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An improved class of estimators for finite population variance

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Abstract

We propose an improved class of estimators in estimating the finite population variance, using the auxiliary information. The expressions for the bias and mean squared error of the proposed class of estimators are derived up to the first order of approximation. Some estimators are also derived from a proposed class by allocating the suitable values of known parameters and identified as particular members of the proposed class of estimators. A numerical study is carried out to demonstrate performances of the estimators. It is observed that the proposed class of estimators is more efficient than the usual sample mean estimator, the regression estimator suggested by Isaki (1983), Shabbir and Gupta (2007), Singh and Solanki (2013*b*), Yadav et al. (2013), Yadav and Kadilar (2014) and Singh and Malik (2014) estimators.

Keywords: Auxiliary variable; bias; mean squared error; population variance; percentage relative efficiency.

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1. Introduction

In this article, an improved class of estimators is proposed in estimating the finite population variance under simple random sampling. Various fields of life like genetics, biology and medical studies have been facing the problem in estimating the finite population variance. An agriculturist requires sufficient knowledge of climatic variation to devise appropriate plan for cultivating his crop. A fair understanding of variability is vitally important for better results in different walks of life. Singh et al. [36] and Das and Tripathi [9] have proposed different estimators for finite population variance $\left(S_y^2\right)$.

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Chaudhury [8] and Mukhopadhyay [27, 28] have made significant contributions in estimating the finite population variance under super population models. Srivastava and Jhajj [47] and Wu [51], have taken the advantage of correlation between the study and the auxiliary variables in estimating the finite population variance. Isaki [17] has proposed ratio and regression type estimators for population variance. Likewise, Singh [37], Searls and Intrapanich [31], and Prasad and Singh [29, 30] have given some estimators for population variance. Singh and Biradar [39], Garcia and Cebrain [10], Cebrain and Garcia [5], Singh and Joarder [40], Upadhyaya and Singh [49], and Ahmed et al. [2] have paid their attention towards the improved estimation or classes of estimators of S_y^2. AL-Jaraha and Ahmed [3] have given some chain ratio-type as well as chain product-type estimators of S_y^2 using double-sampling scheme. Later on Singh and Singh [41], Upadhyaya et al. [50], Chandra and Singh [7], Arcos et al. [1], Kadilar and Cingi [18, 19, 20, 21, 22, 23], Koyuncu and Kadilar [24, 25], Singh and Vishwakarma [43], Turgut and Cingi [48], Gupta and Shabbir [12, 13, 14], Shabbir and Gupta [33, 34], Grover [11], Singh et al. [38, 42, 44], Yadav et al. [52, 53], Yadav and Kadilar [54], Singh and Solanki [45, 46], and Singh and Malik [35] have paid their attention towards the improved estimation of population variance S_y^2.

Motivated by these studies, the present article focuses on improved class of estimators for S_y^2 using the auxiliary information.

The rest of the article is organized as: Section 2 provides the notations and symbols. Section 3 gives a brief review of some existing estimators of S_y^2. Section 4 gives the expressions for the bias and mean squared error (*MSE*) of the proposed class of estimators. The efficiency comparison of different estimators is shown in Section 5. A numerical study is presented in Section 6. Conclusion is given in Section 7.

2. Notations

Consider a finite population $\Omega = \{1, 2, .., i, .., N\}$ having N units. We draw a sample of size n by using simple random sample without replacement (SRSWOR) sampling scheme from this population. Let y_i and x_i be the values of the study variable (y) and the auxiliary variable (x) respectively. Let $\bar{y} = \frac{1}{n}\sum_{i=1}^{n} y_i$ and $\bar{x} = \frac{1}{n}\sum_{i=1}^{n} x_i$ be the sample means, respectively, corresponding to the population means $\bar{Y} = \frac{1}{N}\sum_{i=1}^{N} y_i$ and $\bar{X} = \frac{1}{N}\sum_{i=1}^{N} x_i$. Let $s_y^2 = \frac{1}{n-1}\sum_{i=1}^{n}(y_i - \bar{y})^2$ and $s_x^2 = \frac{1}{n-1}\sum_{i=1}^{n}(x_i - \bar{x})^2$ be the sample variances corresponding to population variances $S_y^2 = \frac{1}{N-1}\sum_{i=1}^{N}\left(y_i - \bar{Y}\right)^2$ and $S_x^2 = \frac{1}{N-1}\sum_{i=1}^{N}\left(x_i - \bar{X}\right)^2$, respectively. Let $\theta_{22} = \frac{\mu_{11}}{\sqrt{\mu_{20}}\sqrt{\mu_{02}}}$, be the covariance between S_y^2 and S_x^2. Let $\beta_{2(y)} = \frac{\mu_{40}}{\mu_{20}^2}$ and $\beta_{2(x)} = \frac{\mu_{04}}{\mu_{02}^2}$ be the population coefficients of kurtosis of y and x, respectively, where $\mu_{rs} = \frac{1}{N-1}\sum_{i=1}^{N}(y_i - \bar{Y})^r\left(x_i - \bar{X}\right)^s$ and $\gamma = 1/n$. We ignored the finite population correction(fpc) term because of ease of computation. In order to get biases and *MSEs* of the considered estimators, we use the following relative error terms.

Let $\delta_0 = \frac{s_y^2 - S_y^2}{S_y^2}$ and $\delta_1 = \frac{s_x^2 - S_x^2}{S_x^2}$ such that $E(\delta_i) = 0$ for $i = 0, 1$.

Let $E\left(\delta_0^2\right) = \gamma\left(\beta_{2(y)} - 1\right) \cong V_{20}$, $E\left(\delta_1^2\right) = \gamma\left(\beta_{2(x)} - 1\right) \cong V_{02}$, and $E(\delta_0\delta_1) = \gamma(\theta_{22} - 1) \cong V_{11}$, where $V_{rs} = E\left\{\frac{(s_y^2 - S_y^2)^r(s_x^2 - S_x^2)^s}{(S_y^2)^r(S_x^2)^s}\right\}$.

3. Some existing estimators

We discuss the following estimators.

(i) The variance of the usual unbiased variance estimator $\left(\hat{S}_y^2\right)$, is given by

(3.1) $$Var\left(\hat{S}_y^2\right) = S_y^4 V_{20}.$$

(*ii*) Isaki [17] suggested the following regression estimator for S_y^2, given by

(3.2) $$\hat{S}^2_{Reg} = s_y^2 + b_{\left(s_y^2, s_x^2\right)}\left(S_x^2 - s_x^2\right),$$

where $b_{\left(s_y^2, s_x^2\right)}$ is the sample regression coefficient whose population regression coefficient is $\beta = \left(S_y^2 V_{11} / S_x^2 V_{02}\right)$. The variance of $\hat{S}^2_{Reg}$, is given by

(3.3) $$Var(\hat{S}^2_{Reg}) \cong S_y^4 V_{20}\left(1 - \rho^2_{\left(s_y^2, s_x^2\right)}\right) = MSE(\hat{S}^2_{Reg}),$$

where $\rho_{\left(s_y^2, s_x^2\right)} = \left(V_{11}/\sqrt{V_{20}V_{02}}\right)$.

