Hacettepe Journal of Mathematics and Statistics
Volume 44 (4) (2015), 965 – 973

# A multiset based forecasting model for fuzzy time series

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**Abstract**

Since the pioneering work of Song and Chissom ($1993a, b$) on fuzzy time series to model and forecast processes whose values are described by linguistic values, a number of techniques have been proposed by researchers for forecasting. In most of the realistic situation the duplicates of data are significant. This paper presents a new fuzzy time series method, which employs multiset theory. The historical data of daily average temperature in Taipei, Taiwan (central weather bureau 1996) are adopted to illustrate the forecasting process of the proposed method.

**Keywords:** Multiset, Forecasting, Fuzzy time series, Multiset relation, List.

Received 17/01/2013 : Accepted 01/03/2013 Doi : 10.15672/HJMS.2015113314

## 1. Introduction

Forecasting using fuzzy time series has been widely used in many activities. It arises in forecasting the weather, earthquakes, stock fluctuations and any phenomenon indexed by variables that change unpredictably in time. The classical time series methods can not deal with forecasting problems in which the values of time series are linguistic terms represented by fuzzy sets. Therefore in 1993, Song and Chissom proposed the concepts of fuzzy time series and outlined equations and approximate reasoning based on the fuzzy set theory introduced by Zadeh (1965). They also presented the models on time - variant and time invariant fuzzy time series [21], [22], [23] to deal with forecasting problems in which the historical data is represented by linguistic values. They asserted that all traditional forecasting methods fail when the historical enrollment data are composed of linguistic values.Sullivan and Woodall (1994) described a Markov model using linguistic values directly but with membership function of the fuzzy approach replaced by analogus probability function. Instead of complicated maximum - minimum composition operations Chen (1996) used a simple arithmetic operation for time series forecasting.

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Hwang, Chen and Lee (1998) presented a method of forecasting enrollments using fuzzy time series based on the concept that the variation of enrollment of this year is related to the trend of the enrollments of the past years. Huarng (2001) pointed out that the length of intervals will affect the forecasting accuracy rate and a proper choice of length of intervals can greatly improve the forecasting result. He presented the distribution based length approach and the average based length approach to deal with forecasting problems based on the intervals with different lengths. In recent years researchers focused on the research topic of using fuzzy time series for handling forecasting problems. A number of works have been reported on order one [2],[3],[7]-[9], [24],[25], high order [4],[5],[10],[12]-[14],single factor [2], [3], [8], [10], [24],[25], two factor [5], [7], [12]-[14] and multifactor [9] models. The formulation of fuzzy relation is one of the key issues affecting forecasting results. Li and Chen [15] proposed a forecasting model based on the hidden Markov model by enhancing Sullivan and woodall's [24] work to allow handling of two factor forecasting problems. In most of the realistic situation, we have to deal with collection of objects in which repetition of elements is significant. In this situation bags (multisets) are very useful structures. Multiset theory (Bag) was introduced by Cerf et al. [1] in 1971.Peterson [19] and Yager [26] made contributions to it. Further study was carried out by Jena et al. [11]. In this paper, we propose an efficient fuzzy time series forecasting model based on the concept of characterization of Bag to form the intervals of different length. Then based on the obtained intervals and the concept of list we present a method to deal with the temperature prediction. The remainder of this paper is organised as follows. In section 2, the basic concept of fuzzy time series is briefly introduced. In section 3, the concept of multiset is introduced and in section 4, the new forecasting model based on multiset (Bag) is proposed. Section 5 presents a performance evaluation of the model and a comparison of the results. The conclusions are discussed in section 6.

## 2. Fuzzy Time Series :

In the following, we briefly review some basic concepts of fuzzy time series and its forecasting frame work. The definition of fuzzy time series used in this paper was first proposed by Song and Chissom [20].

**2.1. Definition.** Let $Y(t)$ $(t = \dots, 0, 1, 2 \dots)$, a subset of $R$, be the universe of discourse on which fuzzy sets $f_i(t)$ $(i = 1, 2 \dots)$ are defined, and let $F(t)$ be a collection of $f_i(t)$. Then, $F(t)$ is called a fuzzy time series on $Y(t)$ $(t = \dots, 0, 1, 2 \dots)$.

