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Sâlih Zeki's First Lecture in *Dârü'l-fünûn Konferansları:* Locating its Main Source*

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Abstract

Sâlih Zeki presented a series of lectures on mathematics, which were later published in the old Turkish script. They are about certain developments and fields that arose in mathematics in the 19th century. He talks in a concise and historical manner about non-Euclidean geometries and their discovery in the first five lectures of the first volume. In the first lecture, he presents the gist of his views concerning how these geometries were discovered.

Sâlih Zeki's lecture seems to be the first addressing and dealing with this discovery among the available printed materials in Turkish. It, thus, certainly deserves to be examined. My aim is to determine his main source on which he structured his account of this discovery in order to appreciate, and asses better his mathematical, philosophical and methodological concerns.

Keywords

Sâlih Zeki, Dârü'l-fünûn Konferansları, non-Euclidean geometries, discovery of non-Euclidean geometries.

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INTRODUCTION

Sâlih Zeki (1864-1921) presented a series of lectures on mathematics at the Lecture Hall of Istanbul University in the years between 1914 and 1916. These lectures were published in 1915 and in 1916 in the old Turkish script with the title *Dârü'l-fünûn Konferansları* (*DFK*) in two volumes. These lectures are about certain developments and fields that arose in mathematics in the 19th century. He talks in a concise and historical manner about non-Euclidean geometries and their discovery in the first five lectures of the first volume. However, in the first lecture of the first volume he presents the very gist of his views concerning how these geometries were discovered.

Sâlih Zeki's lecture seems to be the first addressing and dealing with this discovery among the available printed materials in Turkish. It, thus, certainly deserves to be examined. My aim in this study is to determine his main source on which he structured his account of this discovery in order to appreciate, and asses better his mathematical, philosophical and methodological concerns.

SOME PRELIMINARIES TO DFK

One could generally describe the lectures in *DFK*[1] in the following way: the first volume has 14 lectures concerning non-Euclidean geometries and their discovery, and the second volume contains 5 lectures about imaginary and complex numbers.

On the first page of the first volume there is a saying from Henri Poincaré reading: "we mathematicians work for physics and philosophy".[2] On the next page appear Sâlih Zeki's introductory notes. It seems that he must have been approached to deliver some talks on mathematics. He immediately adds: the particular mathematical fields ("mesâlik-i hussûsiyye-i riyâziyye"), about which he would talk, arose fully in the 19th century, and were important from both mathematical and philosophical points of view. Although there might be some people objecting the relevance of philosophy to these issues, real scientists were perfectly convinced about their philosophical significance. He says that he would be talking in particular about the new geometry in the meetings during the teaching period of the years 1330-1331 (1914 and 1915). However, he says that in order to provide a basis for the later talks about this particular scientific development, the initial lectures would be historical and introductory. Although he indicates that these lectures were intended for the teachers and lovers of mathematics, even those who were not mathematicians, but philosophers, could easily understand these lectures. The first volume is of 224 pages long.

On the first page of the second volume, the same motto also appears. On the same page, Sâlih Zeki indicates that these lectures were intended for the same audience, and were delivered in the meetings during the teaching period of the years 1331-1332. The second volume is of 112 pages long. On the next page, Sâlih Zeki's introductory notes appear. It seems that he had wanted to deliver his lectures on group theory, in particular continuous groups ("zümre-i mütemadiyye"), but he was requested to deliver his talks on imaginary quantities ("kemiyyât-ı mevhûme") and imaginary numbers ("âddad-ı mevhûm").

LOCATING THE MAIN SOURCE IN THE FIRST LECTURE

In this section, I will trace the main source Sâlih Zeki utilised in providing his account of the discovery of non-Euclidean geometries. We do not know much about Sâlih Zeki's sources he consulted in writing the first lecture. There seems to have been one previous attempt in this respect. This is the one by Dilek Kadıoğlu who, although correctly picks up a certain book as Sâlih Zeki's main source, partly deals with the issue in her Master's thesis (2013: 73-79). I shall refer to her study where the need arises, but will postpone generally considering her attempt in the concluding remarks.

