heat conductor temperature calculation at the outlet of the 1<sup>st</sup> circuit under the set operation mode

The analysis has shown that at operating the double circuit solar installation with the heat conductor there is changed the direction of the heat flow between the solar collector and the environment. Accordingly, the solar collectors principal components, protecting it from the environment impact (translucent cover, thermal insulation, body) lose functional meaning and exert negative effect on the heat exchange process, detaining the heat input from atmospheric air. Refusal from the constructive elements thereof brings to raising the thermal performance of the 1<sup>st</sup> circuit and decreasing materials-output ratio and the solar collector’s cost to 50%.

### 3.3 Theoretical analysis of the 3<sup>rd</sup> circuit

To solve the theoretical analysis of the 3rd circuit let’s formulate the processes complex, which will be expressed by the equation of the circuit’s thermal balance equation:

$$Q_6 + Q_{EL} - Q_7 - Q_8 - Q_9 = C_3 \cdot \frac{dt_6}{d\tau} \tag{29}$$

where:  $C_3$  – 2<sup>nd</sup> circuit’s heat conductor’s thermal capacity kkal/ degree;  $t_6$  – the water current temperature

in the accumulator tank;  $Q_{\text{out}}$ —thermal flow from there serve source;  $Q_7$  and  $Q_8$  -accordingly, the thermal power, outgoing for hot water supply and heating;  $Q_9$  – accumulator tank thermal losses. Through analyzing the equation (29) we reduced it to an expanded form:

$$Q_6 + P_3 \cdot \tau - k_{var} \cdot \sum n_{7i} v_{7i} - V_8 \cdot n_8 - k_{T3} \delta_3 \cdot S_3 \cdot (t_7 - t_8) = C_3 \cdot \frac{dt_7}{dt} \quad (30)$$

where:

–  $k_{var}$  – consumption simultaneity factor is accepted in average as equal to 0,7...0,8;  $n_{7i}$  and  $v_{7i}$  – consumption amount and norm of i consumer of hot water;

–  $V_8$  – square of the heated premise;  $n_8$  – heating norm for a climatic zone.

The output of the solution (30) turned out to be the dependence of accumulator tank current heat conductor’s temperature dependence on the factors complex:

$$t_7 = t_8 + \frac{Q_6 + P_3 \cdot \tau - k_{var} \cdot \sum n_{7i} v_{7i} - V_8 \cdot n_8}{k_{T3} \delta_3 \cdot S_3} \left[ 1 - \exp\left(-\frac{(k_{T3} \delta_3 \cdot S_3) \tau}{C_3}\right) \right] \quad (31)$$

For verification of the formula correctness there used the values  $t_7$  in the characteristic points. For instance, at we obtain the expected value  $t_7 = t_8$ . We obtain the formula for calculating the heat conductor’s temperature at the outlet for the set operation regime:

$$t_7 = t_8 + \frac{Q_6 + P_3 \cdot \tau - k_{var} \cdot \sum n_{7i} v_{7i} - V_8 \cdot n_8}{k_{T3} \delta_3 \cdot S_3} \quad (32)$$

The analysis has proved that for raising the thermal performance the most effective is the mode of constant heat consumption from the accumulator tank at hot water supply and heating. It promoted the maximum heat output  $Q_6$ , and consequently, maintenance of the thermal conductor’s efficient work, securing the thermal conductor’s maximum transformation rate . To increase the thermal performance it is needed as well to secure the accumulator-tank’s heat insulation.

### 3.4 Theoretical analysis of heat pump circuit the 4<sup>rd</sup> circuit

To conduct the theoretical analysis of the thermal pump let’s formulated the thermal pump’s processes aggregate by means of the thermal balance equation:

$$Q_3 + W_{EL} \cdot \eta_{EL} \cdot \eta_k + Q_4 - Q_5 - Q_6 = C_2 \cdot \frac{dt_5}{dt} \quad (33)$$

where:

–  $W_{EL} \cdot \eta_{EL} \cdot \eta_k$ – the power inletting the circuit at the expense of the work carried out by the compressor;

–  $Q_4$  – recoverable heat, generated by compressor in the working process;

–  $Q_5$  and  $Q_6$  – accordingly, heat losses through the condenser’s heat exchanger surface and the energy output by the thermal pump to the accumulator tank;

–  $C_2$  – heat capacity of the condenser circuit’s heat conductor;

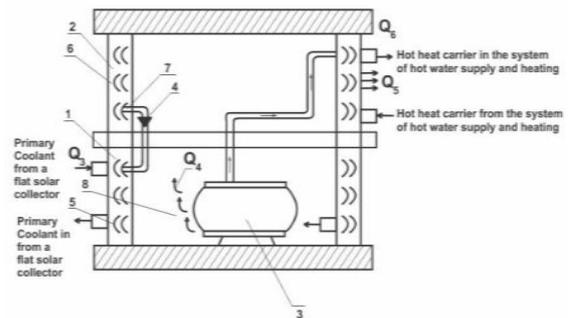
–  $t_5$  – heat conductor’s current temperature at the outlet from the circuit.