(*iii*) Singh et al. [38] considered the following difference type estimator for S_y^2, given by

(3.4) $$\hat{S}_d^2 = k_1 s_y^2 + k_2\left(S_x^2 - s_x^2\right),$$

where k_1 and k_2 are suitably chosen constants.
The bias and minimum *MSE* of $\hat{S}_d^2$, to first order of approximation, at optimum values $k_1^{(opt)} = \frac{V_{02}}{\left(V_{02} + V_{02}V_{20} - V_{11}^2\right)}$ and $k_2^{(opt)} = \frac{S_x^2}{S_y^2}\frac{V_{11}}{\left(V_{02} + V_{02}V_{20} - V_{11}^2\right)}$,
are given by

$$Bias(\hat{S}_d^2) \cong (k_1 - 1)\, S_y^2,$$

and

(3.5) $$MSE_{\min}(\hat{S}_d^2) \cong \frac{Var(\hat{S}^2_{Reg})}{1 + S_y^{-4} Var(\hat{S}^2_{Reg})}.$$

(*iv*) Shabbir and Gupta [32] suggested the following ratio-type exponential estimator S_y^2, given by

(3.6) $$\hat{S}^2_{SG} = \left\{k_3 s_y^2 + k_4\left(S_x^2 - s_x^2\right)\right\}\exp\left(\frac{S_x^2 - s_x^2}{S_x^2 + s_x^2}\right),$$

where k_3 and k_4 are suitably chosen constants.
The bias of $\hat{S}^2_{SG}$, to first order of approximation, is given by

$$Bias\left(\hat{S}^2_{SG}\right) \cong S_y^2(k_3 - 1) + \frac{3}{8}k_3 S_y^2 V_{02} - \frac{1}{2}k_3 S_y^2 V_{11} + \frac{1}{2}k_4 S_x^2 V_{02}.$$

The minimum *MSE* of $\hat{S}^2_{SG}$, to first order of approximation, at optimum values of $k_3^{(opt)} = \frac{V_{02}}{8}\left(\frac{8 - V_{02}}{V_{02} + V_{20}V_{02} - V_{11}^2}\right)$ and $k_4^{(opt)} = \frac{S_y^2}{8S_x^2}\left(\frac{-4V_{02} + V_{02}^2 + 8V_{11}^2 - V_{11}V_{02} + 4V_{20}V_{02} - 4V_{11}^2}{V_{02} + V_{20}V_{02} - V_{11}^2}\right)$,
is given by

(3.7) $$MSE_{\min}\left(\hat{S}^2_{SG}\right) \cong \frac{S_y^4}{64}\left\{\frac{V_{02}^2 + 16\left(V_{02} - 4\right) S_y^{-4} Var(\hat{S}^2_{Reg})}{-1 - S_y^{-4} Var(\hat{S}^2_{Reg})}\right\}.$$

(*v*) Singh and Solanki [46] suggested a difference-in-ratio type estimator for S_y^2, given by

(3.8) $$\hat{S}^2_{SS} = \left\{k_5 s_y^2 + k_6(S_x^2 - s_x^2)\right\}\left(\frac{aS_x^2 + b}{as_x^2 + b}\right),$$

where k_5 and k_6 are suitably chosen constants and $a\,(\neq 0)$ and b are functions of known parameters of the auxiliary variable x.
The bias of $\hat{S}^2_{SS}$, to first order of approximation, is given by

$$Bias\left(\hat{S}^2_{SS}\right) \cong S_y^2(k_5 - 1) + k_5 S_y^2 \tau^2 V_{02} - k_5 S_y^2 \tau V_{11} + k_6 S_x^2 \tau V_{02},$$

where $\tau = aS_x^2/\left(aS_x^2+b\right)$.
The minimum MSE of $\hat{S}_{SS}^2$, to first order of approximation, at optimum values $k_5^{(opt)} = \frac{V_{02}(-1+\tau^2V_{02})}{(-V_{02}-V_{02}V_{20}+\tau^2V_{02}^2+V_{11}^2)}$ and
4 $k_6^{(opt)} = -\frac{S_y^2}{S_x^2}\left(\frac{-V_{02}\tau+V_{02}^2\tau^3+V_{11}-V_{11}V_{02}\tau^2+\tau V_{02}V_{20}-\tau V_{11}^2}{-V_{02}-V_{02}V_{20}+\tau^2V_{02}^2+V_{11}^2}\right)$,
is given by

$$MSE_{\min}\left(\hat{S}_{SS}^2\right) \cong S_y^4\left\{\frac{AS_y^{-4}Var(\hat{S}_{Reg}^2)}{A+S_y^{-4}Var(\hat{S}_{Reg}^2)}\right\}, \tag{3.9}$$

where $A = 1 - V_{02}\tau^2$.

(vi) Yadav et al. [53] suggested a general class of estimators for S_y^2, given by

$$\hat{S}_{YG}^2 = \left\{k_7 s_y^2 + k_8(S_x^2 - s_x^2)\right\}\left\{\lambda\left(\frac{aS_x^2+b}{as_x^2+b}\right) + (1-\lambda)\exp\left(\frac{a\left(S_x^2-s_x^2\right)}{a\left(S_x^2+s_x^2\right)+2b}\right)\right\}, \tag{3.10}$$

where k_7 and k_8 are suitably chosen constants, λ can takes values 0 or 1 and a, b be the population parameters of the auxiliary variables. Shabbir and Gupta [32] estimator in (3.6) and Singh and Solanki [46] estimator in (3.8) can be generated from (3.10) by substituting the suitable choices of λ, a and b. The bias of $\hat{S}_{YG}^2$, to first order of approximation, is given by

$$Bias\left(\hat{S}_{YG}^2\right) \cong S_y^2\left[(k_7-1) + \left(\frac{3+5\lambda}{8}\right)k_7V_{02}\tau^2 + \frac{(1+\lambda)\tau}{2}\left\{\left(\frac{S_x^2}{S_y^2}\right)k_8V_{02} - k_7V_{11}\right\}\right].$$

