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Monte Carlo Evaluation of the Methods Estimating Structural Change Point in Panel Data

Selim Dağlıoğlu*[1] Mehmet Akif Bakır[2]

Abstract

In this study, we investigate the existence of structural break in a panel data consisting of N time series of T unit length, and the estimation performance of Simple Mean Shift Model, Fluctuation Test, Wald Statistic Test, Kim Test which are based on common break assumption are examined to determine the break date. In this context, 108 Monte Carlo simulations are performed, each of which consisted of 3000 repetitions for the factors number of cross-sections, time dimension, break size and break rate factors, which are considered to influence the performance of the tests. As a result of the Monte Carlo simulations, the Simple Mean Shift Model approach predicts the break point with a higher performance than the other methods. In addition, if the breakpoints are at the midpoint of the series, the Wald Statistic and Kim Tests show the highest performances, while the Fluctuation Test shows the highest breakpoint predictive performance if break occur in the third quarter of the series. Generally, as the number of cross-sections increases, the estimation performance of the tests increases, whereas as the time dimension increases, the performance of methods other than the Simple Mean Shift Model decreases. As a final point, it has been observed that there is no significant effect of the break size on the predictive performance of the methods.

Keywords: Panel data, structural break, estimation of break point, Monte Carlo simulations.

1. INTRODUCTION

Structural break(s) are permanent changes in the structure of variables, due to permanent effects of economic or financial shocks, policy changes, cultural and technological changes, etc., on the distribution of variables. Changes in the behaviour of economic time series such as employment, growth and unemployment can occur in the long run due to policy changes and various economic events. However, when the models used in examining the data for such variables are estimated, it is usually assumed that the model parameters do not change over the sampling periods. This assumption makes the analysis relatively simple. However, the assumption that a time series is not subject to a change throughout the sample becomes more difficult to achieve as the length of the series increases. In the case of structural breaks in series, continuing analysis without considering this structural change can lead to incorrect estimations of model parameters. A typical example of this is the investigation of the presence of unit root in Nelson and Plosser data; Nelson and Plosser [1], Perron [2], Zivot and Andrews [3] and Lumsdaine and Papell [4] have achieved different results. Despite the use of the same data set in these

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studies, the results differ depending on whether structural breaks are taken into account and whether structural breaks are included in the model.

The time series consists of observations obtained over a single cross-sectional unit at different times. Policy or technology changes often lead to permanent changes in the structure of the time series. For this reason, structural breaks in time series are generally encountered. However, some difficulties arise when estimating the point of break in the time series. If a structural change occurs at any time point k_0 in the time series y_t, the change point k_0 can't be consistently estimated, regardless of how large the sample is, and the estimator $\hat{k}$ of the breakpoint k_0 is not consistent. For this reason, it is usually attempted to estimate the break fraction instead of estimating the k_0's in which the structural change occurs in the time series. The effectiveness of the approach using a single time series in determining structural break depends on two assumptions: First, the magnitude of structural break (the difference between pre-break mean and post-break mean) is large enough. The second is that the true point of break point k_0 is far enough from the beginning and end of the sample. In a single series it is impossible to identify break point when the regime has a single observation [5, 6]. In the study of both single and multiple structural breakpoints in time series, asymptotic framework is used in which the magnitudes of change(s) asymptotically converge to zero as the sample size increases in order to obtain critical statistics [7]. In other words, obtaining the limit distribution of the test statistics requires the assumption that the size of the structural break decreases as the sample size increases [8]. In the structural break literature this assumption is called the shrinking magnitude of structural break assumption. According to this assumption, as the sample size increases in the time series, the point of change can be determined [9]. The necessity of both the change point inconsistency and the reduced break assumption is related to the problem of defining the break point in time series models. The main reason for these two situations to emerge is that time series can't carry enough information. Additional information is needed in order to determine the actual break point in the time series. This information is tried to be obtained by increasing the sample size. When examining structural break in panel data, the additional information carried by the cross-sectional dimension of panel data eliminate the necessity of artificially increasing the number of observations using the reduced shrinking magnitude of structural break assumption. In addition, panel data can be used to derive asymptotics around the actual break-up date, since it has the cross-sectional dimension as well as the time dimension [9].

