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On some generalized soft mappings

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Abstract

In the present paper, we introduce and explore new form of continuity called soft pu-semi-continuity via soft semi-open set in soft topological spaces. Moreover we introduce the concepts of soft-pu-semi-open and soft pu-semi-closed functions and discuss many of their characterizations and properties. It is interesting to mention that the soft functions define and discuss here are the generalization of soft functions explored in [7][21].

Keywords: Soft topology, Soft semi-open(closed) sets, Soft semi-interior(closure), Soft semi-boundary, Soft pu-semi-continuous function, Soft pu-semi-open(closed) functions.

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1. Introduction

Researchers working in the fields of science including engineering physics, computer sciences, economics, social sciences and medical sciences usually deals with modelling of problems having uncertainty present in data and not clear objects. The difficulty arose, because of failure of classical methods to solve the problems having uncertainties and not enough information. Researches are going on for the development of new theories and ideas day by day and a lot of material is available in the literature.

In [15], Molodtsov introduced soft sets theory as a new general mathematical approach to deal with uncertain data and not clear objects. In soft systems, a very general frame work has been provided with the involvement of parameters. In [16], they applied successfully this approach for modelling the problems having uncertainties. In [13-14], Maji et. al explored the basics of soft set theory and presented its applications in decision making problems. Xiao et. al [20] and Pei et. al [18] discussed the relationship among soft sets and information systems. Using approach of soft sets, Kostek[11] introduced the criteria to measure sound quality. In [17], Mushrif et. al used the notions of soft set

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theory to develop the remarkable method for the classification of natural textures. Many researchers worked on the algebraic structures of soft set theory.
Shabir and Naz [19] explored and discussed the basics of soft topological spaces. After that Hussain [6-7], Hussain and Ahmad [8-9] [1], Aygunoglu et.al [2], Zorlutana et. al [21] continued studying the properties and introduced many interesting concepts in soft topological spaces. Bin Chen [3-4] presented and discussed soft semi-open sets and soft-semi-closed sets in soft topological spaces. S. Hussain [5] continued to add many notions and concepts toward soft semi-open sets and soft semi-closed sets in soft topological spaces.
Kharral and Ahmad[10] and then Zorlutana [21] discussed the mappings of soft classes and their properties in soft topological spaces. Recently, in 2015, S. Hussain[7], established fundamental and important characterizations of soft pu-continuous functions, soft pu-open functions and soft pu-closed functions via soft interior, soft closure, soft boundary and soft derived set.
In the present paper, we introduce and explore new form of continuity called soft pu-semi-continuity via soft semi-open set in soft topological spaces. Moreover we introduce the concepts of soft-pu-semi-open and soft pu-semi-closed functions and discuss many of their characterizations and properties. It is interesting to mention that the soft functions define and discuss here are the generalization of soft functions introduced in [7] [21].

2. Preliminaries

First we recall some definitions and results, which will use in the sequel.

2.1. Definition. [15] Let X be an initial universe and E be a set of parameters. Let $P(X)$ denotes the power set of X and A be a non-empty subset of E. A pair (F, A) is called a soft set over X , where F is a mapping given by $F : A \to P(X)$. In other words, a soft set over X is a parameterized family of subsets of the universe X . For $e \in A$, $F(e)$ may be considered as the set of e-approximate elements of the soft set (F, A). Clearly, a soft set is not a set.

Here we consider only soft sets (F, A) over a universe X in which all the parameters of set A are same. We denote the family of these soft sets by $SS(X)_A$. For soft subsets, soft union , soft intersection, soft complement and their properties and relations to each other; the interested reader is refer to [13-16].

2.2. Definition. [21] The soft set $(F, A)\tilde{\in}SS(X)_A$ is called a soft point in X_A, denoted by e_F, if for the element $e \in A$, $F(e) \neq \phi$ and $F(e^c) = \phi$, for all $e^{^c} \in A - \{e\}$.

2.3. Definition. [21] The soft point e_F is said to be in the soft set (G, A), denoted by $e_F\tilde{\in}(G, A)$, if for the element $e \in A$, $F(e) \subseteq G(e)$.

2.4. Definition. [21] A soft set (F, A) over X is said to be a null soft set, denoted by Φ_A , if for all $e \in A$, $F(e) = \phi$.

2.5. Definition. [21] A soft set (F, A) over X is said to be an absolute soft set, denoted by $\tilde{X}_A$, if for all $e \in A$, $F(e) = X$. Clearly, $\tilde{X}_A^c = \Phi_A$ and $\Phi_A^c = \tilde{X}_A$.

2.6. Definition. [19] Let τ be the collection of soft sets over X, then τ is said to be a soft topology on X, if
(1) Φ, $\tilde{X}$ belong to τ.
(2) the union of any number of soft sets in τ belongs to τ.
(3) the intersection of any two soft sets in τ belongs to τ.
The triplet (X, τ, E) is called a soft topological space over X.

2.7. Definition. [19][8] Let (X,τ,E) be a soft topological space over X and $A \subseteq E$ then
(1) soft interior of soft set (F,A) over X is denoted by $(F,A)^\circ$ and is defined as the union of all soft open sets contained in (F,A). Thus $(F,A)^\circ$ is the largest soft open set contained in (F,A). A soft set (F,A) over X is said to be a soft closed set in X, if its relative complement $(F,A)^c$ belongs to τ.
(2) soft closure of (F,A), denoted by $\overline{(F,A)}$ is the intersection of all soft closed super sets of (F,A). Clearly $\overline{(F,A)}$ is the smallest soft closed set over X which contains (F,A).
(3) soft boundary of soft set (F,A) over X is denoted by $\underline{(F,A)}$ and is defined as $\underline{(F,A)} = \overline{(F,A)} \cap \overline{((F,A)')}$. Obviously $\underline{(F,A)}$ is a smallest soft closed set over X containing (F,A).

For detailed properties of soft interior, soft closure and soft boundary, we refer to [8].

2.8. Definition. [3] Let (X,τ,E) be a soft topological space over X with $A \subseteq E$ and (F,A) be a soft set over X. Then (F,A) is called soft semi-open set if and only if there exists a soft open set (G,A) such that $(G,A)\tilde{\subseteq}(F,A)\tilde{\subseteq}\overline{(G,A)}$. The set of all soft semi-open sets is denoted by $S.S.O(X)$. Note that every soft open set is soft semi-open set. A sot set (F,A) is said to be soft semi-closed if its soft relative complement is soft semi-open . Equivalently there exists a soft closed set (G,A) such that $(G,A)^\circ\tilde{\subseteq}(F,A)\tilde{\subseteq}(G,A)$. Note that every soft closed set is soft semi-closed set.

2.9. Definition. [5] Let (X,τ,E) be a soft topological space over X and $A \subseteq E$. Then
(i) soft semi-interior of soft set (F,A) over X is denoted by $sint^s(F,A)$ and is defined as the union of all soft semi-open sets contained in (F,A).
(ii) soft semi-closure of (F,A) over X is denoted by $scl^s(F,A)$ is the intersection of all soft semi-closed super sets of (F,A).
(3) soft semi-exterior of soft set (F,A) over X is denoted by $sext^s(F,A)$ and is defined as $sext^s(F,A) = sint^s((F,A)^c)$.
(4) soft semi-boundary of soft set (F,A) over X is denoted by $sbd^s(F,A)$ and is defined as $sbd^s(F,A) = (sint^s(F,A) \cup sext^s(F,A))^c$.

For detailed properties of soft semi-interior, soft semi-exterior, soft semi-closure and soft semi-boundary, we refer to [5].

