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Existence of symmetric positive solutions for a semipositone problem on time scales

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Abstract

This paper studies the existence of symmetric positive solutions for a second order nonlinear semipositone boundary value problem with integral boundary conditions by applying the Krasnoselskii fixed point theorem. Emphasis is put on the fact that the nonlinear term f may take negative value. An example is presented to demonstrate the application of our main result.

Keywords: Positive solution, Symmetric solution, Semipositone problems, Fixed point theorems, Time scales.

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1. Introduction

We will be concerned with proving the existence of at least one symmetric positive solution to the semipositone second order nonlinear boundary value problem on a symmetric time scale T given by

$$[g(t)u^{\triangle}(t)]^{\nabla} + \lambda f(t, u(t)) = 0, \qquad t \in (a, b), \tag{1.1}$$

$$\alpha u(a) - \beta \lim_{t \to a^+} g(t)u^{\triangle}(t) = \int_a^b h_1(s)u(s)\nabla s, \tag{1.2}$$

$$\alpha u(b) + \beta \lim_{t \to b^-} g(t)u^{\triangle}(t) = \int_a^b h_2(s)u(s)\nabla s, \tag{1.3}$$

where $\lambda > 0$ is a parameter, $\alpha, \beta > 0$, ∇-differentiable function $g \in C([a, b], (0, \infty))$ is symmetric on $[a, b]$, $h_1, h_2 \in L^1([a, b])$ is nonnegative, symmetric on $[a, b]$ and the continuous function $f : [a, b] \times [0, \infty) \to R$ satisfies $f(b + a - t, u) = f(t, u)$.

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A class of boundary value problems with integral boundary conditions arise naturally in thermal condition problems [4], semiconductor problems [7], and hydrodynamic problems [5]. Such problems include two, three and multi-point boundary conditions and have recently been investigated by many authors [3, 6, 8, 9].

The present work is motivated by recent paper [3]. In this paper, Boucherif considered the following second order boundary value problem with integral boundary conditions

$$x''(t) = f(t, x(t)), \qquad 0 < t < 1, \tag{1.4}$$

$$x(0) - cx'(0) = \int_0^1 g_0(s)x(s)ds, \tag{1.5}$$

$$x(1) - dx'(1) = \int_0^1 g_1(s)x(s)ds, \tag{1.6}$$

where $f : [0, 1] \times R \to R$ is continuous, $g_0, g_1 : [0, 1] \to [0, \infty)$ are continuous and positive, c and d are nonnegative real parameters. The author established some excellent results for the existence of positive solutions to problem $(1.4) - (1.6)$ by using the fixed point theorem in cones.

Throughout this paper T is a symmetric time scale with a, b are points in T. By an interval (a, b), we always mean the intersection of the real interval (a, b) with the given time scale, that is $(a, b) \cap$T. Other types of intervals are defined similarly. For the details of basic notions connected to time scales we refer to [1, 2].

Now, we present some symmetric definition.

1.1. Definition. A time scale T is said to be symmetric if for any given $t \in$ T, we have $b + a - t \in$ T.

1.2. Definition. A function $u :$ T $\to R$ is said to be symmetric on T if for any given $t \in$ T, $u(t) = u(b + a - t)$.

2. The Preliminary Lemmas

In this section we collect some preliminary results that will be used in subsequent section.

Throughout the paper we will assume that the following conditions are satisfied:

(H_1) $\alpha, \beta > 0$,

(H_2) ∇-differentiable function $g \in C([a, b], (0, \infty))$ is symmetric on $[a, b]$,

(H_3) the continuous function $f : [a, b] \times [0, \infty) \to R$ is semipositone, i.e., $f(t, u)$ needn't be positive for all $(t, u) \in [a, b] \times [0, \infty)$ and $f(., u)$ is symmetric on $[a, b]$ for all $u \geq 0$,

(H_4) $h_1, h_2 \in L^1([a, b])$ is nonnegative, symmetric on $[a, b]$ and $A > 0$, where $A = \mu + (\beta - K)v_1 - \beta v_2$, $K = \dfrac{\mu}{\alpha}$, $\mu = 2\alpha\beta + \alpha^2 \displaystyle\int_a^b \frac{\Delta r}{g(r)}$, $v_1 = \displaystyle\int_a^b h_1(\tau)\nabla\tau$, $v_2 = \displaystyle\int_a^b h_2(\tau)\nabla\tau$.

