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Likelihood and Bayesian estimations for step-stress life test model under Type-I censoring

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Abstract

This paper discusses likelihood and Bayesian estimations for partially accelerated step-stress life test model under Type-I censoring assuming Pareto distribution of the second kind. The posterior means and posterior variances are obtained under the squared error loss function using Lindley's approximation procedure. It has been observed that Lindley's method usually provides posterior variances and mean square errors smaller than those of the maximum likelihood estimators. Furthermore, the highest posterior density credible intervals of the model parameters based on Gibbs sampling technique are computed. For illustration, simulation studies and an illustrative example based on a real data set are provided.

Keywords: Reliability, partially accelerated step-stress life test, Bayesian estimation, Gibbs sampling.

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1. Introduction

For most practical tests, it may be not easy to gather data on failure-time of a device under use conditions when this device is a highly reliable. Consequently, such devices should be tested under accelerated (i.e. harsher-than-use) conditions to obtain failures quickly. According to Pathak et al. [33], "the model of acceleration is chosen so that the relationship between the parameters of the failure distribution and the accelerated stress conditions is known. Such relationship is used to extrapolate the accelerated data to the design stress to estimate the life distribution. The tests performed under

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accelerated stress conditions are called fully accelerated life tests (FALT or simply ALT)". Involved persons may refer to "Meeker and Escobar [30] and Nelson [31], which are two comprehensible sources for ALT".

Sometimes, such relationship (the life-stress relationship) may not be known or can't be assumed. So, in this case, ALT can't be applied to predict products' reliability because the cumulative exposure model in this case can't be assumed. Instead, as proposed by DeGroot and Goel [13], "another type of tests called partially accelerated life tests (PALT) is used according to a tampered random variable model".

As Nelson [31] shows, "the stress can be applied in various ways, commonly used method is step-stress. Under step-stress PALT, a test item is first run at use condition and, if it does not fail for a specified time, then it is run at accelerated condition until failure occurs or the test is terminated. Accelerated test stresses involve higher than usual temperature, voltage, pressure, load, humidity, ..., etc., or some combination of them".

Most of literature performed on PALT discussed non-Bayesianl approaches to make some statistical inferences, for example, see Goel [15], Bhattacharyya and Soejoeti [10], Bai and Chung [6], Bai et al. [7], Attia et al. [5], Abdel-Ghaly et al. [2], Madi [28], Abdel-Ghani [3], Aly and Ismail [4], Ismail and Sarhan [24], Ismail [22], Ismail and Abu-Youssef [23], Ismail [20-21] and Abd-Elfattah et al. [1].

Few of Bayesian researches had been made on PALT. Goel [15] "used the Bayesian approach for estimating the acceleration factor and the parameters in the case of step-stress PALT (SSPALT) with complete sampling for items having exponential and uniform distributions". DeGroot and Goel [13] "investigated the optimal Bayesian design of a PALT in the case of the exponential distribution under complete sampling". Abdel-Ghani [3] "considered the Bayesian approach to estimate the parameters of Weibull distribution in SSPALT with censoring". Ismail [19] "considered the Bayesian approach to estimate the parameters of Gompertz distribution with time-censoring".

In this paper, our objective is to apply a Bayesian analysis of SSPALT considering two-parameter Pareto distribution with Type-I censoring assuming the squared error (SE) loss function. The Bayes estimators (BEs) of the acceleration factor and the distribution parameters are derived and compared with the maximum likelihood estimators (MLEs) counterparts by Monte Carlo simulations.

The rest of this paper is organized as follows. In Section 2, the model and test method are described. Approximate BEs of the parameters under consideration are derived in Section 3. In Section 4, BEs derived in Section 3 are obtained numerically using Lindley's approximation and compared with the MLEs. Also, the highest posterior density credible intervals of the model parameters based on Gibbs sampling technique are presented in Section 3. In Section 4 Monte Carlo simulation study is made for investigating and comparing the methods of ML and Bayes estimators. Section 5 considers an illustrative example with real data set. Finally, a conclusion is presented in Section 6.

2. The model and test method

2.1. The Pareto distribution as a lifetime model. The lifetimes of the test items are assumed to follow two-parameter Pareto distribution of the second kind. Pareto [32]" introduced a distribution (Pareto distribution) as a model for the distribution of income". Many authors, for example, Davis and Feldstein [12], Cohen and Whitten [11], Grimshaw [17] among others "studied its models in several different forms". According to Johnson et al. [25], "Pareto distribution of the second kind also know as Lomax or Pearson's Type VI distribution". Bain and Engelhardt [8] said that "it has been found as a good model in biomedical problems, such as survival time following a heart transplant". Using the

Pareto distribution, Dyer [14] "studied annual wage data of production line workers in a large industrial firm". Lomax [27] "used this distribution in the analysis of business failure data". In addition, Bain and Engelhardt [8] indicated that "the length of wire between flaws also follows a Pareto distribution". Moreover, Howlader and Hossain [18] showed that "since Pareto distribution has a decreasing hazard or failure rate, it has often been used to model incomes and survival times".