Sâlih Zeki in *Âsâr-ı Bâkiye* tells us how he developed an interest in the history of mathematical studies conducted in the era of Islam (2003: xv-xix). However, he does not provide any reason for his interest in non-Euclidean geometries and the history of their discovery. There is yet another piece of information that might shed some light on the issue. It is from his memoirs (Sâlih Zeki 1924: 682-707), in which he writes that Hüseyin Tevfik Paşa of Vidin (1832-1901) used to hold evening meetings at his house, but he does not say anything about the date and the frequency of these meetings. But the subject-matter in these meetings is non-Euclidean geometry. What he writes can be summarised in the following way: they got together in the evenings on Fridays, and on those days what Europeans called non-Euclidean geometry or new geometry ("hendese-i cedide") had been one of the important matter of these discussions. More or less the same state of affairs was once pointed out by Felix Klein in the same spirit: non-Euclidean geometry is "one of the few parts of mathematics which is talked about in wide circles, so that any teacher may be asked about it at any moment" (Klein 2004: 185).

Sâlih Zeki requested his host to provide some information about this new geometry. About 15 minutes later the Paşa came in with a collection of books, booklets, articles from journals and newspapers, some of which were hand written, and then began his exposition of the new geometry starting from the beginning, how Nikolai I. Lobachevsky of Russia and Johann Bolyai of Hungary objected Euclid's theory of parallels, and how they argued and proved that the Eleventh Postulate of Euclid had no connection with the other axioms and postulates, how around the same time Carl F. Gauss, independently of these studies, was certainly aware of these issues; and years later, how Bernhard Riemann and Hermann von Helmholtz tried to overcome the problem of parallels. The Paşa was talking about these with confidence and providing references.

Sâlih Zeki writes that he could not describe how much he had enjoyed these friendly discussions because all the detailed information the Paşa supplied could not be found in one single book. One should appreciate the level of the Paşa's interest in and enthusiasm for the subject. He collected all the writings of anybody who had written on the subject including Lobachevsky, Bolyai, Gauss, Riemann, Helmholtz, William Kingdon Clifford, Arthur Cayley and others. It appeared to Sâlih Zeki to be an impossible task to obtain all these books and articles published in different countries; but, the Paşa must have also read them all and constructed his views on the issue as well. What Sâlih Zeki writes in his memoirs can be taken as at least one of the motives for him becoming interested in and studying the subject. However, Sâlih Zeki does not say anything in particular about the books and articles the Paşa brought in, nor whether he used any of these printed materials in writing his university lectures.

Given the fact that Sâlih Zeki was not a mathematician or a philosopher by education, he must have consulted the works of other people in writing these lectures. Even if he had been, it would still have been very hard for him to develop his own views about the subject; for even most mathematicians and philosophers at the time were not interested in the subject. To the most of mathematicians then the relevance of non-Euclidean geometries to mathematics was as that of metaphysics to physics. There was a handful of mathematicians and philosophers then being aware of the mathematical and philosophical significance of non-Euclidean geometries.[3]

There were several studies published in the years between 1878-1911 compiling bibliographies of the works concerning non-Euclidean geometries, the last of which was produced by Duncan M. Y. Sommerville (1911):

> The titles of which Dr. Sommerville takes into account are over 4,000 in number. They may be roughly classified as follows: theory of parallels 700, non-Euclidean geometry and the foundations of geometry 1,600, n dimensions 1,800. (Archibald 1912: 255)

But Sommerville's compilation is not to be regarded as complete (cf. Archibald 1912: 258). Subsequently, Sâlih Zeki's main source should be among these 1600 or so works. Thus, searching for his main source by itself would not be an easy task to achieve. One would need some heuristics to narrow down the possible candidates for his main source.

There is an interesting remark by Sâlih Zeki that can be considered a good starting point. What he writes there (1331:12-13) amounts to the following: of famous mathematicians, Klein divides the studies concerning non-Euclidean geometries into three periods, each of which was conducted with respect to a particular point of view. The first period begins with Gauss, Lobachevsky and Bolyai, and goes to Riemann. The main aim of this period was to establish a new geometry without entailing any inconsistencies on the basis of Euclid's postulates except the postulate of parallels. The second period starting with Riemann had a totally different character; for, what Riemann's paper, which was read in an almost secret-like-meeting held at the Philosophy Faculty Committee, addressed was neither the theory of parallels nor directly non-Euclidean geometry. The main issue of this paper was the relationship among the postulates of geometry, or particularly, the problem of space. This paper involves another new non-Euclidean geometry, coined Riemann's geometry, though the discovery of this new non-Euclidean geometry was a necessary result of the paper. Because of this feature, this second period was important not only from a mathematical, but a philosophical point of view as well.