2.10. Definition. [10] Let $SS(X)_A$ and $SS(Y)_B$ be families of soft sets. $u : X \to Y$ and $p : A \to B$ be mappings. Then a function $f_{pu} : SS(X)_A \to SS(Y)_B$ defined as :
(1) Let (F,A) be a soft set in $SS(X)_A$. The image of (F,A) under f_{pu} , written as $f_{pu}(F,A) = (f_{pu}(F), p(A))$, is a soft set in $SS(Y)_B$ such that

$$f_{pu}(F)(y) = \begin{cases} \bigcup_{x \in p^{-1}(y)\cap A} u(F(x)), & p^{-1}(y) \cap A \neq \phi \\ \phi, & \text{otherwise} \end{cases},$$

for all $y \in B$.
(2) Let (G,B) be a soft set in $SS(Y)_B$. Then the inverse image of (G,B) under f_{pu} , written as $f_{pu}^{-1}(G,B) = (f_{pu}^{-1}(G), p^{-1}(B))$, is a soft set in $SS(X)_A$ such that

$$f_{pu}^{-1}(G)(x) = \begin{cases} u^{-1}(G(p(x))), & p(x) \in B \\ \phi, & \text{otherwise} \end{cases},$$

for all $x \in A$.

The soft function f_{pu} is called soft surjective, if p and u are surjective. The soft function f_{pu} is called soft injective, if p and u are injective.

For detailed properties of soft functions, we refer to [7][21].

3. Properties of soft pu-semi-continuous functions

3.1. Definition. Let (X, τ, A) and (Y, τ^*, B) be soft topological spaces over X and Y respectively and $u : X \to Y$ and $p : A \to B$ be mappings. Then the soft function $f_{pu} : SS(X)_A \to SS(Y)_B$ is soft pu-semi-continuous if and only if for any soft open set (G, B) in $SS(Y)_B$, $f_{pu}^{-1}(G, B)$ is a soft semi-open set in $SS(X)_A$.
Clearly it follows from the definition that the soft function $f_{pu} : SS(X)_A \to SS(Y)_B$ is soft pu-semi-continuous if and only if for any soft closed set (G, B) in $SS(Y)_B$, $f_{pu}^{-1}(G, B)$ is a soft semi-closed set in $SS(X)_A$.

3.2. Theorem. *Let (X, τ, A) and (Y, τ^*, B) be soft topological spaces over X and Y respectively and soft point $e_F \tilde{\in} \tilde{X}_A$. Then the soft function $f_{pu} : SS(X)_A \to SS(Y)_B$ is soft pu-semi-continuous if and only if for each soft open set (G, B) in $SS(Y)_B$ such that $f_{pu}(e_F) \tilde{\subseteq} (G, B)$, there exists soft semi-open set (F, A) in $SS(X)_A$ such that $e_F \tilde{\in} (F, A)$ and $f_{pu}(F, A) \tilde{\subseteq} (G, B)$.*

Proof. ($\Rightarrow$) Let f_{pu} is soft pu-semi-continuous function. Then for soft open set (G, B) in $SS(Y)_B$, $f_{pu}^{-1}(G, B)$ is soft semi-open in $SS(X)_A$. We show that for each soft open set (G, B) containing $f_{pu}(e_F)$, there exists soft semi-open set (F, A) in $SS(X)_A$ such that $e_F \tilde{\in} (F, A)$ and $f_{pu}(F, A) \tilde{\subseteq} (G, B)$. Let $e_F \tilde{\in} f_{pu}^{-1}(G, B)$, which is soft semi-open and $(F, A) \tilde{=} f_{pu}^{-1}(G, B)$. Then $e_F \tilde{\in} (F, A)$ and for soft open set (G, B),
$f_{pu}(F, A) \tilde{\subseteq} f_{pu}(f_{pu}^{-1}(G, B)) \tilde{\subseteq} (G, B)$, where (G, B) is soft open.
($\Leftarrow$) Suppose that (G, B) be a soft open set in $SS(Y)_B$. We prove that inverse image of soft open set in $SS(Y)_B$ is soft semi-open set in $SS(X)_A$. Let $e_F \tilde{\in} f_{pu}^{-1}(G, B)$. Then $f_{pu}(e_F) \tilde{\in} (G, B)$. Thus there exists soft semi-open set (F, A_{e_F}) such that $e_F \tilde{\in} (F, A_{e_F})$ and $f_{pu}(F, A_{e_F}) \tilde{\subseteq} (G, B)$. Then $e_F \tilde{\in} (F, A_{e_F})$
$\tilde{\subseteq} f_{pu}^{-1}(G, B)$ and $f_{pu}^{-1}(G, B) \tilde{=} \bigcup_{e_F \tilde{\in} f_{pu}^{-1}(G,B)} (F, A_{e_F})$. This follows that $f_{pu}^{-1}(G, B)$ is soft semi-open, by Theorem 3.2[3]. Hence f_{pu} is soft pu-semi-continuous. □

3.3. Theorem. *Suppose $f_{pu} : SS(X)_A \to SS(Y)_B$ be a soft function. Then the following statements are equivalent:*
(1) f_{pu} is soft pu-semi-continuous.
(2) For any soft subset (G, B) of $SS(Y)_B$, $sbd^s(f_{pu}^{-1}(G, B)) \tilde{\subseteq} f_{pu}^{-1}(\underline{(G, B)})$.
(3) For any soft subset (F, A) of $SS(X)_A$, $f_{pu}(scl^s(F, A)) \tilde{\subseteq} \overline{(f_{pu}(F, A))}$.

Proof. (1) $\Rightarrow$ (2) Let f_{pu} is soft pu-semi-continuous and (G, B) be a soft set in $SS(Y)_B$. So, $sbd^s(f_{pu}^{-1}(G, B)) \tilde{=} scl^s(f_{pu}^{-1}(G, B)) \cap scl^s((f_{pu}^{-1}(G, B))^c)$
$\tilde{\subseteq} scl^s(f_{pu}^{-1}(\overline{(G, B)}) \tilde{\cap} scl^s(f_{pu}^{-1}(\overline{((G, B)^c)}) \tilde{=} f_{pu}^{-1}(\overline{(G, B)}) \tilde{\cap} f_{pu}^{-1}(\overline{(G, B)})$
$\tilde{=} f_{pu}^{-1}(\overline{(G, B)} \tilde{\cap} \overline{((G, B)^c)}) \tilde{=} f_{pu}^{-1}(\underline{(G, B)})$. Hence $sbd^s(f_{pu}^{-1}(G, B)) \tilde{\subseteq} f_{pu}^{-1}(\underline{(G, B)})$.
(2) $\Rightarrow$ (1) Suppose that (G, B) be a soft closed set in $SS(Y)_B$. We prove that $f_{pu}^{-1}(G, B)$ is soft semi-closed in $SS(X)_A$. As $sbd^s(f_{pu}^{-1}(G, B)) \tilde{\subseteq} f_{pu}^{-1}(\underline{(G, B)})$
$\tilde{\subseteq} f_{pu}^{-1}(G, B)$. This follows that $sbd^s(f_{pu}^{-1}(G, B)) \tilde{\subseteq} f_{pu}^{-1}(G, B)$. Therefore $f_{pu}^{-1}(G, B)$ is soft semi-closed in $SS(X)_A$. This gives that f_{pu} is soft pu-semi-continuous.
(1) $\Rightarrow$ (3) Suppose (F, A) be any soft set in $SS(X)_A$. As $\overline{(f_{pu}(F, A))}$ is soft closed in $SS(Y)_B$. Therefore, f_{pu} is soft pu-semi-continuous implies that $f_{pu}^{-1}(\overline{(f_{pu}(F, A))})$ is soft semi-closed in $SS(X)_A$ with $(F, A) \tilde{\subseteq} f_{pu}^{-1}(\overline{(f_{pu}(F, A))})$. This implies that
$scl^s(F, A) \tilde{\subseteq} scl^s(f_{pu}^{-1}(\overline{(f_{pu}(F, A))})) \tilde{=} f_{pu}^{-1}(\overline{(f_{pu}(F, A))})$. Which implies that
$f_{pu}(scl^s(F, A)) \tilde{\subseteq} f_{pu}(f_{pu}^{-1}(\overline{(f_{pu}(F, A))}))$. Hence $f_{pu}(scl^s(F, A)) \tilde{\subseteq} \overline{(f_{pu}(F, A))}$.
(3) $\Rightarrow$ (1) Suppose (G, B) be soft closed in $SS(Y)_B$. We prove that $f_{pu}^{-1}(G, B)$ is soft semi-closed. By (3), $f_{pu}(scl^s(f_{pu}^{-1}(G, B))) \tilde{\subseteq} \overline{(f_{pu}(f_{pu}^{-1}(G, B)))}$
$\tilde{\subseteq} \overline{(G, B)} \tilde{=} (G, B)$. This follows that $scl^s(f_{pu}^{-1}(G, B)) \tilde{\subseteq} f_{pu}^{-1} f_{pu}(scl^s(f_{pu}^{-1}(G, B))) \tilde{\subseteq}$

