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New ranked set sampling for estimating the population mean and variance

Ehsan Zamanzade*† and Amer Ibrahim Al-Omari‡

Abstract

The purpose of this study is to suggest a new modification of the usual ranked set sampling (RSS) method, namely; neoteric ranked set sampling (NRSS) for estimating the population mean and variance. The performances of the empirical mean and variance estimators based on NRSS are compared with their counterparts in ranked set sampling and simple random sampling (SRS) via Monte Carlo simulation. Simulation results indicate that the NRSS estimators perform much better than their counterparts using RSS and SRS designs when the ranking is perfect. When the ranking is imperfect, the NRSS estimators are still superior to their counterparts in ranked set sampling and simple random sampling methods. These findings show that the NRSS provides a uniform improvement over RSS without any additional costs. Finally, an illustrative example of a real data is provided to show the application of the new method in practice.

Keywords: Ranked set sampling, Imperfect rankings, Simple random sampling, Monte Carlo simulation.

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*Department of Statistics, University of Isfahan, Isfahan 81746-73441, Iran.
Email: E.zamanzade@sci.ui.ac.ir; Ehsanzamanzadeh@yahoo.com
†Corresponding Author.
‡Department of Mathematics, Faculty of Science, Al al-Bayt University, P.O. Box 130095, Mafraq 25113, Jordon.
Email: alomari_ amer@yahoo.com

1. Introduction

The ranked set sampling (RSS) was first proposed by McIntyre [15] as an efficient sampling scheme for estimating the population mean of pasture and forage yields. This sampling scheme is suitable in situations where the ranking of observations can be easily done based on an auxiliary variable correlated with the variable of interest or any inexpensive method. The RSS has wide applications in many scientific problems, especially in environmental and ecological studies where the main focus is on economical and efficient sampling strategies. For example, assume that the Environmental Protection Agency wants to assure that the gasoline stations in metropolitan areas are distributing gasoline which complies with air clean regulations. However, the chemical parameters of gasoline can be easily ranked right after the collection at the gasoline pump by some crude field techniques which are cheap and easy. While bringing the sample units to the laboratory and use actual laboratory techniques to measure its chemical parameters is expensive. For similar applications of RSS in environmental studies, we refer to Cobby et al. [9], Halls and Dell [12], Martin et al. [14], and Ozturk et al. [17].

The standard ranked set sampling design can be described as follows:

I. Select a simple random sample of size k^2 units from the target population and divide them into k samples each of size k.

II. Rank the units within each sample in increasing magnitude by using personal judgment, eye inspection or based on a concomitant variable.

III. Select the ith ranked unit from the ith $(i = 1, \cdots, k)$ sample for actual quantification.

IV. The above Steps I through III can be repeated n times (cycles) if needed to obtain a ranked set sample of size $N = nk$.

Let $Y_1, \cdots, Y_k$ be a simple random sample of size k, then the measured ranked set sample units are denoted by $\left\{Y_{[i]j}, i = 1, \cdots, k, j = 1, \cdots, n\right\}$, where $Y_{[i]j}$ is the ith ranked unit from the jth cycle. It is of interest to note here that $Y_{[i]j}$ $(i = 1, \cdots, k)$ are independent random variables, and they follow the distribution of the ith order statistic of a sample of size k based on perfect ranking in the jth cycle, $j = 1, \cdots, n$. The cumulative distribution function (cdf) of $Y_{[i]}$ is given by $F_{[i]}(y) = i\binom{k}{i}\int_0^{F(y)} w^{i-1}(1-w)^{k-i}dw$, and its probability density function (pdf) is defined as $f_{[i]}(y) = i\binom{k}{i}\left[F(y)\right]^{i-1}\left[1 - F(y)\right]^{k-i}$. The mean and the variance of $Y_{[i]}$ are $\mu_{[i]} = \int_{-\infty}^{+\infty} y f_{[i]}(y)dy$ and $\sigma_{[i]}^2 = \int_{-\infty}^{+\infty}(y-\mu_{[i]})^2 f_{[i]}(y)dy$, respectively.

Under imperfect ranking, the $Y_{[i]j}$'s follow the distribution of the ith judgment order statistic. McIntyre [15] used the empirical estimator of the mean based on RSS to estimate the population mean and deduced that his estimator is more efficient than its SRS counterpart via Monte Carlo simulation based on the same number of measured units. The RSS empirical mean estimator is defined as

$$\bar{Y}_{RSS} = \frac{1}{nk}\sum_{j=1}^{n}\sum_{i=1}^{k} Y_{[i]j}, \tag{1.1}$$

with variance

$$Var\left(\bar{Y}_{RSS}\right) = \frac{\sigma^2}{nk} - \frac{1}{nk^2}\sum_{i=1}^{k}\left(\mu_{[i]} - \mu\right)^2. \tag{1.2}$$

Takahasi and Wakimoto [21] introduced the same method independently and was the first who proved mathematically that, $\bar{Y}_{RSS}$ is an unbiased estimator and has smaller variance than its counterpart in SRS regardless of the issue of ranking. They proved that

$$1 \leq \frac{Var\left(\bar{Y}_{SRS}\right)}{Var\left(\bar{Y}_{RSS}\right)} \leq \frac{k+1}{2},$$

where $\bar{Y}_{SRS} = \frac{1}{nk}\sum_{j=1}^{n}\sum_{i=1}^{k} Y_{ij}$ is the SRS estimator of the population mean with $Var\left(\bar{Y}_{SRS}\right) = \frac{\sigma^2}{nk}$.

The lower bound is attained if and only if the parent distribution is degenerate when the ranking is perfect, while the upper bound is attained if and only if the parent distribution is rectangular.

Bouza [7] and Al-Omari and Bouza [4] considered the problem of estimation of population mean in the RSS with missing values. Al-Saleh and Al-Omari [5], Al-Omari and Al-Saleh [3] and Al-Omari [1], [2] proposed some mean estimators in other variations of the RSS.