Sâlih Zeki's presentation of Klein's classification can be taken such a heuristic, since his account of the discovery of this new geometry is based upon this division. So, Sâlih Zeki takes this division of Klein's as a historical-mathematical-philosophical basis for his account. However, Sâlih Zeki does not cite any reference to Klein's division. Klein writes in his lecture of 1893 (1893: 79-80):

> There are three points of view from which non-Euclidean geometry has been considered.
>
> (i) First, we have the point of view of elementary geometry, of which Lobachevsky and Bolyai themselves are representatives. Both begin with simple geometrical constructions, proceeding just like Euclid, except that they substitute another axiom for the axiom of parallels. …
>
> (ii) From the point of view of projective geometry. …
>
> (iii) Finally, we have the point of view of Riemann and Helmholtz. Riemann starts with the idea of the element of distance.

So, given the fact that Sâlih Zeki knew only French, the question is: were these lectures translated into French? They were so by Léonce Laugel (1898: 85-93). So, it seems possible that he might have read its French translation. Nonetheless, one should not immediately jump to the conclusion that Sâlih Zeki read Klein's lecture; for there is a certain difference between Klein's own division and Sâlih Zeki's presentation of Klein's distinction. The difference lies in the way they each present the distinction. In Klein's division, the first is the point of view of elementary geometry, the second one is the point of view of projective geometry, and the last one is the point of views of Riemann and Helmholtz. In Sâlih Zeki's division, the first is the point of elementary geometry and the second one is the point of view of Riemann and Helmholtz. The second point of view in Klein's distinction does not appear in Sâlih Zeki's first lecture at all. However, it does so in his 4th lecture of the first volume. So, there appears not only a change of the order of the periods, but the point of view of projective geometry is omitted. The presence of these differences by itself of course does not necessarily nullify the possibility that Sâlih Zeki might have read the French translation. However, it may also be taken as an indication that Sâlih Zeki might have got Klein's classification from somewhere else. Because of these changes in his presentation, one would tend to think that he must have got it from another book.

In his translation, Laugel mentions a book by Bertrand Russell (1898: 122-123). In the *Preface* of this book, Russell writes:

> My chief obligation is to Professor Klein. Throughout the first chapter, I have found his "Lectures on non-Euclidean Geometry" an invaluable guide; I have accepted from him the division of Metageometry into three periods...

Russell also provides a more detailed exposition of Klein's distinction later in his book (1897: 7-9, 10-53). So, Russell too employs Klein's distinction in his study as a historical basis. However, this is not the only reason for Russell to refer to Klein. He also takes for granted Klein's study on the foundations of geometry that projective geometry is the foundation for any kind of geometry. Russell argues on the basis of this mathematically established claim for maintaining his neo-Kantian view that projective geometry is the *a priori* basis of our knowledge.

So, the other possible source mentioned above might be Russell's book; for both Sâlih Zeki and Russell employ the division exactly in the same way. However, Sâlih Zeki did not have any English. But one might wonder if its French translation had been available then. As it turns out, its French translation was available (Russell 1901). This is a revised edition and translation, which has a number of corrections and additions not to be found in the English edition. So, the question is: which of Russell's works –the English or the French edition- is the one Sâlih Zeki was using as his main source? It will presently suffice to say that Sâlih Zeki appeals to Russell's book in English. However, I shall postpone to the concluding remarks to deal with the question of which edition he used. So, Russell's works appear to be the best candidate for Sâlih Zeki's presentation of Klein's division. Since Sâlih Zeki's account is based upon this distinction and Russell's book seems to be Sâlih Zeki's main source, one would naturally expect to come across more references to these sources in his lecture with increasing frequency.

Sâlih Zeki begins his lecture by writing (1331: 4):

> The strong fortress ("hısn-ı metin") constructed by Euclid in the 4th BC had remained invincible for about twenty some centuries. In particular, it had been the only secure place for rationalists/idealist ("hayâliyyûn") during the war between empiricists ("ihtibâriyyûn") and rationalists throughout the 17th and 18th centuries.