Song and Chissom employed a fuzzy relational equation to develop their forecasting model under the assumption that the observations at time t are dependent only upon the accumulated results of the observation at previous times, which is defined as follows.

**2.2. Definition.** If, for any $f_j(t) \in F(t)$ , where $j \in J$, there exist an $f_i(t-1) \in F(t-1)$, where $i \in I$, and a fuzzy relation $R_{ij}(t, t-1)$, such that $f_j(t) = f_i(t-1) \circ \mathrm{R}_{ij}(t, t-1)$, let $R(t, t-1) = \bigcup_{i,j} R_{ij}(t, t-1)$, where " $\cup$ " is the union operator and "$\circ$" is the composition. $R(t, t-1)$ is called the fuzzy relation between $F(t)$ and $F(t-1)$, which can be represented using the following fuzzy relational equation: $F(t) = F(t-1) \circ R(t, t-1)$.

**2.3. Definition.** If we suppose that $F(t)$ is caused by $F(t-1)$, $F(t-2)\dots$ or $F(t-m)(m > 0)$, then the first - order model of $F(t)$ can be expressed as $F(t) = F(t-1) \circ R(t, t-1)$(or) $F(t) = (F(t-1) \cup F(t-2) \cup \dots \cup F(t-m)) \circ R_\circ(t, t-m)$

where " $\cup$ " is the union operator and "$o$" is the composition. $R(t, t-1)$ is called the fuzzy relation between $F(t)$ and $F(t-1)$, and $R_\circ(t, t-k)$ is the fuzzy relation that joins $F(t)$ with $F(t-1), F(t-2) \dots, or F(t-k)$, where the subscript " $\circ$ " denotes the relationship "*or*". In the literature, the fuzzy relation $R_{ij}(t, t-1)$ is usually represented by a fuzzy logical relationship rule.

## 3. Multiset(Bag) :

In the following, we briefly review some basic concepts on Multiset (Bag) [11].

**3.1. Definition.** A collection of elements which may contain duplicates is called a Multiset (bag). Formally if $X$ is a set of elements, a bag drawn from the set $X$ is represented by a function count $B$ or $C_B$ defined as $C_B : X \to N$, where $N$ represents the set of non -negative integers. For each $x \in X$, $C_B(x)$ is a **characteristic value** of $x$ in $B$ and indicates the number of occurrences of the elements $x$ in $B$.

**3.2. Definition.** A **list** $L$ drawn from a set $X$ is represented by a position function $P_L$ defined as $P_L : X \to P(N)$, where $P(N)$, denotes the power set of non-negative integers subject to the following conditions:
1. $P_L(x) = \emptyset$ iff $x \notin L$
2. $P_L(x) \cap P_L(y) = \emptyset$ for all $x \neq y \in L$
3. $P_{[\,]}(x) = \emptyset$ for each $x \in X$, where$[\,]$ is an empty set.

**3.3. Definition.** For any finite list $L$ drawn from $X$, we define $|P_L(x)|$ is the number of occurrences of the element $x$ in $L$.

**Note :** The notion of list can be considered as a generalization of notion of bag in the sense that the order of occurrence of elements is unimportant in the case of bag whereas incase of a list, it is significant.

**3.4. Notation.** Let $M$ be a multiset from which $x$ appearing $n$ times in $M$ and denoted by $x \in {}^nM$. The counts of the members of the domain and codomain vary in relation to the counts of the $x$ co-ordinate and $y$ co-ordinate in $(m|x, n|y)|k$, where $(m|x, n|y)|k$ denotes that $x$ is repeated $m$-times, $y$ is repeated $n$-times and the pair $(x, y)$ is repeated $k$ times. Let $C_1(x, y)$ and $C_2(x, y)$ be the count of the first and second co-ordinate in the ordered pair $(x, y)$ respectively.

**3.5. Definition.** Let $M_1$ and $M_2$ be two multisets drawn from a set $X$. Then the **Cartesian product** of $M_1$ and $M_2$ is defined as
$M_1 \times M_2 = \{(m|x, n|y)|mn : x \in {}^mM_1, y \in {}^nM_2\}$.