Stokes [20] suggested an estimator of the population variance based on RSS and showed that it is asymptotically ($n \to \infty$ or $k \to \infty$) unbiased of the population variance and has greater efficiency than the sample variance using SRS regardless of the issue of ranking. The variance estimator of Stokes [20] is given by

$$S^2_{Stokes} = \frac{1}{nk-1}\sum_{j=1}^{n}\sum_{i=1}^{k}\left(Y_{[i]j} - \bar{Y}_{RSS}\right)^2. \tag{1.3}$$

Recently, an unbiased estimator of variance is proposed by MacEachern et al. [13] as

$$S^2_M = \frac{1}{2n^2k^2}\sum_{i\neq j}^{k}\sum_{r=1}^{n}\sum_{s=1}^{n}\left(Y_{[i]r} - Y_{[j]s}\right)^2 + \frac{1}{2n(n-1)k^2}\sum_{i=1}^{k}\sum_{r=1}^{n}\sum_{s=1}^{n}\left(Y_{[i]r} - Y_{[i]s}\right)^2. \tag{1.4}$$

They showed that this estimator is more efficient than S^2_{Stokes} , especially when the ranking is perfect. However, S^2_M can be applied if the number of cycles is $n \geq 2$.

Perron and Sinha [18] demonstrated that S^2_M has the minimum variance among all unbiased estimators of the form $\sum_i\sum_j\sum_r\sum_s \gamma_{i,j,r,s}Y_{[i]r}Y_{[j]s}$, where the coefficients $\{\gamma_{i,j,r,s}\}$ satisfy $\gamma_{i,j,r,s} = \gamma_{j,i,r,s}$.

Another estimator of variance when the RSS is applied by measuring a concomitant variable is proposed by Zamanzade and Vock [22]. Their estimator was obtained by conditioning on observed concomitant values and using nonparametric kernel regression. Zamanzade and Vock [22]'s simulation results indicated that their proposed estimator considerably improves the estimation of variance when the rankings are fairly good. However, since our interest here is not about using values of concomitant variable, we do not consider their estimator for more investigations.

Biswas et al. [6] considered the problem of estimation of variance in finite population setting using jackknife method. Chen and Lim [8] considered the problem of estimation of variances of strata in a balanced ranked set sample. Sengupta and Mukhuti [19] proposed some unbiased variance estimators when the parent distribution is known to be simple exponential.

The rest of this paper is organized as follows: In Section 2, the suggested sampling scheme is explained and discussed for estimating the population mean and variance. In Section 3, we compare the performance of the mean and variance estimators using NRSS with their counterparts in RSS and SRS methods. In Section 4, a real data example is provided to show the application of the new sampling strategy in practice. Some concluding remarks are provided in Section 5.

2. Neoteric Ranked Set Sampling

Similar to the RSS, neoteric ranked set sampling (NRSS) is suggested to apply in situations where the ranking of the sample observations is much easier than obtaining

their precise values. The NRSS scheme can be described as follows:
I. Select a simple random sample of size k^2 units from the target population.
II. Rank the k^2 selected units in an increasing magnitude based on a concomitant variable, personal judgment or any inexpensive method.
III. If k is an odd, then select the $\left[\frac{k+1}{2} + (i-1)\,k\right]$th ranked unit for $i = 1, \cdots, k$. But if k is an even, then select the $[l + (i-1)k]$th ranked unit, where $l = \frac{k}{2}$ if i is an even and $l = \frac{k+2}{2}$ if i is an odd for $i = 1, \cdots, k$.
IV. Repeat Steps I through III n times (cycles) if needed to obtain a neoteric ranked set sample of size $N = nk$.
To illustrate the NRSS method, let us consider the following special case of univariate observations.

Let $Y_{ij}, Y_{2j}, \cdots, Y_{k^2j}$ be k^2 simple random units selected from the population of interest, and let $Y_{[i]j}, Y_{[2]j}, \cdots, Y_{[k^2]j}$ be the order statistics of $Y_{ij}, Y_{2j}, \cdots, Y_{k^2j}$ for $j = 1, \cdots, n$.
1) Using NRSS
Assume that $k = 3$ and $n = 1$, then we have to select $k^2 = 9$ units as

$$Y_{11}, Y_{21}, Y_{31}, Y_{41}, Y_{51}, Y_{61}, Y_{71}, Y_{81}, Y_{91}.$$

Now, rank the units based on personal judgment or eye inspection to get

$$Y_{[1]1}, Y_{[2]1}, Y_{[3]1}, Y_{[4]1}, Y_{[5]1}, Y_{[6]1}, Y_{[7]1}, Y_{[8]1}, Y_{[9]1}.$$

Using NRSS method, we have to choose the units with the rank 2, 5, 8 for actual quantification as

$$\left\{Y_{[1]1}, \boxed{Y_{[2]1}}, Y_{[3]1}, Y_{[4]1}, \boxed{Y_{[5]1}}, Y_{[6]1}, Y_{[7]1}, \boxed{Y_{[8]1}}, Y_{[9]1}\right\}.$$

Then the measured NRSS units are $\left\{Y_{[2]1}, Y_{[5]1}, Y_{[8]1}\right\}$, where their mean and the variance are considered as estimators of the population mean and variance, respectively.
2) Using RSS

Now, using RSS method, we have to select 9 units:

$$\begin{bmatrix} Y_{11}, & Y_{12}, & Y_{13} \\ Y_{21}, & Y_{22}, & Y_{23} \\ Y_{31}, & Y_{32}, & Y_{33} \end{bmatrix}.$$

We then rank the units within each set with respect to a variable of interest and then select the ith ranked unit of the ith sample as:

$$\begin{bmatrix} \boxed{Y_{1[1]}}, & Y_{1[2]}, & Y_{1[3]} \\ Y_{2[1]}, & \boxed{Y_{2[2]}}, & Y_{2[3]} \\ Y_{3[1]}, & Y_{3[2]}, & \boxed{Y_{3[3]}} \end{bmatrix}.$$

The measured RSS units are $\left\{Y_{1[1]}, Y_{2[2]}, Y_{3[3]}\right\}$.
It is of interest to note here, that even if we select k^2 units in both methods RSS and NRSS, we only measure k units. Also, in RSS we rank k units in each of the k sets, while in the NRSS, we rank all the k^2 selected units at the same time.

