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Multi-stage multi-objective solid transportation problem for disaster response operation with type-2 triangular fuzzy variables

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Abstract

In this paper, for the first time we formulate and solve multi-stage solid transportation problem (MSSTP) to minimize the total cost and time with type-2 fuzzy transportation parameters. During transportation period, loading, unloading cost and time, volume and weight for each item, limitation of volume and weight for each vehicle are normally imprecise and taken into account to formulate the models. To remove the uncertainty of the type-2 fuzzy transportation parameters from objective functions and constraints, we apply CV-Based reduction methods and generalized credibility measure. Disasters are unexpected situations that require significant logistical deployment to transport equipment and humanitarian goods in order to help and provide relief to victims and sometime this transportation is not possible directly from supply point to destination. Again, the availabilities at supply points and requirements at destinations are not known precisely due to disaster. For this reason, we formulate the multi-stage solid transportation problems under uncertainty (type-2 fuzzy). The models are illustrated with a numerical example. Finally, generalized reduced gradient technique (LINGO.13.0 software) is used to solve the models.

Keywords: Multi-stage solid transportation problem, type-2 fuzzy variable, CV-based reduction methods, generalized credibility, goal programming approach.

2000 AMS Classification: 90B06, 90C05, 90C29

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1. Introduction

1.1. Literature review and the main work of the research: The transportation problem originally developed by Hitchcock [7] is one of the most common combinatorial problems involving constraints. The solid transportation problem (STP) was first stated by Shell [8]. Haley [5, 6] developed the solution procedure of a solid transportation problem and made a comparison between the STP and the classical transportation problem. Geoffrion and Graves [19] were the first researchers studied on two-stage distribution problem. After that so many researchers(Pirkul and Jayaraman [20], Heragu [17], Hindi et al. [21], Syarif and Gen [22], Amiri [23], Gen et al. [24]) study two-stage TP. Mahapatra et al. [25] applied fuzzy multi-objective mathematical programming technique on a reliability optimization model. A type-2 fuzzy variable is a map from a fuzzy possibility space to the real number space; it is an appropriate tool for describing type-2 fuzziness. The concept of a type-2 fuzzy set was first proposed in Qin et al. [1] as an extension of an ordinary fuzzy set. Mitchell [10] used the concept of an embedded type-1 fuzzy number. Liang and Mendel [11] proposed the concept of an interval type-2 fuzzy set. Karnik and Mendel [2], Liu [3], Qin et al. [1], Yang et al. [30], Liu et al. [29], Yang et al. [30] worked on type-2 fuzzy set. In the literature, data envelopment analysis (DEA) technology was first proposed in [12]. Sengupta [22] incorporated stochastic input and output variations; Banker [14] incorporated stochastic variables into DEA; Cooper et al. [15] and Land et al. [16] developed a chance-constrained programming to accommodate the stochastic variations in the data. Qin et al. [4] proposes three noble methods of reduction for a type-2 fuzzy variable. Here, we present in tabular form a scenario of literature development made on transportation problem in Table-1.

Table-1: Some remarkable research works on TP/STP

Author(s), Ref.	Objective	Nature	Additional function	Environments	Solution Techniques
Kundu et al.	Multi-objective	STP	Multi-item	fuzzy	LINGO
Yang and Feng	Single-objective	STP	fixed charges	stochastic	Tabu search algorithm
Kundu et al.	Single-objective	TP	Fixed charge	type-2 fuzzy variable	LINGO
Baidya et al.	Single-objective	STP	Safety factor	Fuzzy, Stochastic, Interval	LINGO
Gen et al.	Single-objective	TP	Two-stage	Deterministic	Genetic algorithms
Proposed	Multi-objective	STP	Multi-stage	Triangular type-2 fuzzy	LINGO

In spite of the above developments, there are so many gaps in the literature. Some of these omissions which are used to formulate the model with type-2 triangular fuzzy number are as follows:

- So many ([26], [27], [28], [31], [32], [33],) solid transportation problems exist in the literature to minimize the total transportation cost only but nobody can formulate any STP to minimize the total transportation time, purchasing cost, loading and unloading cost at a time.

- In spite of the above developments, very few can minimized the time objective function which involves total transportation time, loading and unloading time at a time.

- Lots of two stage transportation problems ([19]-[25]) exist in the literature where the transportation cost is minimized. But nobody formulate and solved a multi-stage multi-item multi-objective solid transportation problem to minimize the "total cost" which involves transportation cost, purchasing cost, loading and unloading cost and "total time" which involves transportation time, loading and unloading time.

- Sometimes, the value of the transportation parameters are not known to us precisely but at that time some imprecise data are known to us. For this reason, lots of researchers solved so many transportation problems with fuzzy (triangular fuzzy number, trapezoidal fuzzy number, type-1 fuzzy number, type-2 fuzzy number, interval type-2 fuzzy number etc.) transportation parameters. But nobody solved any multi-stage multi-item multi-objective (cost and time) STP with transportation parameters as type-2 triangular fuzzy number.

- So many STPs are developed in the literature to minimize the total transportation cost and time subjected to the supply constraints, demand constraints and conveyances capacity constraints, budget constraint, safety constraint etc. but nobody formulated any STP subjected to the weights constraints and volumes constraints during transportation.

In this paper, a multi-item multi-stage solid transportation problem is formulated and solved. Type-2 fuzzy theory is an appropriate field for research. To formulate the model, we consider unit transportation cost, time, supplies, demands, conveyances capacities, loading and unloading cost and time, volume and weights for each and every item, volume and weight capacities for each conveyances as type-2 triangular fuzzy variables. The objective functions for the respective transportation model is to minimize the total cost and time. To defuzzify the constraints and objective functions, we apply CV-based reduction method. The goal programming approach is used to solve the multi-objective programming problem. The deterministic problems so obtained are then solved by using the standard optimization solver - LINGO 13.0 software. We have provided numerical examples illustrating the proposed model and techniques. Some sensitivity analyzes for the model are also presented.
The paper is organized as follows. Problem descriptions are includes in the section 2. In section 3 we briefly introduce some fundamental concepts. The assumptions and notations to construct the model are put in the section 4. In section 5, multi-stage multi-objective solid transportation model with type-2 triangular fuzzy variable is formulated. In section 6, we discuss about the methodology and defuzzification method that used to solve the model. A numerical example put in the section 7 is to illustrate the model numerically. The results of solving the model numerically are put in the section 8. A sensitivity analysis of the model is discussed in the section 9. In section 9, we discuss the results obtained by solving the numerical example. The comparisons of the work with the earlier research are discussed in the section 10. The conclusion and future extension of the research work are discussed in the section 11. The references which are used to prepare this manuscript are put in the last. In this work, we formulate and solve a multi-stage multi-item solid transportation problem to minimize the total cost and time under type-2 fuzzy environment. The real life applications of the research work are as follows:

- Basically to provide some relief to the survived peoples in disaster, we developed our MSSTP model. In our paper, we consider all the transportation parameters as type-2 fuzzy variables since after disaster, it is very difficult to define all transportation parameters precisely. Since due to disaster roads, bridges, towers etc. are damaged, thus it is not possible to survive the peoples smoothly with the help of direct transportation network. This is the reason to formulate an n-stage solid transportation model

- To get the permit of a vehicle is a difficult task for the owner of the vehicle. Some vehicles are permitted to driver on a particular state or country. This permit restricts us to carry the goods from one state to another or one country to another. So in the transportation period, it is important to load and unload the goods so many times at the destination centers (the destination centers are lies between supply points and customers).So to overcome these transportation difficulties we can apply this newly developed model.

1.2. Motivations: The motivation for this research dated back to September 2014, the Kashmir region witnessed disastrous floods across majority of its districts caused by torrential rainfall. The Indian administrated Jammu and Kashmir, as well as Azad Kashmir, Gilgit-Baltistan and Punjab in Pakistan, were affected by these floods. By September 24, 2014, nearly 277 people in India and 280 people in Pakistan died due to the floods and more than 1.1 million were affected by the floods. During this period, it is tedious to send the necessary foods to the survived peoples. For this reason, it is important to impose so many destination centers in between supply point and customers.

2. Problem Description:

Disaster (earthquick, flood etc.) is an extra ordinary situation for any country or state and to provide relief to the survived person which is a risky and tedious task to us. But at that time transportation is required to serve the foods, clothes etc. to the peoples. Also due to the disaster, it is not possible to deliver the necessary things directly to the survived people. It required some destination centers in between source point and survived peoples such that the total cost and time should be minimized. This also motivated us to formulate a multi-stage multi-objective solid transportation problem. The exact figure of the survived peoples due to disaster is not known to us exactly. For this reason, the transportation parameters are also remains unknown to us. Since all the transportation parameters are not known to us precisely, so we consider the transportation parameters as type-2 triangular fuzzy variables. In this multi-stage transportation network, destination center for stage-1 is reduced to the supply point for stage-2 and destination center for stage-2 is reduced to the supply point for stage-3 and similarly the destination center for the stage-(n-1) is converted to the supply point to the stage-n. The pictorial representation of the multi-stage solid transportation problem is as follows:

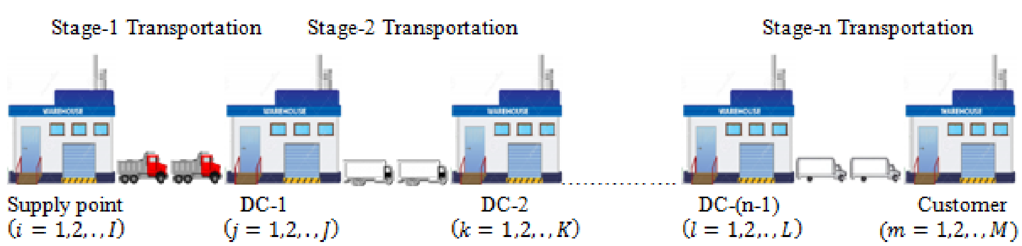

Figure 1. *Multi-stage solid transportation network*

3. Fundamental Concepts

Definition 1. (Regular Fuzzy Variable)
Let Γ be the universe of discourse. An ample field [32] $\mathcal{A}$ on Γ is a class of subsets of Γ that is closed under arbitrary unions, intersections, and complements in Γ.
Let $Pos : \mathcal{A} \to [0,1]$ be a set function on the amplefield Γ. Pos is said to be a possibility measure [32] if it satisfies the following conditions:
(p1) $Pos(\varphi) = 0$ and $Pos(\Gamma) = 1$.
(p2) For any subclass $\{A_i | i \in I\}$ of $\mathcal{A}$ (finite, counter or uncountable), $Pos(\bigcup_{i\in I}(A_i)) = Sup_{i\in I} Pos(A_i)$
The triplet $(\Gamma, \mathcal{A}, Pos)$ is referred to as a possibility space, in which a credibility measure [33] is defined as $Cr(A) = \frac{1}{2}(1 + Pos(A) - Pos(A^c)), A \in \mathcal{A}$.
If $(\Gamma, \mathcal{A}, Pos)$ is a possibility space, then an m-ary regular fuzzy vector $\xi = (\xi_1, \xi_2, ..., \xi_m)$ is defined as a membership map from Γ to the space $[0,1]^m$ in the sense that for every $t = (t_1, t_2, ..., t_m) \in [0,1]^m$, one has
$\{\gamma \in \Gamma | \xi(\gamma) \le t\} = \{\gamma \in \Gamma | \xi_1(\gamma) \le t_1, \xi_2(\gamma) \le t_2, ..., \xi_m(\gamma) \le t_m\} \in \mathcal{A}$
When $m = 1, \xi$ is called a regular fuzzy variable (RFV).

3.1. Critical values for RFVs. .

Definition 2.(Qin et al. [1]) Let ξ be an RFV. Then the optimistic CV of ξ, denoted by $CV^*[\xi]$, defined as $CV^*[\xi] = \underbrace{Sup\{\alpha \wedge Pos\{\xi \ge \alpha\}\}}_{\alpha\in[0,1]}$, while the pessimistic CV of ξ, denoted by $CV_*[\xi]$, is defined as $CV_*[\xi] = \underbrace{Sup\{\alpha \wedge Nec\{\xi \ge \alpha\}\}}_{\alpha\in[0,1]}$.
The CV of ξ , denoted by $CV[\xi]$, is defined as $CV[\xi] = \underbrace{Sup\{\alpha \wedge Cr\{\xi \ge \alpha\}\}}_{\alpha\in[0,1]}$.

Theorem 1. (Qin et al. [1]) Let $\xi = (r_1, r_2, r_3, r_4)$ be a trapezoidal RFV. Then we have

(i) The optimistic CV of ξ is $CV^*[\xi] = \frac{r_4}{1+r_4-r_3}$.
(ii) The pessimistic CV of ξ is $CV_*[\xi] = \frac{r_2}{1+r_2-r_1}$.
(iii) The CV of ξ is $CV[\xi] = \begin{cases} \frac{2r_2-r_1}{1+2(r_2-r_1)}, & \text{if } r_2 > \frac{1}{2} \\ \frac{1}{2}, & \text{if } r_2 \le \frac{1}{2} \le r_3 \\ \frac{r_4}{1+2(r_4-r_3)}, & r_3 \le \frac{1}{2} \end{cases}$

3.2. Methods of reduction for type-2 fuzzy variables (CV-Based Reduction Methods).

Due to the fuzzy membership function of a type-2 fuzzy number, the computation complexity is very high in practical applications. To avoid this difficulty, some defuzzification methods have been proposed in the literature (see [6-8]). In this section, we propose some new methods of reduction for a type-2 fuzzy variable. Compared with the existing methods, the new methods are very much easier to implement when we employ them to build a mathematical model with type-2 fuzzy coefficients.
Let $(\Gamma, \mathcal{A}, \tilde{Pos})$ be a fuzzy possibility space and $\tilde{\xi}$ a type-2 fuzzy variable with a known secondary possibility distribution function $\mu_{\tilde{\xi}}(x)$. To reduce the type-2 fuzziness, one approach is to give a representing value for RFV $\mu_{\tilde{\xi}}(x)$. For this purpose, we suggest employing the CVs of $\tilde{Pos}\{\gamma | \tilde{\xi}(\gamma) = x\}$ as the representing values. This methods the CV-based methods for the type-2 fuzzy variable $\tilde{\xi}$
Theorem 2.(Qin et al. [1]) Let $\tilde{\xi}$ be a type-2 triangular fuzzy variable defined as $\tilde{\xi} = (\tilde{r}_1, \tilde{r}_2, \tilde{r}_3; \theta_l, \theta r)$. Then we have

(i) Using the optimistic CV reduction method, the reduction ξ_1 of $\tilde{\xi}$ has the following possibility distribution:

$$\mu_{\xi_1}(x) = \begin{cases} \frac{(1+\theta_r)(x-r_1)}{(r_2-r_1+\theta_r(x-r_1))}, & \text{if } x \in [r_1, \frac{r_1+r_2}{2}] \\ \frac{(1-\theta_r)x+\theta_r r_2-r_1}{(r_2-r_1+\theta_l(r_2-x))}, & \text{if } x \in [\frac{r_1+r_2}{2}, r_2] \\ \frac{(-1+\theta_r)x-\theta_r r_2+r_3}{r_3-r_2+\theta_r(x-r_2)}, & \text{if } x \in [r_2, \frac{r_2+r_3}{2}] \\ \frac{(1+\theta_r)(r_3-x)}{r_3-r_2+\theta_r(r_3-x)}, & \text{if } x \in [\frac{r_2+r_3}{2}, r_3] \end{cases}$$

(ii) Using the pessimistic CV reduction method, the reduction ξ_2 of $\tilde{\xi}$ has the following possibility distribution:

$$\mu_{\xi_2}(x) = \begin{cases} \frac{x-r_1}{r_2-r_1+\theta_l(x-r_1)}, & \text{if } x \in [r_1, \frac{r_1+r_2}{2}] \\ \frac{x-r_1}{r_2-r_1+\theta_l(r_2-x)}, & \text{if } x \in [\frac{r_1+r_2}{2}, r_2] \\ \frac{r_3-x}{r_3-r_2+\theta_l(x-r_2)}, & \text{if } x \in [r_2, \frac{r_2+r_3}{2}] \\ \frac{r_3-x}{r_3-r_2+\theta_l(r_3-x)}, & \text{if } x \in [\frac{r_2+r_3}{2}, r_3] \end{cases}$$

(iii) Using the CV reduction method, the reduction ξ_3 of $\tilde{\xi}$ has the following possibility distribution:

$$\mu_{\xi_3}(x) = \begin{cases} \frac{(1+\theta_r)(x-r_1)}{r_2-r_1+2\theta_r(x-r_1)}, & \text{if } x \in [r_1, \frac{r_1+r_2}{2}] \\ \frac{(1-\theta_l)x+\theta_l r_2-r_1}{r_2-r_1+2\theta_l(r_2-x)}, & \text{if } x \in [\frac{r_1+r_2}{2}, r_2] \\ \frac{(-1+\theta_l)x-\theta_l r_2+r_3}{r_3-r_2+2\theta_l(x-r_2)}, & \text{if } x \in [r_2, \frac{r_2+r_3}{2}] \\ \frac{(1+\theta_r)(r_3-x)}{r_3-r_2+2\theta_r(r_3-x)}, & \text{if } x \in [\frac{r_2+r_3}{2}, r_3] \end{cases}$$

3.3. Generalized credibility and its properties. Suppose ξ is a general fuzzy variable with the distribution μ. The generalized credibility measure $\tilde{Cr}$ of the event $\{\xi \geq \alpha\}$ is defined by
$\tilde{Cr}(\{\xi \geq \alpha\}) = \frac{1}{2}(Sup_{x\in\Re}\mu(x) + Sup_{x\geq r}\mu(x) - Sup_{x<r}\mu(x)), r \in \Re$.
Therefore, if ξ is normalized, it is easy to check that $Cr(\xi \geq \alpha) + Cr(\xi < \alpha) = Sup_{x\in\Re}\mu_\xi(x) = 1$; then $\tilde{Cr}$ coincides with the usual credibility measure. The concept of independence for normalized fuzzy variables and its properties were discussed in [35]. In the following, we also need to extend independence to general fuzzy variables. The general fuzzy variables $\xi_1, \xi_2, \xi_2 ..., \xi_n$ are said to be mutually independent if and only if $\tilde{Cr}\{\xi_i \in B_i, i = 1, 2, ...n\} = Min_{1\leq i\leq n}\tilde{Cr}\{\xi_i \in B_i\}$ for any subsets $B_i, i = 1, 2, ...n$ of $\Re$
Like the α-optimistic value of the normalized fuzzy variable [36], the α-optimistic value of general fuzzy variables can be defined through the generalized credibility measure. Let ξ be a fuzzy variable (not necessary normalized). Then $\xi_{Sup}(\alpha) = Sup\{r|\tilde{Cr}\{\xi \geq r\} \geq \alpha\}, \alpha \in [0, 1]$ is called the α-optimistic value of ξ, while $\xi_{inf} = Inf\{r|\tilde{Cr}\{\xi \leq r\} \geq \alpha\}, \alpha \in [0, 1]$, is called the $\alpha-$ pessimistic value of ξ.
Theorem 3. (Qin et al. [1]) Let ξ_i be the reduction of the type-2 fuzzy variable$\tilde{\xi}_i = (\tilde{r}_1^i, \tilde{r}_2^i, \tilde{r}_3^i; \theta_{l,i}, \theta_{r,i})$ obtained by the CV reduction method for $i = 1, 2, ...n$. Suppose $\xi_1, \xi_2, ..., \xi_n$ are mutually independent, and $k_i \geq o$ for $i = 1, 2, ...n$.
Case-I: If $\alpha \in (0, 0.25]$, then eqnarray $\tilde{Cr}\{\sum_{i=1}^n k_i\xi_i \leq t\} \geq \alpha$ is equivalent to

$$\sum_{i=1}^{n} \frac{(1-2\alpha+(1-4\alpha)\theta_{r,i})k_i r_1^i + 2\alpha k_i r_2^i}{1+(1-4\alpha)\theta_{r,i}}$$