**3.6. Definition.** A sub multiset $R$ of $M \times M$ is said to be a **multiset relation** on $M$ if every member $(m|x, n|y)$ of $R$ has a count $C_1(x, y)$. $C_2(x, y)$. We denote $m|x$ related to $n|y$ by $m|xRn|y$ .

**3.7. Example.** For example, let $M_1 = \{1, 2, 2\}$ and $M_2 = \{3, 3, 3, 3\}$. $M_1 \times M_2 = \{(1|1, 4|3)|4, (2|2, 4|3)|8\}$. Consider $(1|1, 4|3)$. $C_1(x, y) = 1$ and $C_2(x, y) = 4$. Then $(1|1, 4|3)$ has a count 4.

## 4. The Multiset Based Forecasting Model :

In this section we present a new method for forecasting daily average temperature based on the multiset concepts. The proposed method is now presented as follows :
**Step 1 :** Let $U$ be the universe of discourse, defined by $U = [D_{min} - D_1, D_{max} + D_2]$ where $D_1$ and $D_2$ are two proper positive numbers. Partition the universe of discourse into seven intervals $u_1, u_2, \ldots, u_7$ with equal length [17].
**Step 2 :** Collect the historical data into a crisp set $X$. Construct a multiset $B$ with all entries in ascending order. Find the characteristic value of each $x \in B$.
**Step 3 :** Choose $u_i$, $i = 1$ to 7 and $x_j \in X$ which lies in $u_i$. If $C_B(x_j) = 1$then there

is no change in the interval where $x_j$ lies. If $C_B(x_j) > 1$ and $k = \sum_j C_B(x_j)$ then re divide the interval where $x_j$ lies into $k$ intervals with equal length. Rename the obtained intervals as $v_1, v_2, \ldots, v_n$ where $v_1, v_2, \ldots, v_n$ are of different length.
**Step 4 :** Construct the fuzzy sets $A_i$ in accordance with the intervals in step 3. Fuzzify the historical data. For $n$ fuzzy sets, $A_1, A_2, \ldots, A_n$ can be defined on U [20] as follows:

$A_i = \sum_{j=1}^{n} \frac{\mu_{ij}}{v_j}$ where $\mu_{ij}$ is the membership degree of $A_i$ belonging to $v_j$ and is defined by

$$\mu_{ij} = \begin{cases} 1, & \text{if } j = i \\ 0.5, & \text{if } j = i-1 \text{ or } i+1 \\ 0, & \text{if otherwise} \end{cases}$$

Then, for a given historical datum $Y_t$, its membership degree belonging to interval $v_i$ is determined by the following heuristic rules.

**Rule 1 :** if $Y_t$ is located at $v_1$, the membership degrees are 1 for $v_1$, 0.5 for $v_2$ and 0 otherwise.

**Rule 2 :** if $Y_t$ belongs to $v_i$, $1 < i < n$, then the degrees are 1, 0.5 and 0.5 for $v_i$, $v_{i-1}$ and $v_{i+1}$, respectively and 0 otherwise.

**Rule 3 :** if $Y_t$ is located at $v_n$, the membership degrees are 1 for $v_n$, 0.5 for $v_{n-1}$ and 0 otherwise. Then, $Y_t$ is fuzzified as $A_j$, where the membership degree in interval $j$ is maximal

**Step 5 :** Collect the fuzzy sets into a set $Y$. Form a list with all entries as in the fuzzy time series. Find the position function $P_L$ defined as $P_L : Y \to P(N)$ where $P(N)$ denotes the power set of non - negative integers.
**Step 6 :** Construct the multiset relation $R$ which is a subset of $L \times L$.
**Step 7 :** Forecast the values by the following principles.

*Principle 1:* If $|P_L(A_i)| = 1$, then for $P_L(A_j) = P_L(A_i) + 1$ the forecasted value of $A_j$ is the midvale of $v_j$.