In general, the resulting neoteric ranked set sample is denoted by $\left\{Y_{[(i-1)k+l]j}; i = 1, \cdots, k, j = 1, \cdots, n\right\}$, where $Y_{[(i-1)k+l]j}$ is the $[(i-1)k + l]$th measured unit from the jth cycle, and $l = \frac{k+1}{2}$ if k is odd, $l = \frac{k}{2}$ if k and i are both even and $l = \frac{k}{2} + 1$ if k is even but i is odd. Unlike RSS, NRSS measured units $\{Y_{[(i-1)k+l]j}; i = 1, \cdots, k\}$ are dependent, and they follow the distribution of $[(i-1)k+l]$th order statistics of a sample of size k^2 based on perfect ranking for $j = 1, \cdots, n$. In the case of imperfect rankings, the $\{Y_{[(i-1)k+l]j}; j = 1, \cdots, n\}$ follow distribution of judgment order statistics of a sample of size k^2.

To simplify the notations, if the sample size k is odd, then the measured units will be denoted by $Y_{[\frac{k+1}{2}]}, Y_{[\frac{3k+1}{2}]}, Y_{[\frac{5k+1}{2}]}, \cdots, Y_{[\frac{2k^2-k+1}{2}]}$. But if the sample size k is even, then the measured units are denoted by $Y_{[\frac{k+2}{2}]}, Y_{[\frac{3k}{2}]}, Y_{[\frac{5k+2}{2}]}, Y_{[\frac{7k}{2}]}, Y_{[\frac{9k+2}{2}]}, \cdots, Y_{[\frac{2k^2-k}{2}]}$.

The suggested estimator of the population mean using NRSS is defined by

$$\bar{Y}_{NRSS} = \frac{1}{nk} \sum_{j=1}^{n} \sum_{i=1}^{k} Y_{[(i-1)k+l]j}, \tag{2.1}$$

with variance

$$Var\left(\bar{Y}_{NRSS}\right) = \frac{1}{nk^2} \sum_{i=1}^{k} Var\left(Y_{[(i-1)k+l]1}\right) + \frac{2}{nk^2} \sum_{i<j}^{k} Cov\left(Y_{[(i-1)k+l]1}, Y_{[(j-1)k+l]1}\right). \tag{2.2}$$

In the following theorem, we prove that the proposed mean estimator is unbiased for symmetric distributions.

2.1. Theorem. *$\bar{Y}_{NRSS}$ is an unbiased estimator of population mean if the rankings are perfect and the parent distribution is symmetric.*

Proof. Without loss of generality, we may suppose that $n = 1$.
If k is odd, then the NRSS estimator of the population mean can be written as

$$\bar{Y}_{NRSS} = \frac{1}{k} \sum_{i=1}^{\frac{k-1}{2}} \left(Y_{[\frac{2ik-k+1}{2}]} + Y_{[\frac{2k^2-ik+1}{2}]} \right) + Y_{[\frac{k^2+1}{2}]}.$$

Take its expectation to have

$$E\left(\bar{Y}_{NRSS}\right) = E\left[\frac{1}{k} \sum_{i=1}^{\frac{k-1}{2}} \left(Y_{[\frac{2ik-k+1}{2}]} + Y_{[\frac{2k^2-ik+1}{2}]} \right) + Y_{[\frac{k^2+1}{2}]} \right]$$

$$= \frac{1}{k} \sum_{i=1}^{\frac{k-1}{2}} \left(E\left(Y_{[\frac{2ik-k+1}{2}]} \right) + E\left(Y_{[\frac{2k^2-ik+1}{2}]} \right) \right) + E\left(Y_{[\frac{k^2+1}{2}]} \right).$$

From symmetric assumption about μ , we have $Y_{[i]} - \mu \overset{d}{=} \mu - Y_{[i]}$, see for example David and Nagaraja [11]. Thus, $\mu - \mu_{[\frac{2ik-k+1}{2}]} = \mu_{\frac{2k^2-ik+1}{2}} - \mu$, and then $\mu_{[\frac{2ik-k+1}{2}]} + \mu_{\frac{2k^2-ik+1}{2}} = 2\mu$. Also, $E\left(Y_{[\frac{k^2+1}{2}]} \right) = \mu$ since it is the median of the chosen sample of size k^2. Therefore,

$$E\left(\bar{Y}_{NRSS}\right) = \frac{1}{k} \sum_{i=1}^{\frac{k-1}{2}} \left(\mu_{[\frac{2ik-k+1}{2}]} + \mu_{[\frac{2k^2-ik+1}{2}]} \right) + \mu_{[\frac{k^2+1}{2}]}$$

$$\frac{1}{k} \left[\frac{k-1}{2} (2\mu) + \mu \right] = \mu.$$

The case of the even sample size can be proved by rewriting $\bar{Y}_{NRSS}$ as:

$$\bar{Y}_{NRSS}=\frac{1}{k}\sum_{i=1}^{\frac{k}{4}}\left(Y_{\left[\frac{4ik-3k}{2}\right]}+Y_{\left[\frac{2k^2-4ik+3k+2}{2}\right]}\right)+\frac{1}{k}\sum_{i=1}^{\frac{k}{4}}\left(Y_{\left[\frac{4ik+k+2}{2}\right]}+Y_{\left[\frac{2k^2-4ik-k+4}{2}\right]}\right).$$

□

Let us consider the following two cases of symmetric and asymmetric distributions under perfect ranking.