The lemmas in this section are based on the boundary value problem

$$-[g(t)u^{\triangle}(t)]^{\nabla} = p(t), \qquad t \in (a, b) \tag{2.1}$$

with boundary conditions $(1.2) - (1.3)$.

To prove the main result, we will employ following lemmas.

2.1. Lemma. *Let $(H_1), (H_2)$ hold and $A \neq 0$. Then for any $p \in C([a, b])$, the boundary value problem $(2.1) - (1.2) - (1.3)$ has a unique solution u given by*

$$u(t) = \int_a^b H(t,s)p(s)\nabla s,$$

where

$$H(t,s) = G(t,s) + B_1 \int_a^b G(s,\tau)h_1(\tau)\nabla\tau + B_2 \int_a^b G(s,\tau)h_2(\tau)\nabla\tau \tag{2.2}$$

$$G(t,s) = \frac{1}{\mu}\begin{cases} (\beta + \alpha \int_a^s \frac{\Delta r}{g(r)})(\beta + \alpha \int_t^b \frac{\Delta r}{g(r)}), & a \le s \le t \le b, \\ (\beta + \alpha \int_a^t \frac{\Delta r}{g(r)})(\beta + \alpha \int_s^b \frac{\Delta r}{g(r)}), & a \le t \le s \le b, \end{cases} \tag{2.3}$$

where $\mu = 2\alpha\beta + \alpha^2 \int_a^b \frac{\Delta r}{g(r)}$, $B_1 = \frac{K-\beta}{A}$, $B_2 = \frac{\beta}{A}$.

2.2. Lemma. *Assume that $(H_1), (H_2)$ and (H_4) hold. Then we have*

(i) $H(t,s) > 0$, $\quad G(t,s) > 0$, for $t,s \in [a,b]$,

(ii) $H(b+a-t, b+a-s) = H(t,s)$, $\quad G(b+a-t, b+a-s) = G(t,s)$, for $t,s \in [a,b]$,

(iii) $\frac{1}{\mu}\beta^2\gamma \le H(t,s) \le H(s,s) \le \frac{1}{\mu}\gamma D$ and $\frac{1}{\mu}\beta^2 \le G(t,s) \le G(s,s) \le \frac{1}{\mu}D$, for $t,s \in [a,b]$,

where $D = (\beta + \alpha \int_a^b \frac{\Delta r}{g(r)})^2$, $\gamma = 1 + B_1 v_1 + B_2 v_2$.

Proof. It is clear that (i) hold. Now we prove that (ii) and (iii) hold. First, we consider (ii). If $t \le s$, then $b+a-t \ge b+a-s$. Using (2.3) and the assumption (H_2), we get

$$\begin{aligned} G(b+a-t, b+a-s) &= \frac{1}{\mu}(\beta + \alpha \int_a^{b+a-s} \frac{\Delta r}{g(r)})(\beta + \alpha \int_{b+a-t}^b \frac{\Delta r}{g(r)}) \\ &= \frac{1}{\mu}(\beta + \alpha \int_b^s \frac{\Delta(b+a-r)}{g(b+a-r)})(\beta + \alpha \int_t^a \frac{\Delta(b+a-r)}{g(b+a-r)}) \\ &= \frac{1}{\mu}(\beta + \alpha \int_s^b \frac{\Delta r}{g(r)})(\beta + \alpha \int_a^t \frac{\Delta r}{g(r)}) = G(t,s). \end{aligned}$$

Similarly, we can prove that $G(b+a-t, b+a-s) = G(t,s)$, for $s \le t$. Thus we have $G(b+a-t, b+a-s) = G(t,s)$, for $t,s \in [a,b]$. Now by (2.2), for $t,s \in [a,b]$, we have

$$\begin{aligned} H(b+a-t, b+a-s) &= G(b+a-t, b+a-s) + B_1 \int_a^b G(b+a-s,\tau)h_1(\tau)\nabla\tau \\ &\quad + B_2 \int_a^b G(b+a-s,\tau)h_2(\tau)\nabla\tau \\ &= G(t,s) + B_1 \int_b^a G(b+a-s, b+a-\tau)h_1(b+a-\tau)\nabla(b+a-\tau) \\ &\quad + B_2 \int_b^a G(b+a-s, b+a-\tau)h_2(b+a-\tau)\nabla(b+a-\tau) \\ &= G(t,s) + B_1 \int_a^b G(s,\tau)h_1(\tau)\nabla\tau + B_2 \int_a^b G(s,\tau)h_2(\tau)\nabla\tau \\ &= H(t,s). \end{aligned}$$