*Principle 2:* Consider the weighted factor [Yu (2005)] and the between the actual data and the mid values of the intervals. When $|P_L(Ai)| > 1$ and the multiset relation is of the form $(A_i, A_j)$, $(A_i, A_k)$, $(A_i, A_l)$,..., $(A_i, A_n)$ assign numbers as follows. $j = a_1.k = a_2, l = a_3, \ldots, n = a_n$.
Then the corresponding weights are defined as $w_j = \frac{a_1}{\sum_{i=1}^{n} a_i}$, $w_k = \frac{a_2}{\sum_{i=1}^{n} a_i}$, $w_l = \frac{a_3}{\sum_{i=1}^{n} a_i}$ ,..., $w_n = \frac{a_n}{\sum_{i=1}^{n} a_i}$ where $w_j + w_k + w_l + \cdots + w_n = 1$. Then the forecasted value of $A_j$ is equal to $G_1(t) + G_2(t))$, where $G_1(t) = [m_j, m_k, \ldots, m_n]$ $[w_j, w_k, \ldots w_n]^T$ and $\mathrm{G}_2(t)$ = [Auctual value of $\mathrm{A}_j -$Average of midvalues of $m_j, m_k, \ldots, m_n$] Similarly the forecasted values for $A_k, A_l, \ldots, A_n$.

## 5. Model Verification :

The experiment consisted of forecasting temperature in Taipei, Taiwan, to verify the forecasting performance of the proposed model. We compare with some existing models. In the following, we apply the proposed method to forecast the daily average temperature based on multiset context. Based on the daily average temperature from June 1, 1996 to June 30, 1996 shown in Table 5, the universe of discourse of the daily average temperature is

$U$ = [26, 31] and the seven intervals with equal length are as follows.
$u_1$ = [26, 26.71) ; $u_2$ = [26.71, 27.42) ; $u_3$ = [27.42, 28.13)
$u_4$ = [28.13, 28.84); $u_5$ = [28.84, 29.55); $u_6$ = [29.55, 30.26)
$u_7$ = [30.26, 31].
We have $X$ = {26.1, 27.1, 27.4, 27.5, 27.6, 27.7, 27.8, 28.4, 28.5, 28.7, 28.8, 29, 29.3, 29.4, 29.5, 29.7, 30, 30.2, 30.3, 30.5, 30.8, 30.9}
and the bag
$B$ = {26.1, 27.1, 27.4, 27.5, 27.6, 27.7, 27.8, 27.8, 28.4, 28.5, 28.7, 28.7, 28.8, 28.8, 29, 29, 29, 29.3, 29.4, 29.4,29.5, 29.5, 29.7, 30, 30.2, 30.2, 30.3, 30.5, 30.8, 30.9}
The characteristic values of $x_j$ in $X$ are given in Table 1. By step 3, the intervals of different length and the midpoints of the intervals are given in Table 2. Now the fuzzy sets are defined and the time series is fuzzified by step 4 are given in Table 3.
Based on the fuzzy time series presented in Table 3 and step 5
we have $Y = \{A_1, A_2, A_3, A_4, A_6, A_7, A_8, A_{10}, A_{13}, A_{14}, A_{15}, A_{16}, A_{17}, A_{18}\}$ and the list as
$L = [A_1, A_3, A_{10}, A_{18}, A_{17}, A_{15}, A_{16}, A_{14}, A_8, A_{14}, A_{13}, A_7, A_8, A_3, A_{15}, A_8, A_{10}, \mathrm{A}_{18}, A_{17}, A_{18}, A_{18}, A_8, A_4, A_2, A_3, A_2, A_6, A_4, A_{10}, A_{17}]$
The position functions $P_L$ are as follows:
$P_L(A_1) = \{1\}$; $P_L(A_2) = \{24, 26\}$; $P_L(A_3) = \{2, 14, 25\}$;
$P_L(A_4) = \{23, 28\}$; $P_L(A_6) = \{27\}$; $P_L(A_7) = \{12\}$; $P_L(A_8) = \{9, 13, 16, 22\}$; $P_L(A_{10}) = \{3, 17, 29\}$; $P_L(A_{13}) = \{11\}$; $P_L(A_{14}) = \{8, 10\}$
$P_L(A_{15}) = \{6, 15\}$; $P_L(A16) = \{7\}$; $P_L(A_{17}) = 5, 19, 30$;
$P_L(A_{18}) = \{4, 18, 20, 21\}$
The multiset relation of order 1 obtained from step 6 is