The minimum MSE of $\hat{S}_{YG}^2$, to first order of approximation, at optimum values

$$k_7^{(opt)} = \frac{V_{02}}{2}\left\{\frac{8 - V_{02}\tau^2\left(1+3\lambda+4\lambda^2\right)}{4V_{02} - V_{02}^2\lambda\tau^2\left(1+3\lambda\right) + 4V_{02}V_{20} - 4V_{11}^2}\right\},$$

and

$$k_8^{(opt)} = \frac{S_y^2}{2S_x^2}\left[\frac{\left\{\begin{array}{c} 8V_{11} + 3V_{02}^2\lambda\tau^2\left(1+\lambda\right) - 4V_{02}\tau\left(1+\lambda\right) \\ +V_{02}^2\tau^3\left(1+\lambda^3\right) - V_{02}V_{11}\tau^2\left(1+3\lambda+4\lambda^2\right) \\ +4V_{02}V_{20}\tau\left(1+\lambda\right) - 4V_{11}^2\left(1+\lambda\right)\end{array}\right\}}{4V_{02}\left(1-V_{02}\tau^2\lambda^2\right) - V_{02}^2\tau^2\lambda\left(1+\lambda\right) + 4V_{02}V_{20} - 4V_{11}^2}\right],$$

is given by

$$MSE_{\min}\left(\hat{S}_{YG}^2\right) \cong \frac{S_y^4}{16}\left[\frac{\begin{array}{c}\left\{(1+\lambda)^2 - 4\lambda^2\left(1+\lambda-\lambda^2\right)\right\}V_{02}^2\tau^4 \\ +16S_y^{-4}Var(\hat{S}_{Reg}^2)\left\{(1+\lambda)^2V_{02}\tau^2 - 4\right\}\end{array}}{-4+3\lambda^2\tau^2V_{02}+\lambda\tau^2V_{02}-4S_y^{-4}Var(\hat{S}_{Reg}^2)}\right]. \tag{3.11}$$

(vii) Yadav and Kadilar [54] suggested two parameters ratio-product-ratio type estimator for S_y^2, given by

$$\hat{S}_{YK}^2 = s_y^2\left[\alpha_1\left\{\frac{(1-\beta_1)s_x^2+\beta_1S_x^2}{\beta_1s_x^2+(1-\beta_1)S_x^2}\right\} + (1-\alpha_1)\left\{\frac{\beta_1s_x^2+(1-\beta_1)S_x^2}{(1-\beta_1)s_x^2+\beta_1S_x^2}\right\}\right], \tag{3.12}$$

where α_1 and β_1 are suitably chosen constants.
The bias and MSE of $\hat{S}^2_{YK}$, to first order of approximation, are given by

$$Bias\left(\hat{S}^2_{YK}\right) \cong S^2_y \left\{V_{02}\left(1-\alpha_1-3\beta_1+2\alpha_1\beta_1+2\beta_1^2\right)-V_{11}\left(1-2\alpha_1-2\beta_1+4\alpha_1\beta_1\right)\right\}-S^2_y,$$

and

$$MSE\left(\hat{S}^2_{YK}\right) \cong S^4_y \left\{\begin{array}{c}\left(V_{02}+V_{20}-2V_{11}\right)+16\alpha_1\beta_1V_{02}\left(1-\alpha_1-\beta_1+\alpha_1\beta_1\right)\\ +4V_{11}\left(\alpha_1-\beta_1\right)^2+4V_{02}\left(-\alpha_1-\beta_1+\alpha_1^2+\beta_1^2\right)\end{array}\right\}.$$

Solving above for minimum MSE of $\hat{S}^2_{YK}$, to first order of approximation at $(\alpha_1,\beta_1)=(1/2,1/2)$, is

$$(3.13)\qquad MSE_{\min}\left(\hat{S}^2_{YK}\right) \cong Var\left(\hat{S}^2_y\right),$$

and at $(\alpha_1,\beta_1)=\left\{\left(V_{02}-V_{11}\right)/2V_{02},0\right\}$, we have

$$(3.14)\qquad MSE_{\min}\left(\hat{S}^2_{YK}\right) \cong Var(\hat{S}^2_{Reg}).$$

($viii$) Recently Singh and Malik [35] suggested an improved estimator for S^2_y, given by

$$(3.15)\qquad \hat{S}^2_{SM}=s^2_y\left\{k_9+k_{10}(S^2_x-s^2_x)\right\}\exp\left(\psi_1\frac{(aS^2_x+b)-(as^2_x+b)}{(aS^2_x+b)+(as^2_x+b)}\right),$$

where k_9 and k_{10} are suitably chosen constants. Here ψ_1 is the scalar quantity which takes the values $+1$ and -1 for ratio and product type estimators respectively.
The bias of $\hat{S}^2_{SM}$, to first order of approximation, is given by

$$\begin{aligned}Bias\left(\hat{S}^2_{SM}\right) &\cong S^2_y\left(k_9-1\right)+\frac{1}{4}S^2_yk_9\psi_1\tau^2V_{02}+\frac{1}{8}S^2_yk_9\gamma_1^2\tau^2V_{02}\\ &+\frac{1}{2}S^2_yS^2_xk_{10}\psi_1\tau V_{02}-\frac{1}{2}S^2_yS^2_xk_9\psi_1\tau V_{11}-S^2_yS^2_xk_{10}V_{11}.\end{aligned}$$

The minimum MSE of $\hat{S}^2_{SM}$, to first order of approximation, at optimum values

$$k_9^{(opt)}=\frac{1}{4}\left(\frac{-12\psi_1\tau V_{02}V_{11}+3\psi_1^2\tau^2V_{02}^2+16V_{11}^2-8V_{02}-2\psi_1\tau^2V_{02}^2}{\psi_1^2\tau^2V_{02}^2-4\psi_1\tau V_{02}V_{11}+8V_{11}^2-2V_{02}V_{20}-2V_{02}-\psi_1\tau^2V_{02}^2}\right)$$

and $k_{10}^{(opt)}=-\frac{1}{4S^2_x}\left(\frac{-6\psi_1^2\tau^2V_{02}V_{11}+\psi_1^2\tau^3V_{02}^2+8\psi_1\tau V_{11}^2-4\psi_1\tau V_{02}+8V_{11}-8V_{02}V_{11}+4\psi_1\tau V_{02}V_{20}}{\psi_1^2\tau^2V_{02}^2-4\psi_1\tau V_{02}V_{11}+8V_{11}^2-2V_{02}V_{20}-2V_{02}-\psi_1\tau^2V_{02}^2}\right)$, is given by $MSE_{\min}\left(\hat{S}^2_{SM}\right) \cong S^4_y\left\{\frac{}{32\left(\psi_1^2\tau^2V_{02}^2-4\psi_1\tau V_{02}V_{11}+8V_{11}^2-2V_{02}V_{20}-2V_{02}-\psi_1\tau^2V_{02}^2\right)}\right\}$,
or, at $\tau=\psi_1=1$, we have

(3.16)

$$MSE_{\min}\left(\hat{S}^2_{SM}\right) \cong \frac{S^4_y}{64}\left[\frac{V_{02}\left\{V_{02}(V_{02}+8V_{11})+16(V_{02}-4)Var(\hat{S}^2_{Reg})+16V_{11}(V_{11}-V_{02})\right\}}{-V_{02}(1+V_{02}+2V_{11})+4V_{11}^2}\right].$$