Case-II: If $\alpha \in (0.25, 0.50]$, then eqnarray $\tilde{Cr}\{\sum_{i=1}^n k_i\xi_i \leq t\} \geq \alpha$ is equivalent to

$$\sum_{i=1}^{n} \frac{(1-2\alpha)k_i r_1^i + (2\alpha+(4\alpha-1)\theta_{l,i})k_i r_2^i}{1+(1-4\alpha)\theta_{l,i}}$$

Case-III: If $\alpha \in (0.50, 0.75]$, then eqnarray $\tilde{Cr}\{\sum_{i=1}^{n} k_i\xi_i \leq t\} \geq \alpha$ is equivalent to

$$\sum_{i=1}^{n} \frac{(2\alpha-1)k_i r_3^i + (2(1-\alpha)+(3-4\alpha)\theta_{l,i})k_i r_2^i}{1+(3-4\alpha)\theta_{l,i}}$$

Case-IV: If $\alpha \in (0.75, 1]$, then eqnarray $\tilde{Cr}\{\sum_{i=1}^{n} k_i\xi_i \leq t\} \geq \alpha$ is equivalent to

$$\sum_{i=1}^{n} \frac{(2\alpha-1+(4\alpha-3)\theta_{r,i})k_i r_3^i + 2(1-\alpha)k_i r_2^i}{1+(4\alpha-3)\theta_{r,i}}$$

3.4. Goal programming Method. .
The goal programming method is used to solve the multi-objective programming problem (MOPP). A general MOPP is of the following form:

$$(3.1)\begin{cases} \text{Find the values of L decision variables}\, x_1, x_2, ..., x_L \text{which minimizes} \\ F(x) = (f_1(x), f_2(x), ..., f_Q(x))^T \\ \text{subject to}\, x \in X \end{cases}$$

Where, $X = \{x = (x_1, x_2, ..., x_L)$ such that $g_t(x) \leq 0, x_l \geq 0, t = 1, 2, ..., T; l = 1, 2, ..., L\}$ and $f_1(x), f_2(x), ..., f_Q(x)$ are $Q(\geq 2)$ objective functions.
The different steps of the goal programming method are as follows:
Step-1: Solve the multi-objective programming problem (1) as a single objective problem using only one objective at a time ignoring the others, and determine the ideal objective vector, say $f_1^{min}, f_2^{min}, ..., f_Q^{min}$.
Step-2: Formulate the following GP problem using the ideal objective vector obtained is Step-1,

$$Min\{\sum_{q=1}^{Q}[(d_q^+)^p + (d_q^-)^p]\}^{\frac{1}{p}}$$

subject to $f_q(x) + d_q^+ - d_q^- = f_q^{min}, d_q^+ \geq 0, d_q^- \geq 0, d_q^+ d_q^- = 0 (q = 1, 2, ..., Q)$, for all $x \in X$.
Step-3: Now, solve the above single objective problem described in Step-2 by GRG method and obtain the compromise solution.

4. Notations and assumptions for the proposed model

(i) $\tilde{C}_{ijk_1q}^1, \tilde{C}_{jkk_2q}^2, \tilde{C}_{lmk_nq}^n$ = Fuzzy unit transportation cost is to transport the q-th item from i-th plant to j-th DC by k_1-th vehicle, j-th plant to k-th DC k_2-th vehicle and l-th plant to m-th customer k_n-th vehicle respectively.

(ii) $\tilde{t}_{ijk_1q}^1, \tilde{t}_{jkk_2q}^2, \tilde{t}_{lmk_nq}^n$ = Fuzzy unit transportation time is to transport the q-th item from i-th plant to j-th DC by k_1-th vehicle, j-th plant to k-th DC k_2-th vehicle and l-th plant to m-th customer k_n-th vehicle respectively.

(iii) $\tilde{x}_{ijk_1q}^1, \tilde{x}_{jkk_2q}^2, \tilde{x}_{lmk_nq}^n, \tilde{x}_{ulk_{(n-1)}q}^n, \tilde{x}_{lmk_nq}^n$ = Unknown quantities which is to be transported from i-th plant to j-th DC of q-th item by k_1-th vehicle for stage-1, j-th plant to k-th DC of q-th item by k_2-th vehicle for stage-2, u-th plant to l-th DC of q-th item by $k_{(n-1)}$-th vehicle for stage-$(n-1)$, and l-th plant to m-th customer of q-th item by k_n-th vehicle for stage-n respectively.

(iv) $\tilde{PC}$ = Fuzzy purchasing cost of q-th item at i-th source.

(v) $\tilde{LO}_i^1, \tilde{LO}_j^2, ..., \tilde{LO}_l^n$ = Fuzzy loading cost at i-th plant of stage-1, j-th plant of stage-2 and l-th plant of stage-n respectively.

(vi) $\tilde{UD}_j^1, \tilde{UD}_k^2, ..., \tilde{UD}_m^n$ =Fuzzy unloading cost at j-th DC of stage-1, k-th DC of stage-2 and m-th customer of stage-n respectively.

(vii) $L\tilde{TO}_i^1, L\tilde{TO}_j^2, ..., L\tilde{TO}_l^n$ = Fuzzy loading time at i-th plant of stage-1, j-th plant of stage-2 and l-th plant of stage-n respectively.

(viii) $U\tilde{TD}_j^1, U\tilde{TD}_k^2, ..., U\tilde{TD}_m^n$ =Fuzzy unloading time at j-th DC of stage-1, k-th DC of stage-2 and m-th customer of stage-n respectively.

(ix) $y_{ijk_1q}^1 = \begin{cases} 1, if\ x_{ijk_1q}^1 > 0 \\ 0,\ otherwise \end{cases}$, $y_{jkk_2q}^2 = \begin{cases} 1, if\ x_{jkk_2q}^2 > 0 \\ 0,\ otherwise \end{cases}$, $y_{lmk_nq}^n = \begin{cases} 1, if\ x_{lmk_nq}^n > 0 \\ 0,\ otherwise \end{cases}$

5. Formulation of solid transportation problem with transportation parameters as type-2 triangular fuzzy variables

Let us consider $'I'$ supply points (or sources), $'J'$ destination centers, K_1 conveyances for stage-1 transportation; $'J'$ supply points (or sources), $'K'$ destination centers, K_2 conveyances for stage-2 transportation; $'U'$ supply points, $'L'$ destination centers, $k_{(n-1)}$ conveyances for stage-$(n-1)$ transportation; $'L'$ supply points (or sources), $'M'$ destination centers, K_n conveyances for stage-n transportation. Also we consider that Q be the number of items which is to be transported from plants to DC by different modes of conveyances.

$$
\begin{aligned}
(5.1) \qquad Minf_1 = & \sum_{q=1}^{Q}\sum_{i=1}^{I}\sum_{j=1}^{J}\sum_{k_1=1}^{K_1}(\tilde{C}_{ijk_1q}^1 + \tilde{LO}_i^1 + \tilde{UD}_j^1 + \tilde{PC}_{iq}^1)x_{ijk_1q}^1 \\
& + \sum_{q=1}^{Q}\sum_{j=1}^{J}\sum_{k=1}^{K}\sum_{k_2=1}^{K_2}(\tilde{C}_{jkk_2q}^2 + \tilde{LO}_j^2 + \tilde{UD}_k^2 + \tilde{PC}_{jq}^2)x_{jkk_2q}^2 + ... \\
& + \sum_{q=1}^{Q}\sum_{l=1}^{L}\sum_{m=1}^{M}\sum_{k_n=1}^{K_n}(\tilde{C}_{lmk_nq}^n + \tilde{LO}_l^n + \tilde{UD}_m^n + \tilde{PC}_{lq}^n)x_{lmk_nq}^n
\end{aligned}
$$

$$
\begin{aligned}
(5.2) \qquad Minf_2 = & \sum_{q=1}^{Q}\sum_{i=1}^{I}\sum_{j=1}^{J}\sum_{k_1=1}^{K_1}(\tilde{t}_{ijk_1q}^1 + L\tilde{TO}_i^1 + U\tilde{TD}_j^1)y_{ijk_1q} \\
& + \sum_{q=1}^{Q}\sum_{j=1}^{J}\sum_{k=1}^{K}\sum_{k_2=1}^{K_2}(\tilde{t}_{jkk_2q}^2 + L\tilde{TO}_j^2 + U\tilde{TD}_k^2)y_{jkk_2q}^2 + ... \\
& + \sum_{q=1}^{Q}\sum_{l=1}^{L}\sum_{m=1}^{M}\sum_{k_n=1}^{K_n}(\tilde{t}_{lmk_nq}^n + L\tilde{TO}_l^n + U\tilde{TD}_m^n)y_{lmk_nq}^n
\end{aligned}
$$

$$(5.3)\quad \sum_{j=1}^{J}\sum_{k_1=1}^{K_1} x^1_{ijk_1q} \le \tilde{a}^1_{iq}, i=1,2,...,I; q=1,2,...,Q,$$

$$(5.4)\quad \sum_{i=1}^{I}\sum_{k_1=1}^{K_1} x^1_{ijk_1q} \ge \tilde{b}^1_{jq}, j=1,2,...,J; q=1,2,...,Q,$$

$$(5.5)\quad \sum_{q=1}^{Q}\sum_{i=1}^{I}\sum_{j=1}^{J} x^1_{ijk_1q} \le \tilde{e}^1_{k_1}, k_1=1,2,...,K_1,$$

$$(5.6)\quad \sum_{q=1}^{Q}\sum_{i=1}^{I}\sum_{j=1}^{J} \tilde{w}_q x^1_{ijk_1q} \le \tilde{W}^1_{k_1}, k_1=1,2,...,K_1,$$

$$(5.7)\quad \sum_{q=1}^{Q}\sum_{i=1}^{I}\sum_{j=1}^{J} \tilde{v}_q x^1_{ijk_1q} \le \tilde{V}^1_{k_1}, k_1=1,2,...,K_1,$$

$$(5.8)\quad \sum_{k=1}^{K}\sum_{k_2=1}^{K_2} x^2_{jkk_2q} \le \sum_{i=1}^{I}\sum_{k_1=1}^{K_1} x^1_{ijk_1q}, j=1,2,...,J; q=1,2,...,Q,$$

$$(5.9)\quad \sum_{j=1}^{J}\sum_{k_2=1}^{K_2} x^2_{jkk_2q} \ge \tilde{b}^2_{kq} k=1,2,...K; q=1,2,...,Q,$$

$$(5.10)\sum_{q=1}^{Q}\sum_{j=1}^{J}\sum_{k=1}^{K} x^2_{jkk_2q} \le \tilde{e}^2_{k_2}, k_2=1,2,...,K_2,$$

$$(5.11)\quad \sum_{q=1}^{Q}\sum_{j=1}^{J}\sum_{k=1}^{K} \tilde{w}_q x^2_{jkk_2q} \le \tilde{W}^2_{k_2}, k_2=1,2,...,K_2,$$

$$(5.12)\sum_{q=1}^{Q}\sum_{j=1}^{J}\sum_{k=1}^{K} \tilde{v}_q x^2_{jkk_2q} \le \tilde{V}^2_{k_2}, k_2=1,2,...,K_2,$$

....

$$(5.13)\quad \sum_{m=1}^{M}\sum_{k_n=1}^{K_n} x^n_{lmk_nq} \le \sum_{u=1}^{U}\sum_{k_{(n-1)}=1}^{K_{(n-1)}} x^{(n-1)}_{ulk_{(n-1)}q}, l=1,2,...,L; q=1,2,...,Q,$$

$$(5.14)\ \sum_{l=1}^{L}\sum_{k_n=1}^{K_1} x^n_{lmk_nq} \ge \tilde{b}^n_{mq}, m=1,2,...M; q=1,2,...,Q,$$

$$(5.15)\sum_{q=1}^{Q}\sum_{l=1}^{L}\sum_{m=1}^{M} x^n_{lmk_nq} \le \tilde{e}^n_{k_n}, k_n=1,2,...,K_n,$$

$$(5.16)\quad \sum_{q=1}^{Q}\sum_{l=1}^{L}\sum_{m=1}^{M} \tilde{w}_q x^n_{lmk_nq} \le \tilde{W}^n_{k_n}, k_n=1,2,...,K_n,$$

$$(5.17)\quad \sum_{q=1}^{Q}\sum_{l=1}^{L}\sum_{m=1}^{M} \tilde{v}_q x^n_{lmk_nq} \le \tilde{V}^n_{k_n}, k_n=1,2,...,K_n,$$

$$x^1_{ijk_1q} \ge 0, x^2_{jkk_2q} \ge 0, ..., x^{(n-1)}_{ulk_{(n-1)}q} \ge 0, x^n_{lmk_nq} \ge 0,$$

for all $i,j,k,u,l,m,q,k_1,k_2,k_{(n-1)},k_n$.

where, $\tilde{W}^1_{k_1}, \tilde{W}^2_{k_2}, \tilde{W}^n_{k_n}$ are the fuzzy weight capacity of k_1-th vehicle of stage-1, k_2-th vehicle of stage-2, k_n-th vehicle of stage-n. $\tilde{V}^1_{k_1}, \tilde{V}^2_{k_2}, \tilde{V}^n_{k_n}$ are the fuzzy volume capacity of k_1-th vehicle of stage-1, k_2-th vehicle of stage-2, k_n-th vehicle of stage-n. $\tilde{w}_q, \tilde{v}_q$ are the fuzzy weight and volume of the q-th item. Also $\tilde{a}^1_{iq}$ be the fuzzy availabilities of the q-th item at i-th source of stage-1. b^1_{jq}, b^2_{kq}, and b^n_{mq} are the fuzzy demands of q-th item at j-th DC, k-th DC and m-th customer for stage-1, stage-2 and stage-3 respectively. Also, $\tilde{e}^1_{k_1}\tilde{e}^2_{k_2}, \tilde{e}^n_{k_n}$ are the fuzzy conveyances capacities of the k_1-th, k_2-th,k_n-th conveyances for stage-1, stage-2 and stage-n respectively. In this model formulation, we are to minimize two objective functions as total cost and time under supply, demand, conveyance capacity, weight and volume constraints. Here the first summation of the first objective indicates the total cost for stage-1 transportation. Similarly, second and last summation of the first objective function indicates the total cost for stage-2 and stage-n transportation respectively. Also the three summations of second objective denotes the total time in transportation respectively for stage-1, stage-2 and stage-n respectively. We formulate the model in such a way that the goods are loaded at the supply point and it is unloaded at the DC for stage-1 transportation. Since due to disaster, it is not possible to move the vehicle directly to the survived people so after unloading at the first DC it again loaded to another vehicle and goes to the next DC and it is unloaded again in second DC for stage-2. In this way the necessary goods are transported to the survived peoples or customers. For this reason, we impose the loading and unloading cost and time for each stage. Again purchasing cost is also imposed in our model.

6. Methodology and defuzzification technique used to solve the Model

6.1. Methodology. The world has become more complex and almost every important real-world problem involves more than one objective. In such cases, decision makers find imperative to evaluate best possible approximate solution alternatives according to multiple criteria. To solve such multi-objective programming problem we apply goal programming method. Using CV-based reduction method and generalized credibility measure we find the deterministic form of type-2 fuzzy transportation parameters. Finally generalized reduced gradient technique (LINGO 13.0 optimization software) is used to solve the developed model.