The used PDF is expressed by

$$f(t;\theta,\alpha)=\frac{\alpha\theta^{\alpha}}{(\theta+t)^{\alpha+1}},t\geq 0,\theta>0,\alpha>0, \tag{2.1}$$

Its reliability function is given by

$$R(t)=\frac{\theta^{\alpha}}{(\theta+t)^{\alpha}}, \tag{2.2}$$

and its failure-rate function is

$$h(t)=\frac{\alpha}{\theta+t}. \tag{2.3}$$

McCune and McCune [29] indicated that "Pareto distribution has classically been used in economic studies of income, size of cities and firms, service time in queuing systems and so on". Also, according to Davis and Feldstein [12], "it has been used in connection with reliability theory and survival analysis".

2.2. The Test Method. Fundamental Assumptions

(1) Two levels of stress x_1 and x_2 (normal and severe) are applied.
(2) The distribution is Pareto for each stress level.
(3) The total lifetime Y of an item is given by

$$Y=\begin{cases} T, & \text{if } T\leq t\\ \tau+\beta^{-1}(T-\tau), & \text{if } T>\tau, \end{cases} \tag{2.4}$$

where T is the lifetime of an item under normal condition. According to the litrature, "DeGroot Goel [13] proposed this model which is called a tampered random variable (TRV) model". For the tampered random variable models, the readers may also refer to Tang et al. [35].
(4) The failure times y_i; $i=1,\ldots,n$ are i.i.d. r.v.'s.

Test Process

(1) Each of the n test items is first operate under design stress.
(2) If it does not fail by a pre-specified time τ then it is put on severe condition and run until it fails or the experiment is ended.

The PDF of total lifetime Y of an item under SSPALT is expressed by

$$Y=\begin{cases} 0, & \text{if } y\leq 0\\ f_1(y)\equiv f_T(t;\theta,\alpha)=\frac{\alpha\theta^{\alpha}}{(\theta+y)^{\alpha+1}}, & \text{if } 0<y\leq\tau\\ f_2(y)=\frac{\beta\alpha\theta^{\alpha}}{(\theta+\tau+\beta(y-\tau))^{\alpha+1}}, & \text{if } y>\tau, \end{cases} \tag{2.5}$$

where $\theta>0$ and $\alpha>0$.

3. Bayesian estimation

3.1. Posterior means and posterior variances. In this section, the SE loss function is used. Under SE loss function, the Bayes estimator of a parameter is its posterior expectation. The Bayes estimators can't be given in explicit forms. Approximate Bayes estimators will be discussed under the assumption of non-informative priors using Lindley's approximation. Basu et al. [9] showed that " in many practical situations, the information about the parameters are available in an independent manner". Thus, here it is assumed that the parameters are independent a priori and let the non-informative prior (NIP) for each parameter be represented by the limiting form of the appropriate natural conjugate prior.

Therefore, the joint NIP of the three parameters can be expressed by

$\pi(\beta,\theta,\alpha)(\beta\theta\alpha)^{-1}$, $\beta > 1$, $\theta > 0$, $\alpha > 0$. (3.1)

The observed values of the total lifetime Y are given by

$$y_{(1)} \leq \ \dots \ \leq \ y_{(nu)} \leq \tau \leq \ y_{(nu+1)} \leq \ \dots \ \leq \ y_{(nu+na)} \leq \eta$$

where n_u is the number of items failed at use condition and n_a is the number of items failed at accelerated condition.

Since the total lifetimes $y_1, \dots, y_n$ of n items are independent and identically distributed random variables, then the total likelihood function for them is given by

$$L(\underline{y}\,|\beta,\theta,\alpha) = \prod_{i=1}^{n_u} [\tfrac{\alpha\theta^{\alpha}}{(\theta+y_{\{(i)\}})^{\alpha+1}}].\prod_{i=1}^{n_a} [\tfrac{\beta\alpha\theta^{\alpha}}{(\theta+\tau+\beta(y_{\{(i+n_u)\}}-\tau))^{\alpha+1}}].\prod_{i=1}^{n_c} [\tfrac{\theta^{\alpha}}{(\theta+\tau+\beta(\eta-\tau))^{\alpha}}], \quad (3.2)$$

where $n_c = n$ - n_u - n_a.