So (ii) is established. Now we show that (iii) holds. In fact, if $t \le s$, from (2.3) and the assumption (H_2), then we get

$$G(t,s) = \frac{1}{\mu}(\beta + \alpha\int_a^t \frac{\Delta r}{g(r)})(\beta + \alpha\int_s^b \frac{\Delta r}{g(r)}) \le \frac{1}{\mu}(\beta + \alpha\int_a^s \frac{\Delta r}{g(r)})(\beta + \alpha\int_s^b \frac{\Delta r}{g(r)})$$
$$= G(s,s)$$
$$\le \frac{1}{\mu}(\beta + \alpha\int_a^b \frac{\Delta r}{g(r)})(\beta + \alpha\int_a^b \frac{\Delta r}{g(r)}) = \frac{1}{\mu}(\beta + \alpha\int_a^b \frac{\Delta r}{g(r)})^2 = \frac{1}{\mu}D.$$

Similarly, we can prove that $G(t,s) \le G(s,s) \le \frac{1}{\mu}D$ for $s \le t$.

Therefore $G(t,s) \le G(s,s) \le \frac{1}{\mu}D$, for $t, s \in [a,b]$. And then, by (2.2), we have

$$H(t,s) = G(t,s) + B_1\int_a^b G(s,\tau)h_1(\tau)\nabla\tau + B_2\int_a^b G(s,\tau)h_2(\tau)\nabla\tau$$
$$\le G(s,s) + B_1\int_a^b G(\tau,\tau)h_1(\tau)\nabla\tau + B_2\int_a^b G(\tau,\tau)h_2(\tau)\nabla\tau$$
$$\le \frac{1}{\mu}D + \frac{1}{\mu}DB_1\int_a^b h_1(\tau)\nabla\tau + \frac{1}{\mu}DB_2\int_a^b h_2(\tau)\nabla\tau = \frac{1}{\mu}D(1 + B_1v_1 + B_2v_2)$$
$$= \frac{1}{\mu}D\gamma.$$

On the other hand, for $t, s \in [a,b]$, we have

$$G(t,s) \ge \frac{1}{\mu}(\beta + \alpha\int_a^a \frac{\Delta r}{g(r)})(\beta + \alpha\int_b^b \frac{\Delta r}{g(r)}) = \frac{1}{\mu}\beta^2.$$

And then, we get

$$H(t,s) = G(t,s) + B_1\int_a^b G(s,\tau)h_1(\tau)\nabla\tau + B_2\int_a^b G(s,\tau)h_2(\tau)\nabla\tau$$
$$\ge \frac{1}{\mu}\beta^2 + \frac{1}{\mu}\beta^2 B_1\int_a^b h_1(\tau)\nabla\tau + \frac{1}{\mu}\beta^2 B_2\int_a^b h_2(\tau)\nabla\tau = \frac{1}{\mu}\beta^2\gamma.$$

Thus for $t, s \in [a,b]$, we have

$$\frac{1}{\mu}\beta^2\gamma \le H(t,s) \le H(s,s) \le \frac{1}{\mu}\gamma D \text{ and } \frac{1}{\mu}\beta^2 \le G(t,s) \le G(s,s) \le \frac{1}{\mu}D.$$

This completes the proof.

2.3. Lemma. *Let w be the unique positive solution of the boundary value problem*

$$[g(t)u^{\triangle}(t)]^{\nabla} + 1 = 0 \tag{2.4}$$

with the boundary condition (1.2) − (1.3). *Then,*

$$w(t) \le C\delta, \quad t \in [a,b],$$

where

$$\delta = \frac{\beta^2}{D}, \quad C = \frac{b-a}{\mu\beta^2}D^2\gamma \tag{2.5}$$

Proof. Using Lemma 2.2, for all $t \in [a,b]$, we have

$$w(t) = \int_a^b H(t,s)\nabla s \leq \frac{1}{\mu}\gamma D \int_a^b \nabla s = C\delta.$$

The proof is complete.

Let E denote the Banach space $C[a,b]$ with the norm $\|u\| = \max_{t\in[a,b]} |u(t)|$. Define the cone $P \subset E$ by $P = \{u \in E : u(t)$ is symmetric and $u(t) \geq \delta\|u\|$ for $t \in [a,b]\}$.