$f_{pu}^{-1}(G,B)$. Consequently $scl^s(f_{pu}^{-1}(G,B))\tilde{\subseteq} f_{pu}^{-1}(G,B)$. This shows that $f_{pu}^{-1}(G,B)$ is soft semi-closed in $SS(X)_A$. Thus f_{pu} is soft pu-semi-continuous. □

3.4. Theorem. *A soft function $f_{pu}: SS(X)_A \to SS(Y)_B$ is soft pu-semi-continuous if and only if $scl^s(f_{pu}^{-1}(G,B))\tilde{\subseteq} f_{pu}^{-1}(\overline{(G,B)})$, for any soft set (G,B) in $SS(Y)_B$.*

Proof. ($\Rightarrow$) Suppose that f_{pu} is soft pu-semi-continuous. Then by above Theorem 3.3, we get

$$f_{pu}(scl^s(F,A))\tilde{\subseteq}\overline{(f_{pu}(F,A))} \quad \ldots \text{(A)}$$

Suppose (G,B) be any soft set in $SS(Y)_B$. Take
$(F,A)\tilde{=} f_{pu}^{-1}(G,B)$ then $f_{pu}(scl^s(f_{pu}^{-1}(G,B)))\tilde{\subseteq}\overline{(f_{pu}f_{pu}^{-1}(G,B))}\tilde{\subseteq}\overline{(G,B)}$. This follows that $f_{pu}(scl^s(f_{pu}^{-1}(G,B)))\tilde{\subseteq}\overline{(G,B)}$.
($\Leftarrow$) Suppose that $f_{pu}(scl^s(f_{pu}^{-1}(G,B)))\tilde{\subseteq}\overline{(G,B)}$, for any soft subset (G,B) in $SS(Y)_B$. Let $(G,B)\tilde{=} f_{pu}(F,A)$, for any soft set (F,A) in $SS(X)_A$. This gives
$scl^s(F,A)\tilde{\subseteq} scl^s(f_{pu}^{-1}(G,B))\tilde{\subseteq} f_{pu}^{-1}(\overline{(f_{pu}(F,A)}))$. This follows that $f_{pu}(scl^s(F,A))\tilde{\subseteq}$ $\overline{(f_{pu}(F,A))}$. Hence by above Theorem 3.3, f_{pu} is soft pu-semi-continuous. □

3.5. Lemma. [3][5] *The following properties of soft set (F,A) in $SS(X)_A$ are equivalent:*
(1) (F,A) is soft semi-closed.
(2) $(\overline{(F,A)})^\circ \tilde{\subseteq}(F,A)$.
(3) $(F,A)^c$ is soft semi-open.

3.6. Theorem. *Let $f_{pu}: SS(X)_A \to SS(Y)_B$ be a soft function. Then the following statements are equivalent:*
(1) f_{pu} is soft pu-semi-continuous.
(2) For any soft set (G,B) in $SS(Y)_B$, $\{\overline{(f_{pu}^{-1}(G,B))}\}^\circ \tilde{\subseteq} f_{pu}^{-1}(\overline{(G,B)})$.
(3) For any soft set (F,A) in $SS(X)_A$, $f_{pu}(\{\overline{(F,A)}\}^\circ)\tilde{\subseteq}\overline{(f_{pu}(F,A))}$.

Proof. (1) $\Rightarrow$ (2) Suppose (G,B) be any soft set in $SS(Y)_B$. Then using soft pu-semi-continuity of f_{pu}, $f_{pu}^{-1}(\overline{(G,B)})$ is a soft semi-closed. Using Lemma 3.5 and $(G,B)\tilde{\subseteq}\overline{(G,B)}$, we have $f_{pu}^{-1}(\overline{(G,B)})\tilde{\supseteq}(\overline{(f_{pu}^{-1}(\overline{(G,B)}))})^\circ \tilde{\supseteq}(\overline{(f_{pu}^{-1}(G,B))})^\circ$. This follows that $\{\overline{(f_{pu}^{-1}(G,B))}\}^\circ \tilde{\subseteq} f_{pu}^{-1}(\overline{(G,B)})$.
(2) $\Rightarrow$ (3) Suppose that (F,A) be any soft set in $SS(X)_A$. Take $(G,B)\tilde{=} f_{pu}(F,A)$. Then $(F,A)\tilde{\subseteq} f_{pu}^{-1}(G,B)$. Using our supposition, we get $(\overline{(F,A)})^\circ \tilde{\subseteq}(\overline{(f_{pu}^{-1}(G,B))})^\circ$
$\tilde{\subseteq} f_{pu}^{-1}(\overline{(G,B)})$. This implies that $f_{pu}((\overline{(F,A)})^\circ)\tilde{\subseteq} f_{pu}f_{pu}^{-1}(\overline{(G,B)})\tilde{\subseteq}\overline{(G,B)}$
$\tilde{=}\overline{(f_{pu}(F,A))}$.
(3) $\Rightarrow$ (1) Suppose that(G,B) be any soft closed set in $SS(Y)_B$. Take $(F,A)\tilde{=}$ $f_{pu}^{-1}(G,B)$, then $f_{pu}(F,A)\tilde{\subseteq}(G,B)$. Using our supposition, we get

$$f_{pu}((\overline{(F,A)})^\circ)\tilde{\subseteq}\overline{(f_{pu}(F,A))}\tilde{\subseteq}\overline{(G,B)}\tilde{=}(G,B) \text{ ..... (B)}$$

By (B), we have $(\overline{(F,A)})^\circ \tilde{\subseteq} f_{pu}^{-1}f_{pu}((\overline{(F,A)})^\circ)\tilde{\subseteq} f_{pu}^{-1}(\overline{(f_{pu}(F,A))})\tilde{\subseteq} f_{pu}^{-1}(\overline{(G,B)})\tilde{=}$ $f_{pu}^{-1}(G,B)$. This gives $(\overline{(F,A)})^\circ \tilde{\subseteq} f_{pu}^{-1}(G,B)\tilde{=}(F,A)$. Lemma 3.5 implies that $f_{pu}^{-1}(G,B)\tilde{=}(F,A)$ is a soft semi-closed set. Hence f_{pu} is soft pu-semi-continuous. □

3.7. Definition. Let (X,τ,A) be a soft topological spaces over X, (F,A) be a soft set in $SS(X)_A$ and soft point $e_F\tilde{\in}\tilde{X}_A$. Then e_F is called a soft semi-limit point of a soft set (F,A), if $(H,A)\tilde{\cap}((F,A)\tilde{-}\{e_F\})\tilde{\neq}\tilde{\phi}$, for any soft semi-open set (H,A) such that $e_F\tilde{\in}(H,A)$. The set of all soft semi-limit point of (F,A) is called as soft semi-derived set of (F,A) and is denoted by $sd^s(F,A)$.