The main element of the circuit being researched is the thermal pump. Figure 2 shows the scheme of the new technological solution of the thermal pump in section. Offered device allows utilizing the heat output by the compressor in the operation process and simultaneously cooling.

### 3.5. Main constructive parameters of the thermal pump

The main element of the investigated circuit is a heat pump. Figure 5 shows a diagram of the new technical solution of the heat pump in the section. The proposed device allows to utilize the heat emitted by the compressor during operation and at the same time to cool it.

**Table 1.** The main design parameters of the heat pump



1 – evaporator heat exchanger; 2 – condenser’s heat exchanger; 3 - compressor; 4 – throttle valve; 5 and 6 – pipes for evaporator and condenser’s coolant; 7- hole for pipes input into the condenser body; 8 – netted barrel sheet-divider of air streams

**Figure 5.** Design scheme of a new technological solution of thermal pump for double circuit solar station and principal constructive parameters

Evaporator 1 and condenser 2 heat exchangers are manufactured in the form of ring-shaped vessels with internal and external cylindrical vessels with radiuses  $R_1$  and  $R_2$  and installed coaxially one over another, evaporator from below, condenser from above, forming the internal cylindrical air cavity. In the evaporator’s heat exchanger cavity there is located a compressor 3. To maintain optimal heat exchange from the compressor to evaporator in the gap between them there is placed the barrel sheet-divider 8. Inside the ring-shaped vessels there circulated heat conductors of the 1<sup>st</sup> and 2<sup>nd</sup> circuits, taking off the heat from the tubes 5 and 6. Thus, a compressor actually is inside the «cold pot», the walls of which are cooled with the pipes’ coolant. 5. In the result the heat, yielded by a compressor is absorbed by the heat

conductor evaporator, increasing its performance, and a compressor is simultaneously cooled without a ventilator. Based on the equation (33) there was compiled the balance's expanded equation:

$$Q_3 + W_{EL} \cdot \eta_{EL} \cdot \eta_k + Q_3 \cdot (1 - \eta_{EL}) - k_{T2} \cdot S_2 \cdot (t_5 - t_8) - W_{EL} \cdot [a - b \cdot (t_5 - t_1)] = C_2 \cdot \frac{dt_5}{dt} \tag{34}$$

where:

-  $t_8$  - ambient temperature;

-  $k_{T2}$  - thermal transmission factor of the thermal conductor's heat exchanger thermal insulation of the thermal conductor, kkal/degree.

Derived the dependence of the thermal conductor temperature:

$$t_5 = \frac{k_{T2} S_2 \cdot t_8 + [Q_3 + W_{EL} \eta_k + Q_3 \cdot (1 - \eta_k) + W_{EL}(1 - \eta_{EL}) - W_{EL} a - W_{EL} \cdot b \cdot t_1] \cdot \left\{ 1 - \exp\left(-\frac{[k_{T2} S_2 + W_{EL}(a-b)] \cdot \tau}{C_2}\right) \right\}}{W_{EL} \cdot b \cdot \exp\left(-\frac{[k_{T2} S_2 + W_{EL}(a-b)] \cdot \tau}{C_2}\right) + k_{T2} S_2 + W_{EL} \cdot b} \tag{35}$$

$$t_5 = \frac{k_{T2} S_2 \cdot t_8 + [Q_3 + W_{EL} \eta_k + Q_3 \cdot (1 - \eta_k) + W_{EL}(1 - \eta_{EL}) - W_{EL} a - W_{EL} \cdot b \cdot t_1]}{k_{T2} S_2 - W_{EL} \cdot b} \tag{36}$$

Analysis has shown that the new solution, without account of power expenses for the cooling ventilator drive, promotes raising the thermal performance of the double circuit solar installation.

#### 4 CONCLUSION

Executed theoretical and mathematical analysis of the double-circuit solar installation with the thermo siphon circulation is broken down into four conventional units, represented by thermal-hydraulic circuits: 1<sup>st</sup>, formed by thermo siphon and thermal condenser; 2<sup>nd</sup>, thermal collector and heat conductor evaporator; 3<sup>rd</sup>, formed by the heat conductor condenser and accumulator tank and the 4<sup>th</sup> in the center, formed by the heat conductor itself.

Proceeding from the theoretical analysis of the 1<sup>st</sup> circuit we obtained the equation of the thermo siphon circuit, which shows how to define all parameters of the system, as well as efficiency coefficient of the solar collector and overall system. Proceeding from the 2<sup>nd</sup> circuit analysis we obtained the dependence of the 2<sup>nd</sup> circuit's heat conductor current temperature, which specifies the thermal flow direction between solar collector and environment, solar collector principal elements, protecting it from external impact (translucent cover, thermal insulation, body). Analyzing the 4<sup>th</sup> circuit we obtained the formula for heat conductor design temperature calculation at the outlet. In the 4<sup>th</sup> circuit the new solution, without regard to expenses on the power

for the cooling ventilator drive, promotes the double circuit solar station's thermal performance rise.

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