1. Uniform distribution. Suppose that the random variable Y has a uniform $U(0,1)$ distribution. Therefore, the mean and variance of the ith ranked unit $Y_{[i]}$, respectively, are given by $E\left(Y_{[i]}\right)=\frac{i}{k+1}$ and $\operatorname{Var}\left(Y_{[i]}\right)=\frac{i(k-i+1)}{(k+1)^2(k+2)}$.

For $k=6$, we have to select 36 units from the population and then measure only 6 units of them to be a neoteric ranked set sample which are $Y_{[4]},Y_{[9]},Y_{[16]},Y_{[21]},Y_{[28]},Y_{[33]}$. The NRSS mean estimator can be obtained as

$$\bar{Y}_{NRSS}=\frac{1}{6}\left[Y_{[4]}+Y_{[9]}+Y_{[16]}+Y_{[21]}+Y_{[28]}+Y_{[33]}\right].$$

The expectation of this estimator is

$$E\left(\bar{Y}_{NRSS}\right)=\frac{1}{6}\left[E\left(Y_{[4]}\right)+E\left(Y_{[9]}\right)+E\left(Y_{[16]}\right)+E\left(Y_{[21]}\right)+E\left(Y_{[28]}\right)+E\left(Y_{[33]}\right)\right]$$

$$=\frac{1}{6}\left(\frac{4}{37}+\frac{9}{37}+\frac{16}{37}+\frac{21}{37}+\frac{28}{37}+\frac{33}{37}\right)=\frac{1}{6}\left(\frac{111}{37}\right)=0.5,$$

which is an unbiased estimator of the true population mean, $\mu=0.5$. Recall that,

$$Var\left(\bar{Y}_{NRSS}\right)=\frac{1}{k^2}\sum_{i=1}^{k}Var\left(Y_{[(i-1)k+s]}\right)+\frac{2}{k^2}\sum_{i<j}^{k}Cov\left(Y_{[(i-1)k+s]},Y_{[(j-1)k+s]}\right),$$

where for the uniform distribution

$$Cov\left(Y_{[j]},Y_{[i]}\right)=E\left(Y_{[j]}.Y_{[i]}\right)-E\left(Y_{[j]}\right)E\left(Y_{[i]}\right)=\frac{j(k+1-i)}{(k+1)^2(k+2)}.$$

Therefore,

$$\begin{aligned}
&Var\left(\bar{Y}_{NRSS}\right)=\tfrac{1}{36}\left(Var\left(Y_{[4]}\right)+Var\left(Y_{[9]}\right)+Var\left(Y_{[16]}\right)+Var\left(Y_{[21]}\right)+Var\left(Y_{[28]}\right)+Var\left(Y_{[33]}\right)\right)\\
&+\tfrac{2}{36}\begin{pmatrix}Cov\left(Y_{[4]},Y_{[9]}\right)+Cov\left(Y_{[4]},Y_{[16]}\right)+Cov\left(Y_{[4]},Y_{[21]}\right)+Cov\left(Y_{[4]},Y_{[28]}\right)+Cov\left(Y_{[4]},Y_{[33]}\right)+\\
Cov\left(Y_{[9]},Y_{[16]}\right)+Cov\left(Y_{[9]},Y_{[21]}\right)+Cov\left(Y_{[9]},Y_{[28]}\right)+Cov\left(Y_{[9]},Y_{[33]}\right)+Cov\left(Y_{[16]},Y_{[21]}\right)+\\
Cov\left(Y_{[16]},Y_{[28]}\right)+Cov\left(Y_{[16]},Y_{[33]}\right)+Cov\left(Y_{[21]},Y_{[28]}\right)+Cov\left(Y_{[21]},Y_{[33]}\right)+Cov\left(Y_{28},Y_{[33]}\right)\end{pmatrix}\\
&=\frac{1}{36}\left(\frac{66}{26011}+\frac{126}{26011}+\frac{168}{26011}+\frac{168}{26011}+\frac{126}{26011}+\frac{66}{26011}\right)\\
&+\frac{2}{36}\left[\begin{array}{l}\left(\frac{56}{26011}+\frac{42}{26011}+\frac{32}{26011}+\frac{18}{26011}+\frac{8}{26011}\right)+\left(\frac{189}{52022}+\frac{72}{26011}+\frac{81}{52022}+\frac{18}{26011}\right)+\\ \left(\frac{128}{26011}+\frac{72}{26011}+\frac{32}{26011}\right)+\left(\frac{189}{52022}+\frac{42}{26011}\right)+\frac{56}{26011}\end{array}\right]\\
&=\frac{7}{2812}.
\end{aligned}$$

Now, the variance of mean estimator based on a simple random sample of size $k=6$ is $Var\left(\bar{Y}_{SRS}\right)=\frac{\sigma^2}{k}=\frac{1}{12(6)}=\frac{1}{72}$. Therefore, the relative efficiency (RE) of the NRSS estimator with respect to SRS estimator is $RE_1\left(\bar{Y}_{NRSS},\bar{Y}_{SRS}\right)=\frac{MSE\left(\bar{Y}_{SRS}\right)}{MSE\left(\bar{Y}_{NRSS}\right)}=5.5794$, and the RE of the RSS estimator with respect to its SRS counterpart is $RE_2\left(\bar{Y}_{RSS},\bar{Y}_{SRS}\right)=\frac{MSE\left(\bar{Y}_{SRS}\right)}{MSE\left(\bar{Y}_{RSS}\right)}=3.5$.

Exponential distribution. If Y has an exponential distribution with mean 1, then the mean and variance of the ith order statistic, $Y_{[i]}$ are given by

$E\left(Y_{[i]}\right) = \sum_{w=k-i+1}^{k} \frac{1}{w}$, and $\operatorname{Var}\left(Y_{[i]}\right) = \sum_{w=k-i+1}^{k} \frac{1}{w^2}$.