Forming the product of (3.1) and (3.2), the joint posterior density function of β , θ and α given the data can be written as

$$g(\beta,\theta,\alpha\,|\underline{y}) \propto L(\underline{y}\,|\beta,\theta,\alpha).\Pi(\beta,\theta,\alpha)$$
$$\propto \tfrac{\beta^{n_a-1}\theta^{\alpha n-1}\alpha^{n_u+n_a-1}}{(\theta+\tau+\beta(\eta-\tau))^{\alpha n_c}}[\prod_{i=1}^{n_u} \tfrac{1}{(\theta+y_{\{(i)\}})^{\alpha+1}}].[\prod_{i=n_u+1}^{n_u+n_a} \tfrac{1}{(\theta+\tau+\beta(y_{\{(i)\}}-\tau))^{\alpha+1}}]. \quad (3.3)$$

According to Lindley [26], "an approximation via an asymptotic expansion of the ratio of two non-tractable integrals is used to obtain approximate Bayes estimators".

Now, let Θ be a set of parameters $\{\Theta_1, \Theta_2, \dots, \Theta_m\}$, where m is the number of parameters, then the posterior expectation of an arbitrary function $u(\Theta)$ can be asymptotically estimated by

$$\mathrm{E}(u(\Theta)) = \frac{\int_{\Theta} u(\Theta)\pi(\Theta)e^{ln\mathrm{L}(\mathrm{y}|\Theta)}d\Theta}{\int_{\Theta} \pi(\Theta)e^{ln\mathrm{L}(\mathrm{y}|\Theta)}d\Theta}$$
$$\approx [u + (1/2)\sum_{i,j} (u_{ij}^{(2)} + 2u_i^{(1)}\rho_j^{(1)})\sigma_{\mathrm{ij}} + (1/2)\sum_{i,j,k,s} L_{ijk}^{(3)}\sigma_{ij}\sigma_{ks}u_s^{(1)}] \downarrow \hat{\Theta}, \quad (3.4)$$

which is the Bayes estimator of $u(\Theta)$ under a squared error loss function, where $\pi(\Theta)$ is the prior distribution of Θ

,$u \equiv u(\Theta)$, $L \equiv L(\Theta)$ is the likelihood function, $\rho \equiv \rho(\Theta) = log\ \pi(\Theta)$, σ_{ij} are the elements of the inverse of the asymptotic Fisher's information matrix of β , θ and α , and

$$\mathrm{u}_i^{(1)} = \frac{\partial u}{\partial \Theta_i}, u_{ij}^{(2)} = \frac{\partial^2 u}{\partial \Theta_i \partial \Theta_j}, \rho_j^{(1)} = \frac{\partial log \pi(\Theta)}{\partial \Theta_j} \text{and} L_{ijk}^{(3)} = \frac{\partial^3 lnL(y|\Theta)}{\partial \Theta_i \partial \Theta_j \partial \Theta_k}$$

According to Green [16], "the linear Bayes estimator in (3.4) is a very good and operational approximation for the ratio of multi-dimension integrals". Also, as pointed out by Sinha [34], "it has led to many useful applications".

Bayesian interval estimators, called credible intervals, for the model parameters are derived from their posterior distributions. We propose the following Markov Chain Monte Carlo (MCMC) method to draw samples from the posterior density function and then to compute the Bayes estimates and the highest posterior density (HPD) credible intervals. We use the Gibbs sampling procedure to compute HPD credible intervals.

3.2. Credible intervals using Gibbs sampling. Assume that the priori are Gamma distributions and that they are independent. Therefore, samples of β , θ and α can be easily generated using any of the gamma generating routines. We use the Gibbs sampling procedure to generate a sample from the posterior density function and then to compute the Bayes estimates and HPD credible intervals. To run the Gibbs sampler algorithm, it is appropriate to start with the approximate BEs. The following algorithm is used for this purpose.

Step 1: Start with an($\theta^{(0)} = \tilde{\theta}$ and $\beta^{(0)} = \tilde{\beta}$) and set I = 1.

Step 2: Generate $\alpha^{(\mathrm{I})}$ from the conditional Gamma distribution $(g(\alpha \left| \theta^{(\mathrm{I}-1)}, \beta^{(\mathrm{I}-1)}, \underline{y}\right.))$

Step 3: Generate $\theta^{(\mathrm{I})}$ from the conditional Gamma distribution $(g(\theta \left| \alpha^{(\mathrm{I}-1)}, \beta^{(\mathrm{I}-1)}, \underline{y}\right.))$

Step 4: Generate $\beta^{(\mathrm{I})}$ from the conditional Gamma distribution $(g(\beta \left| \theta^{(\mathrm{I}-1)}, \alpha^{(\mathrm{I}-1)}, \underline{y}\right.))$

Step 5: Set I = I +1.