> Those philosophers who held that an exact science independent of experience and yet applicable to the real world was possible needed only to point to Euclid's geometry for this purpose. It is because no one dared to doubt its soundness and its relevance to the real world.
>
> Finally, the great philosopher of that time Kant came almost to claim that if geometry has an apodeictic certainty, its matter, that is, space, must be theoretical/speculative ("nazarî"), and if, conversely, space is just theoretical, then geometry must have an apodeictic certainty.

The term "nazarî" (theoretical or speculative) Sâlih Zeki employs here to refer to Kant's notion of space. However, Kant never employs either term to describe his understanding of space. The terms he employs are *a priori* and pure form of intuition (see Kant 1985, 1987). So, Sâlih Zeki's term seems to be really missing the gist of Kant's theory of space and geometry. Two possibilities are available for us here: either he did not understand at all Kant's theory of space and geometry, or he was misled by reading someone else.

Sâlih Zeki was not only misled by, but borrowed his first three paragraphs from the same source without citing any reference to it (Russell 1897: 1):

> Geometry, throughout the 17th and 18th centuries, remained, in the war against empiricism, an impregnable fortress of the idealist. Those who held –as was generally held on the Continent- that certain knowledge, independent of experience, was possible about the real world, had only to point to Geometry: none but a madman, they said, would throw doubt on it validity, and none but a fool would deny its objective reference.

And

> It was only through Kant, the creator of modern Epistemology, that the geometrical problem received a modern form. He reduced the question to the following hypotheticals: If geometry has apodeictic certainty, its matter, i.e. space, must be a priori, and as such must be purely subjective; and conversely, if space is purely subjective, Geometry must have apodeictic certainty.

The two passages above are from Russell's book. As it is clear, there seems to be an almost perfect match between Sâlih Zeki's passages and Russell's passages. After writing that the major weakness of this "impregnable fortress" was the postulate of parallels[4], Sâlih Zeki keeps on that there had been so many people in history to fix this problem and none was successful. However, he does not state any reasons why no attempt was successful. One of these people who tried to reform and put geometry into proper order he says is Legendre (1331: 5). In what follows, Sâlih Zeki describes generally Legendre's study, but he does not say where Legendre does this.

Sâlih Zeki then provides Legendre's definition of parallel lines: "Parallels are straight lines in the same plane, such that, if a third line cuts them, it makes the sum of the interior angles on the same side equal to two right angles" (1331: 5). He goes on:

> Even though in the light of this definition the proposition whether or not two parallel lines can cut one another can be proved, this proof cannot be good enough to prove the proposition that two parallels in the same plane must cut one another. Because of this, Legendre attempted to prove the corollary ("netice-i lazıme") without the parallel postulate: the sum of three angles of a rectilineal triangle ("müselles-i müstakîmü'l-adla'") is equal to two right angles. However, the only conclusion he establishes is that the sum of the three angles of a rectilineal triangle cannot exceed two right angles. If he could have established at the same time that they could not be smaller, he could have proven the parallel postulate. But he failed in this. Instead he proved that if he could prove that the sum of the three angles of a rectilineal triangle is equal to two right angles, the sum of three angles of all triangles would equal to two right angles. But he was not able to prove the existence of such a triangle of which the sum of the three angles is equal to two right angles. (1331: 5-6)

Although this is an inference, it is still possible that Sâlih Zeki might have come to this conclusion by reading Legendre's book of 1794 or his work of 1833 (1833: 367-411). Or it is also possible that he might have got this information from someone else's study.

Russell (1897: 7) makes some observations concerning Legendre's study:

> Parallels are defined by Legendre as lines in the same plane, such that, if a third line cut them, it makes the sum of the interior and opposite angles equal to two right angles. He proves without difficulty that such lines would not meet, but is unable to prove that non-parallel lines in a plane must meet. Similarly, he can prove that the sum of the angles of a triangle cannot exceed two right angles, and that if any triangle

> has a sum equal to two right angles, all triangles have the same sum; but he is unable to prove the existence of this one triangle.

As is obvious, there is almost exact correspondence between Sâlih Zeki's treatment of Legendre's study and that of Russell's. Sâlih Zeki's passage is Turkish translation of Russell's one. Sâlih Zeki again does not cite any reference here.