$$\begin{aligned} R =\{&(\mathrm{A}_1, A_3), (A_2, A_6), (A_2, A_3), (A_3, A_{10}), (A_3, A_2), (A_3, A_{15}), (A_4, A_2),\\ &(\mathrm{A}_4, A_{10}), (A_6, A_4), (A_7, A_8), (A_8, A_{14}), (A_8, A_3), (A_8, A_{10}), (A_8, A_4),\\ &(\mathrm{A}_{10}, A_{18}), (A_{10}, A_{18}), (A_{10}, A_{17}), (A_{13}, A_7), (A_{14}, A_8), (A_{14}, A_{13}),\\ &(\mathrm{A}_{15}, A_{16}), (A_{15}, A_8), (A_{16}, A_{14}), (A_{17}, A_{15}), (A_{17}, A_{18}), (A_{18}, A_{17}),\\ &(\mathrm{A}_{18}, A_{17}), (A_{18}, A_{18}), (A_{18}, A_8)\}. \end{aligned}$$

By the principles in step 7, the forecasted values are given in Table 3.

Evaluate the performance of the proposed fuzzy time series model with the forecasting results by predicting the temperature and comparing with the models of Lee et al's (2006) and Li et al (2010). The average forecasting error rate (AFER) and Mean square error (M.S.E) are used in this section to compare the forecasted accuracy rate of the daily average temperature of the proposed method with the existing models where the historical data is shown in Table 5.

Average forecasting error rate (AFER) $= \frac{\frac{|A_i - F_i|}{A_i}}{n} * 100$

Mean square error (M.S.E) $= \frac{\sum_{i=1}^{n} [A_i - F_i]^2}{n}$ where $A_i$ denotes the actual temperature of day $i$ , $F_i$ denotes the forecasting temperature of day $i$ and $n$ is the number of errors respectively.

**Table 3 :** Forecasting Results of the Proposed Model for June 1 to June 30,1996.

| Day | Actual Temp. | Fuzzified Temp. | Forecasted Temp. | $(\mathrm{A}_i - F_i)^2$ | $\frac{\lvert A_i - F_i \rvert}{A_i}$*100 |
|---|---|---|---|---|---|
| 1 | 26.1 | $A_1$ | - | - | - |
| 2 | 27.6 | $A_3$ | 27.5975 | 0.000006 | 0.00905 |
| 3 | 29.0 | $A_{10}$ | 29.6151 | 0.378469 | 0.12137 |
| 4 | 30.5 | $A_{18}$ | 30.5068 | 0.000047 | 0.02258 |
| 5 | 30.0 | $A_{17}$ | 30.1797 | 0.032300 | 0.59908 |
| 6 | 29.5 | $A_{15}$ | 29.5514 | 0.002647 | 0.17440 |
| 7 | 29.7 | $A_{16}$ | 29.8627 | 0.026484 | 0.54795 |
| 8 | 29.4 | $A_{14}$ | 29.3978 | 0.000005 | 0.00726 |
| 9 | 28.8 | $A_8$ | 28.8649 | 0.004212 | 0.22536 |
| 10 | 29.4 | $A_{14}$ | 29.8212 | 0.177451 | 1.43282 |
| 11 | 29.3 | $A_{13}$ | 29.3649 | 0.004212 | 0.22151 |
| 12 | 28.5 | $A_7$ | 28.5737 | 0.005439 | 0.25877 |
| 13 | 28.7 | $A_8$ | 28.7512 | 0.002627 | 0.17857 |
| 14 | 27.5 | $A_3$ | 27.9212 | 0.177451 | 1.53182 |
| 15 | 29.5 | $A_{15}$ | 30.1151 | 0.378469 | 2.08542 |
| 16 | 28.8 | $A_8$ | 28.9627 | 0.026485 | 0.56507 |
| 17 | 29.0 | $A_{10}$ | 29.4212 | 0.177451 | 1.45259 |
| 18 | 30.3 | $A_{18}$ | 30.3068 | 0.000047 | 0.02273 |
| 19 | 30.2 | $A_{17}$ | 30.3797 | 0.032301 | 0.59511 |
| 20 | 30.9 | $A_{18}$ | 30.9514 | 0.002647 | 0.16650 |
| 21 | 30.8 | $A_{18}$ | 30.9797 | 0.032301 | 0.58352 |
| 22 | 28.7 | $A_8$ | 28.8797 | 0.032301 | 0.62621 |
| 23 | 27.8 | $A_4$ | 28.2212 | 0.177451 | 1.51529 |
| 24 | 27.4 | $A_2$ | 28.0423 | 0.412585 | 2.34426 |
| 25 | 27.7 | $A_3$ | 27.8331 | 0.017729 | 0.48069 |
| 26 | 27.1 | $A_2$ | 27.7151 | 0.378469 | 2.27010 |
| 27 | 28.4 | $A_6$ | 28.5331 | 0.017729 | 0.46884 |
| 28 | 27.8 | $A_4$ | 27.9525 | 0.023256 | 0.54856 |
| 29 | 29.0 | $A_{10}$ | 29.6423 | 0.412585 | 2.21492 |
| 30 | 30.2 | $A_{17}$ | 30.2068 | 0.000047 | 0.02280 |
| | | | | $M.S.E$ = 0.10115 | $AFER$ = 0.80323 |