Step 6: Repeat steps 2-4 M times and obtain α_i, θ_i and β_i for i=1,..., M.

Step 7: The Bayes MCMC point estimates of β , θ and α with respect to the squared error function are then

$\tilde{\beta} = \tilde{E}(\beta\, |data\,) = \frac{1}{M}\sum\limits_{k=1}^{M} \beta_k$, $\tilde{\theta} = \tilde{E}(\theta\, |data\,) = \frac{1}{M}\sum\limits_{k=1}^{M} \theta_k$ and $\tilde{\alpha} = \tilde{E}(\alpha\, |data\,) = \frac{1}{M}\sum\limits_{k=1}^{M} \alpha_k$.

Step 8: The posterior variances of β , θ and α are

$\tilde{V}(\beta\, |data\,) = \frac{1}{M}\sum\limits_{k=1}^{M} \left\{\beta_k - \tilde{E}(\beta\, |data\,)\right\}^2$, $\tilde{V}(\theta\, |data\,) = \frac{1}{M}\sum\limits_{k=1}^{M} \left\{\theta_k - \tilde{E}(\theta\, |data\,)\right\}^2$ and $\tilde{V}(\alpha\, |data\,) = \frac{1}{M}\sum\limits_{k=1}^{M} \left\{\alpha_k - \tilde{E}(\alpha\, |data\,)\right\}^2$.

Step 9: To compute the credible intervals (CRIs) of $\phi_l\, (\phi_1 = \alpha, \phi_2 = \theta \text{and}\, \phi_3 = \beta)$, the quantiles of the sample is usually taken as the endpoints of the intervals. Order $\phi_l^{(1)}, \phi_l^{(2)}, ..., \phi_l^{(M)}, as\, \phi_{l(1)}, \phi_{l(2)}, ..., \phi_{l(M)}$.

Then, the $100(1-2\gamma)\%$ CRIs for ϕ_l become $(\phi_{l(\gamma M)}, \phi_{l((1-\gamma)M)})$.

4. Simulation results and discussion

Simulation results are made for comparing the methods of ML and Bayes estimators, using a SE loss function. The posterior means and posterior variances of the model parameters are obtained suggesting a NIP for each parameter under a SE loss function with time-censored data. Since the BEs of the model parameters can't be found in closed form, approximate BEs are determined numerically using Lindley techniuqe. The performance of the approximate BEs is assessed and compared with the MLEs in Tables 1 and 2 via their variances, MSEs and average confidence interval lengths (CIL) for different settings of true parameter values and sample sizes.

95% confidence intervals of the model parameters are constructed with average CIL presented in Tables 1 and 2. It is shown from the results presented in Tables 1 and 2 that the CRIs obtained under Bayes method (via Gibbs sampling approach) are narrower than those obtained using the ML approach. Also, we observed that the computed coverage probabilities (CP) of the CRIs for each parameter are very close to the nominal level. On the other hand, it was found that these CP using the ML approach are lower than the nominal level in general.

Table 1: Average values of MLEs, BEs, variances and MSEs, when β =2, θ = 0.2, α = 0.5, τ = 3 and η = 7

n	*parameter*	*Method*	*estimate*	*MSE*	*variance*	*CIL*	*CP*
30	β	*ML*	2.4633	0.1569	0.0843	1.1382	92
		Bayes	2.2863	0.1328	0.0669	0.9824	94.1
	θ	*ML*	0.5154	0.0264	0.0376	0.7601	92.3
		Bayes	0.4289	0.0185	0.0115	0.3214	94.3
	α	*ML*	0.7232	0.0172	0.0169	0.5096	93
		Bayes	0.6821	0.0093	0.0081	0.2972	94.4
50	β	*ML*	2.3954	0.1143	0.0548	0.9176	93
		Bayes	2.1627	0.0881	0.0333	0.6302	94.3
	θ	*ML*	0.3653	0.0784	0.0218	0.5788	94
		Bayes	0.3102	0.0113	0.0071	0.2733	94.4
	α	*ML*	0.5563	0.0132	0.0074	0.3372	94.5
		Bayes	0.5382	0.0047	0.0034	0.1765	94.8
75	β	*ML*	2.2233	0.0911	0.0273	0.6477	94.1
		Bayes	2.0793	0.0578	0.0099	0.3140	94.7
	θ	*ML*	0.2977	0.0546	0.0079	0.3484	94.3
		Bayes	0.2286	0.0078	0.0052	0.2261	94.8
	α	*ML*	0.4796	0.0082	0.0025	0.1960	94.6
		Bayes	0.4836	0.0023	0.0015	0.1103	95.1
100	β	*ML*	2.1143	0.0366	0.0057	0.2960	94.3
		Bayes	2.0371	0.0206	0.0046	0.1874	94.9
	θ	*ML*	0.2384	0.0281	0.0047	0.2687	94.4
		Bayes	0.2178	0.0037	0.0016	0.1210	94.9
	α	*ML*	0.4885	0.0026	0.0011	0.1300	94.8
		Bayes	0.4913	0.0014	0.0006	0.0411	95.0