To obtain the a positive solution of BVP $(1.1)-(1.3)$, the following fixed point theorem is essential.

2.4. Theorem. *Let $E = (E, \|.\|)$ be a Banach space, and let $P \subset E$ be a cone in B. Assume Ω_1, Ω_2 are bounded open subsets of E with $0 \in \Omega_1, \overline{\Omega_1} \subset \Omega_2$, and let*
$S : P \cap (\overline{\Omega_2}\backslash\Omega_1) \to P$
be a continuous and completely continuous operator such that, either
(a) $\|Su\| \leq \|u\|, u \in P \cap \partial\Omega_1$, *and* $\|Su\| \geq \|u\|, u \in P \cap \partial\Omega_2$, *or*
(b) $\|Su\| \geq \|u\|, u \in P \cap \partial\Omega_1$, *and* $\|Su\| \leq \|u\|, u \in P \cap \partial\Omega_2$.
Then S has a fixed point in $P \cap (\overline{\Omega_2}\backslash\Omega_1)$.

3. Main Results

In this section, we apply the Krasnoselskii fixed point theorem to obtain the existence of at least one symmetric positive solution for the nonlinear boundary value problem $(1.1) - (1.3)$.

The main result of this paper is following:

3.1. Theorem. *Let $(H_1) - (H_4)$ hold. Assume that*

(C_1) There exists a constant $M > 0$ such that $f(t,u) \geq -M$ for all $(t,u) \in [a,b] \times [0,\infty)$,

(C_2) There exist $t_1, t_2 \in (a,b)$ such that
$$\lim_{u\to\infty} \frac{f(t,u)}{u} = \infty$$
uniformly on $[t_1,t_2]$,

(C_3) r is a given positive real number and the parameter λ satisfies

$$0 < \lambda \leq \eta := \min\{\frac{r}{M_1\|w\|}, \frac{r}{2MC}\} \tag{3.1}$$

where $M_1 = \max\{f(t,u) + M : (t,u) \in [a,b] \times [0,r]\}$.

Then the boundary value problem $(1.1) - (1.3)$ has at least one symmetric positive solution u such that $\|u\| \geq \frac{r}{2}$.

Proof. Let $x(t) = \lambda M w(t)$, where w is the unique solution of the boundary value problem $(2.4) - (1.2) - (1.3)$.

We shall show that the following boundary value problem

$$[g(t)y^{\triangle}(t)]^{\nabla} + \lambda F(t, y(t) - x(t)) = 0,\ t \in (a,b), \tag{3.2}$$

$$\alpha y(a) - \beta \lim_{t\to a^+} g(t)y^{\triangle}(t) = \int_a^b h_1(s)y(s)\nabla s, \tag{3.3}$$

$$\alpha y(b) + \beta \lim_{t\to b^-} g(t)y^{\triangle}(t) = \int_a^b h_2(s)y(s)\nabla s, \tag{3.4}$$

where

$$F(t,z) = \begin{cases} f(t,z) + M, & z \geq 0, \\ f(t,0) + M, & z \leq 0, \end{cases}$$

has at least one positive solution. Thereafter we shall obtain at least one positive solution for the boundary value problem (1.1) – (1.3).

It is well known that the existence of positive solution to the boundary value problem (3.2) – (3.4) is equivalent to the existence of fixed point of the operator S. So we shall seek a fixed point of S in our cone P where the operator $S : E \to E$ is defined by

$$Sy(t) = \lambda \int_a^b H(t,s)F(s, y(s) - x(s))\nabla s, \quad t \in [a,b].$$

First, it is obvious that S is continuous and completely continuous.

Now we shall prove that $S(P) \subseteq P$. Let $y \in P$. Then, using Lemma 2.2, we get for $t \in [a,b]$,

$$Sy(t) = \lambda \int_a^b H(t,s)F(s, y(s) - x(s))\nabla s \leq \frac{\lambda}{\mu}\gamma D \int_a^b F(s, y(s) - x(s))\nabla s,$$

and so

$$\|Sy\| \leq \frac{\lambda}{\mu}\gamma D \int_a^b F(s, y(s) - x(s))\nabla s. \tag{3.5}$$

Now, using Lemma 2.2 and (3.5), we obtain for $t \in [a,b]$,

$$\begin{aligned} Sy(t) &= \lambda \int_a^b H(t,s)F(s, y(s) - x(s))\nabla s \geq \frac{\lambda}{\mu}\beta^2\gamma \int_a^b F(s, y(s) - x(s))\nabla s \\ &= \frac{\lambda}{\mu}\delta\gamma D \int_a^b F(s, y(s) - x(s))\nabla s \geq \delta\|Sy\|. \end{aligned}$$