In Table 4 the forecasted daily average temperatures of the proposed method is compared with the existing methods.

**Table 4 :**(AFER)(In percentage)

| Month | Lee et.al(2006) | Li.et.al(2010) | Proposed Method |
|---|---|---|---|
| June | 1.44 | 2.2669 | 0.8052 |
| July | 1.59 | 2.5855 | 0.8196 |
| August | 1.26 | 2.5352 | 1.0243 |
| September | 1.89 | 2.9656 | 0.8163 |

**Table 1:** Characteristic value of $x_j \in X$

| $x_j$ | $C_B(x_j)$ | $x_j$ | $C_B(x_j)$ |
|---|---|---|---|
| 26.1 | 1 | 29.0 | 3 |
| 27.1 | 1 | 29.3 | 1 |
| 27.4 | 1 | 29.4 | 2 |
| 27.5 | 1 | 29.5 | 2 |
| 27.6 | 1 | 29.7 | 1 |
| 27.7 | 1 | 30.0 | 1 |

| $x_j$ | $C_B(x_j)$ | $x_j$ | $C_B(x_j)$ |
|---|---|---|---|
| 27.8 | 2 | 30.2 | 2 |
| 28.4 | 1 | 30.3 | 1 |
| 28.5 | 1 | 30.5 | 1 |
| 28.7 | 2 | 30.8 | 1 |
| 28.8 | 2 | 30.9 | 1 |

**Table 2 :**The intervals of different length and the midpoints of the intervals

| Intervals | Midpoints | Intervals | Midpoints |
|---|---|---|---|
| $v_1$=[26,26.71) | $m_1$=26.355 | $v_{10}$=[28.94143,29.04286) | $m_{10}$=28.992145 |
| $v_2$=[27.71,27.42) | $m_2$=27.065 | $v_{11}$=[29.04286,29.14429) | $m_{11}$=29.093575 |
| $v_3$=[27.42,27.775) | $m_3$=27.5975 | $v_{12}$=[29.14429,29.24572) | $m_{12}$=29.195005 |
| $v_4$=[27.775,28.13) | $m_4$=27.9525 | $v_{13}$=[29.24572,29.34715) | $m_{13}$=29.296435 |
| $v_5$=[28.13,28.3075) | $m_5$=28.21875 | $v_{14}$=[29.34715,29.44858) | $m_{14}$=29.397865 |
| $v_6$=[28.3075,28.485) | $m_6$=28.39625 | $v_{15}$=[29.44858,29.55) | $m_{15}$=29.49929 |
| $v_7$=[28.485,28.6625) | $m_7$=28.57375 | $v_{16}$=[29.55,29.905) | $m_{16}$=29.7275 |
| $v_8$=[28.6625,28.84) | $m_8$=28.75125 | $v_{17} = [29.905, 30.26)$ | $m_{17}$=30.0825 |
| $v_9$=[28.84,28.94143) | $m_9$=28.890715 | $v_{18}$=[30.26,31) | $m_{18}$=30.63 |

## 6. Conclusion

In this paper, we have presented a new method for forecasting the daily average temperature of the Taipei, Taiwan in which duplicates of data are significant, given in Table 5, based on the characterization of bag and multiset relations. First we compute the intervals of different length using characterization of bag. Then the daily average temperature value using list and multiset relation are forecasted. From the experimental results the proposed method provides the smallest AFER and MSE and improves on other methods using fuzzy times series forecasting methods.