For $m = 5$, we have to select 25 units from the population and then measure only 5 units of them to be a neoteric ranked set sample, which are $Y_{[3]}, Y_{[8]}, Y_{[13]}, Y_{[18]}, Y_{[23]}$. Therefore, the NRSS mean estimator is

$$\bar{Y}_{NRSS} = \frac{1}{5}\left[Y_{[3]} + Y_{[8]} + Y_{[13]} + Y_{[18]} + Y_{[23]}\right].$$

The expectation of this estimator is

$$E\left(\bar{Y}_{NRSS}\right) = \frac{1}{5}\left[E\left(Y_{[3]}\right) + E\left(Y_{[8]}\right) + E\left(Y_{[13]}\right) + E\left(Y_{[18]}\right) + E\left(Y_{[23]}\right)\right]$$

$$= \frac{1}{5}\left(\frac{1727}{13800} + \frac{22798213}{60568200} + \frac{19081066231}{26771144400} + \frac{10914604807}{8923714800} + \frac{20666950267}{8923714800}\right)$$

$$= \frac{1}{5}\left(\frac{5090112581}{1070845776}\right) = 0.950671,$$

where $E\left(Y_{[3]}\right) = \sum_{w=23}^{25} \frac{1}{w} = \frac{1727}{13800}$, $E\left(Y_{[8]}\right) = \sum_{w=18}^{25} \frac{1}{w} = \frac{22798213}{60568200}$, $E\left(Y_{[13]}\right) = \sum_{w=13}^{25} \frac{1}{w} = \frac{19081066231}{26771144400}$, $E\left(Y_{[18]}\right) = \sum_{w=8}^{25} \frac{1}{w} = \frac{10914604807}{8923714800}$, $E\left(Y_{[23]}\right) = \sum_{w=3}^{25} \frac{1}{w} = \frac{20666950267}{8923714800}$.

It can be seen that this estimator is biased with $Bias\left(\bar{Y}_{NRSS}\right) = -0.0493285$, which is very quite close to the bias value -0.05 obtained in Table 2, when $\rho = 1$.

The suggested NRSS estimator of the population variance is given by

$$(2.3) \qquad S^2_{NRSS} = \frac{1}{nk-1} \sum_{j=1}^{n} \sum_{i=1}^{k} \left(Y_{[(i-1)k+l]j} - \bar{Y}_{NRSS}\right)^2$$

It is of interest to note here that S^2_{NRSS} has a negligible bias of the population variance, which approaches to zero in most cases.

3. Monte Carlo Comparison

In this section, the performances of the proposed mean and variance estimators based on NRSS are compared with their counterparts using RSS and SRS methods. As we mentioned before, we only measure on $N = nk$ units using NRSS and RSS methods, to compare them with N units using SRS method.

For Monte Carlo simulation, we have used the model of imperfect ranking suggested by Dell and Clutter (1972), assuming (Z, X) follows a standard bivariate normal distribution with correlation coefficient ρ. Then, we take $Y = Z$, $\Phi(Z)$, $\log\left[\frac{\Phi(Z)}{1-\Phi(Z)}\right]$, $-\log\left[\Phi(Z)\right]$ and $\left[\Phi(Z)\right]^5$ as the variable of interest, where $\Phi(.)$ is the cdf of the standard normal distribution. Therefore, we allow the relation between the interest variable (Y) and the auxiliary variable (X) to be linear or non-linear, and the parent distributions to be Normal (0,1), Uniform (0,1), Logistic (0,1), Exponential (1) and Beta (0.2,1), respectively. Thus we have considered both symmetric and asymmetric distributions with bounded and unbounded supports in our simulation study.

The values of ρ are 0, 0.2, 0.4, 0.6, 0.8, 1. Without loss of generality, we assumed that the ranking is based on X. Therefore, as ρ gets large to 1, the ranking approaches to

Table 1. The relative efficiencies of NRSS mean estimator to SRS mean estimator (RE_1) and RSS mean estimator to SRS mean estimator (RE_2) for different values of (N, k).

Parent Distribution		Normal(0,1)		Uniform(0,1)		Logistic(0,1)		Exponential(1)		Beta(0.2,1)	
(N,k)	ρ	RE1	RE2	RE1	RE2	RE1	RE2	RE1	RE2	RE1	RE2
(10,5)	0	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
	0.2	1.04	1.03	1.03	1.03	1.03	1.03	1.03	1.02	1.02	1.01
	0.4	1.14	1.11	1.13	1.12	1.15	1.11	1.14	1.11	1.1	1.08
	0.6	1.40	1.28	1.34	1.30	1.42	1.29	1.38	1.24	1.29	1.21
	0.8	2.02	1.68	1.94	1.71	2.05	1.66	1.92	1.51	1.71	1.47
	1	4.75	2.78	4.68	3.00	4.88	2.56	4.37	2.16	4.05	2.14
(10,10)	0	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
	0.2	1.03	1.03	1.03	1.03	1.03	1.03	1.03	1.03	1.03	1.02
	0.4	1.17	1.14	1.16	1.14	1.18	1.14	1.14	1.11	1.12	1.10
	0.6	1.47	1.4	1.43	1.39	1.51	1.39	1.43	1.32	1.32	1.27
	0.8	2.37	2.03	2.25	2.03	2.38	1.99	2.19	1.77	1.94	1.71
	1	9.78	4.82	9.71	5.50	9.99	4.2	9.00	3.43	8.94	3.53
(20,5)	0	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
	0.2	1.03	1.02	1.03	1.03	1.03	1.03	1.03	1.02	1.02	1.01
	0.4	1.15	1.11	1.12	1.11	1.16	1.12	1.13	1.09	1.10	1.08
	0.6	1.41	1.31	1.35	1.27	1.42	1.29	1.37	1.23	1.29	1.21
	0.8	2.03	1.68	1.90	1.7	2.06	1.64	1.90	1.53	1.68	1.47
	1	4.74	2.78	4.66	3.00	4.89	2.58	3.99	2.19	3.93	2.12
(20,10)	0	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
	0.2	1.03	1.03	1.05	1.04	1.03	1.02	1.03	1.02	1.02	1.03
	0.4	1.16	1.13	1.15	1.14	1.17	1.13	1.15	1.12	1.11	1.09
	0.6	1.48	1.40	1.44	1.39	1.49	1.39	1.43	1.32	1.32	1.27
	0.8	2.35	2.02	2.24	2.03	2.37	1.96	2.16	1.79	1.93	1.70
	1	9.71	4.79	9.74	5.50	9.92	4.21	8.00	3.41	8.84	3.52