Note that if $(F,A)\tilde{\subseteq}(H,A)$ then $sd^s(F,A)\tilde{\subseteq}sd^s(H,A)$..... (C)

3.8. Remark. Clearly e_F is a soft semi-limit point of (F,A) if and only if $e_F\tilde{\in}scl^s((F,A)\tilde{-}\{e_F\})$.

In the following theorem, we discuss the properties of soft semi-derived set "sd^s".

3.9. Theorem. *Let (X,τ,A) be a soft topological spaces over X and (F,A) be a soft set in $SS(X)_A$. Then*
(1) $scl^s(F,A)\tilde{=}(F,A)\tilde{\cup}sd^s(F,A)$.
(2) $sd^s((F,A)\tilde{\cup}(H,A))\tilde{=}sd^s(F,A)\tilde{\cup}sd^s(H,A)$.
In general,
(3) $\bigcup_i sd^s(F,A_i)\tilde{=}sd^s(\bigcup_i(F,A_i))$.
(4) $sd^s(sd^s(F,A))\tilde{\subseteq}sd^s(F,A)$.
(5) $scl^s(sd^s(F,A))\tilde{=}sd^s(F,A)$.

Proof. (1) Suppose $e_F\tilde{\in}scl^s(F,A)$. Then for any soft semi-closed set (K,A) such that $(F,A)\tilde{\subseteq}(K,A)$, we have $e_F\tilde{\in}(K,A)$. Now we consider two cases:
Case (i) If $e_F\tilde{\in}(F,A)$, then $e_F\tilde{\in}(F,A)\tilde{\cup}sd^s(F,A)$.
Case (ii) If $e_F\tilde{\notin}(F,A)$, then we prove that $e_F\tilde{\in}scl^s(F,A)$.
For this consider (L,A) is a soft semi-open set such that $e_F\tilde{\in}(L,A)$. Then $(L,A)\tilde{\cap}(F,A)\tilde{\neq}\tilde{\phi}$. If not, then , $(F,A)\tilde{\subseteq}(L,A)^c\tilde{=}(K,A)$, where (K,A) is a soft semi-closed soft superset of (F,A) such that $e_F\tilde{\notin}(F,A)$. Which is contradiction to the fact that e_F soft belongs to every soft semi-closed soft superset (K,A) of (F,A). This follows that $e_F\tilde{\in}sd^s(F,A)$. This implies that $e_F\tilde{\in}(F,A)\tilde{\cup}sd^s(F,A)$.
Conversely, suppose that $e_F\tilde{\in}(F,A)\tilde{\cup}sd^s(F,A)$, we prove that $e_F\tilde{\in}scl^s(F,A)$. If $e_F\tilde{\in}(F,A)$ then $e_F\tilde{\in}scl^s(F,A)$. If $e_F\tilde{\in}sd^s(F,A)$, then we show that e_F is in every soft semi-closed soft superset of (F,A). Contrarily suppose that there is a soft semi-closed soft superset (K,A) of (F,A) such that $e_F\tilde{\notin}(F,A)$. This follows that $e_F\tilde{\in}(K,A)^c\tilde{=}(L,A)$(say), which is soft semi-open and $(L,A)\tilde{\cap}(F,A)\tilde{=}\tilde{\phi}$. This gives $e_F\tilde{\notin}sd^s(F,A)$. This contradiction proves that $e_F\tilde{\in}scl^s(F,A)$. Hence $scl^s(F,A)\tilde{=}(F,A)\tilde{\cup}sd^s(F,A)$. This completes the proof of (1).
(2) First we prove that $sd^s((F,A)\tilde{\cup}(H,A))\tilde{\subseteq}sd^s(F,A)\tilde{\cup}sd^s(H,A)$.
Suppose $e_F\tilde{\in}sd^s((F,A)\tilde{\cup}(H,A))$. Then $e_F\tilde{\in}scl^s(((F,A)\tilde{\cup}(H,A))\tilde{-}\{e_F\})$ or $e_F\tilde{\in}scl^s(((F,A)\tilde{-}\{e_F\})\tilde{\cup}((H,A)\tilde{-}\{e_F\})$ implies $e_F\tilde{\in}scl^s((F,A)\tilde{-}\{e_F\})$ or $e_F\tilde{\in}scl^s((H,A)\tilde{-}\{e_F\})$. This gives $e_F\tilde{\in}sd^s(F,A)$ or $e_F\tilde{\in}sd^s(H,A)$. Therefore $e_F\tilde{\in}sd^s(F,A)\tilde{\cup}sd^s(H,A)$. This proves $sd^s((F,A)\tilde{\cup}(H,A))\tilde{\subseteq}sd^s(F,A)\tilde{\cup}sd^s(H,A)$. The reverse inclusion follows form property(C).
(3) This directly follows from property (C).
(4) Suppose that $e_F\tilde{\notin}sd^s(F,A)$. Then $e_F\tilde{\notin}scl^s((F,A)\tilde{-}\{e_F\})$. This follows that there is a soft semi-open set (L,A) such that $e_F\tilde{\in}(L,A)$ with $(L,A)\tilde{\cap}((F,A)\tilde{-}\{e_F\})$ $\tilde{=}\tilde{\phi}$. We show that $e_F\tilde{\notin}sd^s(sd^s(F,A))$. Contrarily suppose that $e_F\tilde{\in}sd^s(sd^s(F,A))$. This implies that $e_F\tilde{\in}scl^s(sd^s(F,A)\tilde{-}\{e_F\})$. $e_F\tilde{\in}(L,A)$ follows that $(L,A)\tilde{\cap}(sd^s(F,A)\tilde{-}\{e_F\})\tilde{\neq}\tilde{\phi}$. Thus there exists a $q_F\tilde{\neq}e_F$ such that $q_F\tilde{\in}(L,A)$ $\tilde{\cap}(sd^s(F,A))$. This implies that $q_F\tilde{\in}((L,A)\tilde{-}\{e_F\})\tilde{\cap}(sd^s(F,A)\tilde{-}\{e_F\})$. Therefore $((L,A)\tilde{-}\{e_F\})\tilde{\cap}(sd^s(F,A)\tilde{-}\{e_F\})\tilde{\neq}\tilde{\phi}$. This is contradiction to the fact that $((L,A)\tilde{\cap}(sd^s(F,A)\tilde{-}\{e_F\})\tilde{=}\tilde{\phi}$. This follows that $e_F\tilde{\notin}sd^s(sd^s(F,A))$. Hence $sd^s(sd^s(F,A))\tilde{\subseteq}sd^s(F,A)$. This proves (4).
(5) The proof follows form (1), (2) and (4). □

3.10. Theorem. *Let $f_{pu}:SS(X)_A\to SS(Y)_B$ be a soft function. Then the following statements are equivalent:*

(1) f_{pu} *is soft pu-semi-continuous.*
(2) For any soft set (F,A) *in* $SS(X)_A$, $f_{pu}(sd^s(F,A))\tilde{\subseteq}\overline{(f_{pu}(F,A))}$.