**Table 5 :** Historical data of the daily average temperature from June 1, 1996 to September 30, 1996 in Taipei, Taiwan($unit :^0 C$)

(Central weather bureau, 1996)

| | Month | | | |
|---|---|---|---|---|
| Day | June | July | August | September |
| 1 | 26.1 | 29.9 | 27.1 | 27.5 |
| 2 | 27.6 | 28.4 | 28.9 | 26.8 |
| 3 | 29.0 | 29.2 | 28.9 | 26.4 |
| 4 | 30.5 | 29.4 | 29.3 | 27.5 |
| 5 | 30.0 | 29.9 | 28.8 | 26.6 |
| 6 | 29.5 | 29.6 | 28.7 | 28.2 |
| 7 | 29.7 | 30.1 | 29.0 | 29.2 |
| 8 | 29.4 | 29.3 | 28.2 | 29.0 |
| 9 | 28.8 | 28.1 | 27.0 | 30.3 |
| 10 | 29.4 | 28.9 | 28.3 | 29.9 |
| 11 | 29.3 | 28.4 | 28.9 | 29.9 |
| 12 | 28.5 | 29.6 | 28.1 | 30.5 |
| 13 | 28.7 | 27.8 | 29.9 | 30.2 |
| 14 | 27.5 | 29.1 | 27.6 | 30.3 |
| 15 | 29.5 | 27.7 | 26.8 | 29.5 |
| 16 | 28.8 | 28.1 | 27.6 | 28.3 |
| 17 | 29.0 | 28.7 | 27.9 | 28.6 |
| 18 | 30.3 | 29.9 | 29.0 | 28.1 |
| 19 | 30.2 | 30.8 | 29.2 | 28.4 |
| 20 | 30.9 | 31.6 | 29.8 | 28.3 |
| 21 | 30.8 | 31.4 | 29.6 | 26.4 |
| 22 | 28.7 | 31.3 | 29.3 | 25.7 |
| 23 | 27.8 | 31.3 | 28.0 | 25.0 |
| 24 | 27.4 | 31.3 | 28.3 | 27.0 |
| 25 | 27.7 | 28.9 | 28.6 | 25.8 |
| 26 | 27.1 | 28.0 | 28.7 | 26.4 |
| 27 | 28.4 | 28.6 | 29.0 | 25.6 |
| 28 | 27.8 | 28.0 | 27.7 | 24.2 |
| 29 | 29.0 | 29.3 | 26.2 | 23.3 |
| 30 | 30.2 | 27.9 | 26.0 | 23.5 |
| 31 | - | 26.9 | 27.7 | - |

## References


[1] Cerf, V., Fernandez, E., Gostelow, K., Volausky, S. *Formal control and low properties of a model of computation*, Report ENG 7178, Computer Science Department ,University of California, Los Angeles,CA, p.81, December, 1971.

[2] Chen, S.M, and Hau.C.C. *A new method to forecast enrollments using fuzzy time series*, Int.J. Appl. Sci. Eng., vol.2, no.3, pp.234-244, 2004.

[3] Chen,S.M., *Forecasting enrollments based on Fuzzy time series*, "Fuzzy Sets Syst.,vol.81,no.3,pp.311-319,Aug.1996.

[4] Chen,S.M., *Forecasting enrollments based on high-order fuzzy time series*, Cybern.Syst.,Int.J.,vol.33,no.1,pp.1-16,Jan.2002.

[5] Chen,S.M., and Hwang,J,R. *Temperature prediction using fuzzy time series*,"IEEE Transactions System, Man, Cybern.B.Cybern.,vol.30,no.2,pp.263-275, Apr.2000.

[6] Girish, K.P., Sunil Jacob john. *Relations and functions in multiset context*, "Information sciences 179,pp.758-768, 2009.