6.2. Defuzzification. The deterministic form of the objective functions and constraints obtained by using CV-based reduction method and generalized credibility measure are as follows:

$$
\begin{aligned}
(6.1)\quad & Cr\{(\sum_{q=1}^{Q}\sum_{i=1}^{I}\sum_{j=1}^{J}\sum_{k_1=1}^{K_1}(\tilde{C}^1_{ijk_1q}+\tilde{LO}^1_i+\tilde{UD}^1_j+\tilde{PC}^1_{iq})x^1_{ijk_1q} \\
& +\sum_{q=1}^{Q}\sum_{j=1}^{J}\sum_{k=1}^{K}\sum_{k_2=1}^{K_2}(\tilde{C}^2_{jkk_2q}+\tilde{LO}^2_j+\tilde{UD}^2_k+\tilde{PC}^2_{jq})x^2_{jkk_2q}+\ldots \\
& +\sum_{q=1}^{Q}\sum_{l=1}^{L}\sum_{m=1}^{M}\sum_{k_n=1}^{K_n}(\tilde{C}^n_{lmk_nq}+\tilde{LO}^n_l+\tilde{UD}^n_m+\tilde{PC}^n_{lq})x^n_{lmk_nq})\geq f_1\}\leq\alpha_c
\end{aligned}
$$

$$(6.2)\quad Cr\{(\sum_{q=1}^{Q}\sum_{i=1}^{I}\sum_{j=1}^{J}\sum_{k_1=1}^{K_1}(\tilde{t}^1_{ijk_1q}+L\tilde{T}O^1_i+U\tilde{T}D^1_j)y^1_{ijk_1q}$$
$$+\sum_{q=1}^{Q}\sum_{j=1}^{J}\sum_{k=1}^{K}\sum_{k_2=1}^{K_2}(\tilde{t}^2_{jkk_2q}+L\tilde{T}O^2_j+U\tilde{T}D^2_k)y^2_{jkk_2q}+...$$
$$+\sum_{q=1}^{Q}\sum_{l=1}^{L}\sum_{m=1}^{M}\sum_{k_n=1}^{K_n}(\tilde{t}^n_{lmk_nq}+L\tilde{T}O^n_l+U\tilde{T}D^n_m)y^n_{lmk_nq})\geq f_2\}\leq\alpha_t$$

$$(6.3)\quad Cr(\sum_{j=1}^{J}\sum_{k_1=1}^{K_1}x^1_{ijk_1q}\leq\tilde{a}^1_{iq})\geq\alpha_{avail.}, i=1,2,...,I; q=1,2,...,Q,$$

$$(6.4)\quad Cr(\sum_{i=1}^{I}\sum_{k_1=1}^{K_1}x^1_{ijk_1q}\geq\tilde{b}^1_{jq})\geq\alpha_{demand}, j=1,2,...,J; q=1,2,...,Q,$$

$$(6.5)\quad Cr(\sum_{q=1}^{Q}\sum_{i=1}^{I}\sum_{j=1}^{J}x^1_{ijk_1q}\leq\tilde{e}^1_{k_1})\geq\alpha_{con.cap.}, k_1=1,2,...,K_1,$$

$$(6.6)\quad Cr(\sum_{q=1}^{Q}\sum_{i=1}^{I}\sum_{j=1}^{J}\tilde{w}_qx^1_{ijk_1q}\leq\tilde{W}^1_{k_1})\geq\alpha_{weight}, k_1=1,2,...,K_1,$$

$$(6.7)\quad Cr(\sum_{q=1}^{Q}\sum_{i=1}^{I}\sum_{j=1}^{J}\tilde{v}_qx^1_{ijk_1q}\leq\tilde{V}^1_{k_1})\geq\alpha_{volume}, k_1=1,2,...,K_1,$$

$$(6.8)\quad Cr(\sum_{j=1}^{J}\sum_{k_2=1}^{K_2}x^2_{jkk_2q}\geq\tilde{b}^2_{kq})\geq\alpha_{demand}, k=1,2,...K; q=1,2,...,Q,$$

$$(6.9)\quad Cr(\sum_{q=1}^{Q}\sum_{j=1}^{J}\sum_{k=1}^{K}x^2_{jkk_2q}\leq\tilde{e}^2_{k_2})\geq\alpha_{con.cap.}, k_2=1,2,...,K_2,$$

$$(6.10)\quad Cr(\sum_{q=1}^{Q}\sum_{j=1}^{J}\sum_{k=1}^{K}\tilde{w}_qx^2_{jkk_2q}\leq\tilde{W}^2_{k_2})\geq\alpha_{weight}, k_2=1,2,...,K_2,$$

$$(6.11)\quad Cr(\sum_{q=1}^{Q}\sum_{j=1}^{J}\sum_{k=1}^{K}\tilde{v}_qx^2_{jkk_2q}\leq\tilde{V}^2_{k_2})\geq\alpha_{volume}, k_2=1,2,...,K_2,$$

....

$$(6.12)\quad Cr(\sum_{l=1}^{L}\sum_{k_n=1}^{K_1}x^n_{lmk_nq}\geq\tilde{b}^n_{mq})\geq\alpha_{demand}, m=1,2,...M; q=1,2,...,Q,$$

$$(6.13)\quad Cr(\sum_{q=1}^{Q}\sum_{l=1}^{L}\sum_{m=1}^{M}x^n_{lmk_nq}\leq\tilde{e}^n_{k_n})\geq\alpha_{con.cap.}v, k_n=1,2,...,K_n,$$

$$(6.14)\quad Cr(\sum_{q=1}^{Q}\sum_{l=1}^{L}\sum_{m=1}^{M}\tilde{w}_qx^n_{lmk_nq}\leq\tilde{W}^n_{k_n})\geq\alpha_{weight}, k_n=1,2,...,K_n,$$

$$(6.15)\quad Cr(\sum_{q=1}^{Q}\sum_{l=1}^{L}\sum_{m=1}^{M}\tilde{v}_qx^n_{lmk_nq}\leq\tilde{V}^n_{k_n})\geq\alpha_{volume}, k_n=1,2,...,K_n,$$

Let us consider $\alpha_c, \alpha_t, \alpha_{avail.}, \alpha_{demand}, \alpha_{con.cap.}, \alpha_{weight}, \alpha_{volume}$ be the credibility level for cost, time, availabilities, demands, conveyances capacities, weights, volume respectively for stage-1, stage-2,...,stage-n.
The crisp conversion of the constraints (6.1)-(6.15) are as follows:

$$
\begin{aligned}
Minf_1 = &\sum_{q=1}^{Q}\sum_{i=1}^{I}\sum_{j=1}^{J}\sum_{k_1=1}^{K_1}(S_{\tilde{C}^1_{ijk_1q}} + S_{\tilde{LO}^1_i} + S_{\tilde{UD}^1_j} + S_{\tilde{PC}^1_{iq}})x^1_{ijk_1q}\\
&+\sum_{q=1}^{Q}\sum_{j=1}^{J}\sum_{k=1}^{K}\sum_{k_2=1}^{K_2}(S_{\tilde{C}^2_{jkk_2q}} + S_{\tilde{LO}^2_j} + S_{\tilde{UD}^2_k} + S_{\tilde{PC}^2_{jq}})x^2_{jkk_2q} + ...\\
&+\sum_{q=1}^{Q}\sum_{l=1}^{L}\sum_{m=1}^{M}\sum_{k_n=1}^{K_n}(S_{\tilde{C}^n_{lmk_nq}} + S_{\tilde{LO}^n_l} + S_{\tilde{UD}^n_m} + S_{\tilde{PC}^n_{lq}})x^n_{lmk_nq}\\
Minf_2 = &\sum_{q=1}^{Q}\sum_{i=1}^{I}\sum_{j=1}^{J}\sum_{k_1=1}^{K_1}(S_{\tilde{t}^1_{ijk_1q}} + S_{L\tilde{T}O^1_i} + S_{U\tilde{T}D^1_j})y_{ijk_1q}\\
&+\sum_{q=1}^{Q}\sum_{j=1}^{J}\sum_{k=1}^{K}\sum_{k_2=1}^{K_2}(S_{\tilde{t}^2_{jkk_2q}} + S_{L\tilde{T}O^2_j} + S_{U\tilde{T}D^2_k})y^2_{jkk_2q} + ...\\
&+\sum_{q=1}^{Q}\sum_{l=1}^{L}\sum_{m=1}^{M}\sum_{k_n=1}^{K_n}(S_{\tilde{t}^n_{lmk_nq}} + S_{L\tilde{T}O^n_l} + S_{U\tilde{T}D^n_m})y^n_{lmk_nq}
\end{aligned}
$$

$$\sum_{j=1}^{J}\sum_{k_1=1}^{K_1} x^1_{ijk_1q} \le S_{\tilde{a}^1_{iq}}, i = 1, 2, ..., I; q = 1, 2, ..., Q,$$

$$\sum_{i=1}^{I}\sum_{k_1=1}^{K_1} x^1_{ijk_1q} \ge S_{\tilde{b}^1_{jq}}, j = 1, 2, ..., J; q = 1, 2, ..., Q,$$

$$\sum_{q=1}^{Q}\sum_{i=1}^{I}\sum_{j=1}^{J} x^1_{ijk_1q} \le S_{\tilde{e}^1_{k_1}}, k_1 = 1, 2, ..., K_1,$$

$$\sum_{q=1}^{Q}\sum_{i=1}^{I}\sum_{j=1}^{J} S_{\tilde{w}_q} x^1_{ijk_1q} \le S_{\tilde{W}^1_{k_1}}, k_1 = 1, 2, ..., K_1,$$

$$\sum_{q=1}^{Q}\sum_{i=1}^{I}\sum_{j=1}^{J} S_{\tilde{v}_q} x^1_{ijk_1q} \le S_{\tilde{V}^1_{k_1}}, k_1 = 1, 2, ..., K_1,$$

$$\sum_{j=1}^{J}\sum_{k_2=1}^{K_2} x^2_{jkk_2q} \geq S_{\tilde{b}^2_{kq}} k=1,2,...K; q=1,2,...,Q,$$

$$\sum_{q=1}^{Q}\sum_{j=1}^{J}\sum_{k=1}^{K} x^2_{jkk_2q} \leq S_{\tilde{e}^2_{k_2}}, k_2=1,2,...,K_2,$$

$$\sum_{q=1}^{Q}\sum_{j=1}^{J}\sum_{k=1}^{K} S_{\tilde{w}_q} x^2_{jkk_2q} \leq S_{\tilde{W}^2_{k_2}}, k_2=1,2,...,K_2,$$

$$\sum_{q=1}^{Q}\sum_{j=1}^{J}\sum_{k=1}^{K} S_{\tilde{v}_q} x^2_{jkk_2q} \leq S_{\tilde{V}^2_{k_2}}, k_2=1,2,...,K_2,$$

....

$$\sum_{l=1}^{L}\sum_{k_n=1}^{K_1} x^n_{lmk_nq} \geq S_{\tilde{b}^n_{mq}}, m=1,2,...M; q=1,2,...,Q,$$

$$\sum_{q=1}^{Q}\sum_{l=1}^{L}\sum_{m=1}^{M} x^n_{lmk_nq} \leq S_{\tilde{e}^n_{k_n}}, k_n=1,2,...,K_n,$$

$$\sum_{q=1}^{Q}\sum_{l=1}^{L}\sum_{m=1}^{M} S_{\tilde{w}_q x^n_{lmk_nq}} \leq S_{\tilde{W}^n_{k_n}}, k_n=1,2,...,K_n,$$

$$\sum_{q=1}^{Q}\sum_{l=1}^{L}\sum_{m=1}^{M} S_{\tilde{v}_q} x^n_{lmk_nq} \leq S_{\tilde{V}^n_{k_n}}, k_n=1,2,...,K_n,$$

Where $S_{\tilde{C}^1_{ijk_1q}}$, $S_{\tilde{C}^2_{jkk_2q}}$, $S_{\tilde{C}^n_{lmk_nq}}$, $S_{\tilde{t}^1_{ijk_1q}}$, $S_{\tilde{t}^2_{jkk_2q}}$, $S_{\tilde{t}^n_{lmk_nq}}$ $S_{\tilde{LO}^1_i}$, $S_{\tilde{LO}^2_j}$, $S_{\tilde{LO}^n_l}$, $S_{\tilde{UD}^1_j}$, $S_{\tilde{UD}^2_k}$, $S_{\tilde{UD}^n_m}$, $S_{\tilde{PC}^1_{iq}}$, $S_{\tilde{PC}^2_{jq}}$, $S_{\tilde{PC}^n_{lq}}$, $S_{\tilde{LTO}^1_i}$, $S_{\tilde{LTO}^2_j}$, $S_{\tilde{LTO}^n_l}$, $S_{\tilde{UTD}^1_j}$, $S_{\tilde{UTD}^2_k}$, $S_{\tilde{UTD}^n_m}$,, $S_{\tilde{a}^1_{iq}}$, $S_{\tilde{b}^1_{jq}}$, $S_{\tilde{b}^2_{kq}}$, $S_{\tilde{b}^n_{mq}}$, $S_{\tilde{e}^1_{k_1}}$, $S_{\tilde{e}^2_{k_2}}$, $S_{\tilde{e}^n_{k_n}}$, $S_{\tilde{w}_q}$, $S_{\tilde{W}^1_{k_1}}$, $S_{\tilde{v}_q}$, $S_{\tilde{V}^1_{k_1}}$, $S_{\tilde{W}^2_{k_2}}$, $S_{\tilde{V}^2_{k_2}}$, $S_{\tilde{W}^n_{k_n}}$, $S_{\tilde{V}^n_{k_n}}$ are equivalent crisp form of fuzzy parameters respectively and given as follows:

$$S_{\tilde{C}^1_{ijk_1q}} = \begin{cases} \frac{(1-2\alpha+(1-4\alpha_c)\theta_{r,\tilde{C}^1_{ijk_1q}})r_1^{\tilde{C}^1_{ijk_1q}}+2\alpha_c r_2^{\tilde{C}^1_{ijk_1q}}}{(1+(1-4\alpha_c)\theta_{r,\tilde{C}^1_{ijk_1q}})}, & \text{if } 0<\alpha_c\leq 0.25 \\ \frac{(1-2\alpha_c)r_1^{\tilde{C}^1_{ijk_1q}}+(2\alpha_c+(4\alpha_c-1)\theta_{l,\tilde{C}^1_{ijk_1q}})r_2^{\tilde{C}^1_{ijk_1q}}}{(1+(1-4\alpha_c)\theta_{l,\tilde{C}^1_{ijk_1q}})}, & \text{if } 0.25<\alpha_c\leq 0.50 \\ \frac{(2\alpha_c-1)r_3^{\tilde{C}^1_{ijk_1q}}+(2(1-\alpha_c)+(3-4\alpha_c)\theta_{l,\tilde{C}^1_{ijk_1q}})r_2^{\tilde{C}^1_{ijk_1q}}}{(1+(3-4\alpha_c)\theta_{l,\tilde{C}^1_{ijk_1q}})}, & \text{if } 0.50<\alpha_c\leq 0.75 \\ \frac{(2\alpha_c-1+(4\alpha_c-3)\theta_{r,\tilde{C}^1_{ijk_1q}})r_3^{\tilde{C}^1_{ijk_1q}}+2(1-\alpha_c)r_2^{\tilde{C}^1_{ijk_1q}}}{(1+(4\alpha_c-3)\theta_{r,\tilde{C}^1_{ijk_1q}})}, & \text{if } 0.75<\alpha_c\leq 1 \end{cases}$$

$$S_{\tilde{LO}^1_i} = \begin{cases} \frac{(1-2\alpha_c+(1-4\alpha_c)\theta_{r,\tilde{LO}^1_i})r_1^{\tilde{LO}^1_i}+2\alpha_c r_2^{\tilde{LO}^1_i}}{(1+(1-4\alpha_c)\theta_{r,\tilde{LO}^1_i})}, & \text{if } 0<\alpha_c\leq 0.25 \\ \frac{(1-2\alpha_c)r_1^{\tilde{LO}^1_i}+(2\alpha_c+(4\alpha_c-1)\theta_{l,\tilde{LO}^1_i})r_2^{\tilde{LO}^1_i}}{(1+(1-4\alpha_c)\theta_{l,\tilde{LO}^1_i})}, & \text{if } 0.25<\alpha_c\leq 0.50 \\ \frac{(2\alpha_c-1)r_3^{\tilde{LO}^1_i}+(2(1-\alpha_c)+(3-4\alpha_c)\theta_{l,\tilde{LO}^1_i})r_2^{\tilde{LO}^1_i}}{(1+(3-4\alpha_c)\theta_{l,\tilde{LO}^1_i})}, & \text{if } 0.50<\alpha_c\leq 0.75 \\ \frac{(2\alpha_c-1+(4\alpha_c-3)\theta_{r,\tilde{LO}^1_i})r_3^{\tilde{LO}^1_i}+2(1-\alpha_c)r_2^{\tilde{LO}^1_i}}{(1+(4\alpha_c-3)\theta_{r,\tilde{LO}^1_i})}, & \text{if } 0.75<\alpha_c\leq 1 \end{cases}$$

$$S_{\widetilde{UD}_j^1}=\begin{cases}\frac{(1-2\alpha_c+(1-4\alpha_c)\theta_{r,\widetilde{UD}_j^1})r_1^{\widetilde{UD}_j^1}+2\alpha_c r_2^{\widetilde{UD}_j^1}}{(1+(1-4\alpha_c)\theta_{r,\widetilde{UD}_j^1})}, & \text{if } 0<\alpha_c\leq 0.25\\ \frac{(1-2\alpha_c)r_1^{\widetilde{UD}_j^1}+(2\alpha_c+(4\alpha_c-1)\theta_{l,\widetilde{UD}_j^1})r_2^{\widetilde{UD}_j^1}}{(1+(1-4\alpha_c)\theta_{l,\widetilde{UD}_j^1})}, & \text{if } 0.25<\alpha_c\leq 0.50\\ \frac{(2\alpha_c-1)r_3^{\widetilde{UD}_j^1}+(2(1-\alpha_c)+(3-4\alpha_c)\theta_{l,\widetilde{UD}_j^1})r_2^{\widetilde{UD}_j^1}}{(1+(3-4\alpha_c)\theta_{l,\widetilde{UD}_j^1})}, & \text{if } 0.50<\alpha_c\leq 0.75\\ \frac{(2\alpha_c-1+(4\alpha_c-3)\theta_{r,\widetilde{UD}_j^1})r_3^{\widetilde{UD}_j^1}+2(1-\alpha_c)r_2^{\widetilde{UD}_j^1}}{(1+(4\alpha_c-3)\theta_{r,\widetilde{UD}_j^1})}, & \text{if } 0.75<\alpha_c\leq 1\end{cases}$$

$$S_{\widetilde{PC}_{iq}}=\begin{cases}\frac{(1-2\alpha_c+(1-4\alpha_c)\theta_{r,\widetilde{PC}_{iq}})r_1^{\widetilde{PC}_{iq}}+2\alpha_c r_2^{\widetilde{PC}_{iq}}}{(1+(1-4\alpha_c)\theta_{r,\widetilde{PC}_{iq}})}, & \text{if } 0<\alpha_c\leq 0.25\\ \frac{(1-2\alpha_c)r_1^{\widetilde{PC}_{iq}}+(2\alpha_c+(4\alpha_c-1)\theta_{l,\widetilde{PC}_{iq}})r_2^{\widetilde{PC}_{iq}}}{(1+(1-4\alpha_c)\theta_{l,\widetilde{PC}_{iq}})}, & \text{if } 0.25<\alpha_c\leq 0.50\\ \frac{(2\alpha_c-1)r_3^{\widetilde{PC}_{iq}}+(2(1-\alpha_c)+(3-4\alpha_c)\theta_{l,\widetilde{PC}_{iq}})r_2^{\widetilde{PC}_{iq}}}{(1+(3-4\alpha_c)\theta_{l,\widetilde{PC}_{iq}})}, & \text{if } 0.50<\alpha_c\leq 0.75\\ \frac{(2\alpha_c-1+(4\alpha_c-3)\theta_{r,\widetilde{PC}_{iq}})r_3^{\widetilde{PC}_{iq}}+2(1-\alpha_c)r_2^{\widetilde{PC}_{iq}}}{(1+(4\alpha_c-3)\theta_{r,\widetilde{PC}_{iq}})}, & \text{if } 0.75<\alpha_c\leq 1\end{cases}$$

$$S_{\tilde{C}^2_{jkk_2q}}=\begin{cases}\frac{(1-2\alpha_c+(1-4\alpha_c)\theta_{r,\tilde{C}^2_{jkk_2q}})r_1^{\tilde{C}^2_{jkk_2q}}+2\alpha_c r_2^{\tilde{C}^2_{jkk_2q}}}{(1+(1-4\alpha_c)\theta_{r,\tilde{C}^2_{jkk_2q}})}, & \text{if } 0<\alpha_c\leq 0.25\\ \frac{(1-2\alpha_c)r_1^{\tilde{C}^2_{jkk_2q}}+(2\alpha_c+(4\alpha_c-1)\theta_{l,\tilde{C}^2_{jkk_2q}})r_2^{\tilde{C}^2_{jkk_2q}}}{(1+(1-4\alpha_c)\theta_{l,\tilde{C}^2_{jkk_2q}})}, & \text{if } 0.25<\alpha_c\leq 0.50\\ \frac{(2\alpha_c-1)r_3^{\tilde{C}^2_{jkk_2q}}+(2(1-\alpha_c)+(3-4\alpha_c)\theta_{l,\tilde{C}^2_{jkk_2q}})r_2^{\tilde{C}^2_{jkk_2q}}}{(1+(3-4\alpha_c)\theta_{l,\tilde{C}^2_{jkk_2q}})}, & \text{if } 0.50<\alpha_c\leq 0.75\\ \frac{(2\alpha_c-1+(4\alpha_c-3)\theta_{r,\tilde{C}^2_{jkk_2q}})r_3^{\tilde{C}^2_{jkk_2q}}+2(1-\alpha_c)r_2^{\tilde{C}^2_{jkk_2q}}}{(1+(4\alpha_c-3)\theta_{r,\tilde{C}^2_{jkk_2q}})}, & \text{if } 0.75<\alpha_c\leq 1\end{cases}$$