Sâlih Zeki goes on (1331: 6-7):

> On the one hand Gauss and on the other Lobachevsky and Bolyai, independently of each other, asserted the following thought: 'if Euclid's postulate is logically derivable from the rest of the postulates and axioms, then it is not possible not to have any contradiction within a new geometry that could be constructed by denying the postulate itself and retaining all the rest'.
>
> Each of these three-people constructed a new geometry on the basis of denying Euclid's postulate and keeping all the rest of the postulates and common notions and they did not find any contradiction at all in this geometry.

Russell in talking about the failure of Legendre writes (1897: 7-8):

> Thus Legendre's attempt broke down; but mere failure could prove nothing. A bolder method, suggested by Gauss, was carried out by Lobatchewsky and Bolyai. If the axiom of parallels is logically deducible from the others, we shall, by denying it and maintaining the rest, be led to contradictions. These three mathematicians, accordingly, attacked the problem indirectly: they denied the axiom of parallels, and yet obtained a logically consistent Geometry. They inferred that the axiom was logically independent of the others, and essential to the Euclidean system.[5]

The similarity between the passages from both Sâlih Zeki and Russell is striking, and he must have got it from Russell.

Sâlih Zeki, while talking about Gauss, points out (1331: 7):

> However, he did write almost nothing concerning this new geometry. Although we know some of the results Gauss had obtained today, their proofs are missing. Nevertheless, as I shall explain below, there is no doubt that Gauss was the first to investigate the results that were to be obtained by denying Euclid's postulate. He actually called this

> geometry, which he envisaged to be constructed in this way, non-Euclidean geometry in order to differentiate it from Euclid's one. This is clearly seen in his correspondence with his friends among whom Wolfgang Bolyai, the father of Bolyai, was. The first letter Gauss was talking about it was written in 1795.

Russell also writes (1897: 10):

> The originator of the whole system, Gauss, does not appear, as regards strictly non-Euclidean Geometry, in any of his hitherto published papers, to have given more than results; his proofs remain unknown to us. Nevertheless, he was the first to investigate the consequences of denying the axiom of parallels, and in his letters he communicated these consequences to some of his friends, among whom was Wolfgang Bolyai. The first mention of the subject in his letters occurs when he was only 18; four years later, in 1799, writing to W. Bolyai, he enunciates the important theorem that, in hyperbolic Geometry, ...

It is obvious that this passage also comes from Russell's book too.

The next person Sâlih Zeki mentions is Lobachevsky and some of Lobachevsky's studies. He writes that "[Lobachevsky's] first publication about this begins with the one published in Courier de Kasan in 1829" (1331: 7). So, he states the title of the journal where it was published and the year of its publication, but he does not provide the title of the work.[6]

Sâlih Zeki goes on to say (1331: 7):

> Lobachevsky published a long article in analysis concerning imaginary geometry and its French translation was also published in 1837 in Journel de Crelle. That work did not get any attention until it was 30 years later unearthed by Beltrami, who later provided a very important and meaningful interpretation of this non-Euclidean geometry.

Russel writes (1897: 10-11):

> Lobatchewsky, a professor in the University of Kasan, first published his results, in their native Russian, in the proceedings of that learned body for the years 1829-1830. Owing to this double obscurity of language and place, they attracted little attention, until he translated them into French [in a footnote: Crelle's Journal, 1837] and German; even then, they do not appear to have obtained the notice they deser-

> ved, until, in 1868, Beltrami unearthed the article in Crelle, and made it the theme of a brilliant interpretation.

What Sâlih Zeki writes above must have come again from Russell's book.

Sâlih Zeki next mentions Johann Bolyai's work and provides some information about Bolyai's geometry by saying that it was constructed on the basis of the negation of Euclid's postulate, and that it was published as an appendix to the first volume of his father's book *Tentament* in 1832 (see 1331: 8). Sâlih Zeki goes on (1331:8-9):

> Wolfgang Bolyai, the father of Johann, was a school friend of Gauss. They exchanged letters concerning the theory of parallels. What Bolyai the son did is very similar to what Lobachevsky did. Both independently arrive at the same results and theorems; there is no reason to distinguish their geometries from each other, except that the postulates Bolyai the son accepted are clearer than the ones Lobachevsky accepted.