On the other hand, noticing $y(t), x(t)$ and $f(t,u)$ are symmetric on $[a,b]$, we have

$$\begin{aligned} Sy(b+a-t) &= \lambda \int_a^b H(b+a-t,s)F(s, y(s) - x(s))\nabla s \\ &= \lambda \int_a^b H(b+a-t,s)(f(s, y(s) - x(s)) + M)\nabla s \\ &= \lambda \int_b^a H(b+a-t, b+a-s)(f(s, (y-x)(b+a-s)) + M)\nabla(b+a-s) \\ &= \lambda \int_a^b H(t,s)(f(s, (y-x)(s)) + M)\nabla s \\ &= \lambda \int_a^b H(t,s)F(s, (y-x)(s))\nabla s = Sy(t) \end{aligned}$$

Therefore Sy is symmetric.
So, we get $S(P) \subseteq P$.

Let $\Omega_1 = \{y \in E : \|y\| < r\}$. We shall prove that $\|Sy\| \leq \|y\|$ for $y \in P \cap \partial\Omega_1$. If $y \in P \cap \partial\Omega_1$, then $\|y\| = r$. By definition and (3.1), we find for $t \in [a,b]$,

$$Sy(t) = \lambda \int_a^b H(t,s)F(s, y(s) - x(s))\nabla s \leq \lambda M_1 \int_a^b H(t,s)\nabla s \leq \lambda M_1\|w\| \leq r.$$

Therefore, we get $\|Sy\| \le r = \|y\|$ for $y \in P \cap \partial\Omega_1$.

Let K be a positive real number such that

$$\frac{1}{2}\lambda K(t_2 - t_1)\delta\frac{1}{\mu}\beta^2\gamma > 1. \tag{3.6}$$

In view of (C_2), there exists $N > 0$ such that for all $z \ge N$ and $t \in [t_1, t_2]$,

$$F(t,z) = f(t,z) + M \ge Kz \tag{3.7}$$

Now, set

$$R = r + \frac{2N}{\delta}. \tag{3.8}$$

Let $\Omega_2 = \{y \in E : \|y\| < R\}$. We shall prove that $\|Sy\| \ge \|y\|$ for $y \in P \cap \partial\Omega_2$. If $y \in P \cap \partial\Omega_2$, then $\|y\| = R$. So from Lemma 2.3 and the fact that $y \in P$, we get for $t \in [a, b]$,

$$x(t) = \lambda M w(t) \le \lambda MC\delta \le \lambda MC\frac{y(t)}{R}.$$

This implies for $t \in [a, b]$,

$$y(t) - x(t) \ge (1 - \frac{\lambda MC}{R})y(t) \ge (1 - \frac{\lambda MC}{R})\delta R,$$

and, from (3.1) and (3.8), we get for $t \in [t_1, t_2]$,

$$y(t) - x(t) \ge \frac{1}{2}R\delta \ge N. \tag{3.9}$$

Thus, by (3.7) and (3.9), we see that for $t \in [t_1, t_2]$,

$$F(t, y(t) - x(t)) \ge K(y(t) - x(t)) \ge \frac{1}{2}KR\delta. \tag{3.10}$$

Considering Lemma 2.2 and (3.10), we get for $t \in [a, b]$,

$$\begin{aligned} Sy(t) = \lambda \int_a^b H(t,s)F(s, y(s) - x(s))\nabla s &\ge \lambda\frac{1}{\mu}\beta^2\gamma \int_{t_1}^{t_2} F(s, y(s) - x(s))\nabla s \\ &\ge \frac{1}{2\mu}\lambda KR\delta\beta^2\gamma \int_{t_1}^{t_2} \nabla s \end{aligned}$$

and so by (3.6),

$$\|Sy\| \ge \frac{1}{2\mu}\lambda KR(t_2 - t_1)\delta\beta^2\gamma \ge R.$$

Therefore, we get $\|Sy\| \ge R = \|y\|$ for $y \in P \cap \partial\Omega_2$.