$$S_{\widetilde{LO}_j^2}=\begin{cases}\frac{(1-2\alpha_c+(1-4\alpha_c)\theta_{r,\widetilde{LO}_j^2})r_1^{\widetilde{LO}_j^2}+2\alpha_c r_2^{\widetilde{LO}_j^2}}{(1+(1-4\alpha_c)\theta_{r,\widetilde{LO}_j^2})}, & \text{if } 0<\alpha_c\leq 0.25\\ \frac{(1-2\alpha_c)r_1^{\widetilde{LO}_j^2}+(2\alpha_c+(4\alpha_c-1)\theta_{l,\widetilde{LO}_j^2})r_2^{\widetilde{LO}_j^2}}{(1+(1-4\alpha_c)\theta_{l,\widetilde{LO}_j^2})}, & \text{if } 0.25<\alpha_c\leq 0.50\\ \frac{(2\alpha_c-1)r_3^{\widetilde{LO}_j^2}+(2(1-\alpha_c)+(3-4\alpha_c)\theta_{l,\widetilde{LO}_j^2})r_2^{\widetilde{LO}_j^2}}{(1+(3-4\alpha_c)\theta_{l,\widetilde{LO}_j^2})}, & \text{if } 0.50<\alpha_c\leq 0.75\\ \frac{(2\alpha_c-1+(4\alpha_c-3)\theta_{r,\widetilde{LO}_j^2})r_3^{\widetilde{LO}_j^2}+2(1-\alpha_c)r_2^{\widetilde{LO}_j^2}}{(1+(4\alpha_c-3)\theta_{r,\widetilde{LO}_j^2})}, & \text{if } 0.75<\alpha_c\leq 1\end{cases}$$

$$S_{\tilde{UD}^2_k} = \begin{cases} \frac{(1-2\alpha_c+(1-4\alpha_c)\theta_{r,\tilde{UD}^2_k})r_1^{\tilde{UD}^2_k}+2\alpha_c r_2^{\tilde{UD}^2_k}}{(1+(1-4\alpha_c)\theta_{r,\tilde{UD}^2_k})}, & \text{if } 0<\alpha_c\le 0.25 \\ \frac{(1-2\alpha_c)r_1^{\tilde{UD}^2_k}+(2\alpha_c+(4\alpha_c-1)\theta_{l,\tilde{UD}^2_k})r_2^{\tilde{UD}^2_k}}{(1+(1-4\alpha_c)\theta_{l,\tilde{UD}^2_k})}, & \text{if } 0.25<\alpha_c\le 0.50 \\ \frac{(2\alpha_c-1)r_3^{\tilde{UD}^2_k}+(2(1-\alpha_c)+(3-4\alpha_c)\theta_{l,\tilde{UD}^2_k})r_2^{\tilde{UD}^2_k}}{(1+(3-4\alpha_c)\theta_{l,\tilde{UD}^2_k})}, & \text{if } 0.50<\alpha_c\le 0.75 \\ \frac{(2\alpha_c-1+(4\alpha_c-3)\theta_{r,\tilde{UD}^2_k})r_3^{\tilde{UD}^2_k}+2(1-\alpha_c)r_2^{\tilde{UD}^2_k}}{(1+(4\alpha_c-3)\theta_{r,\tilde{UD}^2_k})}, & \text{if } 0.75<\alpha_c\le 1 \end{cases}$$

$$S_{\tilde{C}^n_{lmk_nq}} = \begin{cases} \frac{(1-2\alpha_c+(1-4\alpha_c)\theta_{r,\tilde{C}^n_{lmk_nq}})r_1^{\tilde{C}^n_{lmk_nq}}+2\alpha_c r_2^{\tilde{C}^n_{lmk_nq}}}{(1+(1-4\alpha_c)\theta_{r,\tilde{C}^n_{lmk_nq}})}, & \text{if } 0<\alpha_c\le 0.25 \\ \frac{(1-2\alpha_c)r_1^{\tilde{C}^n_{lmk_nq}}+(2\alpha_c+(4\alpha_c-1)\theta_{l,\tilde{C}^n_{lmk_nq}})r_2^{\tilde{C}^n_{lmk_nq}}}{(1+(1-4\alpha_c)\theta_{l,\tilde{C}^n_{lmk_nq}})}, & \text{if } 0.25<\alpha_c\le 0.50 \\ \frac{(2\alpha_c-1)r_3^{\tilde{C}^n_{lmk_nq}}+(2(1-\alpha_c)+(3-4\alpha_c)\theta_{l,\tilde{C}^n_{lmk_nq}})r_2^{\tilde{C}^n_{lmk_nq}}}{(1+(3-4\alpha_c)\theta_{l,\tilde{C}^n_{lmk_nq}})}, & \text{if } 0.50<\alpha_c\le 0.75 \\ \frac{(2\alpha_c-1+(4\alpha_c-3)\theta_{r,\tilde{C}^n_{lmk_nq}})r_3^{\tilde{C}^n_{lmk_nq}}+2(1-\alpha_c)r_2^{\tilde{C}^n_{lmk_nq}}}{(1+(4\alpha_c-3)\theta_{r,\tilde{C}^n_{lmk_nq}})}, & \text{if } 0.75<\alpha_c\le 1 \end{cases}$$

$$S_{\tilde{LO}^n_l} = \begin{cases} \frac{(1-2\alpha_c+(1-4\alpha_c)\theta_{r,\tilde{LO}^n_l})r_1^{\tilde{LO}^n_l}+2\alpha_c r_2^{\tilde{LO}^n_l}}{(1+(1-4\alpha_c)\theta_{r,\tilde{LO}^n_l})}, & \text{if } 0<\alpha_c\le 0.25 \\ \frac{(1-2\alpha_c)r_1^{\tilde{LO}^n_l}+(2\alpha_c+(4\alpha_c-1)\theta_{l,\tilde{LO}^n_l})r_2^{\tilde{LO}^n_l}}{(1+(1-4\alpha_c)\theta_{l,\tilde{LO}^n_l})}, & \text{if } 0.25<\alpha_c\le 0.50 \\ \frac{(2\alpha_c-1)r_3^{\tilde{LO}^n_l}+(2(1-\alpha_c)+(3-4\alpha_c)\theta_{l,\tilde{LO}^n_l})r_2^{\tilde{LO}^n_l}}{(1+(3-4\alpha_c)\theta_{l,\tilde{LO}^n_l})}, & \text{if } 0.50<\alpha_c\le 0.75 \\ \frac{(2\alpha_c-1+(4\alpha_c-3)\theta_{r,\tilde{LO}^n_l})r_3^{\tilde{LO}^n_l}+2(1-\alpha_c)r_2^{\tilde{LO}^n_l}}{(1+(4\alpha_c-3)\theta_{r,\tilde{LO}^n_l})}, & \text{if } 0.75<\alpha_c\le 1 \end{cases}$$

$$S_{\tilde{UD}^n_m} = \begin{cases} \frac{(1-2\alpha_c+(1-4\alpha_c)\theta_{r,\tilde{UD}^n_m})r_1^{\tilde{UD}^n_m}+2\alpha_c r_2^{\tilde{UD}^n_m}}{(1+(1-4\alpha_c)\theta_{r,\tilde{UD}^n_m})}, & \text{if } 0<\alpha_c\le 0.25 \\ \frac{(1-2\alpha_c)r_1^{\tilde{UD}^n_m}+(2\alpha_c+(4\alpha_c-1)\theta_{l,\tilde{UD}^n_m})r_2^{\tilde{UD}^n_m}}{(1+(1-4\alpha_c)\theta_{l,\tilde{UD}^n_m})}, & \text{if } 0.25<\alpha_c\le 0.50 \\ \frac{(2\alpha_c-1)r_3^{\tilde{UD}^n_m}+(2(1-\alpha_c)+(3-4\alpha_c)\theta_{l,\tilde{UD}^n_m})r_2^{\tilde{UD}^n_m}}{(1+(3-4\alpha_c)\theta_{l,\tilde{UD}^n_m})}, & \text{if } 0.50<\alpha_c\le 0.75 \\ \frac{(2\alpha_c-1+(4\alpha_c-3)\theta_{r,\tilde{UD}^n_m})r_3^{\tilde{UD}^n_m}+2(1-\alpha_c)r_2^{\tilde{UD}^n_m}}{(1+(4\alpha_c-3)\theta_{r,\tilde{UD}^n_m})}, & \text{if } 0.75<\alpha_c\le 1 \end{cases}$$

$$S_{\tilde{t}^1_{ijk_1q}} = \begin{cases} \frac{(1-2\alpha_t+(1-4\alpha_t)\theta_{r,\tilde{t}^1_{ijk_1q}})r_1^{\tilde{t}^1_{ijk_1q}}+2\alpha_t r_2^{\tilde{t}^1_{ijk_1q}}}{(1+(1-4\alpha_t)\theta_{r,\tilde{t}^1_{ijk_1q}})}, & \text{if } 0<\alpha_t\le 0.25 \\ \frac{(1-2\alpha_t)r_1^{\tilde{t}^1_{ijk_1q}}+(2\alpha_t+(4\alpha_t-1)\theta_{l,\tilde{t}^1_{ijk_1q}})r_2^{\tilde{t}^1_{ijk_1q}}}{(1+(1-4\alpha_t)\theta_{l,\tilde{t}^1_{ijk_1q}})}, & \text{if } 0.25<\alpha_t\le 0.50 \\ \frac{(2\alpha_t-1)r_3^{\tilde{t}^1_{ijk_1q}}+(2(1-\alpha_t)+(3-4\alpha_t)\theta_{l,\tilde{t}^1_{ijk_1q}})r_2^{\tilde{t}^1_{ijk_1q}}}{(1+(3-4\alpha_t)\theta_{l,\tilde{t}^1_{ijk_1q}})}, & \text{if } 0.50<\alpha_t\le 0.75 \\ \frac{(2\alpha_t-1+(4\alpha_t-3)\theta_{r,\tilde{t}^1_{ijk_1q}})r_3^{\tilde{t}^1_{ijk_1q}}+2(1-\alpha_t)r_2^{\tilde{t}^1_{ijk_1q}}}{(1+(4\alpha_t-3)\theta_{r,\tilde{t}^1_{ijk_1q}})}, & \text{if } 0.75<\alpha_t\le 1 \end{cases}$$

$$S_{L\tilde{T}O_i^1} = \begin{cases} \frac{(1-2\alpha_t+(1-4\alpha_t)\theta_{r,L\tilde{T}O_i^1})r_1^{L\tilde{T}O_i^1}+2\alpha_t r_2^{L\tilde{T}O_i^1}}{(1+(1-4\alpha_t)\theta_{r,L\tilde{T}O_i^1})}, & \text{if } 0 < \alpha_t \leq 0.25 \\ \frac{(1-2\alpha_t)r_1^{L\tilde{T}O_i^1}+(2\alpha_t+(4\alpha_t-1)\theta_{l,L\tilde{T}O_i^1})r_2^{L\tilde{T}O_i^1}}{(1+(1-4\alpha_t)\theta_{l,L\tilde{T}O_i^1})}, & \text{if } 0.25 < \alpha_t \leq 0.50 \\ \frac{(2\alpha_t-1)r_3^{L\tilde{T}O_i^1}+(2(1-\alpha_t)+(3-4\alpha_t)\theta_{l,L\tilde{T}O_i^1})r_2^{L\tilde{T}O_i^1}}{(1+(3-4\alpha_t)\theta_{l,L\tilde{T}O_i^1})}, & \text{if } 0.50 < \alpha_t \leq 0.75 \\ \frac{(2\alpha_t-1+(4\alpha_t-3)\theta_{r,L\tilde{T}O_i^1})r_3^{L\tilde{T}O_i^1}+2(1-\alpha_t)r_2^{L\tilde{T}O_i^1}}{(1+(4\alpha_t-3)\theta_{r,L\tilde{T}O_i^1})}, & \text{if } 0.75 < \alpha_t \leq 1 \end{cases}$$

$$S_{U\tilde{T}D_j^1} = \begin{cases} \frac{(1-2\alpha_t+(1-4\alpha_t)\theta_{r,U\tilde{T}D_j^1})r_1^{U\tilde{T}D_j^1}+2\alpha_t r_2^{U\tilde{T}D_j^1}}{(1+(1-4\alpha_t)\theta_{r,U\tilde{T}D_j^1})}, & \text{if } 0 < \alpha_t \leq 0.25 \\ \frac{(1-2\alpha_t)r_1^{U\tilde{T}D_j^1}+(2\alpha_t+(4\alpha_t-1)\theta_{l,U\tilde{T}D_j^1})r_2^{U\tilde{T}D_j^1}}{(1+(1-4\alpha_t)\theta_{l,U\tilde{T}D_j^1})}, & \text{if } 0.25 < \alpha_t \leq 0.50 \\ \frac{(2\alpha_t-1)r_3^{U\tilde{T}D_j^1}+(2(1-\alpha_t)+(3-4\alpha_t)\theta_{l,U\tilde{T}D_j^1})r_2^{U\tilde{T}D_j^1}}{(1+(3-4\alpha_t)\theta_{l,U\tilde{T}D_j^1})}, & \text{if } 0.50 < \alpha_t \leq 0.75 \\ \frac{(2\alpha_t-1+(4\alpha_t-3)\theta_{r,U\tilde{T}D_j^1})r_3^{U\tilde{T}D_j^1}+2(1-\alpha_t)r_2^{U\tilde{T}D_j^1}}{(1+(4\alpha_t-3)\theta_{r,U\tilde{T}D_j^1})}, & \text{if } 0.75 < \alpha_t \leq 1 \end{cases}$$

$$S_{\tilde{t}^2_{jkk_2q}} = \begin{cases} \frac{(1-2\alpha_t+(1-4\alpha_t)\theta_{r,\tilde{t}^2_{jkk_2q}})r_1^{\tilde{t}^2_{jkk_2q}}+2\alpha_t r_2^{\tilde{t}^2_{jkk_2q}}}{(1+(1-4\alpha_t)\theta_{r,\tilde{t}^2_{jkk_2q}})}, & \text{if } 0 < \alpha_t \leq 0.25 \\ \frac{(1-2\alpha_t)r_1^{\tilde{t}^2_{jkk_2q}}+(2\alpha_t+(4\alpha_t-1)\theta_{l,\tilde{t}^2_{jkk_2q}})r_2^{\tilde{t}^2_{jkk_2q}}}{(1+(1-4\alpha_t)\theta_{l,\tilde{t}^2_{jkk_2q}})}, & \text{if } 0.25 < \alpha_t \leq 0.50 \\ \frac{(2\alpha_t-1)r_3^{\tilde{t}^2_{jkk_2q}}+(2(1-\alpha_t)+(3-4\alpha_t)\theta_{l,\tilde{t}^2_{jkk_2q}})r_2^{\tilde{t}^2_{jkk_2q}}}{(1+(3-4\alpha_t)\theta_{l,\tilde{t}^2_{jkk_2q}})}, & \text{if } 0.50 < \alpha_t \leq 0.75 \\ \frac{(2\alpha_t-1+(4\alpha_t-3)\theta_{r,\tilde{t}^2_{jkk_2q}})r_3^{\tilde{t}^2_{jkk_2q}}+2(1-\alpha_t)r_2^{\tilde{t}^2_{jkk_2q}}}{(1+(4\alpha_t-3)\theta_{r,\tilde{t}^2_{jkk_2q}})}, & \text{if } 0.75 < \alpha_t \leq 1 \end{cases}$$

$$S_{L\tilde{T}O_j^2} = \begin{cases} \frac{(1-2\alpha_t+(1-4\alpha_t)\theta_{r,L\tilde{T}O_j^2})r_1^{L\tilde{T}O_j^2}+2\alpha_t r_2^{L\tilde{T}O_j^2}}{(1+(1-4\alpha_t)\theta_{r,L\tilde{T}O_j^2})}, & \text{if } 0 < \alpha_t \leq 0.25 \\ \frac{(1-2\alpha_t)r_1^{L\tilde{T}O_j^2}+(2\alpha_t+(4\alpha_t-1)\theta_{l,L\tilde{T}O_j^2})r_2^{L\tilde{T}O_j^2}}{(1+(1-4\alpha_t)\theta_{l,L\tilde{T}O_j^2})}, & \text{if } 0.25 < \alpha_t \leq 0.50 \\ \frac{(2\alpha_t-1)r_3^{L\tilde{T}O_j^2}+(2(1-\alpha_t)+(3-4\alpha_t)\theta_{l,L\tilde{T}O_j^2})r_2^{L\tilde{T}O_j^2}}{(1+(3-4\alpha_t)\theta_{l,L\tilde{T}O_j^2})}, & \text{if } 0.50 < \alpha_t \leq 0.75 \\ \frac{(2\alpha_t-1+(4\alpha_t-3)\theta_{r,L\tilde{T}O_j^2})r_3^{L\tilde{T}O_j^2}+2(1-\alpha_t)r_2^{L\tilde{T}O_j^2}}{(1+(4\alpha_t-3)\theta_{r,L\tilde{T}O_j^2})}, & \text{if } 0.75 < \alpha_t \leq 1 \end{cases}$$

$$S_{U\tilde{T}D_k^2}=\begin{cases}\frac{(1-2\alpha_t+(1-4\alpha_t)\theta_{r,U\tilde{T}D_k^2})r_1^{U\tilde{T}D_k^2}+2\alpha_t r_2^{U\tilde{T}D_k^2}}{(1+(1-4\alpha_t)\theta_{r,U\tilde{T}D_k^2})}, & \text{if } 0<\alpha_t\leq 0.25\\ \frac{(1-2\alpha_t)r_1^{U\tilde{T}D_k^2}+(2\alpha_t+(4\alpha_t-1)\theta_{l,U\tilde{T}D_k^2})r_2^{U\tilde{T}D_k^2}}{(1+(1-4\alpha_t)\theta_{l,U\tilde{T}D_k^2})}, & \text{if } 0.25<\alpha_t\leq 0.50\\ \frac{(2\alpha_t-1)r_3^{U\tilde{T}D_k^2}+(2(1-\alpha_t)+(3-4\alpha_t)\theta_{l,U\tilde{T}D_k^2})r_2^{U\tilde{T}D_k^2}}{(1+(3-4\alpha_t)\theta_{l,U\tilde{T}D_k^2})}, & \text{if } 0.50<\alpha_t\leq 0.75\\ \frac{(2\alpha_t-1+(4\alpha_t-3)\theta_{r,U\tilde{T}D_k^2})r_3^{U\tilde{T}D_k^2}+2(1-\alpha_t)r_2^{U\tilde{T}D_k^2}}{(1+(4\alpha_t-3)\theta_{r,U\tilde{T}D_k^2})}, & \text{if } 0.75<\alpha_t\leq 1\end{cases}$$