Russell (1897: 11-12):

> Very similar is the system of Johann Bolyai, so similar, indeed, as to make the independence of the two works, though a well-authenticated fact, seem all but incredible. Johann Bolyai first published his results in 1832, in an appendix to a work by his father Wolfgang, entitled; "Appendix, ..." Gauss, whose bosom friend he became at college and remained through life, was, as we have seen, the inspirer of Wolfgang Bolyai, and used to say that the latter was the only man who appreciated his philosophical speculations on the axioms of Geometry; … Both as to method and as to results, the system is very similar to Lobatchewsky's, ... Only the initial postulates, which are more explicit than Lobatchewsky's, demand a brief attention.[7]

It is again Russell's book Sâlih Zeki draws on here.

Sâlih Zeki then mentions Riemann and Helmholtz and their studies. He writes (1331: 12):

> A very important event took place in Germany in 1854. That event was the meeting where Riemann read his paper "Hendesenin esasını teşkil eden faraziyeye dair"[8]. However, its publication was delayed due to Riemann's wish to introduce some corrections. It was published posthumously by Dedekind later in 1866. The content of this study

> had remained obscure. Helmholtz was by coincidence working about the same issue. He had not yet published any of his studies. But when Riemann's paper appeared in publication, Helmholtz also published an article entitled "Hendesenin Müesses Bulunduğu Mebâdiye Da'ir Muhtıra", which involved the results he had earlier obtained. Finally in 1868, he also published another paper "Hendeseye Esas Olan Hadisata Dair Makale" containing a proof of a theorem he had promised in his first paper to publish.

Sâlih Zeki does not provide any information on where they were published. Moreover, he gets wrong the publication date of Riemann's paper. It was published in 1867, not in 1866, and translated into French by Hoüel. In his translation, Hoüel writes that Riemann's paper was published in 1867 posthumously by Dedekind. So, if he was consulting Hoüel's translation, how could he make such a mistake? Is it just a slip of pen or is it a mistake due to something else? At this point it is not easy to claim whether or not he made use of Hoüel's translation; but Russell states (1897: 14):

> Riemann's epoch-making work, "Ueber die Hypothesen, welche der Geometriezu Grunde liegen", was written, and read to a small circle, in 1854; owing, however, to some changes which he desired to make in it, it remained unpublished till 1867, when it was published by his executors.

As for Helmholtz's first work, Sâlih Zeki is not clear when it was published. He just says that Helmholtz, as soon as Riemann's paper was in print, published his work. According to Sâlih Zeki, this work was a presentation, which had been first delivered around the same time of the publication of Riemann's in 1866, and then it was published as an article. He does not provide any reference for these two articles either. The first work Sâlih Zeki mentions of Helmholtz's should be the lecture he gave in 1866 "Ueber die thatsächlichen Grundlagen der Geometrie"; it is because he says that it is a presentation. The second paper he mentions should be Helmholtz's 1868 paper "Über die Thatsachen, die der Geometrie zum Grunde liegen". The confusion comes from the source Sâlih Zeki uses. Russell in his 1897 book writes (1897: 23):

> In this chapter, …, only two of his writings need occupy us, namely the two articles in the *Wissenschaftliche Abhandlungen, Vol. II., entitled respectively "Ueber die thatsachlichen Grundlagen der Geometrie," 1866*

(p. 610 ff.), and "Ueber die Thatsachen, die der Geometrie zum Grunde liegen," 1868 (p. 618 ff.).

Russell a bit further down writes (1897: 24): "The second article, which is mainly mathematical, supplies the promised proof of the arc-formula, which is Helmholtz's most important contribution to Geometry". Russell gives the date of the presentation of the first work as 1866.

Sâlih Zeki next turns to provide some information about Riemann's habilitation paper (1331: 13). Sâlih Zeki paraphrases and translates the first paragraph and the first two sentences of the second paragraph of Riemann's paper. He uses quotation marks to indicate that these passages are from the habilitation. He should have quoted these passages from Hoüel's translation (1870: 280); for this translation employs the term "varieté" for manifold as opposed to the term "multiplicité" employed in Russell's French translation.