Proof. (1) $\Rightarrow$ (2) Let f_{pu} be soft pu-semi-continuous and (F,A) be any soft set in $SS(X)_A$. $\overline{(f_{pu}(F,A))}$ is soft closed implies that $f_{pu}^{-1}(\overline{f_{pu}(F,A)})$ is soft semi-closed in $SS(X)_A$. $(F,A)\tilde{\subseteq}f_{pu}^{-1}(f_{pu}(F,A))\tilde{\subseteq}f_{pu}^{-1}(\overline{(f_{pu}(F,A))})$. This follows that $scl^s(F,A)\tilde{\subseteq}scl^s(f_{pu}^{-1}(\overline{(f_{pu}(F,A))}))\tilde{=}f_{pu}^{-1}(\overline{(f_{pu}(F,A))})$. This implies that $f_{pu}(sd^s(F,A))\tilde{\subseteq}f_{pu}(scl^s(F,A))\tilde{\subseteq}f_{pu}f_{pu}^{-1}(\overline{(f_{pu}(F,A))})\tilde{\subseteq}\overline{(f_{pu}(F,A))}$. Therefore, $f_{pu}(sd^s(F,A))\tilde{\subseteq}\overline{(f_{pu}(F,A))}$.
(2) $\Rightarrow$ (1) Let $f_{pu}(sd^s(F,A))\tilde{\subseteq}\overline{(f_{pu}(F,A))}$, for any soft set (F,A) in $SS(X)_A$. Suppose that (G,B) be any soft closed subset in $SS(Y)_B$. We prove that $f_{pu}^{-1}(G,B)$ is soft semi-closed. By our supposition, $f_{pu}(sd^s(f_{pu}^{-1}(G,B)))\tilde{\subseteq}\overline{(f_{pu}(f_{pu}^{-1}(G,B)))}$ $\tilde{\subseteq}\overline{(G,B)}\tilde{=}(G,B)$. This follows that $f_{pu}(sd^s(f_{pu}^{-1}(G,B)))\tilde{\subseteq}(G,B)$. This implies that $sd^s(f_{pu}^{-1}(G,B))\tilde{\subseteq}f_{pu}^{-1}(G,B)$. This follows that $f_{pu}^{-1}(G,B)$ is soft semi-closed. Hence f_{pu} is soft pu-semi-continuous. $\square$

3.11. Theorem. *Let* $f_{pu}: SS(X)_A \to SS(Y)_B$ *be a soft function. Then* f_{pu} *is soft pu-semi-continuous if and only if for any soft set* (G,B) *in* $SS(Y)_B$, $f_{pu}^{-1}((G,B)^\circ)\tilde{\subseteq}sint^s$ $(f_{pu}^{-1}(G,B))$.

Proof. ($\Rightarrow$) Since for any soft set $G,B)$ in $SS(Y)_B$, $(G,B)^\circ\tilde{=}(\overline{(G,B)^c})^c$ [5]. This follows that $f_{pu}^{-1}((G,B)^\circ)\tilde{=}f_{pu}^{-1}((\overline{(G,B)^c})^c)\tilde{=}(f_{pu}^{-1}(\overline{(G,B)^c}))^c$. Since f_{pu} is soft pu-semi-continuous, by Theorem 3.4, we get $scl^s(f_{pu}^{-1}((G,B)^c)\tilde{\subseteq}$ $f_{pu}^{-1}(\overline{(G,B)^c})$. Therefore $f_{pu}^{-1}((G,B)^\circ)\tilde{\subseteq}scl^s((f_{pu}^{-1}((G,B)^c))^c)$. This implies that $f_{pu}^{-1}((G,B)^\circ)\tilde{\subseteq}(scl^s((f_{pu}^{-1}(G,B))^c))^c$. Thus $f_{pu}^{-1}((G,B)^\circ)\tilde{\subseteq}X-(scl^s(f_{pu}^{-1}(G,B)))^c\tilde{=}$ $sint^s(f_{pu}^{-1}(G,B))$.
($\Leftarrow$) Suppose that (G,B) be any soft open set in $SS(Y)_B$. Then $(G,B)^\circ\tilde{=}(G,B)$. Using our supposition, we get $f_{pu}^{-1}(G,B)\tilde{=}f_{pu}^{-1}((G,B)^\circ)\tilde{\subseteq}sint^s(f_{pu}^{-1}(G,B))$. This follows that $f_{pu}^{-1}(G,B)\tilde{\subseteq}sint^s(f_{pu}^{-1}(G,B))$. But $sint^s(f_{pu}^{-1}(G,B))\tilde{\subseteq}f_{pu}^{-1}(G,B)$. Therefore, $f_{pu}^{-1}(G,B)\tilde{=}sint^s(f_{pu}^{-1}(G,B))$. This shows that $f_{pu}^{-1}(G,B)$ is soft semi-open. Hence f_{pu} is soft pu-semi-continuous. $\square$

4. Properties of soft pu-semi-open functions

4.1. Definition. A soft set (F,A) in $SS(X)_A$ is said to be a soft semi-nbd of a soft point $e_F\tilde{\in}\tilde{X}_A$, if there exists a soft semi-open set (H,A) such that $e_F\tilde{\in}(H,A)\tilde{\subseteq}(F,A)$.

4.2. Definition. Let (X,τ,A) and (Y,τ^*,B) be soft topological spaces over X and Y respectively and $u: X\to Y$ and $p: A\to B$ be mappings. Then the soft function $f_{pu}: SS(X)_A\to SS(Y)_B$ is soft pu-semi-open if and only if for any soft open set (F,A) in $SS(X)_A$, $f_{pu}(F,A)$ is soft semi-open in $SS(Y)_B$.

4.3. Lemma. *If* (F,A) *is soft semi-open and* (H,A) *be any soft set such that* $(F,A)\tilde{\subseteq}(H,A)$. *Then* $(F,A)\tilde{\subseteq}\overline{(H,A)^\circ}$.

Proof. (F,A) is soft semi-open implies that $(F,A)\tilde{\subseteq}\overline{(F,A)^\circ}$[5]. Moreover, $(F,A)\tilde{\subseteq}(H,A)$ implies that $(F,A)^\circ\tilde{\subseteq}(H,A)^\circ$. Thus $\overline{(F,A)^\circ}\tilde{\subseteq}\overline{(H,A)^\circ}$ follows that $(F,A)\tilde{\subseteq}\overline{(H,A)^\circ}$. $\square$

4.4. Theorem. *A soft function* $f_{pu}: SS(X)_A\to SS(Y)_B$ *is soft pu-semi-open if and only if for any soft subset* (F,A) *in* $SS(X)_A$, $f_{pu}((F,A)^\circ)\tilde{\subseteq}\overline{(f_{pu}(F,A))^\circ}$.