$$S_{\tilde{l}^n_{lmk_nq}}=\begin{cases}\frac{(1-2\alpha_t+(1-4\alpha_t)\theta_{r,\tilde{l}^n_{lmk_nq}})r_1^{\tilde{l}^n_{lmk_nq}}+2\alpha_t r_2^{\tilde{l}^n_{lmk_nq}}}{(1+(1-4\alpha_t)\theta_{r,\tilde{l}^n_{lmk_nq}})}, & \text{if } 0<\alpha_t\leq 0.25\\ \frac{(1-2\alpha_t)r_1^{\tilde{l}^n_{lmk_nq}}+(2\alpha_t+(4\alpha_t-1)\theta_{l,\tilde{l}^n_{lmk_nq}})r_2^{\tilde{l}^n_{lmk_nq}}}{(1+(1-4\alpha_t)\theta_{l,\tilde{l}^n_{lmk_nq}})}, & \text{if } 0.25<\alpha_t\leq 0.50\\ \frac{(2\alpha_t-1)r_3^{\tilde{l}^n_{lmk_nq}}+(2(1-\alpha_t)+(3-4\alpha_t)\theta_{l,\tilde{l}^n_{lmk_nq}})r_2^{\tilde{l}^n_{lmk_nq}}}{(1+(3-4\alpha_t)\theta_{l,\tilde{l}^n_{lmk_nq}})}, & \text{if } 0.50<\alpha_t\leq 0.75\\ \frac{(2\alpha_t-1+(4\alpha_t-3)\theta_{r,\tilde{l}^n_{lmk_nq}})r_3^{\tilde{l}^n_{lmk_nq}}+2(1-\alpha_t)r_2^{\tilde{l}^n_{lmk_nq}}}{(1+(4\alpha_t-3)\theta_{r,\tilde{l}^n_{lmk_nq}})}, & \text{if } 0.75<\alpha_t\leq 1\end{cases}$$

$$S_{L\tilde{T}O_l^n}=\begin{cases}\frac{(1-2\alpha_t+(1-4\alpha_t)\theta_{r,L\tilde{T}O_l^n})r_1^{L\tilde{T}O_l^n}+2\alpha_t r_2^{L\tilde{T}O_l^n}}{(1+(1-4\alpha_t)\theta_{r,L\tilde{T}O_l^n})}, & \text{if } 0<\alpha_t\leq 0.25\\ \frac{(1-2\alpha_t)r_1^{L\tilde{T}O_l^n}+(2\alpha_t+(4\alpha_t-1)\theta_{l,L\tilde{T}O_l^n})r_2^{L\tilde{T}O_l^n}}{(1+(1-4\alpha_t)\theta_{l,L\tilde{T}O_l^n})}, & \text{if } 0.25<\alpha_t\leq 0.50\\ \frac{(2\alpha_t-1)r_3^{L\tilde{T}O_l^n}+(2(1-\alpha_t)+(3-4\alpha_t)\theta_{l,L\tilde{T}O_l^n})r_2^{L\tilde{T}O_l^n}}{(1+(3-4\alpha_t)\theta_{l,L\tilde{T}O_l^n})}, & \text{if } 0.50<\alpha_t\leq 0.75\\ \frac{(2\alpha_t-1+(4\alpha_t-3)\theta_{r,L\tilde{T}O_l^n})r_3^{L\tilde{T}O_l^n}+2(1-\alpha_t)r_2^{L\tilde{T}O_l^n}}{(1+(4\alpha_t-3)\theta_{r,L\tilde{T}O_l^n})}, & \text{if } 0.75<\alpha_t\leq 1\end{cases}$$

$$S_{U\tilde{T}D_m^n}=\begin{cases}\frac{(1-2\alpha_t+(1-4\alpha_t)\theta_{r,L\tilde{T}D_m^n})r_1^{L\tilde{T}D_m^n}+2\alpha_t r_2^{L\tilde{T}D_m^n}}{(1+(1-4\alpha_t)\theta_{r,L\tilde{T}D_m^n})}, & \text{if } 0<\alpha_t\leq 0.25\\ \frac{(1-2\alpha_t)r_1^{L\tilde{T}D_m^n}+(2\alpha_t+(4\alpha_t-1)\theta_{l,L\tilde{T}D_m^n})r_2^{L\tilde{T}D_m^n}}{(1+(1-4\alpha_t)\theta_{l,L\tilde{T}D_m^n})}, & \text{if } 0.25<\alpha_t\leq 0.50\\ \frac{(2\alpha_t-1)r_3^{L\tilde{T}D_m^n}+(2(1-\alpha_t)+(3-4\alpha_t)\theta_{l,L\tilde{T}D_m^n})r_2^{L\tilde{T}D_m^n}}{(1+(3-4\alpha_t)\theta_{l,L\tilde{T}D_m^n})}, & \text{if } 0.50<\alpha_t\leq 0.75\\ \frac{(2\alpha_t-1+(4\alpha_t-3)\theta_{r,L\tilde{T}D_m^n})r_3^{L\tilde{T}D_m^n}+2(1-\alpha_t)r_2^{L\tilde{T}D_m^n}}{(1+(4\alpha_t-3)\theta_{r,L\tilde{T}D_m^n})}, & \text{if } 0.75<\alpha_t\leq 1\end{cases}$$

$$S_{\tilde{a}^1_{iq}}=\begin{cases}\frac{(1-2\alpha_{avail.}+(1-4\alpha_{avail.})\theta_{r,\tilde{a}^1_{iq}})r_1^{\tilde{a}^1_{iq}}+2\alpha_{avail.}r_2^{\tilde{a}^1_{iq}}}{(1+(1-4\alpha_{avail.})\theta_{r,\tilde{a}^1_{iq}})}, & \text{if } 0<\alpha_{avail.}\leq 0.25\\ \frac{(1-2\alpha_{avail.})r_1^{\tilde{a}^1_{iq}}+(2\alpha_{avail.}+(4\alpha_{avail.}-1)\theta_{l,\tilde{a}^1_{iq}})r_2^{\tilde{a}^1_{iq}}}{(1+(1-4\alpha_{avail.})\theta_{l,\tilde{a}^1_{iq}})}, & \text{if } 0.25<\alpha_{avail.}\leq 0.50\\ \frac{(2\alpha_{avail.}-1)r_3^{\tilde{a}^1_{iq}}+(2(1-\alpha_{avail.})+(3-4\alpha_{avail.})\theta_{l,\tilde{a}^1_{iq}})r_2^{\tilde{a}^1_{iq}}}{(1+(3-4\alpha_{avail.})\theta_{l,\tilde{a}^1_{iq}})}, & \text{if } 0.50<\alpha_{avail.}\leq 0.75\\ \frac{(2\alpha_{avail.}-1+(4\alpha_{avail.}-3)\theta_{r,\tilde{a}^1_{iq}})r_3^{\tilde{a}^1_{iq}}+2(1-\alpha_{avail.})r_2^{\tilde{a}^1_{iq}}}{(1+(4\alpha_{avail.}-3)\theta_{r,\tilde{a}^1_{iq}})}, & \text{if } 0.75<\alpha_{avail.}\leq 1\end{cases}$$

$$S_{\tilde{b}^1_{jq}} = \begin{cases} \frac{(1-2\alpha_{demand}+(1-4\alpha_{demand})\theta_{r,\tilde{b}^1_{jq}})r_1^{\tilde{b}^1_{jq}}+2\alpha_{demand}r_2^{\tilde{b}^1_{jq}}}{(1+(1-4\alpha_{demand})\theta_{r,\tilde{b}^1_{jq}})}, & \text{if } 0 < \alpha_{demand} \leq 0.25 \\ \frac{(1-2\alpha_{demand})r_1^{\tilde{b}^1_{jq}}+(2\alpha_{demand}+(4\alpha_{demand}-1)\theta_{l,\tilde{b}^1_{jq}})r_2^{\tilde{b}^1_{jq}}}{(1+(1-4\alpha_{demand})\theta_{l,\tilde{b}^1_{jq}})}, & \text{if } 0.25 < \alpha_{demand} \leq 0.50 \\ \frac{(2\alpha_{demand}-1)r_3^{\tilde{b}^1_{jq}}+(2(1-\alpha_{demand})+(3-4\alpha_{demand})\theta_{l,\tilde{b}^1_{jq}})r_2^{\tilde{b}^1_{jq}}}{(1+(3-4\alpha_{demand})\theta_{l,\tilde{b}^1_{jq}})}, & \text{if } 0.50 < \alpha_{demand} \leq 0.75 \\ \frac{(2\alpha_{demand}-1+(4\alpha_{demand}-3)\theta_{r,\tilde{b}^1_{jq}})r_3^{\tilde{b}^1_{jq}}+2(1-\alpha_{demand})r_2^{\tilde{b}^1_{jq}}}{(1+(4\alpha_{demand}-3)\theta_{r,\tilde{b}^1_{jq}})}, & \text{if } 0.75 < \alpha_{demand} \leq 1 \end{cases}$$

$$S_{\tilde{e}^1_{k_1}} = \begin{cases} \frac{(1-2\alpha_{con.cap.}+(1-4\alpha_{con.cap.})\theta_{r,tildee^1_{k_1}})r_1^{tildee^1_{k_1}}+2\alpha_{con.cap.}r_2^{tildee^1_{k_1}}}{(1+(1-4\alpha_{con.cap.})\theta_{r,tildee^1_{k_1}})}, & \text{if } 0 < \alpha_{con.cap.} \leq 0.25 \\ \frac{(1-2\alpha_{con.cap.})r_1^{tildee^1_{k_1}}+(2\alpha_{con.cap.}+(4\alpha_{con.cap.}-1)\theta_{l,tildee^1_{k_1}})r_2^{tildee^1_{k_1}}}{(1+(1-4\alpha_{con.cap.})\theta_{l,tildee^1_{k_1}})}, & \text{if } 0.25 < \alpha_{con.cap.} \leq 0.50 \\ \frac{(2\alpha_{con.cap.}-1)r_3^{tildee^1_{k_1}}+(2(1-\alpha_{con.cap.})+(3-4\alpha_{con.cap.})\theta_{l,tildee^1_{k_1}})r_2^{tildee^1_{k_1}}}{(1+(3-4\alpha_{con.cap.})\theta_{l,tildee^1_{k_1}})}, & \text{if } 0.50 < \alpha_{con.cap.} \leq 0.75 \\ \frac{(2\alpha_{con.cap.}-1+(4\alpha_{con.cap.}-3)\theta_{r,tildee^1_{k_1}})r_3^{tildee^1_{k_1}}+2(1-\alpha_{con.cap.})r_2^{tildee^1_{k_1}}}{(1+(4\alpha_{con.cap.}-3)\theta_{r,tildee^1_{k_1}})}, & \text{if } 0.75 < \alpha_{con.cap.} \leq 1 \end{cases}$$

$$S_{\tilde{w}_q} = \begin{cases} \frac{(1-2\alpha_{weight}+(1-4\alpha_{weight})\theta_{r,\tilde{w}_q})r_1^{\tilde{w}_q}+2\alpha_{weight}r_2^{\tilde{w}_q}}{(1+(1-4\alpha_{weight})\theta_{r,\tilde{w}_q})}, & \text{if } 0 < \alpha_{weight} \leq 0.25 \\ \frac{(1-2\alpha_{weight})r_1^{\tilde{w}_q}+(2\alpha_{weight}+(4\alpha_{weight}-1)\theta_{l,\tilde{w}_q})r_2^{\tilde{w}_q}}{(1+(1-4\alpha_{weight})\theta_{l,\tilde{w}_q})}, & \text{if } 0.25 < \alpha_{weight} \leq 0.50 \\ \frac{(2\alpha_{weight}-1)r_3^{\tilde{w}_q}+(2(1-\alpha_{weight})+(3-4\alpha_{weight})\theta_{l,\tilde{w}_q})r_2^{\tilde{w}_q}}{(1+(3-4\alpha_{weight})\theta_{l,\tilde{w}_q})}, & \text{if } 0.50 < \alpha_{weight} \leq 0.75 \\ \frac{(2\alpha_{weight}-1+(4\alpha_{weight}-3)\theta_{r,\tilde{w}_q})r_3^{\tilde{w}_q}+2(1-\alpha_{weight})r_2^{\tilde{w}_q}}{(1+(4\alpha_{weight}-3)\theta_{r,\tilde{w}_q})}, & \text{if } 0.75 < \alpha_{weight} \leq 1 \end{cases}$$

$$S_{\tilde{W}^1_{k_1}} = \begin{cases} \frac{(1-2\alpha_{weight}+(1-4\alpha_{weight})\theta_{r,\tilde{W}^1_{k_1}})r_1^{\tilde{W}^1_{k_1}}+2\alpha_{weight}r_2^{\tilde{W}^1_{k_1}}}{(1+(1-4\alpha_{weight})\theta_{r,\tilde{W}^1_{k_1}})}, & \text{if } 0 < \alpha_{weight} \leq 0.25 \\ \frac{(1-2\alpha_{weight})r_1^{\tilde{W}^1_{k_1}}+(2\alpha_{weight}+(4\alpha_{weight}-1)\theta_{l,\tilde{W}^1_{k_1}})r_2^{\tilde{W}^1_{k_1}}}{(1+(1-4\alpha_{weight})\theta_{l,\tilde{W}^1_{k_1}})}, & \text{if } 0.25 < \alpha_{weight} \leq 0.50 \\ \frac{(2\alpha_{weight}-1)r_3^{\tilde{W}^1_{k_1}}+(2(1-\alpha_{weight})+(3-4\alpha_{weight})\theta_{l,\tilde{W}^1_{k_1}})r_2^{\tilde{W}^1_{k_1}}}{(1+(3-4\alpha_{weight})\theta_{l,\tilde{W}^1_{k_1}})}, & \text{if } 0.50 < \alpha_{weight} \leq 0.75 \\ \frac{(2\alpha_{weight}-1+(4\alpha_{weight}-3)\theta_{r,\tilde{W}^1_{k_1}})r_3^{\tilde{W}^1_{k_1}}+2(1-\alpha_{weight})r_2^{\tilde{W}^1_{k_1}}}{(1+(4\alpha_{weight}-3)\theta_{r,\tilde{W}^1_{k_1}})}, & \text{if } 0.75 < \alpha_{weight} \leq 1 \end{cases}$$

$$S_{\tilde{v}_q} = \begin{cases} \frac{(1-2\alpha_{volume}+(1-4\alpha_{volume})\theta_{r,\tilde{v}_q})r_1^{\tilde{v}_q}+2\alpha_{volume}r_2^{\tilde{v}_q}}{(1+(1-4\alpha_{volume})\theta_{r,\tilde{v}_q})}, & \text{if } 0 < \alpha_{volume} \leq 0.25 \\ \frac{(1-2\alpha_{volume})r_1^{\tilde{v}_q}+(2\alpha_{volume}+(4\alpha_{volume}-1)\theta_{l,\tilde{v}_q})r_2^{\tilde{v}_q}}{(1+(1-4\alpha_{volume})\theta_{l,\tilde{v}_q})}, & \text{if } 0.25 < \alpha_{volume} \leq 0.50 \\ \frac{(2\alpha_{volume}-1)r_3^{\tilde{v}_q}+(2(1-\alpha_{volume})+(3-4\alpha_{volume})\theta_{l,\tilde{v}_q})r_2^{\tilde{v}_q}}{(1+(3-4\alpha_{volume})\theta_{l,\tilde{v}_q})}, & \text{if } 0.50 < \alpha_{volume} \leq 0.75 \\ \frac{(2\alpha_{volume}-1+(4\alpha_{volume}-3)\theta_{r,\tilde{v}_q})r_3^{\tilde{v}_q}+2(1-\alpha_{volume})r_2^{\tilde{v}_q}}{(1+(4\alpha_{volume}-3)\theta_{r,\tilde{v}_q})}, & \text{if } 0.75 < \alpha_{volume} \leq 1 \end{cases}$$

$$S_{\tilde{V}^1_{k_1}} = \begin{cases} \frac{(1-2\alpha_{volume}+(1-4\alpha_{volume})\theta_{r,\tilde{V}^1_{k_1}})r_1^{\tilde{V}^1_{k_1}}+2\alpha_{volume}r_2^{\tilde{V}^1_{k_1}}}{(1+(1-4\alpha_{volume})\theta_{r,\tilde{V}^1_{k_1}})}, & \text{if } 0 < \alpha_{volume} \leq 0.25 \\ \frac{(1-2\alpha_{volume})r_1^{\tilde{V}^1_{k_1}}+(2\alpha_{volume}+(4\alpha_{volume}-1)\theta_{l,\tilde{V}^1_{k_1}})r_2^{\tilde{V}^1_{k_1}}}{(1+(1-4\alpha_{volume})\theta_{l,\tilde{V}^1_{k_1}})}, & \text{if } 0.25 < \alpha_{volume} \leq 0.50 \\ \frac{(2\alpha_{volume}-1)r_3^{\tilde{V}^1_{k_1}}+(2(1-\alpha_{volume})+(3-4\alpha_{volume})\theta_{l,\tilde{V}^1_{k_1}})r_2^{\tilde{V}^1_{k_1}}}{(1+(3-4\alpha_{volume})\theta_{l,\tilde{V}^1_{k_1}})}, & \text{if } 0.50 < \alpha_{volume} \leq 0.75 \\ \frac{(2\alpha_{volume}-1+(4\alpha_{volume}-3)\theta_{r,\tilde{V}^1_{k_1}})r_3^{\tilde{V}^1_{k_1}}+2(1-\alpha_{volume})r_2^{\tilde{V}^1_{k_1}}}{(1+(4\alpha_{volume}-3)\theta_{r,\tilde{V}^1_{k_1}})}, & \text{if } 0.75 < \alpha_{volume} \leq 1 \end{cases}$$

$$S_{\tilde{b}^2_{kq}} = \begin{cases} \frac{(1-2\alpha_{demand}+(1-4\alpha_{demand})\theta_{r,\tilde{b}^2_{kq}})r_1^{\tilde{b}^2_{kq}}+2\alpha_{demand}r_2^{\tilde{b}^2_{kq}}}{(1+(1-4\alpha_{demand})\theta_{r,\tilde{b}^2_{kq}})}, & \text{if } 0 < \alpha_{demand} \leq 0.25 \\ \frac{(1-2\alpha_{demand})r_1^{\tilde{b}^2_{kq}}+(2\alpha_{demand}+(4\alpha_{demand}-1)\theta_{l,\tilde{b}^2_{kq}})r_2^{\tilde{b}^2_{kq}}}{(1+(1-4\alpha_{demand})\theta_{l,\tilde{b}^2_{kq}})}, & \text{if } 0.25 < \alpha_{demand} \leq 0.50 \\ \frac{(2\alpha_{demand}-1)r_3^{\tilde{b}^2_{kq}}+(2(1-\alpha_{demand})+(3-4\alpha_{demand})\theta_{l,\tilde{b}^2_{kq}})r_2^{\tilde{b}^2_{kq}}}{(1+(3-4\alpha_{demand})\theta_{l,\tilde{b}^2_{kq}})}, & \text{if } 0.50 < \alpha_{demand} \leq 0.75 \\ \frac{(2\alpha_{demand}-1+(4\alpha_{demand}-3)\theta_{r,\tilde{b}^2_{kq}})r_3^{\tilde{b}^2_{kq}}+2(1-\alpha_{demand})r_2^{\tilde{b}^2_{kq}}}{(1+(4\alpha_{demand}-3)\theta_{r,\tilde{b}^2_{kq}})}, & \text{if } 0.75 < \alpha_{demand} \leq 1 \end{cases}$$