4. Proposed estimator

Bahl and Tuteja [4] exponential type estimators for population variance $\left(S^2_y\right)$, are given by

$$(4.1)\qquad \hat{S}^2_R=s^2_y\exp\left(\frac{S^2_x-s^2_x}{S^2_x+s^2_x}\right),$$

and

$$(4.2)\qquad \hat{S}^2_{Pr}=s^2_y\exp\left(\frac{s^2_x-S^2_x}{S^2_x+s^2_x}\right).$$

Following Haq and Shabbir [15, 16], the average of the ratio and the product-type exponential estimators given in (4.1) and (4.2) is

$$\hat{S}_A^2 = s_y^2 \frac{1}{2}\left[\exp\left\{\left(\frac{S_x^2 - s_x^2}{S_x^2 + s_x^2}\right)\right\} + \exp\left\{\left(\frac{s_x^2 - S_x^2}{S_x^2 + s_x^2}\right)\right\}\right].$$

A generalized form of $\hat{S}_A^2$, is given by

$$\hat{S}_A^2 = s_y^2 \frac{1}{2}\left[\exp\left\{\frac{a\left(S_x^2 - s_x^2\right)}{a\left(s_x^2 + S_x^2\right) + 2b}\right\} + \exp\left\{\frac{a\left(s_x^2 - S_x^2\right)}{a\left(s_x^2 + S_x^2\right) + 2b}\right\}\right], \tag{4.3}$$

where $(a \neq 0)$ and b are functions of known parameters of the auxiliary variable. Motivated by Singh and Solanki [46] and Haq and Shabbir [15, 16], replacing s_y^2 given in (4.3) by $\left(\hat{S}_{SS}^2\right)$ given in (3.8), we propose the following class of estimators for estimating the finite population variance S_y^2, given by

$$\hat{S}_P^2 = \left\{k_{11} s_y^2 + k_{12}(S_x^2 - s_x^2)\right\}\left(\frac{aS_x^2 + b}{as_x^2 + b}\right)\hat{S}_{A_1}^2, \tag{4.4}$$

where k_{11} and k_{12} are suitably chosen constants, and
$\hat{S}_{A_1}^2 = \frac{1}{2}\left[\exp\left\{\frac{a\left(S_x^2 - s_x^2\right)}{a\left(s_x^2 + S_x^2\right) + 2b}\right\} + \exp\left\{\frac{a\left(s_x^2 - S_x^2\right)}{a\left(s_x^2 + S_x^2\right) + 2b}\right\}\right]$.
Expressing (4.4) in term of $\delta' s$ and keeping terms up to power two, we have

$$\hat{S}_P^2 = \left\{k_{11} S_y^2 (1 + \delta_0) - k_{12} S_x^2 \delta_1\right\}\left(1 - \tau\delta_1 + \tau^2\delta_1^2\right)\left(1 + \frac{\tau^2\delta_1^2}{8}\right). \tag{4.5}$$

Solving (4.5), up to first order of approximation, we get

$$\begin{aligned}\hat{S}_P^2 - S_y^2 \cong{}& k_{11}S_y^2 - k_{11}S_y^2\tau\delta_1 + k_{11}S_y^2\delta_0 - k_{12}S_x^2\delta_1 + \frac{9}{8}k_{11}S_y^2\tau^2\delta_1^2 \\ & - k_{11}S_y^2\tau\delta_0\delta_1 + k_{12}S_x^2\tau\delta_1^2 - S_y^2.\end{aligned} \tag{4.6}$$

Using (4.6), the bias and MSE of $\hat{S}_P^2$, to first order of approximation are, respectively given by

$$Bias(\hat{S}_P^2) \cong (k_{11} - 1)\,S_y^2 + \frac{9}{8}k_{11}S_y^2\tau^2 V_{02} - k_{11}S_y^2\tau V_{11} + k_{12}S_x^2\tau V_{02}, \tag{4.7}$$

and

$$\begin{aligned}MSE(\hat{S}_P^2) \cong{}& k_{11}^2 S_y^4 - 2k_{11}S_y^4 + k_{11}^2 S_y^4 V_{20} + 2k_{11}S_y^4\tau V_{11} - 2k_{12}S_y^2 S_x^2\tau V_{02} \\ & + k_{12}^2 S_x^4 V_{02} + 4k_{11}k_{12}S_y^2 S_x^2\tau V_{02} - 2k_{11}k_{12}S_y^2 S_x^2 V_{11} \\ & - 4k_{11}^2 S_y^4\tau V_{11} - \frac{9}{4}k_{11}S_y^4\tau^2 V_{02} + \frac{13}{4}k_{11}^2 S_y^4\tau^2 V_{02} + S_y^4.\end{aligned} \tag{4.8}$$

Differentiating (4.8), with respect to k_{11} and k_{12}, we get the optimum values of k_{11} and k_{12} as

$$k_{11}^{(opt)} = \frac{V_{02}}{2}\left(\frac{1 + 7A}{V_{02}^2\tau^2 + 4V_{02}A + 4V_{02}V_{20} - 4V_{11}^2}\right),$$

and

$$k_{12}^{(opt)} = \frac{S_y^2}{2S_x^2}\left(\frac{V_{11} + 7V_{11}A - 8V_{02}\tau A + 8V_{02}V_{20} - 8V_{11}^2}{V_{02}^2\tau^2 + 4V_{02}A + 4V_{02}V_{20} - 4V_{11}^2}\right),$$

where A, is defined earlier.
Substituting the optimum values of k_{11} and k_{12} in (4.8), we get $MSE_{\min}(\hat{S}_P^2)$ as

$$MSE_{\min}(\hat{S}_P^2) \cong \frac{S_y^4}{16}\left\{\frac{64AS_y^{-4}Var(\hat{S}_{Reg}^2) - V_{02}^2\tau^4}{V_{02}\tau^2 + 4A + 4S_y^{-4}Var(\hat{S}_{Reg}^2)}\right\}. \tag{4.9}$$

4.1. Some members of the proposed class of estimators. Different estimators can be generated from the proposed estimator given in (4.4) by substituting the suitable choices of a, and b. Some generated estimators are listed in Table 1.