Then it follows from Theorem 2.1 that S has a fixed point $y \in P$ such that

$$r \le \|y\| \le R. \tag{3.11}$$

Moreover, using (3.1), (3.11) and Lemma 2.3, we obtain for $t \in [a, b]$,

$$y(t) \geq \delta\|y\| \geq r\delta \geq 2\lambda MC\delta \geq 2\lambda Mw(t) = 2x(t). \tag{3.12}$$

Hence,

$u(t) = y(t) - x(t) \geq 0, \quad t \in [a, b].$

On the other hand, $u(t)$ is symmetric on $[a, b]$ since y and x are symmetric.
Now, we shall prove that u is a positive solution of the boundary value problem (1.1) − (1.3). Since y is a fixed point of the operator S,

$Sy(t) = y(t), \quad t \in [a, b],$

or

$$y(t) = Sy(t) = \lambda \int_a^b H(t,s)F(s, y(s) - x(s))\nabla s$$
$$= \lambda \int_a^b H(t,s)(f(s, y(s) - x(s)) + M)\nabla s$$

Noticing that,

$$w(t) = \int_a^b H(t,s)\nabla s$$

we have for $t \in [a, b]$,

$$y(t) = \lambda \int_a^b H(t,s)f(s, y(s) - x(s))\nabla s + \lambda Mw(t),$$

or

$$y(t) - x(t) = \lambda \int_a^b H(t,s)f(s, y(s) - x(s))\nabla s,$$

and hence

$$u(t) = \lambda \int_a^b H(t,s)f(s, u(s))\nabla s.$$

This shows that u is a symmetric positive solution of the boundary value problem of (1.1) − (1.3). In addition, from (3.11) and (3.12), it follows that

$$\|u\| \geq \frac{\|y\|}{2} \geq \frac{r}{2}.$$

3.2. Example. Let T $= Z$. Consider the following boundary value problem

$$[\frac{100}{t^2+1}u^{\triangle}(t)]^{\nabla} + \lambda(be^u \cos^2 t - t^2) = 0, \ t \in (-3, 3), \tag{3.13}$$

$$25u(-3) - 5 \lim_{t \to -3^+} \frac{100}{t^2+1}u^{\triangle}(t) = \int_{-3}^{3} u(s)\cosh s \nabla s, \tag{3.14}$$

$$25u(3) + 5 \lim_{t \to 3^-} \frac{100}{t^2+1}u^{\triangle}(t) = \int_{-3}^{3} u(s)\cosh s \nabla s, \tag{3.15}$$

where $b > 0$, $\alpha = 25, \beta = 5, h_1(t) = h_2(t) = \cosh t, g(t) = \dfrac{100}{t^2+1}, f(t, u(t)) = be^u \cos^2 t - t^2$. It is obvious that f satisfies the conditions (C_2) and (H_3).

Now we shall obtain the constants M and M_1. Clearly, for all $(t, u) \in [-3, 3] \times [0, \infty)$, we get

$f(t, u) = be^u \cos^2 t - t^2 \geq -t^2 \geq -9$ and so we can choose the constant $M = 9$.

$$M_1 = \max_{(t,u)\in[-3,3]\times[0,r]} be^u \cos^2 t - t^2 + M = be^r + M.$$

It follows from a direct calculation that

$v_1 = v_2 = \int_{-3}^{3} h_1(s)\nabla s \cong 21.5, \mu = 2\alpha\beta + \alpha^2 \int_{-3}^{3} \frac{\Delta r}{g(r)} \cong 406.2,$

$D = (\beta + \alpha \int_{-3}^{3} \frac{\Delta r}{g(r)})^2 \cong 126.6, A = \mu + (\beta - K)v_1 - \beta v_2 \cong 56,87,$

$B_1 = \frac{K-\beta}{A} \cong 0.198, B_2 = \frac{\beta}{A} \cong 0.088,\ \gamma = 1 + B_1 v_1 + B_2 v_2 \cong 7.15,$

$C = \frac{6}{\mu\beta^2} D^2 \gamma \cong 67.71.$

Then by Theorem 3.1, we see that the boundary value problem $(3.13) - (3.15)$ has at least one symmetric positive solution u such that $\|u\| \geq \frac{r}{2}$ for any $\lambda \in (0, \eta]$ where $\eta := \min\{\frac{r}{M_1\|w\|}, \frac{r}{2MC}\}$, r is a given positive number and w is the unique positive solution of the boundary value problem $[\frac{100}{t^2+1} u^{\triangle}(t)]^{\nabla} + 1 = 0$ with the boundary condition $(3.14) - (3.15)$.

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