$$S_{\tilde{e}^2_{k_2}} = \begin{cases} \frac{(1-2\alpha_{con.cap.}+(1-4\alpha_{con.cap.})\theta_{r,\tilde{e}^2_{k_2}})r_1^{\tilde{e}^2_{k_2}}+2\alpha_{con.cap.}r_2^{\tilde{e}^2_{k_2}}}{(1+(1-4\alpha_{con.cap.})\theta_{r,\tilde{e}^2_{k_2}})}, & \text{if } 0 < \alpha_{con.cap.} \leq 0.25 \\ \frac{(1-2\alpha_{con.cap.})r_1^{\tilde{e}^2_{k_2}}+(2\alpha_{con.cap.}+(4\alpha_{con.cap.}-1)\theta_{l,\tilde{e}^2_{k_2}})r_2^{\tilde{e}^2_{k_2}}}{(1+(1-4\alpha_{con.cap.})\theta_{l,\tilde{e}^2_{k_2}})}, & \text{if } 0.25 < \alpha_{con.cap.} \leq 0.50 \\ \frac{(2\alpha_{con.cap.}-1)r_3^{\tilde{e}^2_{k_2}}+(2(1-\alpha_{con.cap.})+(3-4\alpha_{con.cap.})\theta_{l,\tilde{e}^2_{k_2}})r_2^{\tilde{e}^2_{k_2}}}{(1+(3-4\alpha_{con.cap.})\theta_{l,\tilde{e}^2_{k_2}\tilde{e}^2_{k_2}})}, & \text{if } 0.50 < \alpha_{con.cap.} \leq 0.75 \\ \frac{(2\alpha_{con.cap.}-1+(4\alpha_{con.cap.}-3)\theta_{r,\tilde{e}^2_{k_2}})r_3^{\tilde{e}^2_{k_2}}+2(1-\alpha_{con.cap.})r_2^{\tilde{e}^2_{k_2}}}{(1+(4\alpha_{con.cap.}-3)\theta_{r,\tilde{e}^2_{k_2}})}, & \text{if } 0.75 < \alpha_{con.cap.} \leq 1 \end{cases}$$

$$S_{\tilde{W}^2_{k_2}} = \begin{cases} \frac{(1-2\alpha_{weight}+(1-4\alpha_{weight})\theta_{r,\tilde{W}^2_{k_2}})r_1^{\tilde{W}^2_{k_2}}+2\alpha_{weight}r_2^{\tilde{W}^2_{k_2}}}{(1+(1-4\alpha_{weight})\theta_{r,\tilde{W}^2_{k_2}})}, & \text{if } 0 < \alpha_{avail.} \leq 0.25 \\ \frac{(1-2\alpha_{weight})r_1^{\tilde{W}^2_{k_2}}+(2\alpha_{weight}+(4\alpha_{weight}-1)\theta_{l,\tilde{W}^2_{k_2}})r_2^{\tilde{W}^2_{k_2}}}{(1+(1-4\alpha_{weight})\theta_{l,\tilde{W}^2_{k_2}})}, & \text{if } 0.25 < \alpha_{avail.} \leq 0.50 \\ \frac{(2\alpha_{weight}-1)r_3^{\tilde{W}^2_{k_2}}+(2(1-\alpha_{weight})+(3-4\alpha_{avail.})\theta_{l,\tilde{W}^2_{k_2}})r_2^{\tilde{W}^2_{k_2}}}{(1+(3-4\alpha_{weight})\theta_{l,\tilde{W}^2_{k_2}})}, & \text{if } 0.50 < \alpha_{avail.} \leq 0.75 \\ \frac{(2\alpha_{weight}-1+(4\alpha_{weight}-3)\theta_{r,\tilde{W}^2_{k_2}})r_3^{\tilde{W}^2_{k_2}}+2(1-\alpha_{weight})r_2^{\tilde{W}^2_{k_2}}}{(1+(4\alpha_{weight}-3)\theta_{r,\tilde{W}^2_{k_2}})}, & \text{if } 0.75 < \alpha_{avail.} \leq 1 \end{cases}$$

$$S_{\tilde{V}^2_{k_2}} = \begin{cases} \frac{(1-2\alpha_{volume}+(1-4\alpha_{volume})\theta_{r,\tilde{V}^2_{k_2}})r_1^{\tilde{V}^2_{k_2}}+2\alpha_{volume}r_2^{\tilde{V}^2_{k_2}}}{(1+(1-4\alpha_{volume})\theta_{r,\tilde{V}^2_{k_2}})}, & \text{if } 0<\alpha_{volume}\leq 0.25 \\ \frac{(1-2\alpha_{volume})r_1^{\tilde{V}^2_{k_2}}+(2\alpha_{volume}+(4\alpha_{volume}-1)\theta_{l,\tilde{V}^2_{k_2}})r_2^{\tilde{V}^2_{k_2}}}{(1+(1-4\alpha_{volume})\theta_{l,\tilde{V}^2_{k_2}})}, & \text{if } 0.25<\alpha_{volume}\leq 0.50 \\ \frac{(2\alpha_{volume}-1)r_3^{\tilde{V}^2_{k_2}}+(2(1-\alpha_{volume})+(3-4\alpha_{volume})\theta_{l,\tilde{V}^2_{k_2}})r_2^{\tilde{V}^2_{k_2}}}{(1+(3-4\alpha_{volume})\theta_{l,\tilde{V}^2_{k_2}})}, & \text{if } 0.50<\alpha_{volume}\leq 0.75 \\ \frac{(2\alpha_{volume}-1+(4\alpha_{volume}-3)\theta_{r,\tilde{V}^2_{k_2}})r_3^{\tilde{V}^2_{k_2}}+2(1-\alpha_{volume})r_2^{\tilde{V}^2_{k_2}}}{(1+(4\alpha_{volume}-3)\theta_{r,\tilde{V}^2_{k_2}})}, & \text{if } 0.75<\alpha_{volume}\leq 1 \end{cases}$$

$$S_{\tilde{b}^n_{mq}} = \begin{cases} \frac{(1-2\alpha_{demand}+(1-4\alpha_{demand})\theta_{r,\tilde{b}^n_{mq}})r_1^{\tilde{b}^n_{mq}}+2\alpha_{demand}r_2^{\tilde{b}^n_{mq}}}{(1+(1-4\alpha_{demand})\theta_{r,\tilde{b}^n_{mq}})}, & \text{if } 0<\alpha_{demand}\leq 0.25 \\ \frac{(1-2\alpha_{demand})r_1^{\tilde{b}^n_{mq}}+(2\alpha_{demand}+(4\alpha_{demand}-1)\theta_{l,\tilde{b}^n_{mq}})r_2^{\tilde{b}^n_{mq}}}{(1+(1-4\alpha_{demand})\theta_{l,\tilde{b}^n_{mq}})}, & \text{if } 0.25<\alpha_{demand}\leq 0.50 \\ \frac{(2\alpha_{demand}-1)r_3^{\tilde{b}^n_{mq}}+(2(1-\alpha_{demand})+(3-4\alpha_{demand})\theta_{l,\tilde{b}^n_{mq}})r_2^{\tilde{b}^n_{mq}}}{(1+(3-4\alpha_{demand})\theta_{l,\tilde{b}^n_{mq}})}, & \text{if } 0.50<\alpha_{demand}\leq 0.75 \\ \frac{(2\alpha_{demand}-1+(4\alpha_{demand}-3)\theta_{r,\tilde{b}^n_{mq}})r_3^{\tilde{b}^n_{mq}}+2(1-\alpha_{demand})r_2^{\tilde{b}^n_{mq}}}{(1+(4\alpha_{demand}-3)\theta_{r,\tilde{b}^n_{mq}})}, & \text{if } 0.75<\alpha_{demand}\leq 1 \end{cases}$$

$$S_{\tilde{e}^n_{k_n}} = \begin{cases} \frac{(1-2\alpha_{con.cap.}+(1-4\alpha_{con.cap.})\theta_{r,\tilde{e}^n_{k_n}})r_1^{\tilde{e}^n_{k_n}}+2\alpha_{con.cap.}r_2^{\tilde{e}^n_{k_n}}}{(1+(1-4\alpha_{con.cap.})\theta_{r,\tilde{e}^n_{k_n}})}, & \text{if } 0<\alpha_{con.cap.}\leq 0.25 \\ \frac{(1-2\alpha_{con.cap.})r_1^{\tilde{e}^n_{k_n}}+(2\alpha_{con.cap.}+(4\alpha_{con.cap.}-1)\theta_{l,\tilde{e}^n_{k_n}})r_2^{\tilde{e}^n_{k_n}}}{(1+(1-4\alpha_{con.cap.})\theta_{l,\tilde{e}^n_{k_n}})}, & \text{if } 0.25<\alpha_{con.cap.}\leq 0.50 \\ \frac{(2\alpha_{con.cap.}-1)r_3^{\tilde{e}^n_{k_n}}+(2(1-\alpha_{con.cap.})+(3-4\alpha_{con.cap.})\theta_{l,\tilde{e}^n_{k_n}})r_2^{\tilde{e}^n_{k_n}}}{(1+(3-4\alpha_{con.cap.})\theta_{l,\tilde{e}^n_{k_n}})}, & \text{if } 0.50<\alpha_{con.cap.}\leq 0.75 \\ \frac{(2\alpha_{con.cap.}-1+(4\alpha_{con.cap.}-3)\theta_{r,\tilde{e}^n_{k_n}})r_3^{\tilde{e}^n_{k_n}}+2(1-\alpha_{con.cap.})r_2^{\tilde{e}^n_{k_n}}}{(1+(4\alpha_{con.cap.}-3)\theta_{r,\tilde{e}^n_{k_n}})}, & \text{if } 0.75<\alpha_{con.cap.}\leq 1 \end{cases}$$

$$S_{\tilde{W}^n_{k_n}} = \begin{cases} \frac{(1-2\alpha_{weight}+(1-4\alpha_{weight})\theta_{r,\tilde{W}^n_{k_n}})r_1^{\tilde{W}^n_{k_n}}+2\alpha_{weight}r_2^{\tilde{W}^n_{k_n}}}{(1+(1-4\alpha_{weight})\theta_{r,\tilde{W}^n_{k_n}})}, & \text{if } 0<\alpha_{weight}\leq 0.25 \\ \frac{(1-2\alpha_{weight})r_1^{\tilde{W}^n_{k_n}}+(2\alpha_{weight}+(4\alpha_{weight}-1)\theta_{l,\tilde{W}^n_{k_n}})r_2^{\tilde{W}^n_{k_n}}}{(1+(1-4\alpha_{weight})\theta_{l,\tilde{W}^n_{k_n}})}, & \text{if } 0.25<\alpha_{weight}\leq 0.50 \\ \frac{(2\alpha_{weight}-1)r_3^{\tilde{W}^n_{k_n}}+(2(1-\alpha_{weight})+(3-4\alpha_{weight})\theta_{l,\tilde{W}^n_{k_n}})r_2^{\tilde{W}^n_{k_n}}}{(1+(3-4\alpha_{weight})\theta_{l,\tilde{W}^n_{k_n}})}, & \text{if } 0.50<\alpha_{weight}\leq 0.75 \\ \frac{(2\alpha_{weight}-1+(4\alpha_{weight}-3)\theta_{r,\tilde{W}^n_{k_n}})r_3^{\tilde{W}^n_{k_n}}+2(1-\alpha_{weight})r_2^{\tilde{W}^n_{k_n}}}{(1+(4\alpha_{weight}-3)\theta_{r,\tilde{W}^n_{k_n}})}, & \text{if } 0.75<\alpha_{weight}\leq 1 \end{cases}$$

$$S_{\tilde{V}^n_{k_n}} = \begin{cases} \frac{(1-2\alpha_{volume}+(1-4\alpha_{volume})\theta_{r,\tilde{V}^n_{k_n}})r_1^{\tilde{V}^n_{k_n}}+2\alpha_{volume}r_2^{\tilde{V}^n_{k_n}}}{(1+(1-4\alpha_{volume})\theta_{r,\tilde{V}^n_{k_n}})}, & \text{if } 0 < \alpha_{volume} \le 0.25 \\ \frac{(1-2\alpha_{volume})r_1^{\tilde{V}^n_{k_n}}+(2\alpha_{volume}+(4\alpha_{volume}-1)\theta_{l,\tilde{V}^n_{k_n}})r_2^{\tilde{V}^n_{k_n}}}{(1+(1-4\alpha_{volume})\theta_{l,\tilde{V}^n_{k_n}})}, & \text{if } 0.25 < \alpha_{volume} \le 0.50 \\ \frac{(2\alpha_{volume}-1)r_3^{\tilde{V}^n_{k_n}}+(2(1-\alpha_{volume})+(3-4\alpha_{volume})\theta_{l,\tilde{V}^n_{k_n}})r_2^{\tilde{V}^n_{k_n}}}{(1+(3-4\alpha_{volume})\theta_{l,\tilde{V}^n_{k_n}})}, & \text{if } 0.50 < \alpha_{volume} \le 0.75 \\ \frac{(2\alpha_{volume}-1+(4\alpha_{volume}-3)\theta_{r,\tilde{V}^n_{k_n}})r_3^{\tilde{V}^n_{k_n}}+2(1-\alpha_{volume})r_2^{\tilde{V}^n_{k_n}}}{(1+(4\alpha_{volume}-3)\theta_{r,\tilde{V}^n_{k_n}})}, & \text{if } 0.75 < \alpha_{volume} \le 1 \end{cases}$$

7. Numerical Example

A firm produces two types of food as Bread and Biscuit and stored at two plants which are the supply points of our problem. The goods are delivered to two destination centers (DCs) from these supply points then finally these products are transported to the final destination centers or customers or survived peoples on disaster via the first DCs. That is the transportation happened in two stages. Due to disaster, the requirements, availabilities and other transportation parameters are not known to us precisely. For this reason, we consider all the transportation parameters as type-2 triangular fuzzy numbers. The type-2 triangular fuzzy inputs for unit transportation costs and times, availabilities, demands, conveyances capacities, purchasing cost, loading and unloading cost and time, weights and volumes etc. for stage-1 and stage-2 are as follows:

Type-2 fuzzy unit transportation cost, time for stage-1 and stage-2:
$C^1_{1111} = (11, 12, 14; .4, .6)$, $C^1_{1211} = (12, 13, 14; .2, .3)$, $C^1_{1121} = (11, 13, 14; .2, .3)$, $C^1_{1221} = (12, 14, 16; .1, .2)$,
$C^1_{2111} = (13, 15, 16; .6, .7)$, $C^1_{2211} = (4, 5, 6; .3, .5)$, $C^1_{2121} = (13, 15, 16; .3, 1.2)$, $C^1_{2221} = (14, 16, 17; .2, .5)$,
$C^1_{1112} = (13, 14, 17; .4, .6)$,$C^1_{1212} = (13, 14, 16; .2, .8)$,$C^1_{1122} = (11, 15, 18; .2, .7)$,$C^1_{1222} = (12, 14, 16; .5, 1.2)$,
$C^1_{2112} = (11, 15, 17; .6, .9)$,$C^1_{2212} = (12, 13, 19; .3, .5)$,$C^1_{2122} = (13, 17, 19; .3, .9)$,$C^1_{2222} = (15, 16, 18; .4, .5)$,
$t^1_{1111} = (2, 3, 5; .4, .6)$,$t^1_{1211} = (3, 4, 7; .7, .9)$,$t^1_{1121} = (7, 9, 12; .9, 1)$,$t^1_{1221} = (2, 4, 6; .1, .2)$,
$t^1_{2111} = (7, 10, 13; .6, .9)$,$t^1_{2211} = (5, 7, 8; .8, 1)$,$t^1_{2121} = (4, 5, 8; .8, 1.3)$,$t^1_{2221} = (6, 7, 9; .7, 1.5)$,
$t^1_{1112} = (5, 9, 13; .2, .8)$,$t^1_{1212} = (5, 7, 9; .8, 1.4)$,$t^1_{1122} = (5, 8, 9; .9, 1.9)$,$t^1_{1222} = (4, 5, 6; .5, .7)$,
$t^1_{2112} = (4, 6, 9; .4, .7)$,$t^1_{2212} = (2, 3, 9; .3, .5)$,$t^1_{2122} = (3, 7, 10; .3, .9)$,$t^1_{2222} = (5, 6, 8; .4, 1.5)$,
$C^2_{1111} = (8, 9, 11; .4, .6)$,$C^2_{1211} = (12, 13, 14; .2, 1)$,$C^2_{1121} = (11, 13, 14; .2, 1.3)$,$C^2_{1221} = (13, 15, 16; .1, .7)$,
$C^2_{2111} = (13, 15, 16; .6\ 1.9)$,$C^2_{2211} = (14, 15, 16; .3, .5)$,$C^2_{2121} = (13, 15, 16; .3, 1.2)$,$C^2_{2221} = (14, 16, 17; .6, 1.5)$,
$C^2_{1112} = (13, 14, 17; .4, .6)$,$C^2_{1212} = (14, 15, 17; .2, 1)$, $C^2_{1122} = (11, 15, 18; .2, .7)$,$C^2_{1222} = (3, 4, 7; .7, .9)$,
$C^2_{2112} = (11, 15, 17; .6, .9)$,$C^2_{2212} = (12, 17, 19; .3, .5)$,$C^2_{2122} = (16, 17, 19; .3, .9)$,$C^2_{2222} = (5, 7, 8; .8, 1)$,
$t^2_{1111} = (2, 4, 8; .6, .7)$,$t^2_{1211} = (3, 5, 7; .4, .8)$,$t^2_{1121} = (2, 3, 4; .6, 1.7)$,$t^2_{1221} = (2, 4, 6; .1, .2)$,
$t^2_{2111} = (7, 8, 11; .2, 1.1)$,$t^2_{2211} = (3, 7, 9; .8, 1.1)$,$t^2_{2121} = (2, 5, 9; .8, 1.9)$,$t^2_{2221} = (2, 6, 8; .7, 1.5)$,
$t^2_{1112} = (3, 9, 12; .3, .8)$,$t^2_{1212} = (2, 3, 4; .5, 1.1)$,$t^2_{1122} = (3, 4, 11; .7, 1.4)$,$t^2_{1222} = (4, 5, 6; .7, 1.3)$,
$t^2_{2112} = (3, 6, 8; .3, .4)$, $t^2_{2212} = (2, 6, 9; .2, .8)$, $t^2_{2122} = (7, 9, 13; .2, .7)$, $t^2_{2222} = (2, 6, 9; .9, 1.2)$.