Table 1. Some members of proposed class of estimators $\hat{S}_{Pj}^2$ $(j = 1, 2, ..., 6)$

a	b	Estimator
1	0	$\hat{S}_{P1}^2$
N	$-S_x^2$	$\hat{S}_{P2}^2$
N	$-\bar{X}^2$	$\hat{S}_{P3}^2$
n	$-S_x^2$	$\hat{S}_{P4}^2$
n^2	$-\bar{X}^2$	$\hat{S}_{P5}^2$
n^2	$-S_x^2$	$\hat{S}_{P6}^2$

5. Efficiency comparisons

In this section, we compare the propose estimator with existing estimators.
Condition i: By (3.1) and (4.9),

$$Var\left(\hat{S}_y^2\right) - MSE_{\min}\left(\hat{S}_P^2\right) > 0, \quad \text{if}$$

$$\frac{S_y^4}{16}\left\{\frac{V_{02}\tau^2\left(16V_{20} + V_{02}\tau^2\right) + 64S_y^{-4}Var(\hat{S}_{Reg}^2) + \frac{64AV_{11}^2}{V_{02}}}{4A + V_{02}\tau^2 + 4S_y^{-4}Var(\hat{S}_{Reg}^2)}\right\} > 0.$$

Condition ii: By [(3.3) or (3.14)] and (4.9),

$$\left[Var\left(\hat{S}_{Reg}^2\right) \quad \text{or} \quad MSE_{\min}\left(\hat{S}_{YK}^2\right)\right] - MSE_{\min}\left(\hat{S}_P^2\right) > 0, \quad \text{if}$$

$$\frac{S_y^4}{16}\left\{\frac{\begin{array}{c}16V_{02}^2\tau^2S_y^{-4}Var(\hat{S}_{yReg}^2) + V_{02}^3\tau^4\\ +64V_{02}\left(S_y^{-4}Var(\hat{S}_{yReg}^2)\right)^2\end{array}}{1 + 3A + 4S_y^{-4}Var(\hat{S}_{yReg}^2)}\right\} > 0.$$

Condition iii: By (3.5) and (4.9),

$$MSE_{\min}\left(\hat{S}_d^2\right) - MSE_{\min}\left(\hat{S}_P^2\right) > 0, \quad \text{if}$$

$$\frac{S_y^4\tau^2}{16}\left[\frac{S_y^{-4}Var(\hat{S}_{yReg}^2)\left\{V_{02}\left(16 + V_{02}\tau^2\right) + 64V_{02}S_y^{-4}Var(\hat{S}_{yReg}^2)\right\} + V_{02}^2\tau^2}{\left(1 + S_y^{-4}Var(\hat{S}_{yReg}^2)\right)\left(4 + 4S_y^{-4}Var(\hat{S}_{yReg}^2) + V_{02}\tau\right)}\right] > 0.$$

Condition iv: By (3.7) and (4.9),

$$MSE_{\min}\left(\hat{S}_{SG}^2\right) - MSE_{\min}\left(\hat{S}_P^2\right) > 0, \quad \text{if}$$

$$\frac{S_y^4 V_{02}}{64}\left[\frac{\begin{array}{r} 3V_{02}\tau^2\left\{V_{02}+16S_y^{-4}Var(\hat{S}^2_{Reg})\right\} \\ +4V_{02}\left\{(1+S_y^{-4}Var(\hat{S}^2_{Reg}))(\tau^2+1)(\tau^2-1)\right\} \\ -64S_y^{-4}Var(\hat{S}^2_{Reg})\left\{(1-\tau^2)+(1-4\tau^2)S_y^{-4}Var(\hat{S}^2_{Reg})\right\}\end{array}}{(1+3A+4S_y^{-4}Var(\hat{S}^2_{Reg}))(1+4S_y^{-4}Var(\hat{S}^2_{Reg}))}\right]>0.$$

Condition v: By (3.9) and (4.9),

$$MSE_{\min}\left(\hat{S}^2_{SS}\right)-MSE_{\min}\left(\hat{S}^2_{P}\right)>0, \quad \text{if}$$

$$S_y^4\tau^2V_{02}^2\left\{\frac{\left(AV_{02}\tau^2+(1+15A)\,S_y^{-4}Var(\hat{S}^2_{Reg})\right)}{\left(A+S_y^{-4}Var(\hat{S}^2_{Reg})\right)\left(1+3A+4S_y^{-4}Var(\hat{S}^2_{Reg})\right)}\right\}>0.$$

Condition vi: By (3.11) and (4.9), when using $\lambda=1$.

$$MSE_{\min}\left(\hat{S}^2_{YG}\right)-MSE_{\min}\left(\hat{S}^2_{P}\right)>0, \quad \text{if}$$

$$S_y^4\tau^2V_{02}^2\left\{\frac{\left(AV_{02}\tau^2+(1+15A)\,S_y^{-4}Var(\hat{S}^2_{Reg})\right)}{\left(A+S_y^{-4}Var(\hat{S}^2_{Reg})\right)\left(1+3A+4S_y^{-4}Var(\hat{S}^2_{Reg})\right)}\right\}>0.$$

Condition vii: By (3.16) and (4.9), when using $\tau=\psi_1=1$.

$$MSE_{\min}\left(\hat{S}^2_{SM}\right)-MSE_{\min}\left(\hat{S}^2_{P}\right)>0, \quad \text{if}$$

$$\frac{S_y^4}{32}\left[\frac{\left\{\begin{array}{r} 512V_{11}^2\,(V_{20}-V_{11})+48V_{02}V_{20}\,(1+4V_{02}V_{20})-60V_{02}V_{11}^2\left(V_{02}+V_{11}^2\right) \\ -960V_{02}V_{11}^2S_y^{-4}Var(\hat{S}^2_{Reg})+256V_{20}V_{11}\left\{(V_{02}V_{11})+(V_{02}-V_{11})\,S_y^{-4}Var(\hat{S}^2_{Reg})\right\} \\ 3\left(V_{02}^4+96V_{02}^3V_{11}^2-156\frac{V_{11}^4}{V_{02}}\right)\end{array}\right\}}{2\left\{S_y^{-4}Var(\hat{S}^2_{Reg})+(1+4V_{11})-3\frac{V_{11}^2}{V_{02}}\right\}\left\{4S_y^{-4}Var(\hat{S}^2_{Reg})+(4-3V_{02})\right\}}\right]>0.$$

Note that the proposed estimator $(\hat{S}^2_P)$ is more efficient than the existing estimators $\hat{S}^2_i\ (i=y,Reg,d,SG,SS,YG,YK,SM)$, when above conditions are satisfied.

6. Numerical study

In this section, we consider the following data sets for numerical comparisons.

Population I: [Source: Kadilar and Cingi [20]]

Let y = Level of apple production (1 unit=100 tones) and x = Number of apple trees (1 unit=100 trees).

$N=104$, $n=20$, $\gamma=0.05$, $\bar{Y}=6.24064$, $\bar{X}=13929.899$, $S_y=11.670$, $S_x=23026.133$, $\rho_{yx}=0.865$,

$\rho_{(S_y^2,S_x^2)}=0.83675$, $\beta_{2(y)}=16.523$, $\beta_{2(x)}=17.516$, $\theta_{22}=14.398$, $V_{20}=0.77615$, $V_{02}=0.8258$, $V_{11}=0.6699$.