[7] Hsu,Y.Y., Tse,S.M., and Wu,B. *A new approach of bivariate fuzzy time series analysis to the forecasting of a stock index*, Int.J. Uncertain Fuzziness Knowl. Based Syst., vol.11,no.6, pp.671-690, 2003.

[8] Huarng, K. *Heuristic models of fuzzy time series for forecasting*, Fuzzy Sets Syst.,vol.123,no.3,pp.369-386,Nov.2001

[9] Huarng,K.H., Yu,T.H.K., and Hsu,Y.W. *A multi variate heuristic model for fuzzy times series forecasting*, IEEE Trans. Sys., Man, Cybern. B. Cybern., vol.37, no.4, pp.836-846, Aug.2007.

[10] Hwang, J.R., Chen,S.M., and Lee,C.H. *Handling forecasting problems using fuzzy time series*, Fuzzy Sets Syst.,vol.100,no.1-3,pp.217-228,Nov.1998.

[11] Jena,S.P., Ghosh,S.K. Tripathy,B.K, *On the theory of bags and lists*, Information Sciences 132, pp 241-254, 2001.

[12] Lee,L.W., Wang,L.H., Chen,S.M., and Leu,Y.H. *Handling forecasting problems based on two-factors high-order fuzzy time series*, IEEE Trans. Fuzzy Syst., vol.14, no.3, pp.468-477, Jun.2006

[13] Lee,L.W., Wang,L.H., and Chen,S.M. *Temperature prediction and TAIFEX forcasting based on fuzzy logical relationships and genetic algorithms*, Expert Syst.Appl.,vol.33,no.3,pp.539-550,Oct.2007.

[14] Li,S.T.,and Cheng,Y.C. *Deterministic fuzzy time series model for forecasting enrollments*, Comput.Math.Appl.,vol.53,no.12,pp.1904-1920, Jun.2007.
[15] Li - Wei Lee, Li - Hui Wang, Shyi - Ming Chen, and Yung - Ho Leu, *Handling forecasting problems Based on Two- factors High - order Fuzzy Time series*, IEEE Transactions on fuzzy systems, vol. 14, No. 3, June 2006.
[16] Li,Y, *Hidden markov models with states depending on observations*, Pattern Recognit.Lett.,vol.26, no.7, pp.977-984, May2005.
[17] Miller,G.A. *The magical number seven, plus or minus two some limits on our capacity of processing information*, The Psychological Review 63, 81 - 97, 1956.
[18] Nai - Yi Wang , Shyi - Ming chen. *Temperature prediction and TAIFEX fore casting based on automatic clustering techniques and two - factors high - order Fuzzy time series*, Expert systems with Applications, 36, pp. 2143 - 2154, 2009.
[19] Peterson,J. *Computation sequence sets*, Journal of Computer System Science 13(1) pp 1-24., 1976.
[20] Sheng - Tun Li and Yi - Chung cheng. *A stochastic HMM Based Forcasting model for fuzzy time series*, IEEE transactions on systems, man, and cybernetics - PART B., cybernetics, vol 40, No. 5, 2010.
[21] Song, Q and Chissom, B.S. *Forecasting enrollments with fuzzy time series - part I*, Fuzzy sets syst., Vol.54, no.1, pp1-9, Feb 1993.
[22] Song,Q and Chissom B,S. *Fuzzy time series and its models*, Fuzzy sets syst., vol.54, no.3, pp.269-277, Mar. 1993.
[23] Song, Q and Chissom B,S. *Forecasting enrollments with fuzzy time series - part II*, Fuzzy sets Syst., vol.62, no.1 pp.1-8, Feb.1994.
[24] Sullivan,J and Woodall,W.H. *A comparison of fuzzy forecasting and Markov modeling*, Fuzzy Sets Syst., vol,64, no.3, pp.279-293, Jun.1994.
[25] Tsaur,R.C.,Yang,J.C.,and Wang,H.-F. *Fuzzy relation analysis in fuzzy time series model*, Compute.Math.Appl.,vol.49,no.4, pp.539-548,Feb.2005.
[26] Yager,R.R. *On the theory of bags*, International Journal General System 13, pp 23-37, 1986.
[27] Yu,H.K. *A refined fuzzy time - series model for forecasting*, Phys. A, vol.346, no.3/4, pp 657-681, Feb.2005.