Table-2: Type-2 fuzzy availabilities, demands, conveyances capacities, loading and unloading time and cost, weights and volume for stage-1 and stage-2

Availabilities	$\tilde{a}_{11}^1 = (60, 66, 67; .2, .5)$,$\tilde{a}_{21}^1 = (54, 56, 60; .1, .2)$,$\tilde{a}_{12}^1 = (42, 47, 55; .2, .4)$,$\tilde{a}_{22}^1 = (47, 53, 55; .5, .6)$
Demands for stage-1	$\tilde{b}_{11}^1 = (19, 26, 30; .1, .3)$,$\tilde{b}_{21}^1 = (21, 24, 25; .2, .3)$,$\tilde{b}_{12}^1 = (20, 21, 22; .7, 2.1)$,$\tilde{b}_{22}^1 = (22, 23, 25; .5, 1.2)$
Demands for stage-2	$\tilde{b}_{11}^2 = (18, 20, 23; .2, .3)$,$\tilde{b}_{21}^2 = (17, 18, 25; .1, .3)$,$\tilde{b}_{12}^2 = (15, 16, 17; .1, .4)$,$\tilde{b}_{22}^2 = (12, 14, 16; .9, 1.3)$
Conveyances Capacities	$\tilde{e}_1^1 = (52, 54, 56; .2, .3)$,$\tilde{e}_2^1 = (53, 55, 57; .6, .9)$,$\tilde{e}_1^2 = (42, 44, 49; 1.8, 2.3)$,$\tilde{e}_2^2 = (45, 49, 50; .4, .9)$
Loading Cost	$\tilde{LO}_1^1 = (2, 4, 6; .2, .3)$,$\tilde{LO}_2^1 = (5, 6, 7; .2, .3)$,$\tilde{LO}_1^2 = (5, 6, 9; .2, .6)$,$\tilde{LO}_2^2 = (2, 9, 10; .3, .4)$
Unloading Cost	$\tilde{UD}_1^1 = (2, 3, 4; .4, .6)$,$\tilde{UD}_2^1 = (3, 7, 8, .6; .7,)$,$\tilde{UD}_1^2 = (2, 3, 5; .4, .9)$,$\tilde{UD}_2^2 = (6, 8, 9; .6, 1)$
Loading Time	$L\tilde{T}O_1^1 = (3, 7, 9; 1.2, 1.3)$,$L\tilde{T}O_2^1 = (1, 2, 3; 1.1, 1.2)$,$L\tilde{T}O_1^2 = (4, 6, 8; .2, .3)$,$L\tilde{T}O_2^2 = (1, 3, 6; .1, .2)$
Unloading Time	$U\tilde{T}D_1^1 = (3, 7, 10; .3, .8)$,$U\tilde{T}D_2^1 = (2, 9, 11; .1, .4)$,$U\tilde{T}D_1^2 = (2, 4, 5; .1, .5)$,$U\tilde{T}D_2^2 = (4, 5, 6; .5, .6)$
Purchasing Cost	$\tilde{PC}_{11} = (10, 11, 12; 1.2, 1.4)$,$\tilde{PC}_{12} = (13, 15, 16; 1.1, 1.4)$,$\tilde{PC}_{21} = (11, 12, 13; 1.1, 1.2)$,$\tilde{PC}_{22} = (14, 16, 17; .9, 1.3)$
Weights capacity	$\tilde{W}_1^1 = (185, 190, 225; .6, .7)$,$\tilde{W}_2^1 = (288, 320, 400; .7, .8)$, $\tilde{W}_1^2 = (310, 320, 331; .1, .9)$,$\tilde{W}_2^2 = (368, 340, 345; .6, .7)$
Volumes capacity	$\tilde{V}_1^1 = (334, 360, 370; 1.2, 1.3)$,$\tilde{V}_2^1 = (231, 294, 370; .6, .7)$, $\tilde{V}_1^2 = (393, 395, 399; .6, .8)$,$\tilde{V}_2^2 = (353, 354, 356; .8, .9)$
weight,volume of items	$\tilde{w}_1 = (1, 5, 7; .4, .5)$,$\tilde{w}_2 = (1, 4, 5; .7, .9)$,$\tilde{v}_1 = (1, 2, 6; 1.1, 1.3)$,$\tilde{v}_2 = (1, 3, 8; .2, .5)$

8. Results

The fuzzy multi-stage STP is converted to its equivalent crisp problem by using CV-based reduction method and generalized credibility measure. Then using LINGO.13.0 optimization software, we obtain the optimal solution of the deterministic STP. The values of the credibility level for the transportation parameters are sometime lies in the interval (0, 0.25] or (0.25, 0.5] or (0.5, 0.75] or (0.75, 1]. For this reason, we obtain the optimal solution of the newly developed model with the four limitations of the credibility level. A sensitivity analysis is taken into consideration to show the change of the optimal values of the objective functions and the transported amounts with respect to the credibility level of availabilities, demands and conveyances capacities.

Table-3: Changes of optimum cost and transported amount for different credibility levels

Credibility Level	Item-1	Item-2	Item-1	Item-2	Stage-1	Stage-2	Opt. cost	Opt. time
$\alpha_c = 0.07$ $\alpha_t = 0.10$ $\alpha_{avail.} = 0.13$ $\alpha_{demand} = 0.16$ $\alpha_{con.cap.} = 0.19$ $\alpha_{weight} = 0.22$ $\alpha_{volume} = 0.25$	41.92	42.40	35.87	27.71	$x^1_{1111} = 21.02$ $x^1_{2211} = 21.90$ $x^1_{1122} = 20.20$ $x^1_{1222} = 22.20$	$x^2_{1111} = 18.58$ $x^2_{2221} = 17.29$ $x^2_{2112} = 15.28$ $x^2_{2212} = 6.88$ $x^2_{1222} = 5.49$ $x^2_{2222} = 0.06$	3482.99	105.86
$\alpha_c = 0.26$ $\alpha_t = 0.29$ $\alpha_{avail.} = 0.31$ $\alpha_{demand} = 0.33$ $\alpha_{con.cap.} = 0.35$ $\alpha_{weight} = 0.37$ $\alpha_{volume} = 0.40$	51.40	64.08	40.84	41.44	$x^1_{1111} = 13.81,$ $x^1_{2211} = 26.19,$ $x^1_{1121} = 11.45,$ $x^1_{2212} = 14.30,$ $x^1_{1122} = 32.68,$ $x^1_{1222} = 17.10,$	$x^2_{1111} = 22,$ $x^2_{1211} = 3.25,$ $x^2_{2211} = 15.59,$ $x^2_{2112} = 14.7,$ $x^2_{1122} = 2.04,$ $x^2_{1222} = 24.7$	5730.85	188.26
$\alpha_c = 0.56$ $\alpha_t = 0.59$ $\alpha_{avail.} = 0.61$ $\alpha_{demand} = 0.63$ $\alpha_{con.cap.} = 0.65$ $\alpha_{weight} = 0.67$ $\alpha_{volume} = 0.69$	40.45	44.62	40.45	30.60	$x^1_{2211} = 24.58,$ $x^1_{1121} = 15.87,$ $x^1_{1112} = 14.56,$ $x^1_{1122} = 6.64,$ $x^1_{1222} = 23.42$	$x^2_{1111} = 15.87,$ $x^2_{2121} = 4.84,$ $x^2_{2221} = 19.74,$ $x^2_{1112} = 4.43,$ $x^2_{2112} = 9.48,$ $x^2_{1212} = 14.36,$ $x^2_{1122} = 2.33.$	4912.23	186.70
$\alpha_c = 0.76$ $\alpha_t = 0.79$ $\alpha_{avail.} = 0.83$ $\alpha_{demand} = 0.86$ $\alpha_{con.cap.} = 0.90$ $\alpha_{weight} = 0.95$ $\alpha_{volume} = 0.98$	53.76	46.48	45.53	32.41	$x^1_{2211} = 4.04,$ $x^1_{1121} = 29.01,$ $x^1_{1221} = 20.71,$ $x^1_{1112} = 21.85,$ $x^1_{1212} = 17.62,$ $x^1_{1222} = 7.01$	$x^2_{1111} = 22.26$ $x^2_{1211} = 6.75,$ $x^2_{2211} = 12.57,$ $x^2_{2221} = 3.95,$ $x^2_{1212} = 6.24,$ $x^2_{2122} = 16.76,$ $x^2_{1222} = 9.41$	6055.26	284.63

8.1. Particular Case. Let us consider, the credibility level for costs, times, availabilities, demands, conveyances capacities, weights and volume are all equal. i.e., $\alpha_c = \alpha_t = \alpha_{avail.} = \alpha_{demand} = \alpha_{con.cap.} = \alpha_{weight} = \alpha_{volume} = \alpha$, say.

Table-4: Optimal results of the model with same credibility level

Credibility Level	Item-1	Item-2	Item-1	Item-2	Stage-1	Stage-2	Opt. cost	Opt. time
$\alpha = 0.24$	44.74	42.90	36.43	28.38	$x^1_{1111} = 22.32$, $x^1_{2211} = 22.42$, $x^1_{1122} = 20.44$, $x^1_{1222} = 22.46$,	$x^2_{1111} = 18.95$ $x^2_{1111} = 3.37$, $x^2_{1111} = 5.09$, $x^2_{2221} = 9.02$, $x^2_{2112} = 15.47$, $x^2_{1222} = 12.91$	4022.51	115.49
$\alpha = 0.32$	50.60	60.86	40.27	39.02	$x^1_{2211} = 25.70$, $x^1_{1121} = 24.90$, $x^1_{1212} = 19.88$, $x^1_{2212} = 10.19$, $x^1_{1122} = 30.79$,	$x^2_{1111} = 21.61$, $x^2_{1211} = 3.29$, $x^2_{2211} = 15.37$, $x^2_{1212} = 14.24$, $x^2_{2212} = 8.23$, $x^2_{1122} = 16.55$.	7588.72	210.61
$\alpha = 0.72$	42.33	45.24	42.33	31.22	$x^1_{2211} = 25.84$, $x^1_{1121} = 16.49$, $x^1_{1112} = 5.91$, $x^1_{1122} = 15.5$, $x^1_{1222} = 23.83$,	$x^2_{1111} = 16.49$, $x^2_{2111} = 4.8$, $x^2_{2211} = 14.93$, $x^2_{2221} = 6.11$, $x^2_{1112} = 6.61$, $x^2_{2122} = 9.82$, $x^2_{1222} = 14.79$.	5317.32	215.64
$\alpha = 0.80$	53.11	46.07	44.23	31.99	$x^1_{2211} = 8.02$, $x^1_{1121} = 28.49$, $x^1_{1221} = 16.6$, $x^1_{1112} = 21.72$, $x^1_{1212} = 13.12$, $x^1_{1222} = 11.23$,	$x^2_{1111} = 21.87$, $x^2_{1211} = 3.67$, $x^2_{2211} = 18.69$, $x^2_{1212} = 3.4$, $x^2_{2122} = 16.63$, $x^2_{1222} = 11.96$.	6037.55	268.83

8.2. Sensitivity Analysis of the availabilities and demands of the model. We know that the sensitivity analysis is used to analyze the outputs with the given inputs data. For this reason, in Table-5 and -6 we analyze some inputs data and outputs as sensitivity analysis. Basically the minimization of cost objective and time objective in the STP depend on the values of the transportation parameters such as unit transportation costs, times, demands, supplies etc.

Table-5: Sensitivity analysis on availabilities

α_c	α_t	$\alpha_{avail.}$	α_{demand}	$\alpha_{con.cap.}$	α_{weight}	α_{volume}	Opt. cost	Opt. time	Item-1	Item-2
		0.10					3574.57	101.08	35.93	27.78
		0.12					3584.04	100.11	35.99	27.84
0.11	0.15	0.16	0.17	0.20	0.22	0.25	3584.13	91.03	36.00	27.85
		0.19					3587.07	99.96	36.02	27.87
		0.25					3601.32	109.94	36.06	27.92
		0.26					5541.09	201.52	40.27	39.02
		0.29					5600.76	189.59	40.27	39.02
0.26	0.29	0.32	0.32	0.35	0.38	0.40	5556.28	211.09	40.27	39.02
		0.35					5544.85	197.75	40.27	39.02
		0.38					5508.48	163.44	40.27	39.02
		0.53					4821.18	216.28	39.85	30.45
		0.58					4811.73	180.94	40.24	30.56
0.52	0.56	0.63	0.60	0.64	0.68	0.72	4857.09	185.68	40.85	30.72
		0.68					4905.02	195.65	40.46	30.91
		0.73					4938.51	179.64	41.72	31.07
		0.77					6093.96	256.79	45.73	32.46
		0.83					6139.95	259.16	46.11	32.56
0.77	0.83	0.87	0.87	0.91	0.95	0.99	6148.66	277.31	46.49	32.66
		0.92					6173.94	280.47	46.83	32.95
		0.98					6187.11	280.50	46.93	33.04

Table-6: Sensitivity analysis on demands

α_c	α_t	$\alpha_{avail.}$	α_{demand}	$\alpha_{con.cap.}$	α_{weight}	α_{volume}	Opt. cost	Opt. time	Item-1	Item-2
			0.13				3539.57	101.08	35.68	27.54
			0.16				3565.36	100.18	35.87	27.71
0.11	0.15	0.17	0.19	0.19	0.22	0.25	3594.08	102.09	36.06	27.92
			0.22				3626.59	101.59	36.27	28.18
			0.25				3664.37	100.28	36.50	28.50
			0.26				4534.36	184.54	37	29.69
			0.29				4962.18	151.40	38.6	33.82
0.26	0.29	0.32	0.32	0.35	0.38	0.40	5563.87	186.38	40.27	39.02
			0.35				6193.2	201.54	42.01	45.83
			0.38				7072.83	199.66	43.83	55.23
			0.53				4751.82	157.77	38.54	30.12
			0.58				4798.8	164.77	39.47	30.35
0.52	0.56	0.60	0.63	0.64	0.68	0.72	4833.92	195.21	40.44	30.61
			0.68				4913.07	200.54	41.47	30.92
			0.73				5013.48	184.95	42.55	31.32
			0.77				5921.35	255.83	43.5	31.71
			0.83				6049.3	273.49	44.89	32.22
0.77	0.83	0.87	0.87	0.91	0.95	0.99	6213.72	234.95	45.74	32.46
			0.92				6237.77	266.98	46.67	32.69
			0.98				6271.87	233.34	47.69	32.93

8.3. Pictorial representation of the sensitivity analysis. The Pictorial representation of the sensitivity analysis are shown in the figure-2 - figure-17 and those are given below:

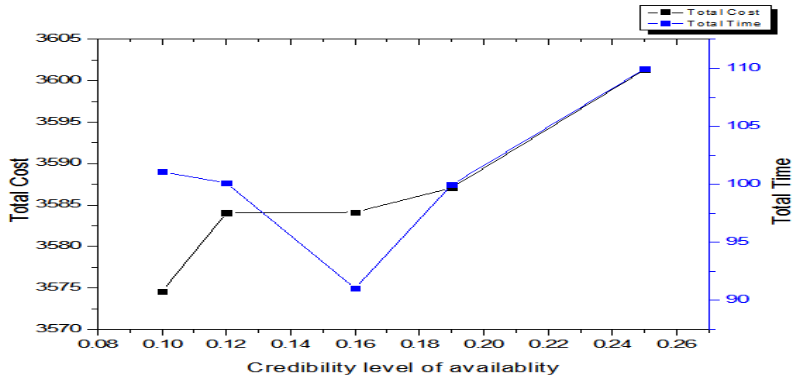

Figure 2. *Change of total optimum cost and time with Credibility level of availability,* $\alpha_{avail.} \in (0, 0.25]$

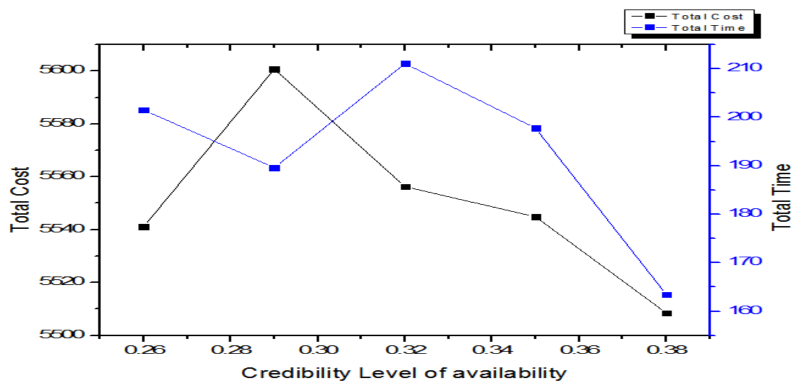

Figure 3. *Change of total optimum cost and time with Credibility level of availability* $\alpha_{avail.} \in (0.25, 0.50]$

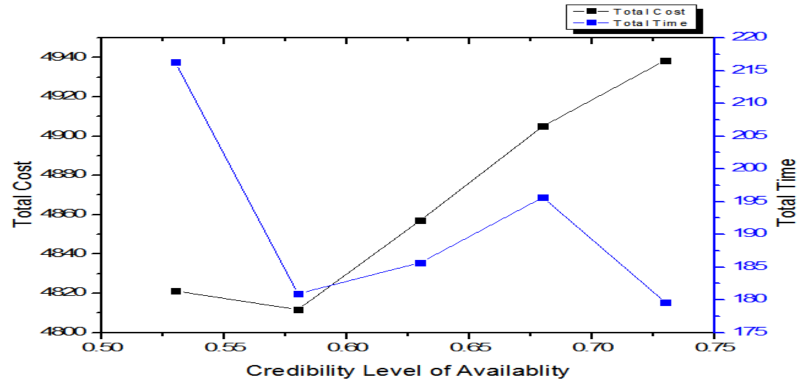

Figure 4. *Change of total optimum cost and time with Credibility level of availability,* $\alpha_{avail.} \in (0.50, 0.75]$

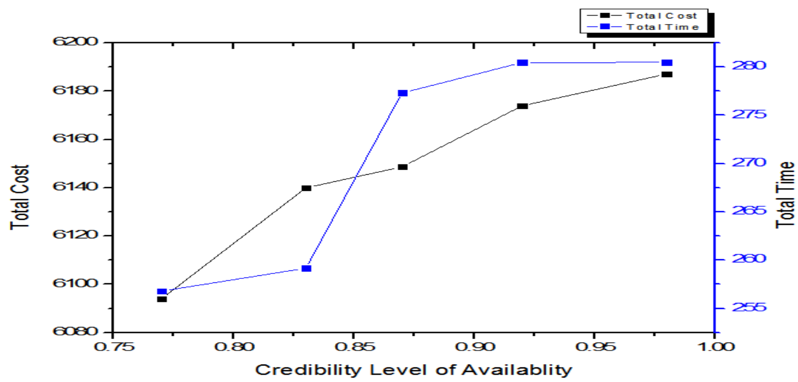

Figure 5. *Change of total optimum cost and time with Credibility level of availability,* $\alpha_{avail.} \in (0.t5, 1]$