Population II: [Source: Cochran [6], p.34]
Let $y =$ The weekly expenditure on food and $x =$ The weekly family income.
$N = 33$, $n = 10$, $\gamma = 0.1$, $\bar{Y} = 27.491$, $\bar{X} = 72.547$, $S_y = 10.131$, $S_x = 10.577$, $\rho_{yx} = 0.432738$, $\rho_{(S_y^2, S_x^2)} = -0.474028$, $\beta_{2(y)} = 5.38276$, $\beta_{2(x)} = 2.015035$, $\theta_{22} = 0.000187$, $V_{20} = 0.43827$, $V_{02} = 0.101503$,
$V_{11} = -0.099981$.

Population III: [Source: Murthy [26], p.399]
Let $y =$ Area of wheat in 1964 and $x =$ Area of wheat in 1963.
$N = 80$, $n = 10$, $\gamma = 0.1$, $\bar{Y} = 5182.638$, $\bar{X} = 285.125$, $S_y = 1835.638$, $S_x = 270.429$, $\rho_{yx} = 0.988421$,
$\rho_{(S_y^2, S_x^2)} = 0.73198$, $\beta_{2(y)} = 2.2665$, $\beta_{2(x)} = 3.5808$, $\theta_{22} = 2.32338$, $V_{20} = 0.12665$, $V_{02} = 0.25808$, $V_{11} = 0.13234$.

We use the following expression for Percentage Relative Efficiency (PRE) and the Absolute Bias (AB).

$$PRE(\hat{S}_y^2, \hat{S}_i^2) = \frac{Var(\hat{S}_y^2)}{MSE_{\min}\left(\hat{S}_i^2\right) \quad \text{or} \quad MSE\left(\hat{S}_i^2\right)},$$

and

$$AB = |Bias\left(\hat{S}_i^2\right)|, \quad \text{for } i{=}Reg, d, SG, SS, YG, YK, SM, P.$$

MSE, PRE and AB values based on Populations I, II and III are given in Tables 2–4.

Table 2. *MSE*, *PRE* and *AB*, values of different estimators for Population I.

a, b		$\hat{S}_y^2$	$\hat{S}_{Reg}^2$	$\hat{S}_d^2$	$\hat{S}_{SG}^2$	$\hat{S}_{SS}^2$	$\hat{S}_{YG}^2$	$\hat{S}_{YK}^2$	$\hat{S}_{SM}^2$	$\hat{S}_P^2$
	MSE	14395.58	4316.32	3501.46	2618.27	1847.80	1847.80	4316.32	3996.51	**903.65**
1, 0	*PRE*	100	333.51	411.13	549.81	779.07	779.07	333.51	360.20	**1593.04**
	AB	—	—	25.71	19.22	13.57	13.57	27.83	29.34	**6.63**
	MSE	14395.58	4316.32	3501.46	2618.27	1939.24	1939.24	4316.32	4017.78	**1006.65**
104, 0.530E+9	*PRE*	100	333.51	411.13	549.81	742.33	742.33	333.51	358.30	**1430.05**
	AB	—	—	25.71	19.22	12.82	12.82	27.83	29.18	**5.82**
	MSE	14395.58	4316.32	3501.46	2618.27	1882.38	1882.38	4316.32	4004.41	**942.17**
104, 0.194E+9	*PRE*	100	333.51	411.13	549.81	764.75	764.75	333.51	359.49	**1527.91**
	AB	—	—	25.71	19.22	13.30	13.30	27.83	29.29	**6.34**
	MSE	14395.58	4316.32	3501.46	2618.27	2239.62	2239.62	4316.32	4097.01	**1372.23**
20, 0.530E+9	*PRE*	100	333.51	411.13	549.81	642.76	642.76	333.51	351.36	**1049.06**
	AB	—	—	25.71	19.22	8.47	8.47	27.83	28.41	**1.67**
	MSE	14395.58	4316.32	3501.46	2618.27	1856.92	1856.92	4316.32	3998.58	**913.76**
400, 0.194E+9	*PRE*	100	333.51	411.13	549.81	775.24	775.24	333.51	360.02	**1575.41**
	AB	—	—	25.71	19.22	13.50	13.50	27.83	29.33	**6.56**
	MSE	14395.58	4316.32	3501.46	2618.27	1872.50	1872.50	4316.32	4002.13	**931.12**
400, 0.530E+9	*PRE*	100	333.51	411.13	549.81	768.78	768.78	333.51	359.69	**1546.05**
	AB	—	—	25.71	19.22	13.38	13.38	27.83	29.30	**6.42**

Bold numbers indicate least values.

Table 3. *MSE*, *PRE* and *AB*, values of different estimators for Population II.

a, b		$\hat{S}^2_y$	$\hat{S}^2_{Reg}$	$\hat{S}^2_d$	$\hat{S}^2_{SG}$	$\hat{S}^2_{SS}$	$\hat{S}^2_{YG}$	$\hat{S}^2_{YK}$	$\hat{S}^2_{SM}$	$\hat{S}^2_P$
	MSE	4616.57	3579.21	2671.46	2602.41	2597.05	2597.05	3579.21	3060.015	**2543.56**
1, 0	*PRE*	100	128.98	172.81	177.396	177.761	177.761	128.98	150.86	**181.50**
	AB	—	—	26.03	25.35	25.30	25.30	**10.03**	29.81	24.78
	MSE	4616.57	3579.21	2671.46	2602.41	2601.70	2601.70	3579.21	3059.885	**2551.47**
33, 111.88	*PRE*	100	128.98	172.81	177.396	177.44	178.11	128.98	150.87	**180.94**
	AB	—	—	26.03	25.35	25.25	25.30	**10.03**	29.815	24.70
	MSE	4616.57	3579.21	2671.46	2602.41	2659.62	2659.62	3579.21	3035.34	**2650.94**
33, 5263.08	*PRE*	100	128.98	172.81	177.396	173.58	173.58	128.98	152.10	**174.15**
	AB	—	—	26.03	25.35	19.67	19.67	**10.02**	22.40	16.11
	MSE	4616.57	3579.21	2671.46	2602.41	2610.83	2610.83	3579.21	3059.22	**2567.04**
20, 111.88	*PRE*	100	128.98	172.81	177.396	176.82	176.82	128.98	150.91	**179.83**
	AB	—	—	26.03	25.35	25.11	25.11	**10.02**	29.80	24.46
	MSE	4616.57	3579.21	2671.46	2602.41	2638.50	2638.50	3579.21	3052.20	**2614.48**
400, 5263.08	*PRE*	100	128.98	172.81	177.396	174.97	174.97	128.98	151.25	**176.5**
	AB	—	—	26.03	25.35	22.75	22.75	**10.02**	29.19	20.63
	MSE	4616.57	3579.21	2671.46	2602.41	2598.64	2598.64	3579.21	3059.98	**2546.26**
400, 111.88	*PRE*	100	128.98	172.81	177.40	177.65	177.65	128.98	150.87	**181.31**
	AB	—	—	26.03	25.35	25.28	25.28	**10.02**	29.81	24.75