completely perfect. The relative efficiency (RE) of NRSS and RSS with respect to SRS is defined as

$$RE_1\left(\bar{Y}_{NRSS}, \bar{Y}_{SRS}\right) = \frac{MSE\left(\bar{Y}_{SRS}\right)}{MSE\left(\bar{Y}_{NRSS}\right)}, RE_2\left(\bar{Y}_{RSS}, \bar{Y}_{SRS}\right) = \frac{MSE\left(\bar{Y}_{SRS}\right)}{MSE\left(\bar{Y}_{RSS}\right)},$$

$$RE_3\left(S^2_{Stokes}, S^2_{SRS}\right) = \frac{MSE\left(S^2_{SRS}\right)}{MSE\left(S^2_{Stokes}\right)}, RE_4\left(S^2_M, S^2_{SRS}\right) = \frac{MSE\left(S^2_{SRS}\right)}{MSE\left(S^2_M\right)},$$

$$RE_5\left(S^2_{NRSS}, S^2_{SRS}\right) = \frac{MSE\left(S^2_{SRS}\right)}{MSE\left(S^2_{NRSS}\right)}.$$

where $MSE\left(\hat{\theta}\right) = Var\left(\hat{\theta}\right) + \left[Bias\left(\hat{\theta}\right)\right]^2$.

The values of (N, k) are selected to be $(10, 5), (10, 10), (20, 5), (20, 10)$. So, we can assess the effect of increasing total sample size for fixed k, and the effect of increasing k when the total sample size is fixed. The number of repetitions in the simulation study is set to be 100,000 for each sample size. The results are reported in Tables 1-4 for estimating the population mean and variance.

Table 2. Estimated biases of $\bar{Y}_{NRSS}$ for estimation of the population mean for different values of (N, k).

	(N,k)	(10,5)						(10,10)					
	ρ	0	0.2	0.4	0.6	0.8	1	0	0.2	0.4	0.6	0.8	1
Parent	Exponential (1)	0.00	0.00	-0.01	-0.02	-0.03	-0.05	0	0	-0.01	-0.01	-0.02	-0.04
Distribution	Beta(0.2,1)	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
	(N,k)	(20,5)						(20,10)					
	ρ	0	0.2	0.4	0.6	0.8	1	0	0.2	0.4	0.6	0.8	1
Parent	Exponential (1)	0	0	-0.01	-0.02	-0.03	-0.05	0	0	0.01	-0.01	-0.02	-0.04
Distribution	Beta(0.2,1)	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00

Table 3. The relative efficiencies of S^2_{NRSS} to S^2_{SRS} (RE_3), S^2_{Stokes} to S^2_{SRS} (RE_4) and S^2_M to S^2_{SRS} (RE_5) for different values of (N,k).

Parent Distribution		Normal(0,1)			Uniform(0,1)			Logistic(0,1)			Exponential(1)			Beta(0.2,1)		
(N,k)	ρ	RE3	RE4	RE5	RE3	RE4	RE5	RE3	RE4	RE5	RE3	RE4	RE5	RE3	RE4	RE5
	0	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
	0.2	1.02	1.00	1.00	1.00	1.00	1.00	1.01	1.00	1.00	1.01	1.00	1.00	1.01	1.00	1.00
(10,5)	0.4	1.04	1.01	1.01	1.01	1	1.02	1.09	1	1.03	1.15	1.05	1.07	1.05	1.01	1.04
	0.6	1.18	1.02	1.06	1.09	1.02	1.10	1.25	1.00	1.04	1.36	1.03	1.08	1.13	1.05	1.11
	0.8	1.56	1.03	1.17	1.33	1.08	1.26	1.77	1.01	1.10	2.10	1.03	1.11	1.35	1.11	1.23
	1	4.05	1.17	1.40	3.66	1.32	1.71	4.08	1.10	1.27	5.44	1.09	1.21	2.86	1.26	1.48
	0	1.00	1.00	-	1.00	1.00	-	1.01	1.00	-	1.00	1.00	-	1.00	1.00	-
	0.2	1.01	1.01	-	1.00	1.00	-	1.01	1.00	-	1.00	1.00	-	1.01	1.01	-
(10,10)	0.4	1.02	0.98	-	1.03	1.00	-	1.08	1.00	-	1.05	1.03	-	1.05	1.02	-
	0.6	1.16	1.00	-	1.10	1.03	-	1.23	1.00	-	1.32	1.05	-	1.14	1.07	-
	0.8	1.62	1.11	-	1.39	1.18	-	1.85	1.09	-	2.11	1.10	-	1.45	1.21	-
	1	9.14	1.55	-	5.68	1.9	-	8.52	1.35	-	9.97	1.20	-	6.35	1.69	-
	0	1.01	1.01	1.01	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
	0.2	1.02	1.01	1.02	1.00	1.00	1.00	1.03	1.00	1.00	1.03	1.00	1.01	1.01	1.00	1.00
(20,5)	0.4	1.05	1.02	1.01	1.02	1	1.02	1.08	1.01	1.01	1.14	1.02	1.03	1.06	1.03	1.04
	0.6	1.17	1.01	1.05	1.07	1.02	1.07	1.25	1.02	1.03	1.39	1.04	1.06	1.16	1.08	1.11
	0.8	1.48	1.07	1.14	1.31	1.10	1.2	1.65	1.05	1.11	1.96	1.05	1.09	1.36	1.16	1.22
	1	2.65	1.22	1.34	4.02	1.36	1.55	2.35	1.13	1.21	3.49	1.12	1.18	2.69	1.33	1.44
	0	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
	0.2	1.02	1.01	1.02	1.00	1.00	1.00	1.01	1.01	1.01	1.03	1.01	1.01	1.01	1.01	1.01
(20,10)	0.4	1.04	1.01	1.01	1.04	1.01	1.04	1.07	1.02	1.01	1.11	1.01	1.01	1.06	1.03	1.05
	0.6	1.17	1.02	1.07	1.10	1.04	1.10	1.24	1.03	1.07	1.36	1.05	1.08	1.17	1.11	1.15
	0.8	1.63	1.17	1.27	1.42	1.22	1.34	1.8	1.11	1.19	2.08	1.13	1.19	1.5	1.28	1.37
	1	5.98	1.65	1.85	7.66	2.07	2.49	4.42	1.42	1.55	5.86	1.26	1.35	6.6	1.83	2.03