Proof. ($\Rightarrow$) Suppose that f_{pu} be soft pu-semi-open. Then $f_{pu}((F,A)^{\circ})\tilde{\subseteq} f_{pu}(F,A))$ implies that $f_{pu}((F,A)^{\circ})$ is soft semi-open. Thus by Lemma 4.3,
$f_{pu}((F,A)^{\circ})\tilde{\subseteq}\overline{(f_{pu}(F,A))^{\circ}}$.
($\Leftarrow$) Suppose that (H,A) be any soft open set in $SS(X)_A$. Then
$(f_{pu}(H,A))^{\circ}\tilde{\subseteq} f_{pu}(H,A))\tilde{\subseteq} f_{pu}((H,A)^{\circ})\tilde{\subseteq}\overline{(f_{pu}(H,A))^{\circ}}$. So $f_{pu}(H,A))$ is soft semi-open. Which implies that f_{pu} is soft pu-semi-open. Hence the proof. □

The following theorem can be proved in similar fashion.

4.5. Theorem. *A soft function $f_{pu} : SS(X)_A \to SS(Y)_B$ is soft pu-semi-open if and only if for any soft subset (G,B) in $SS(Y)_B$, $(f_{pu}^{-1}(G,B))^{\circ}\tilde{\subseteq}\overline{f_{pu}^{-1}((G,B)^{\circ})}$.*

4.6. Theorem. *If a soft function $f_{pu} : SS(X)_A \to SS(Y)_B$ be soft pu-semi-continuous and soft pu-semi-open and (F,A) be soft semi-open set in $SS(X)_A$. Then $f_{pu}(F,A)$ is soft semi-open in $SS(Y)_B$.*

Proof. Since (F,A) is soft semi-open, then there exists soft open set (H,A) in $SS(X)_A$ such that $(H,A)\tilde{\subseteq}(F,A)\tilde{\subseteq}\overline{(H,A)}$. This implies that $f_{pu}(H,A)\tilde{\subseteq} f_{pu}(F,A)$
$\tilde{\subseteq} f_{pu}(\overline{(H,A)})\tilde{\subseteq}\overline{(f_{pu}(H,A))}$. Thus $f_{pu}(F,A)$ is soft semi-open in $SS(Y)_B$. This proves as required. □

The proof of the following lemma and proposition is easy and thus omitted.

4.7. Lemma. *Let (F,A) be any soft set and (H,A) be soft semi-closed set in $SS(X)_A$ such that $(F,A)\tilde{\subseteq}(H,A)$, then $sbd^s(F,A)\tilde{\subseteq}(H,A)$.*

4.8. Proposition. *If $(F,A)\tilde{\cap}(H,A)\tilde{=}\tilde{\phi}$ and (F,A) is soft open, then $(F,A)\tilde{\cap}\overline{(H,A)}\tilde{=}\tilde{\phi}$.*

The following lemma directly follows form Proposition 4.8.

4.9. Lemma. *If $(F,A)\tilde{\cap}(H,A)\tilde{=}\tilde{\phi}$ and (F,A) is soft open in $SS(X)_A$, then
$(F,A)\tilde{\cap}\underline{(H,A)}\tilde{=}\tilde{\phi}$.*

4.10. Theorem. *If $f_{pu} : SS(X)_A \to SS(Y)_B$ be a soft bijective, soft function and (G,B) be any soft subset in $SS(Y)_B$. Then f_{pu} is soft pu-semi-open if and only if $f_{pu}^{-1}(sbd^s(G,B)\tilde{\subseteq}\underline{(f_{pu}^{-1}(G,B))}$.*

Proof. ($\Rightarrow$) Let soft function f_{pu} be soft pu-semi-open and (G,B) be soft open set in $SS(Y)_B$. Take

$$(F,A)\tilde{=}(\underline{f_{pu}^{-1}(G,B)})^c \qquad \ldots \text{(D)}$$

This follows that (F,A) is soft open. Therefore $f_{pu}(F,A)$ is soft semi-open in $SS(Y)_B$. This follows that $(f_{pu}(F,A))^c$ is soft semi-closed in $SS(Y)_B$. Therefore by equation (D) and soft bijectivity of soft function f_{pu}, we have $(G,B)\tilde{\subseteq}(f_{pu}(F,A))^c$. Using Lemma 4.7, we get $f_{pu}^{-1}(sbd^s(G,B))\tilde{\subseteq}$
$f_{pu}^{-1}((f_{pu}(F,A))^c)\tilde{\subseteq}(F,A)^c \tilde{=}((\underline{(f_{pu}^{-1}(G,B))})^c)^c\tilde{=}\underline{(f_{pu}^{-1}(G,B))}$. This gives
$f_{pu}^{-1}(sbd^s(G,B))\tilde{\subseteq}\underline{(f_{pu}^{-1}(G,B))}$.
($\Leftarrow$) Let (H,A) be soft open in $SS(X)_A$. Take $(G,B)\tilde{=}(f_{pu}(H,A))^c$. Clearly
$(G,B)\tilde{\cap} f_{pu}(H,A)\tilde{=}\tilde{\phi}$, follows $(H,A)\tilde{\cap} f_{pu}^{-1}(G,B)\tilde{=}\tilde{\phi}$. Lemma 4.9 implies that
$(H,A)\tilde{\cap}\underline{(f_{pu}^{-1}(G,B))}\tilde{=}\tilde{\phi}$. Therefore $f_{pu}^{-1}(sd^s(G,B))\tilde{\subseteq}\underline{(f_{pu}^{-1}(G,B))}$ implies $(H,A)\tilde{\cap}$
$f_{pu}^{-1}(sbd^s(G,B))\tilde{=}\tilde{\phi}$. Thus, $\tilde{\phi}\tilde{=} f_{pu}((H,A)\tilde{\cap} f_{pu}^{-1}(sbd^s(G,B)))\tilde{=} f_{pu}(H,A)\tilde{\cap} sbd^s(G,B)$ gives $sbd^s(G,B)\tilde{\subseteq}(f_{pu}(H,A))^c\tilde{=}(G,B)$. Therefore (G,B) is soft semi-closed. Hence $f_{pu}(H,A)$ is soft semi-open. Hence the proof. □

4.11. Theorem. *Let $f_{pu} : SS(X)_A \to SS(Y)_B$ be a soft function. Then the following statements are equivalent:*
(1) f_{pu} is soft pu-semi-open.
(2) For any soft set (F, A) in $SS(X)_A$, $f_{pu}((F, A)^\circ)\tilde{\subseteq} sint^s(f_{pu}((F, A))$.
(3) For each $e_F \tilde{\in} \tilde{X}_A$ and each soft open-nbd (U, A) of soft point e_F, there exists a soft semi-open-nbd (V, A) of $f_{pu}(e_F)$ such that $(V, A)\tilde{\subseteq} f_{pu}(U, A)$.