Table 4. *MSE*, *PRE* and *AB*, values of different estimators for Population III.

a, b		$\hat{S}_y^2$	$\hat{S}_{Reg}^2$	$\hat{S}_d^2$	$\hat{S}_{SG}^2$	$\hat{S}_{SS}^2$	$\hat{S}_{YG}^2$	$\hat{S}_{YK}^2$	$\hat{S}_{SM}^2$	$\hat{S}_P^2$
	MSE	0.1438E+13	0.667E+12	0.630E+12	0.578E+12	0.618E+12	0.618E+12	0.667E+12	0.586E+12	**0.558E+12**
1, 0	*PRE*	100	215.42	228.09	248.52	232.49	232.49	215.42	245.15	**257.37**
	AB	—	—	187104.52	171720.74	183559.03	183559.03	429103.301	174078.43	**165818.47**
	MSE	0.1438E+13	0.667E+12	0.630E+12	0.578E+12	0.619E+12	0.619E+12	0.667E+12	0.589E+12	**0.561E+12**
80, 73132.06	*PRE*	100	215.42	228.09	248.52	232.32	232.32	215.42	244.05	**256.24**
	AB	—	—	187104.52	171720.74	183438.60	183438.60	429103.301	173252.14	**165044.56**
	MSE	0.1438E+13	0.667E+12	0.630E+12	0.578E+12	0.619E+12	0.619E+12	0.667E+12	0.589E+12	**0.561E+12**
80, 81297.82	*PRE*	100	215.42	228.09	248.52	232.33	232.33	215.42	243.93	**256.12**
	AB	—	—	187104.52	171720.74	183424.74	183424.74	429103.301	173157.49	**164955.42**
	MSE	0.1438E+13	0.667E+12	0.630E+12	0.578E+12	0.621E+12	0.621E+12	0.667E+12	0.505E+12	**0.575E+12**
10, 73132.06	*PRE*	100	215.42	228.09	248.52	231.52	231.52	215.42	237.73	**249.96**
	AB	—	—	187104.52	171720.74	182369.38	182369.38	429103.301	166223.26	**158131.45**
	MSE	0.1438E+13	0.667E+12	0.630E+12	0.578E+12	0.619E+12	0.619E+12	0.667E+12	0.589E+12	**0.561E+12**
100, 81297.82	*PRE*	100	215.42	228.09	248.52	232.37	232.37	215.42	244.17	**256.36**
	AB	—	—	187104.52	171720.74	183452.25	183452.25	429103.301	173345.47	**165132.37**
	MSE	0.1438E+13	0.667E+12	0.630E+12	0.578E+12	0.619E+12	0.619E+12	0.667E+12	0.589E+12	**0.561E+12**
100, 73132.06	*PRE*	100	215.42	228.09	248.52	232.38	232.38	215.42	244.27	**256.45**
	AB	—	—	187104.52	171720.74	183463.21	183463.21	429103.301	173420.45	**165202.84**

7. Conclusion

In this paper, we have suggested an improved class of estimators for estimating the finite population variance using information on the auxiliary variable. Expressions for bias and *MSE* of the proposed class of estimators have been derived to first order of approximation. The proposed class of estimators $\hat{S}_P^2$ is compared with existing estimators both theoretically and numerically. From Tables 2–4, it is observed that the *MSE* of $\hat{S}_P^2$ is smaller as compared to *MSE* of existing estimators for all different choices of a and b considered here. Also bias of $\hat{S}_P^2$ is smaller as compared to all other considered estimators in all three populations except $\hat{S}_{YK}^2$ in population II. Among three populations, the maximum *PRE* gained by proposed estimator is in Population I. So it is preferable to use the estimator $\hat{S}_P^2$.

References

[1] Arcos, A., Rueda, M., Martinez, M. D., Gonzalez S., Roman, Y. *Incorporating the auxiliary information available in variance estimation*, Applied Mathematics and Computation **160(2)**, 387–399, 2005.

[2] Ahmed, M. S., Rahman, M. S., Hossain, M. I. *Some competitive estimators of finite population variance using multivariate auxiliary information*, International Journal of Information and Management Sciences **11(1)**, 49–54, 2000.

[3] AL-Jaraha, J., Ahmed, M. S. *A class of chain estimators of a finite population variance using double-sampling*, International Journal of Information and Management Sciences **13(2)**, 13–18, 2002.

[4] Bahl, S., Tuteja, R. K. *Ratio and product type exponential estimators*, Journal of Information and Optimization Sciences **12**, 159–164, 1991.

[5] Cebrian, A. A., Garcia, M. R. *Variance estimation using auxiliary information– an almost unbiased multivariate ratio estimator*, Metrika **45** , 171–178, 1997.

[6] Cochran, W. G. *Sampling Techniques*: Third Edition. John Wiley & Sons 1977.

[7] Chandra, P., Singh H. P. *A family of estimators for population variance using knowledge of kurtosis of an auxiliary variable in sample survey*, Statistics in Transition **7**, 27–34, 2005.

[8] Chaudhury, A. *On estimating the variance of a finite population*, Metrika **25**, 66–67, 1978.

[9] Das, A. K., Tripathi, T. P. *Use of auxiliary information in estimating the finite population variance*, Sankhya C. **40**, 139–148, 1978.

[10] Garcia, M. R., Cebrian, A. A. *Repeated substitution method-the ratio estimator for population variance*, Metrika **43**, 101–105, 1996.

[11] Grover, L. K. *A correction note on improvement in variance estimation using auxiliary information*, Communication in Statistics –Theory and Methods **39**, 753–764, 2010.

[12] Gupta, S., Shabbir, J. *On the use of transformed auxiliary variables in estimating population mean*, Journal of Statistical Planning and Inference. **137(5)**, 1606–1611, 2007.

[13] Gupta, S., Shabbir, J. *On improvement in estimating the population mean in simple random sampling* Journal of Applied Statistics **35(5)**, 559–566, 2008.

[14] Gupta S., Shabbir J. *Variance estimation in simple random sampling using uxiliary information*, Hacettepe Journal of Mathematics and Statistics **37(1)**, 57 – 67, 2008.

[15] Haq, A., Shabbir, J.*Improved family of ratio estimators in simple and stratified random sampling*, Communication in Statistics – Theory and Methods **42**, 782–799, 2013.

[16] Haq, A., Shabbir, J., Gupta, S. *Improved exponential ratio type estimators in stratified sampling*, Pakistan Journal of Statistics **29(1)**, 13–31, 2013.