Sâlih Zeki then writes about how Riemann defines the conception of a manifold, and applies this notion to determine space as a collection of magnitudes. He also describes how Riemann defines continuous and discrete manifolds and brings into discussion the idea of measurement being independent of position (1331: 14-15). He seems to be summarising above the first section of Riemann's paper (cf. Clifford 1873: 2-3, Hoüel 1870: 282-283). He seems to be most likely consulting Hoüel's translation while writing those passages. Nevertheless, he seems at the same time to be appealing to Russell's book. For sake of brevity, I will supply with the references here for the reader to compare both Sâlih Zeki's and Russell's passages (Sâlih Zeki 1331: 14-15 and Russell 1897: 14-16). Sâlih Zeki next treats the notion of the measure of curvature and its application on surfaces (1331: 15-17), which is not different from Russell's (1897: 16-18) either. On the issue of developable surfaces, Sâlih Zeki (1331: 18) does not deviate from Russell (1897: 18-19) as well. It is obvious that Sâlih Zeki's passage comes from Russell's book as well.

CONCLUDING REMARKS

Throughout the first lecture, Sâlih Zeki mentions some mathematicians and some of their works. However, it is Russell's book he must have used extensively. That is, he constructed his account of the discovery of non-Euclidean geometries on the basis of the structure of Russell's account presented in his book and he seems to have made use of other sources to supply certain information concerning certain actors of this discovery. However, for the sake of brevity, I have to omit those sources in this study.

As pointed above, Kadıoğlu, though correctly picks up Russell's book as the main source, singles out only some passages in Russell's book as being the sources of some certain passages in the first lecture, all of which are covered in this study as well. However, more passages bearing striking similarities in the two studies in question are brought about here, which are not addressed in her study. Moreover, Kadıoğlu does not bring into discussion the presence of the French edition of Russell's book, and thus, does not address the question of which edition of Russell's book Sâlih Zeki was using.

There remains one issue I have indicated above, but left it unresolved. This is the question of which edition of Russell's book Sâlih Zeki was using. There are certain hints in the first lecture that may point to which edition he was using. The first one is the usage of a priori being "purely subjective" in both of Russell's editions. Russell employs this term in psychological sense and in the English edition Russell explains what he means by this term in order not to cause any misunderstanding. Nevertheless, it resulted in some severe criticisms. So, he is at pains in the French edition to explicate the matter further to clear those misunderstandings and criticisms. Since Sâlih Zeki refers to Kant's theory of space as being "speculative" and does not elaborate the matter further, one can securely claim that it should be the English edition he benefited from. The last one is the usage of the term "manifold". Sâlih Zeki writes the French correspondence of this term as "varieté" in the lecture. However, the term "varieté" never appears in the French edition. Instead, it is the term "multiplicité" employed. This case is explained in a footnote by referring to Hoüel's translation of the habilitation; for the usage in the translation of Hoüel is "varieté".[9] So, the English edition appears to be as the most probable candidate.

Furthermore, the following questions need answers: why did Sâlih Zeki develop such a trust in Russell? How did he manage to read Russell's book in English? Sâlih Zeki's high esteem for Russell is pointed out in Halide Edib's chronicle of her visit to London in 1909 (1939, from Çalışlar 2010: 79-80). This means that he must have known about Russell before Halide Edib's visit in 1909. He could not have developed this interest by himself for the obvious reason that he could not read English. This cannot also come through Tevfik Paşa's meetings if these meeting were held around the summer of 1891;[10] for Russell entered Trinity College, Cambridge a year ago and the English edition of Russell's book would not be published until 1897.

However, Sâlih Zeki could still have learnt about Russell and his book through the Paşa if the Paşa's interest in the subject continued until his death in 1901. There are, however, no documents to support this conjecture at all. There is another possibility. In 1901 Poincaré published a very critical review of Russell's book of 1897 (see Poincaré 1899: 251-279). Sâlih Zeki might have learnt about Russell's book through this article. He might have obtained a copy of the book and studied it with the Paşa. Or he might have asked Halide Edib to translate it for him; for there is a passage describing the early days of her marriage with him (Halide Edib 1972: 207):

> I belonged to the new house and its master, gave the best I had, to create a happy home and to help him in his great work. He had begun at this time his colossal work in Turkish –the "Mathematical Dictionary"- and I prepared for him from different English authorities the lives of the great English mathematicians and philosophers.

Nevertheless, all these are again just conjectures at the moment.

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Endnotes

1 Turkish National Library in Ankara holds a copy of each of these volumes: EHT 1946 A1217 and EHT 1946 A1216 respectively. I thank Dilek Kadıoğlu for allowing me to use her transcriptions of these five lectures. Needless to say, some corrections are made at certain places in her transcriptions.