Proof. (1) $\Rightarrow$ (2) Let f_{pu} be soft pu-semi-open and (F, A) be any soft set in $SS(X)_A$. Then $f_{pu}((F, A)^\circ)$ is soft semi-open and $f_{pu}((F, A)^\circ)\tilde{\subseteq} f_{pu}(F, A)$ implies that
$f_{pu}((F, A)^\circ)\tilde{\subseteq} sint^s(f_{pu}((F, A))$.
(2) $\Rightarrow$ (3) Suppose that (U, A) be any soft open-nbd of soft point $e_F \tilde{\in} \tilde{X}_A$. Then there exists a soft open set (O, A) such that $e_F \tilde{\in} (O, A)\tilde{\subseteq}(U, A)$. Using our supposition, we get $f_{pu}(O, A)\tilde{=} f_{pu}((O, A)^\circ)\tilde{\subseteq} sint^s(f_{pu}((O, A))$. This follows that
$f_{pu}(O, A)\tilde{\subseteq} sint^s(f_{pu}((O, A))$. Consequently, $f_{pu}(O, A)$ is soft semi-open-nbd in $SS(Y)_B$ such that $f_{pu}(e_F)\tilde{\in} f_{pu}(O, A)\tilde{\subseteq} f_{pu}(U, A)$.
(3) $\Rightarrow$ (1) Suppose that (U, A) be a soft open set in $SS(X)_A$. For any $q_F \tilde{\in} f_{pu}(U, A)$, by (3), there exists a soft semi-open-nbd (V_{q_F}, A) of $q_F \tilde{\in} \tilde{Y}_B$ such that $(V_{q_F}, A)\tilde{\subseteq}$
$f_{pu}(U, A)$. Since (V_{q_F}, A) is a soft semi-open-nbd of q_F. Then there exists a soft semi-open set (H_{q_F}, A) in $SS(Y)_B$ such that $q_F \tilde{\in} (H_{q_F}, A)\tilde{\subseteq}(V_{q_F}, A)$. This implies that $f_{pu}(U, A)\tilde{=}\tilde{\bigcup}\{(H_{q_F}, A) : q_F \tilde{\in} f_{pu}(U, A)\}$ is a soft semi-open in $SS(Y)_B$ [3]. Consequently, f_{pu} is a soft pu-semi-open function. □

4.12. Lemma. *Let (F, A) and (G, A) be soft sets in $SS(X)_A$. Then*
(1) $\overline{((F, A)\tilde{-}(H, A))}\tilde{\supseteq}\overline{(F, A)}\tilde{-}\overline{(H, A)}$.
(2) $((F, A)\tilde{-}(H, A))^\circ\tilde{\subseteq}(F, A)^\circ\tilde{-}(H, A)^\circ$.
(3) If (F, A) is soft open, then $(F, A)\tilde{\cap}\overline{(H, A)}\tilde{\subseteq}\overline{((F, A)\tilde{\cap}(H, A))}$.

Proof. (1). Suppose that $e_F \tilde{\in} \overline{(F, A)}\tilde{-}\overline{(H, A)}$. Then $e_F \tilde{\in} \overline{(F, A)}$ and $e_F \tilde{\notin} \overline{(H, A)}$. Thus there exists a soft open nbd (K, A) of e_F such that $(K, A)\tilde{\cap}(F, A)\tilde{\neq}\tilde{\phi}$ and $(K, A)\tilde{\cap}(H, A)\tilde{=}\tilde{\phi}$. This follows that $(K, A)\tilde{\cap}((F, A)\tilde{-}(H, A))\tilde{\neq}\tilde{\phi}$. Thus
$e_F \tilde{\in} \overline{((F, A)\tilde{-}(H, A))}$.
(2) This follows directly by (1) and using Demorgan's law.
(3) Given that (F, A) is soft open. Thus $(F, A)\tilde{=}(F, A)^\circ$. Thus $(F, A)\tilde{\cap}\overline{(H, A)}\tilde{=}$
$\overline{(H, A)}\tilde{\cap}(F, A)^\circ\tilde{=}\overline{(H, A)}\tilde{-}((F, A)^\circ)^c\tilde{=}\overline{(H, A)}\tilde{-}\overline{((F, A)^c)}\tilde{\subseteq}\overline{((H, A)\tilde{-}(F, A)^c)}$
$\tilde{=}\overline{((H, A)\tilde{\cap}(F, A))}\tilde{=}\overline{((F, A)\tilde{\cap}(H, A))}$. Consequently, $(F, A)\tilde{\cap}\overline{(H, A)}\tilde{\subseteq}\overline{((F, A)\tilde{\cap}(H, A))}$. □

4.13. Theorem. *Let $f_{pu} : SS(X)_A \to SS(Y)_B$ be a soft bijective, soft function. Then f_{pu} is soft pu-semi-open if and only if for any soft set (G, B) in $SS(Y)_B$,*
$f_{pu}^{-1}(scl^s(G, B))\tilde{\subseteq}\overline{(f_{pu}^{-1}(G, B))}$.

Proof. ($\Rightarrow$) Suppose that (G, B) be any soft set in $SS(Y)_B$. Take

$$(H, A)\tilde{=}(\overline{(f_{pu}^{-1}(G, B))})^c \text{ ...... (E)}$$

This is clear that (H, A) is a soft open set in $SS(X)_A$. Then by our supposition, $f_{pu}(H, A)$ is a soft semi-open set in $SS(Y)_B$, or $(f_{pu}(H, A))^c$ is soft semi-closed set in $SS(Y)_B$. As f_{pu} is soft onto, from (E), it gives $(G, B)\tilde{\subseteq}(f_{pu}(H, A))^c$. Therefore, we get $scl^s(G, B)\tilde{\subseteq}(f_{pu}(H, A))^c$. f_{pu} is soft one-one, implies that
$f_{pu}^{-1}(scl^s(G, B))\tilde{\subseteq}(f_{pu}^{-1}((f_{pu}(H, A))))^c\tilde{=}(f_{pu}^{-1}f_{pu}(H, A))^c\tilde{\subseteq}(H, A)^c\tilde{=}\overline{(f_{pu}^{-1}(G, B))}$.
($\Leftarrow$) Suppose that (H, A) be any soft open set in $SS(X)_A$. Take $(G, B)\tilde{=}(f_{pu}(H, A))^c$. Since f_{pu} is soft bijective, then by our supposition, $f_{pu}(H, A)\tilde{\cap} scl^s(G, B)\tilde{=}$

$f_{pu}((H,A)\tilde{\cap} f_{pu}^{-1}(scl^s(G,B)))\tilde{\subseteq} f_{pu}((H,A)\tilde{\cap}\overline{(f_{pu}^{-1}(G,B))})$. Since (H,A) is soft open, therefore above Lemma 4.12(3) implies that $(H,A)\tilde{\cap}\overline{(f_{pu}^{-1}(G,B))}$ $\tilde{\subseteq}\overline{((H,A)\tilde{\cap} f_{pu}^{-1}(G,B))}$. Furthermore, it is obvious that $(H,A)\tilde{\cap} f_{pu}^{-1}(G,B)\tilde{=}\tilde{\phi}$. This implies that $f_{pu}(H,A)\tilde{\cap}$ $scl^s(G,B)\tilde{=}\tilde{\phi}$ and hence $scl^s(G,B)\tilde{\subseteq}(f_{pu}(H,A))^c\tilde{=}(G,B)$. This follows that (G,B) is a soft semi-closed and hence $f_{pu}(H,A)$ is a soft semi-open set in $SS(Y)_B$. This shows that f_{pu} is a soft pu-semi-open. □

4.14. Theorem. *Let $f_{pu} : SS(X)_A \to SS(Y)_B$ be soft bijective, soft function. Then f_{pu} is soft pu-semi-open if and only if for any soft subset (V,B) in $SS(Y)_B$ and for any soft closed set (F,A) in $SS(X)_A$ such that $f_{pu}^{-1}(V,B)\tilde{\subseteq}(F,A)$, there exists a soft semi-closed set (G,B) in $SS(Y)_B$ with $(V,B)\tilde{\subseteq}(G,B)$ such that $f_{pu}^{-1}(G,B)\tilde{\subseteq}(F,A)$.*