[17] Isaki, C. T. *Variance estimation using auxiliary information*, Journal of the American Statistical Association **78**, 117–123, 1983.

[18] Kadilar, C., Cingi, H. *Ratio estimator in stratified sampling*, Biometrical Journal **45(2)**, 218–225, 2003.

[19] Kadilar, C., Cingi, H. *Ratio estimators in simple random sampling*, Applied Mathematics and Computations **151**, 893–902, 2004.

[20] Kadilar, C., Cingi, H. *A new estimator using two auxiliary variables*, Applied Mathematics and Computations **162**, 901–908, (2005).

[21] Kadilar, C., Cingi, H. *Ratio estimators for the population variance in simple and stratified random sampling*, Applied Mathematics and Computations **173(2)**, 1047–1059, 2006.

[22] Kadilar, C., Cingi, H. *Improvement in variance estimation using auxiliary information*, Hacettepe Journal of Mathematics and Statistics **35(1)**, 111–115, 2006.

[23] Kadilar, C., Cingi, H. *Improvement in variance estimation in simple random sampling*, Communication in Statistics –Theory and Methods **36(11)**, 2075–2081, 2007.

[24] Koyuncu, N., Kadilar, C. *Ratio and product estimators in stratified random sampling*, Journal of Statistical Planning and Inference **139(8)**, 2552–2558, 2009.

[25] Koyuncu, N., Kadilar, C. *On improvement in estimating population mean in stratified random sampling*, Journal of Applied Statistics **37(6)**, 999–1013, 2010.

[26] Murthy, M. N. *Sampling Theory and Methods*, Statistical Publishing Society. Calcutta, India, 1967.

[27] Mukhopadhyay, P. *Estimating a finite population variance under a super population model*, Metrika **25**, 115–122, 1978.

[28] Mukhopadhyay, P. *Optimum strategies for estimating the variance of a finite population under a super population model*, Metrika **29**, 143–158, 1982.

[29] Parsad, B., Singh, H. P. *Some improved ratio-type estimators of finite population variance using auxiliary information in sample surveys*, Communication in Statistics -Theory and Methods **19(3)**, 1127–1139, 1990.

[30] Parsad, B., Singh, H. P.*Unbiased estimators of finite population variance using auxiliary information in sample survey* , Communication in Statistics -Theory and Methods **21(5)**, 1367–1376, 1992.

[31] Searl, D. T, Intarapanich, P. *A note on an estimator for the variance that utilizes the kurtosis*, American Statistician **44(4)**, 295–296, 1990.

[32] Shabbir, J., Gupta, S. *On improvement in variance estimation using auxiliary information*, Communication in Statistics -Theory and Methods **36**, 2177–2185, 2007.

[33] Shabbir, J., Gupta, S. *Some estimators of finite population variance of stratified sample mean*, Communication in Statistics -Theory and Methods **39**, 3001–3008, 2010.

[34] Shabbir, J., Gupta, S. *On estimating the finite population mean in simple and stratified random sampling*, Communication in Statistics - Theory and Methods **140(10)**, 2870–2874, 2011.

[35] Singh, R., Malik, S. *Improved estimation of population variance using information on auxiliary attribute in simple random sampling*, Applied Mathematics and Computation **235**, 43–49, 2014.

[36] Singh et al. *On the utilization of known coefficient of kurtosis in estimation procedure of variance*, Annals of the Institute of Statistical Mathematics **25**, 51–55, 1973.

[37] Singh, H. P. *A note on the estimation of variance of sample mean using the knowledge of coefficient of variation in normal population*, Communication in Statistics - Theory and Methods **15(12)**, 3737–3746, 1986.

[38] Singh, H. P., Upadhyaya, L. N., Namjoshi U. D. *Estimation of finite population variance*, Current Science **57**, 1331–1334, 1988.

[39] Singh, H. P., Biradar, R. S. *Estimation of finite population variance using auxiliary information*, Journal of the Indian Society of Statistics and Operation Research $\mathbf{15(1-4)}$, 47–63, 1994.

[40] Singh, S., Joarder, A. H . *Estimation of finite population variance using random non-response in survey sampling*, Metrika **47**, 241–249, 1998.

[41] Singh, H. P., Singh, R. *Improved ratio type estimators for variance using auxiliary information.*, Journal of the Indian Social and Agriculture Statistics **54(3)**, 276–287, 2001.

[42] Singh, H. P., Tailor, R. *Use of known correlation coefficient in estimating the finite population mean*, Statistics in Transition **6(4)**, 555–560, 2003.

[43] Singh, H. P., Vishvakarama, G. K. *Some families of estimators of variance of stratified random sample mean using auxiliary information*, Journal of Statistical Theory and Practice **2(1)**, 21–43, 2008.

[44] Singh, H. P., Vishvakarama, G. K. *A general procedure for estimating the population mean in stratified random sampling using auxiliary information*, Metron **68(1)**, 47–65, 2010.

[45] Singh, H. P., Solanki, R. S. *A new procedure for variance estimation in simple random sampling*, Statistical Papers **5**, 479–497, 2013 a.

[46] Singh, H. P., Solanki, R. S. *Improved estimation of finite population variance using auxiliary information*, Communication in Statistics – Theory and Methods **42**, 2718–2730, 2013 b.

[47] Srivastava, S. K., Jhajj, H. S. *A class of estimator using auxiliary information for estimating finite population variance*, Sankhya(C) **42**, 87–96, 1980.

[48] Turgut, Y., Cingi, H. *New generalized estimators for the population variance using auxiliary information* , Hacettepe Journal of Mathematics and Statistics **37(2)**, 177–184, 2008.

[49] Upadhyaya, L. N., Singh, H. P. *An estimator of population variance that utilises the kurtosis of an auxiliary variable in sample survey*, Vikram Math Journal **19**, 14–17, 1999.

[50] Upadhyaya, L. N., Singh, H. P., Singh, S. *A class of estimators for estimating the variance of the ratio estimator*, Journal of the Japan Statistical Society **34(1)**, 47–63, 2004.

[51] Wu, C. F. *Estimation of variance of the ratio estimator.*, Biometrika **69(1)**, 183–189, 1982.

[52] Yadav, C. F., Upadhyaya, L. N., Singh, H. P., Chatterjee, S. *A generalized family of transformed ratio-product estimators for variance in sample surveys*, Communication in Statistics – Theory and Methods **42(10)**, 2013.

[53] Yadav, S. K., Kadilar, C., Shabbir, J., Gupta, S. *Improved family of estimators of population variance in Simple Random Sampling*, Journal of Statistical Theory and Practice **9(2)**, 219–226, 2015.

[54] Yadav, S. K., Kadilar, C. *A two parameter variance estimator using auxiliary information*, Applied Mathematics and Computation **226**, 117–122, 2014.