2 All the translations, unless stated otherwise, are mine.

3 Cayley may be cited here as an interesting example. See Gray (2008: 256-259).

4 Russell in his book (1897: 7) writes, "The liquefaction of Euclidean orthodoxy is the axiom of parallels". It is again almost translation in Turkish.

5 Kadıoğlu correctly identifies the similarity between these two passages (cf. 2013: 76-77).

6 The title of Lobachevsky's first publication on the subject is "O nachalakh geometrii" ("On the Principles of Geometry").

7 Kadıoğlu again correctly identifies the similarity between these two passages (see 2013: 77).

8 (Riemann 1867: 133–152). Riemann's paper was translated and published in French by Hoüel (1870: 309-326). Hoüel's translation was reprinted in Laugel (1898: 280-299). It was also translated into English in 1873 by Clifford (1873: 14-17, 36, 37).

9 The term "multiplicité" rather than "varieté" is also employed in, for example, Poincaré's 1899 paper (1899: 254).

10 However, in Sâlih Zeki's recollection of these tutorials with the Paşa, he does not state any date concerning these gatherings. Nevertheless, one can infer it to be the summer of 1891 from what Sâlih Zeki writes in these recollections.

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Sâlih Zeki'nin Dârü'l-fünûn Konferansları'ndaki İlk Konferansı: Ana Kaynağın Tespit Edilmesi*

Samet Bağçe**

Öz

Sâlih Zeki (1864-1921) Dârü'l-fünûn'un Konferans salonunda 1914 ve 1916 yılları arasında matematiğe dair bir dizi konferanslar vermiştir. Bu konferanslar daha sonra iki cilt halinde, sırasıyla 1915 ve 1916 senelerinde eski yazıyla Dârü'l-fünûn Konferansları (DFK) başlığıyla basılmıştır. DFK'nın Birinci Cildinin esas konusu, gayri-Öklidyen geometriler ve onların keşfi meselesidir. Sâlih Zeki, Birinci Cildin ilk konferansında, bu geometrilerin nasıl keşfedildiğine dair olan görüşünü sunmaktadır.

Sâlih Zeki'nin konferansı, Türkçe yazılı materyaller içinde bu yeni geometrilerin keşfinden bahsetme ve bu meselelerle uğraşma hususunda ilk olma özelliğine sahiptir. Sadece bu niteliğinden ötürü bile bu konferanslar üzerine çalışılmağı hak etmektedirler. Burada amacım basit: Sâlih Zeki'nin, gayri-Öklidyen geometrilerin nasıl keşfedildiğine dair bu konferansta sunduğu yorumunun ana kaynağını ortaya çıkarmak. Böylece, Sâlih Zeki'nin yeni geometrilerin keşfine dair olan görüşünü geliştirmesine dayanak olan matematiksel, felsefi ve metodolojik eğilimlerini daha iyi anlayıp değerlendirebileceğiz.

Anahtar Kelimeler

Sâlih Zeki, Dârü'l-fünûn Konferansları, gayri-Öklidyen geometriler, gayri-Öklidyen geometrilerin keşfi

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Первая лекция Салих Зеки Бея в цикле «Даруль-фунун конферанслары»: к вопросу о первоисточнике*

Самет Багче**

Абстракт

В 1914-1916 гг. Салих Зеки (1864-1921) прочитал цикл лекций о геометрии и математике в конференц-зале Даруль-фунун. Впоследствии вышли в свет два тома этих лекций, соответственно в 1915 и в 1916 году, изданные арабским письмом под названием «Даруль-фунун конферанслары» (Лекции Даруль-фунун). Главная тема первого тома «Лекций» - неевклидова геометрия и проблема ее разработки. В первой лекции этого тома Салих Зеки излагает свои взгляды о том, как была открыта неевклидова геометрия.

Лекция Салиха Зеки является первым материалом на турецком языке, в котором излагаются и изучаются вопросы новой геометрии. Уже одно это качество должно стать поводом для изучения данных лекций. Наша цель проста: найти первоисточник толкования, которое приводит Салих Зеки при объяснении открытия неевклидовой геометрии. Таким образом мы сможем лучше понять и оценить те математические, философские и методологические направления, которые послужили основой для развития взглядов Салиха Зеки.

Ключевые слова

Салих Зеки, «Даруль-фунун конферанслары», открытие неевклидовой геометрии.

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