Table 2: Average values of MLEs, BEs, variances and MSEs, when β =3 , θ = 1.5, α = 2, τ = 3 and η = 7

n	*parameter*	*Method*	*estimate*	*MSE*	*variance*	*CIL*	*CP*
30	β	*ML*	3.4943	0.0951	0.0639	0.9909	92.5
		Bayes	3.3511	0.0722	0.0403	0.6820	94.4
	θ	*ML*	1.9411	0.0689	0.0289	0.6664	93
		Bayes	1.7101	0.0522	0.0233	0.5147	94.5
	α	*ML*	2.2677	0.0645	0.0119	0.4276	93.5
		Bayes	2.2153	0.0529	0.0075	0.2655	94.7
50	β	*ML*	3.4461	0.0552	0.0519	0.8930	93.2
		Bayes	3.3289	0.0373	0.0329	0.5918	94.8
	θ	*ML*	1.6533	0.0487	0.0112	0.4149	93.6
		Bayes	1.5790	0.0307	0.0074	0.2413	94.8
	α	*ML*	2.1791	0.0213	0.0043	0.2571	94
		Bayes	2.1342	0.0128	0.0031	0.1643	94.8
75	β	*ML*	3.1731	0.0375	0.0297	0.6756	94.2
		Bayes	3.0944	0.0187	0.0163	0.4101	94.9
	θ	*ML*	1.5832	0.0215	0.0038	0.2416	94.4
		Bayes	1.5346	0.0117	0.0025	0.1386	94.9
	α	*ML*	2.0773	0.0085	0.0020	0.1753	94.5
		Bayes	2.0522	0.0052	0.0016	0.0982	94.9
100	β	*ML*	3.0891	0.0156	0.0112	0.4149	94.6
		Bayes	3.0343	0.0092	0.0074	0.2283	94.9
	θ	*ML*	1.5421	0.0082	0.0022	0.1839	94.7
		Bayes	1.5117	0.0064	0.0014	0.0922	94.9
	α	*ML*	2.0463	0.0036	0.0007	0.1037	94.6
		Bayes	2.0144	0.0028	0.0004	0.0415	95.0

5. Data analysis: A numerical example

To demonstrate the applicability of the methodology introduced in this paper, a numerical example is provided. Pareto model is used to fit the data set. To verify the power of the model, we calculate the Kolmogorov-Smirnov (K-S) distance between the empirical distribution function and the fitted distribution function when the parameters estimates are determined by the maximum likelihood method. The result of K-S test is D=0.0764 with p-value = 0.542. This result obviously shows that the Pareto model provides excellent fit to the data set. So, it can be served successfully for modeling this data set. Assuming Pareto distribution with time-censoring we use n = 76, β =2, θ = 2.5, α = 1.5, τ = 3 and η = 7. The number of failures gained at use and accelerated conditions are n_u=13 and n_a=46, respectively, with censored items n_c=17. The MLEs of the model parameters β,θ and α are respectively 2.09, 2.57 and 1.54, while the BEs are 2.04, 2.53 and 1.52. The MSEs associated with the MLEs of β,θ and α are 0.0241, 0.0207 and 0.0071, respectively, while those associated with the BEs are respectively 0.0156, 0.0111 and 0.0043. In addition, the 95% confidence intervals of β,θ and α using the approaches ML and MCMC are (1.7650, 2.4150), (2.4402, 2.6198), (1.4570, 1.6232) and (1.7976, 2.2824), (2.5040, 2.5760), (1.4467, 1.5933), respectively.

6. Conclusion

In this paper the ML and Bayes estimations of the SSPALT model parameters have been considered. Bayes estimations have been found assuming squared error loss functions and non-informative priors. Lindley approach has been applied to find BEs. It has been seen that the approach acts very well even for small sample sizes. The approach usually provides smaller posterior variances. That is, it gives improved estimates. In the

MCMC approach, it has been noted that the CRIs are shorter than the ML intervals and always include the population parameter values.

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