Proof. ($\Rightarrow$) Suppose that (V,B) be soft set in $SS(Y)_B$ and (F,A) be any soft closed set in $SS(X)_A$ such that $f_{pu}^{-1}(V,B)\tilde{\subseteq}(F,A)$. Take $(G,B)\tilde{=}(f_{pu}((F,A)^c))^c$. f_{pu} is soft pu-semi-open implies that (G,B) is soft semi-closed sets in $SS(Y)_B$. Since f_{pu} is bijective, it follows from $f_{pu}^{-1}(V,B)\tilde{\subseteq}(F,A)$ that $(V,B)\tilde{\subseteq}(G,B)$. By simple calculations, we have $f_{pu}^{-1}(G,B)\tilde{\subseteq}(F,A)$.
($\Leftarrow$) Let (U,A) be soft open set in $SS(X)_A$. Take $(V,B)\tilde{=}(f_{pu}(U,A))^c$. Then $(U,A)^c$ is a soft closed set such that $f_{pu}^{-1}(V,B)\tilde{\subseteq}(U,A)^c$. By hypothesis, there exists a soft semi-closed set (G,B) in $SS(Y)_B$ such that $(V,B)\tilde{\subseteq}(G,B)$ and $f_{pu}^{-1}(G,B)\tilde{\subseteq}$ $(U,A)^c$. On the other hand, it follows from $(V,B)\tilde{\subseteq}(G,B)$ that $f_{pu}(U,A)\tilde{\subseteq}(G,B)^c$. Hence we get $f_{pu}(U,A)\tilde{=}(G,B)^c$, which is soft semi-open. This follows that soft function f_{pu} is soft pu-semi-open. □

5. Properties of soft pu-semi-closed functions

5.1. Definition. Let (X,τ,A) and (Y,τ^*,B) be soft topological spaces over X and Y respectively and $u : X \to Y$ and $p : A \to B$ are mappings. Then the soft function $f_{pu} : SS(X)_A \to SS(Y)_B$ is soft pu-semi-closed if and only if for any soft closed set (F,A) in $SS(X)_A$, $f_{pu}(F,A)$ is soft semi-closed in $SS(Y)_B$.

5.2. Theorem. *Let $f_{pu} : SS(X)_A \to SS(Y)_B$ be a soft function and (F,A) be soft set in $SS(X)_A$. Then f_{pu} is soft pu-semi-closed if and only if* $f_{pu}(\overline{(F,A)})\tilde{\supseteq}\{\overline{(f_{pu}(F,A))}\}^\circ$.

Proof. ($\Rightarrow$) Suppose that f_{pu} is a soft pu-semi-closed function and (F,A) be soft set in $SS(X)_A$. Then $f_{pu}(\overline{(F,A)})$ is soft semi-closed in $SS(Y)_B$. Then by Lemma 3.5, we get $f_{pu}(\overline{(F,A)})\tilde{\supseteq}\{\overline{f_{pu}(\overline{(F,A)})}\}^\circ\tilde{\supseteq}\{\overline{(f_{pu}(F,A))}\}^\circ$. This follows that $f_{pu}(\overline{(F,A)})\tilde{\supseteq}\{\overline{(f_{pu}(F,A))}\}^\circ$.
($\Leftarrow$) Let (F,A) be a soft closed set in $SS(X)_A$. Then by hypothesis, we have $\{\overline{(f_{pu}(F,A))}\}^\circ\tilde{\subseteq} f_{pu}(\overline{(F,A)})\tilde{=}f_{pu}(F,A)$. By Lemma 3.5, $f_{pu}(F)$ is soft semi-closed in $SS(Y)_B$. This implies that f_{pu} is soft pu-semi-closed. □

5.3. Theorem. *Let $f_{pu} : SS(X)_A \to SS(Y)_B$ be a soft function and (F,A) be soft set in $SS(X)_A$. Then f_{pu} is soft semi-closed if and only if $scl^s(F,A)\tilde{\subseteq} f_{pu}(\overline{(F,A)})$.*

Proof. ($\Rightarrow$) Let $f_{pu} : SS(X)_A \to SS(Y)_B$ be a soft function and (F,A) be soft set in $SS(X)_A$. Then $f_{pu}(F,A)$ is soft semi-closed. Since $f_{pu}(F,A)\tilde{\subseteq} f_{pu}(\overline{(F,A)})$, then $scl^s(f_{pu}(F,A))\tilde{\subseteq} f_{pu}(\overline{(F,A)})$. Therefore $scl^s(f_{pu}(F,A))\tilde{\subseteq} f_{pu}(\overline{(F,A)})$.
($\Leftarrow$) This follows from Theorem 5.2. □

5.4. Theorem. *Let $f_{pu}: SS(X)_A \to SS(Y)_B$ be a soft surjective, soft function. Then f_{pu} is soft pu-semi-closed if and only if for any soft subset (G,B) in $SS(Y)_B$ and any soft open set (F,A) in $SS(X)_A$ such that $f_{pu}^{-1}(G,B)\tilde{\subseteq}(F,A)$, there exists a soft semi-open set (V,B) in $SS(Y)_B$ with $(G,B)\tilde{\subseteq}(V,B)$ such that $f_{pu}^{-1}(V,B)\tilde{\subseteq}(F,A)$.*

Proof. $(\Rightarrow)$ Let (G,B) be any soft set in $SS(Y)_B$ and (F,A) be any soft open set in $SS(X)_A$ such that $f_{pu}^{-1}(G,B)\tilde{\subseteq}(F,A)$. Take

$$(V,B)\tilde{=}(f_{pu}((F,A)^c))^c \ ..... \ (\mathrm{F})$$

Then (V,B) is soft semi-open set. Since $f_{pu}^{-1}(G,B)\tilde{\subseteq}(F,A)$. Simple calculations give $(G,B)\tilde{\subseteq}(V,B)$. Moreover, by (F), we have $f_{pu}^{-1}(V,B)\tilde{=}(f_{pu}^{-1}(f_{pu}((F,A)^c))^c$
$\tilde{\subseteq}((F,A)^c)^c\tilde{=}(F,A)$.
$(\Leftarrow)$ Let (F,A) be any soft closed set in $SS(X)_A$ and e_G be an arbitrary soft point in $(f_{pu}(F,A))^c$, then $f_{pu}^{-1}(e_G)\tilde{\subseteq}(f_{pu}^{-1}(f_{pu}(F,A)))^c\tilde{\subseteq}(F,A)^c$, and $(F,A)^c$ is soft open in $SS(X)_A$. Using our supposition, there exists a soft semi-open set (V_{e_G},B) containing e_G such that $f_{pu}^{-1}(V_{e_G},B)\tilde{\subseteq}(F,A)^c$. This follows $e_G\tilde{\in}(V_{e_G},B)\tilde{\subseteq}(f_{pu}(F,A))^c$. This implies that
$(f_{pu}(F,A))^c\tilde{=}\tilde{\bigcup}\{(V_{e_G},B) : e_G\tilde{\in}(f_{pu}(F,A))^c\}$ is soft semi-open in $SS(Y)_B$, since union of any collection of soft semi-open sets is soft semi-open[3]. Hence $f_{pu}(F,A)$ is soft pu-semi-closed. $\square$

Conclusion: In recent years, many researchers worked on the findings of structures of soft sets theory initiated by Molodtsov and applied to many problems having uncertainties. In the present work, we introduced and explored new form of continuity called soft pu-semi-continuity via soft semi-open set in soft topological spaces. Moreover we also introduced the concepts of soft-pu-semi-open and soft pu-semi-closed functions and discussed many of their characterizations and properties. It is interesting to mention that the soft functions defined and discussed here are the generalization of soft functions introduced in [7]. This is need to continue further research in this direction to upgrade the general framework and to explore the practical life applications.

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