Table 4. Estimated biases of variance estimators S^2_{NRSS} and S^2_{Stokes} for different values of (N,k).

Parent Distribution		Normal(0,1)		Uniform(0,1)		Logistic(0,1)		Exponential(1)		Beta(0.2,1)	
(N,k)	ρ	Bias of S^2_{NRSS}	Bias of S^2_{Stokes}	Bias of S^2_{NRSS}	Bias of S^2_{Stokes}	Bias of S^2_{NRSS}	Bias of S^2_{Stokes}	Bias of S^2_{NRSS}	Bias of S^2_{Stokes}	Bias of S^2_{NRSS}	Bias of S^2_{Stokes}
(10,5)	0	0.00	0.00	0.00	0.00	0.00	0.01	0.00	0.00	0.00	0.00
	0.2	0.00	0.00	0.00	0.00	-0.02	0.01	0.00	0.00	0.00	0.00
	0.4	-0.02	0.01	0.00	0.00	-0.07	0.04	-0.03	0.01	0.00	0.00
	0.6	-0.04	0.02	0.00	0.00	-0.18	0.08	-0.07	0.02	0.00	0.00
	0.8	-0.07	0.04	0.00	0.00	-0.35	0.15	-0.14	0.04	0.00	0.00
	1	-0.10	0.07	0.00	0.00	-0.60	0.23	-0.24	0.06	0.00	0.00
(10,10)	0	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
	0.2	0.00	0.00	0.00	0.00	0.00	0.01	0.00	0.00	0.00	0.00
	0.4	-0.01	0.02	0.00	0.00	-0.04	0.03	-0.02	0.01	0.00	0.00
	0.6	-0.02	0.03	0.00	0.00	-0.09	0.09	-0.05	0.02	0.00	0.00
	0.8	-0.03	0.06	0.00	0.00	-0.21	0.19	-0.09	0.05	0.00	0.00
	1	-0.04	0.09	0.00	0.00	-0.39	0.28	-0.17	0.08	0.00	0.00
(20,5)	0	0.00	0.00	0.00	0.00	0.01	0.00	0.00	0.00	0.00	0.00
	0.2	-0.01	0.00	0.00	0.00	-0.02	0.01	-0.01	0.00	0.00	0.00
	0.4	-0.02	0.00	0.00	0.00	-0.10	0.02	-0.04	0.00	0.00	0.00
	0.6	-0.05	0.01	0.00	0.00	-0.23	0.05	-0.08	0.01	0.00	0.00
	0.8	-0.09	0.02	0.00	0.00	-0.43	0.06	-0.16	0.02	0.00	0.00
	1	-0.14	0.03	0.00	0.00	-0.70	0.11	-0.27	0.03	0.00	0.00
(20,10)	0	0.00	0.00	0.00	0.00	-0.01	0.01	0.00	0.00	0.00	0.00
	0.2	0.00	0.00	0.00	0.00	-0.02	0.00	0.00	0.00	0.00	0.00
	0.4	-0.01	0.01	0.00	0.00	-0.07	0.02	-0.02	0.01	0.00	0.00
	0.6	-0.03	0.01	0.00	0.00	-0.15	0.05	-0.06	0.01	0.00	0.00
	0.8	-0.05	0.03	0.00	0.00	-0.30	0.10	-0.12	0.02	0.00	0.00
	1	-0.09	0.04	0.00	0.00	-0.52	0.13	-0.21	0.04	0.00	0.00

Table 1 gives the relative efficiencies of the mean estimators based on NRSS and RSS schemes to SRS mean estimator for different distributions. We observe that when rankings are prefect ($\rho = 1$), the efficiency gain in using NRSS mean estimator is approximately two times higher than the mean estimator based on RSS scheme. Furthermore, the performance of NRSS mean estimator for the symmetric distributions is slightly better than the asymmetric distributions, and the best performance of NRSS mean estimator is for logistic distribution. It is clear from this table that the effect of the imperfect ranking on $\bar{Y}_{NRSS}$ is more than $\bar{Y}_{RSS}$, however, even in the case of imperfect ranking ($\rho \leq 0.8$), the $\bar{Y}_{NRSS}$ is still superior to $\bar{Y}_{RSS}$ for $\rho \geq 0.4$, and it is as efficient as $\bar{Y}_{RSS}$ for $\rho \leq 0.4$. When the rankings are completely random ($\rho = 0$), all estimators have the same performances. This can be justified by the fact that in the case of random rankings, RSS and NRSS schemes are intrinsically the same as SRS design. It is worth mentioning that in all considered cases, the relative efficiencies of $\bar{Y}_{NRSS}$ and $\bar{Y}_{RSS}$ increase as the set size (k) increases for fixed sample size (N).