9. Discussion

Since the credibility level of the availabilities, demands, conveyances capacities, weights and volumes for each transported item and each vehicle are different, so after taking the variation of each transportation parameters, we obtained lots of results of our STP model where all the transportation parameters are type-2 fuzzy variables and which are discussed below: Following Table-3, we see that the least amount of total cost and time are 3482.99 and 105.86 units respectively and these are obtained when the credibility level of the transportation parameters lies within the interval (0, 0.25]. Again, in Table-4, we put some optimal results which are obtained by taking the credibility level of all the transportation parameters are equal. After careful investigation, we found that the total cost and time are least when the credibility levels are same and it is lies in the interval (0, 0.25]. In Table-5 we have the following:

(i) When credibility level of the availabilities increases with the limit (0, 0.25], then the values of the cost objective also increases and the time objective are sometime increases and decreases. The increase or decrease of the total time within the variation of the credibility level is also significant i.e., if we change the credibility level then the allocations

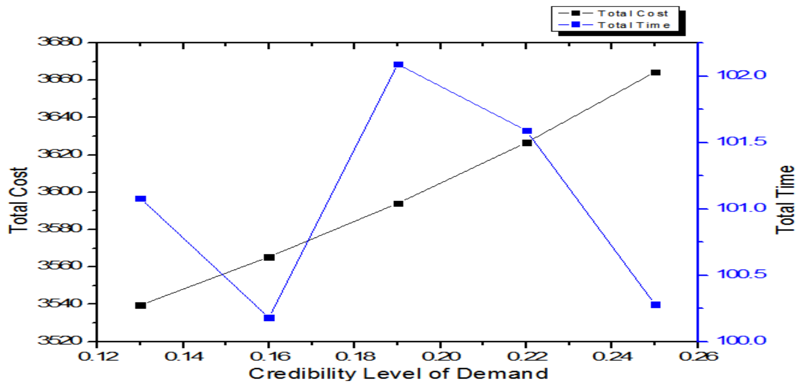

Figure 6. *Change of total optimum cost and time with Credibility level of demand,* $\alpha_{demand} \in (0, 0.25]$

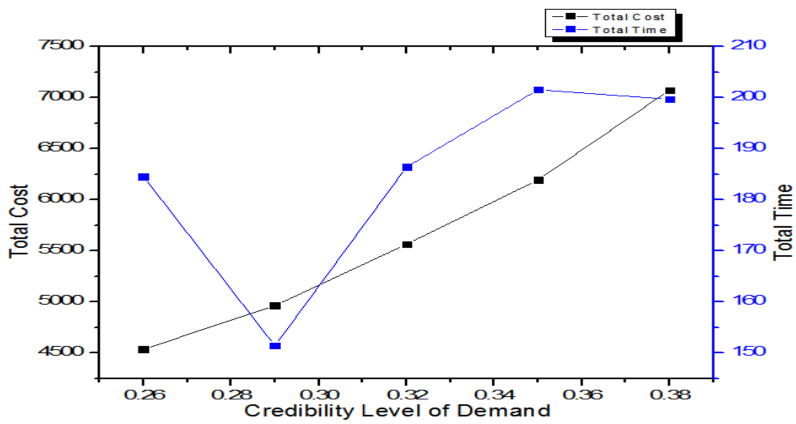

Figure 7. *Change of total optimum cost and time with Credibility level of demand,* $\alpha_{demand} \in (0.25, 0.50]$

are changed and for this reason, it is happening.
(ii) From the third row of the Table-5, we obtained some optimal results of the objectives where the credibility level of availabilities are lies within (0.25, 0.5]. Due to change of credibility level of availabilities, sometimes the value of total cost are increased and sometimes decreased but there is no significant change in time objective. This is found when credibility level increases within the range (0.25, 0.5].
(iii) The value of the cost objective increases when we increase the credibility level of availabilities within the range (0.5, 0.75] but there are some random changes found in the time objective function.
(iv) When credibility level of the availabilities increases within the limit (0.75, 1], then the value of the objectives and transported amounts (item-1 and item-2) are increased.
Again if we can change the value of the credibility level of the demand, then we found some significant changes on the objective functions as well as transported amounts. From Table-6, it is seen that when we increase the credibility level of the demands, then the cost and time objectives are also increases and same type of changes is found on the

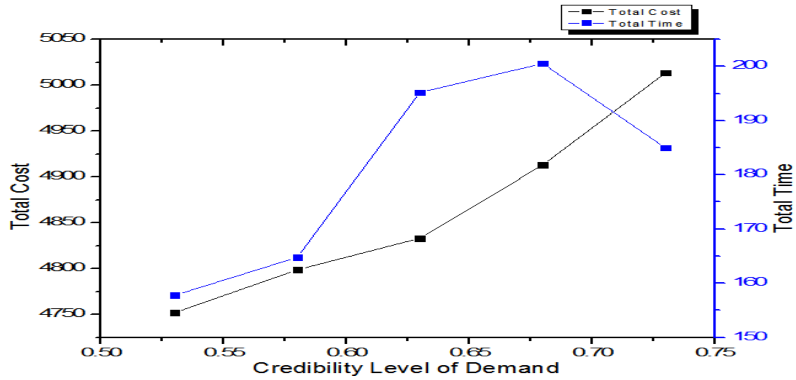

Figure 8. *Change of total optimum cost and time with Credibility level of demand,* $\alpha_{demand} \in (0.50, 0.75]$

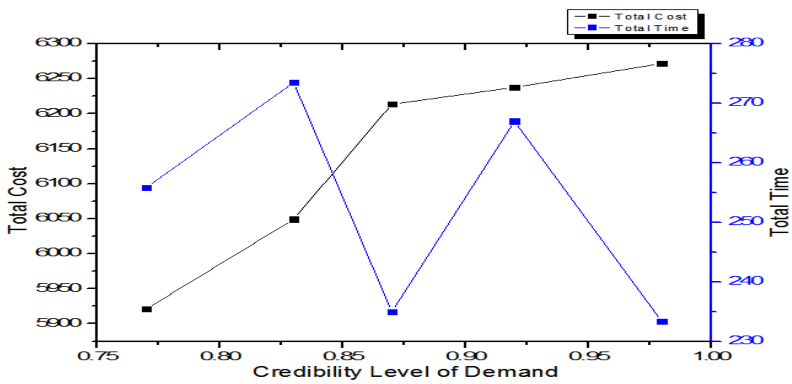

Figure 9. *Change of total optimum cost and time with Credibility level of demand,* $\alpha_{demand} \in (0.75, 1]$

transported amount in the final stages.

10. Comparison with the earlier Research work

Heragu [13] introduced the problem called two stages TP and gave the mathematical model for this problem. The model includes both the inbound and outbound transportation cost and aims to minimize the overall cost. Hindi et al. [12] addressed a two-stage distribution-planning problem. They considered two additional requirements on their problem. First, each customer must be served from a single DC. Second, it must be possible to ascertain the plant origin of each product quantity delivered. A mathematical formulation called PLANWAR presented by Pirkul and Jayaraman [20] to locate a number of sources and destination centers and to design distribution network so that the total operating cost can be minimized. Syarif and Gen [23] considered production/distribution problem formulated as two-stage TP and proposed a hybrid genetic algorithm (GA) for solution. But in our research, we develop a new concept which is totally different from the concept of [20], [13], [12], [23] etc. Here our concept is to supply the commodities from

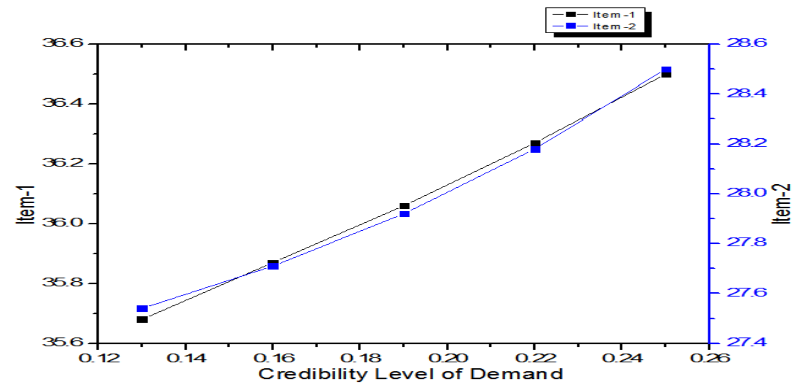

Figure 10. *Change of transported amounts (item-1 and 2) with Credibility level of demand,* $\alpha_{demand} \in (0, 0.25]$

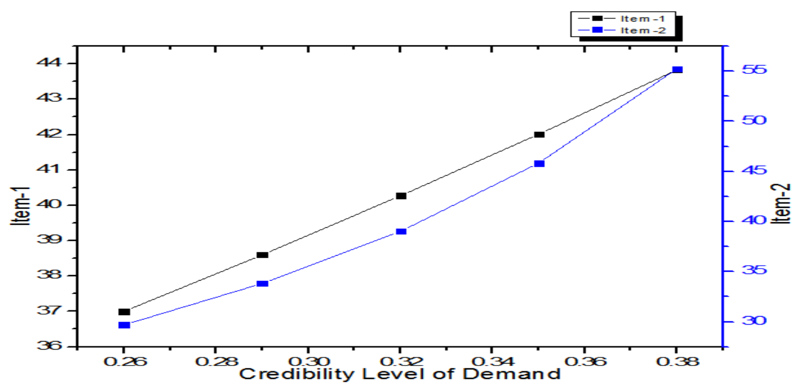

Figure 11. *Change of transported amounts (item-1 and 2) with Credibility level of demand,* $\alpha_{demand} \in (0.25, 0.50]$

sources to destination centers with their requirements in stage-1 and then the transported amounts in stage-1 is converted to the availabilities of the stage-2. The transportation of the stage-2 happened according to requirements of the destination centers of the stage-2 where the availabilities for the stage-2 are the transported amounts for stage-1 and so on for the other stage transportations. So we can't make comparison of our approach to the existing one. But we validate our technique and optimum result by sensitivity analysis.

11. Conclusion and Future Extension of the Research Work

11.1. Conclusion. In this paper, we propose a newly developed STP model under type-2 fuzzy environment. Weight and volume of the transported items and vehicle are more significant in the transportation network. So we add two new additional constraints as weight constraints and volume constraints for each vehicle to handle the STP with different stages. We apply the goal programming method is to solve our multi-objective multi-stage STP since goal programming technique gives the better optimal result of the objective function than the other methods. Here we study four cases of the credibility

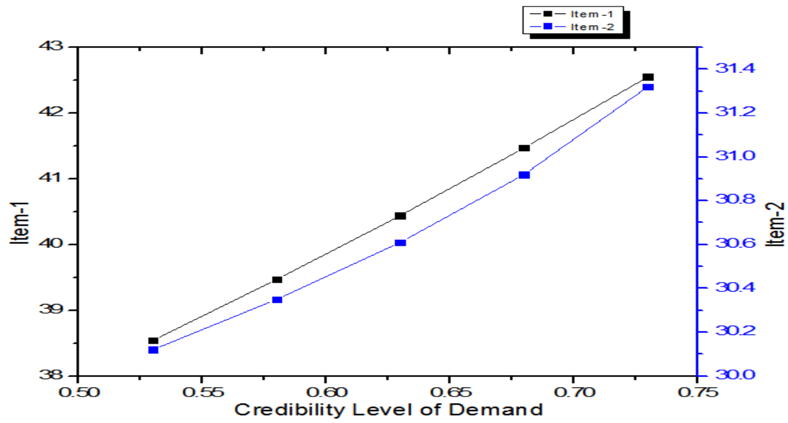

Figure 12. *Change of transported amounts (item-1 and 2) with Credibility level of demand,* $\alpha_{demand} \in (0.50, 0.75]$

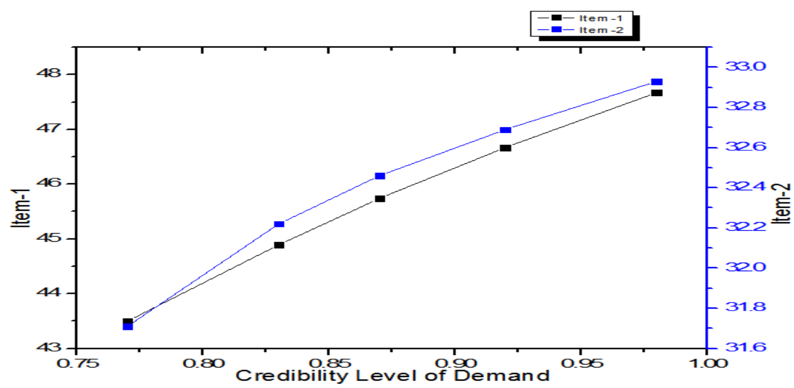

Figure 13. *Change of transported amounts (item-1 and 2) with Credibility level of demand,* $\alpha_{demand} \in (0.75, 1]$

level of the different transportation parameters. Also, after solving the transportation model, we see that the least transportation cost is obtained when the credibility level lies within the range (0, 0.25] and in particular when the credibility level of the transportation parameters are all equal, then a similar type of change is observed in the objective functions. We obtain the optimal solution of the model by using generalized reduced gradient technique (LINGO 13.0 optimization solver) and the results are very effective in real-life sense. So we conclude that, if the credibility levels of the transportation parameters lies within (0, 0.25], then any multi-stage or single stage STP with type-2 fuzzy parameter gives the least value of the objective function.

11.2. Future Extension of the Research Work. The future extensions of our research work are as follows:

• We have formulated the STP model under type-2 fuzzy environment but this model can be developed under fuzzy-rough, fuzzy-random, interval type-2 fuzzy environments

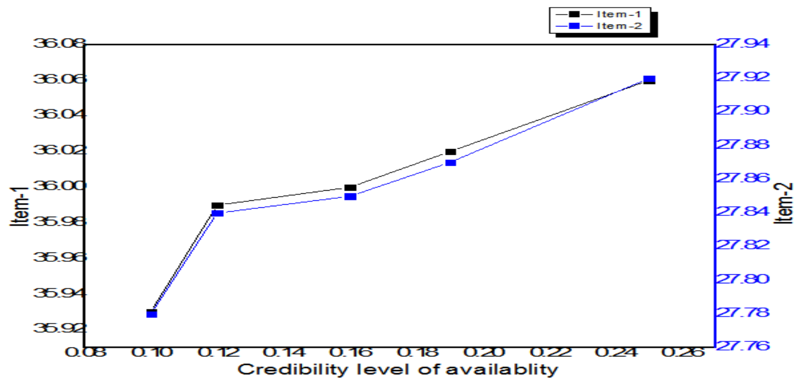

Figure 14. *Change of transported amounts (item-1 and 2) with Credibility level of demand,* $\alpha_{avail.} \in (0, 0.25]$

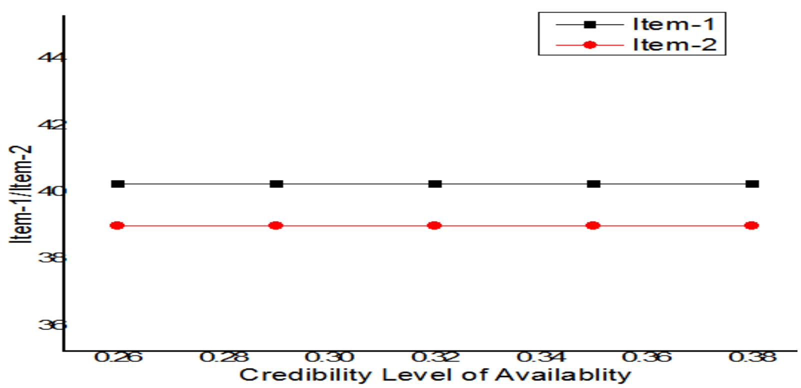

Figure 15. *Change of transported amounts (item-1 and 2) with Credibility level of demand,* $\alpha_{avail.} \in (0.25, 0.50]$

etc.

• In our model we imposed two extra restrictions with the help of weights and volume of each items and vehicles. There is a scope to formulate and solve the model with safety constraints, budget constraint etc.

• In the objective function we considered the unit transportation cost, time, purchasing cost, loading and unloading cost and time etc. but there is a scope to develop the cost objective function of our model with fixed charges, vehicle carrying cost etc.

• In the solution of the imprecise STP model, the transported amounts have been considered as crisp. Hence there is a scope of taking these transported amount as fuzzy also i.e. the models can be formulated as fully fuzzy models.

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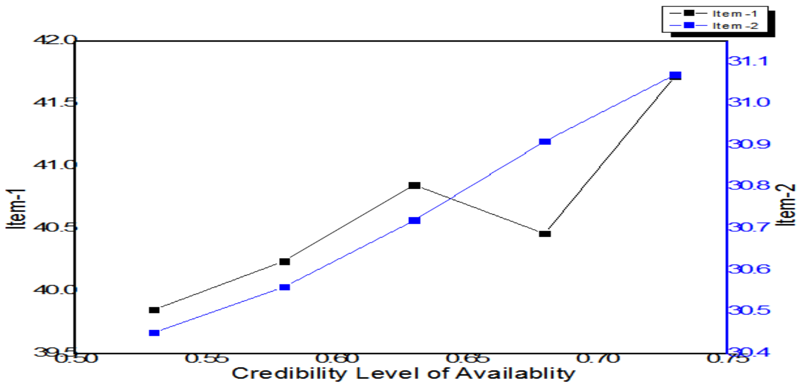

Figure 16. *Change of transported amounts (item-1 and 2) with Credibility level of demand,* $\alpha_{avail.} \in (0.50, 0.75]$

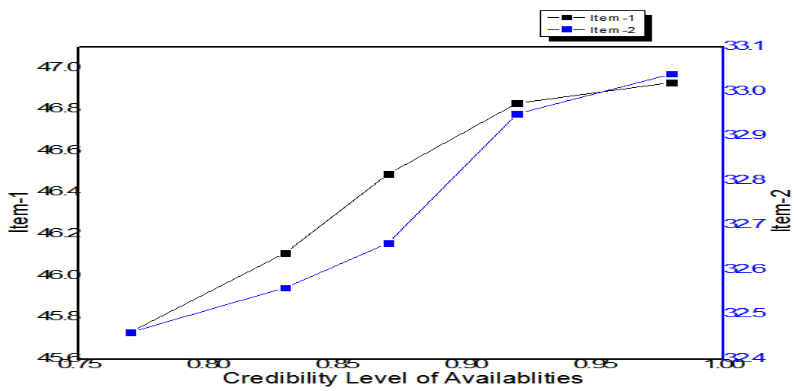

Figure 17. *Change of transported amounts (item-1 and 2) with Credibility level of demand,* $\alpha_{avail.} \in (0.75, 1]$

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