Table 2 presents the estimated biases of the NRSS mean estimator for asymmetric distributions. We observe that the proposed mean estimator slightly underestimates the true population mean when the parent distribution is standard exponential and $\rho \geq 0.4$. Furthermore, the bias of $\bar{Y}_{NRSS}$ decreases in absolute value when set size (k) increases or the correlation of coefficient (ρ) decreases. In the case of the parent distribution being Beta(0.2,1), the NRSS mean estimator is almost unbiased.

The relative efficiencies of different variance estimators S^2_{NRSS} and S^2_{Stokes} to S^2_{SRS} are presented in Table 3. It is clear from this table that the performance of S^2_{NRSS} dominates all other estimators considered here when the rankings are perfect ($\rho = 1$), and S^2_{NRSS} performs at least twice as good as its competitors in RSS scheme. Although the imperfect ranking has more negative effect on S^2_{NRSS} than S^2_{Stokes} and S^2_M, S^2_{NRSS} is still superior to its RSS competitors for $\rho \leq 0.8$. Furthermore, we also observe that the relative efficiencies increase as the set size (k) increases for fixed sample size (N).

The estimated bias values of S^2_{NRSS} and S^2_{Stokes} are given in Table 4. We observe that for standard uniform and Beta(0.2,1) distributions, S^2_{NRSS} and S^2_{Stokes} are almost unbiased. However, for standard normal, standard exponential and standard logistic distributions, S^2_{Stokes} overestimates true population variance and S^2_{NRSS} underestimates σ^2. It is also evident that the bias of S^2_{NRSS} is larger than the bias of S^2_{Stokes} in absolute value. Furthermore, we observe that the biases of S^2_{Stokes} and S^2_{NRSS} decrease in absolute value as ρ decreases.

4. A real data set

In this section, a real data set is considered to illustrate the performance of NRSS method in estimating the population mean and variance. The data set consists of the percentage of body fat determined by underwater weighing and various body circumference measurements for 252 men. For more details about these data, see http://lib.stat.cmu.edu/datasets/bodyfat. We take the percentage of body fat as the interest variable (Y) and abdomen circumference as concomitant variable (X). Sampling with replacement is considered, so the assumption of independence is covered. The mean and variance of the target variable Y in the population are $\mu_Y = 19.15$ and $\sigma^2_Y = 70.03$, respectively, and the correlation of coefficient between the two variables is $\rho_{XY} = 0.81$. To select a sample of size 10, using using both RSS and NRSS designs, the following steps are carry out:

I. Select a bivariate simple random sample of size 25 of (X, Y).

II. On basis of NRSS, rank the X values and use their ordering for Y. Then, select the 3rd, 8th, 13th, 18th and 23rd judgment ranked values of Y for actual quantification to

Table 5. The values of the variable of interest Y using NRSS, RSS and SRS designs.

NRSS	25.5	5.3	19.7	27.2	27.0	15.1	5.7	22.9	26.0	32.3
RSS	27.3	18.5	19.7	27.0	18.5	31.6	10.6	15.2	10.6	15.2
SRS	0.7	29.6	26.7	11.5	19.2	27.3	17.5	16.5	3.0	20.5

constitute a neoteric ranked set sample of size 5.
III. For RSS, divide the 25 SRS observations into 5 sets each of size 5. Then, use the true ranked X values to rank the values of Y within each set of size 5 units. Finally, select the ith judgment ranked values of Y from the ith sample ($i = 1, \cdots, 5$).
IV. Repeat Steps I to III two times to have a sample of size 10 from NRSS and RSS designs.
Also, a simple random sample of size 10 is selected from the same population. The results of measured values in NRSS, RSS and SRS designs are presented in Table 5.
The above results in Table 5 showed that

$$\bar{Y}_{NRSS} = 20.67,\ \bar{Y}_{RSS} = 19.42,\ \bar{Y}_{SRS} = 17.25,$$

$$S^2_{NRSS} = 85.22,,\ S^2_{Stokes} = 51.20,\ S^2_M = 49.89,\ S^2_{SRS} = 96.42.$$

Our results showed that the means of 100000 repeated values of the suggested estimators are all quite close to the real population parameters. For example, $Bias\left(\bar{Y}_{NRSS}\right) = 20.67 - 19.15 = 1.52$, and $Bias\left(S^2_{NRSS}\right) = 85.22 - 70.03 = 15.19$, which are more better than the SRS estimators. Also, the NRSS variance estimator is more efficient than its counterparts in Stokes [20], and MacEachern et al. [13].

5. Conclusion

In this paper, a new modification of the usual RSS is suggested for estimating the population mean and variance. The suggested estimators are compared with their competitors in SRS method. Our simulation results indicate that the suggested empirical mean and variance estimates are strongly better than their competitors in RSS and SRS designs for the same number of measured units with perfect ranking. In the case of imperfect rankings, the NRSS estimators are still superior to their counterparts in the RSS and SRS design and their superiority decrease as the quality of rankings decreases. We prove that the NRSS mean estimator is unbiased when the parent distribution is symmetric. For asymmetric distributions, the simulation results indicate that the NRSS mean estimator is slightly biased. Thus, based on the above observations, the NRSS can be recommended for estimating the population parameters due to its efficiency with respect to SRS and RSS methods.

In this paper, we consider the problem of estimation of mean and variance based on the NRSS. One can use the NRSS scheme for estimation of cumulative distribution function and population quantiles. It is also interesting to investigate the performance of goodness of fit tests based on empirical distribution function (e.g. Kolmogorov-Smirnov, Anderson–Darling, etc) NRSS and compare them with their counterparts in the RSS and SRS designs.

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