Since the panel data has both the cross section and the time dimension, the structural break in the panel data would occur in the cross section dimension as well as in the time dimension. The structural change in the panel data can occur when a group of the panel-forming cross-sectional units has a common equation while the remainder has a different equation. The occurrence of breaks in the time dimension or cross-sectional dimension due to the presence of both cross-section and time dimension of the panel data has led to different studies on the investigation of the presence of structural break and break point in the panel data. Two approaches have been adopted in panel studies in relation to the assumptions made about the position of structural break in the data. The first is the use of the assumption that structural breaks in all series of the panel have emerged in a common date. The second is the use of the random breakpoint assumption that breaks are allowed to occur on a different date for each series depending on the distribution of the random variable. The methods for which the random breakpoint assumption is used are more complicated than the methods considering the common breakpoint hypothesis.

The assumption of the common break point has been used in the studies of Han and Park [10], Joseph and Wolfson [11], Bai [12], Bai et al. [13], Emerson and Kao [14], Bai and Perron [15], Kao et al. [16], Feng et al. [9], Kim [17], Horváth and Hušková [18], Chan et al. [19] and Li et al. [20]. On the other hand, the assumption of random break point is considered in studies such as Joseph and Wolfson [11], Joseph and Wolfson [21], Joseph et al. [22], Joseph et al. [23], Joseph et al. [24] and Liao [6].

While there have been various methods developed in the literature on structural breaks in panel data, no study has been found on the comparison of the performance of these methods in the context of determining breaking point. The contribution of this study is to compare the break point estimation performance of some methods used to determine the structural break date under the assumption of the common breakpoint, according to the factors the number of cross sections, time series dimension, break size and break fraction. In this context, with the aid of Monte Carlo simulations, the Simple Mean Shift Model Method proposed in Bai [5], the Fluctuation Test and the Wald Statistic Test proposed in Emerson and Kao [14] and the Kim Test proposed in Kim [17] performance are evaluated.

In the next section of the study, the performances of the considered methods predicting the breakpoint are discussed. In the third section of the study, the data generating process and the issues considered in determination of factor levels and the assumption of Monte Carlo simulation are explained. In the fourth part of the study, the results obtained by Monte Carlo simulations are given. In the fifth and last part, the results obtained in the study are discussed and some suggestions are made.

2. METHODS OF DETERMINING BREAKPOINT

Bai [5] considers following simple mean shift model:

$$\begin{aligned} y_{it} &= \mu_{i1} + u_{it} \quad t = 1, 2, \dots, k_0 \\ y_{it} &= \mu_{i2} + u_{it} \quad t = k_0 + 1, \dots, T \end{aligned} \tag{1}$$

where $E(u_{it}) = 0$ for all i and t. In this model, each series has a break point at k_0, where k_0 is unknown. The pre-break mean of y_{it} is μ_{i1} and post-break mean is μ_{i2}. For the simple mean shift model, he proposes the OLS estimator of k_0 as follows:

$$\hat{k} = \underset{1 \le k \le T-1}{\operatorname{argmin}} SSR(k) \tag{2}$$

where $SSR_{iT}(k)$ is

$$SSR_{iT}(k) = \begin{cases} SS(y_{i1}) + SS(y_{i2}) & k = 1, \dots, T-1 \\ SS(y_i) & k = T \end{cases} \tag{3}$$

for each $k = 1, \dots, T$. Where $SS(y_{i1})$, $SS(y_{i1})$ and $SS(y_{i1})$ are defined as $SS(y_{i1}) = \sum_{t=1}^{k}(y_{it} - \bar{y}_{i1})^2$, $SS(y_{i2}) = \sum_{t=k+1}^{T}(y_{it} - \bar{y}_{i2})^2$ and $SS(y_i) = \sum_{t=1}^{T}(y_{it} - \bar{y}_i)^2$, respectively. Also, $\bar{y}_i$ is the average of all the observations of the unit of cross-section,

$$\begin{aligned} \bar{y}_{i1} &= \frac{1}{k}\sum_{t=1}^{k} y_{it} \\ \bar{y}_{i2} &= \frac{1}{T-k}\sum_{t=k+1}^{T} y_{it} \end{aligned} \tag{4}$$

and sum of residual squares over all equations is as follows:

$$SSR(k) = \sum_{i=1}^{N} SSR_{iT}(k). \tag{5}$$

Emerson and Kao [14] consider following the one-way random effect panel regression model with the deterministic time trend:

$$y_{it} = \alpha + \beta_t X_t + v_{it} \tag{6}$$

$$v_{it} = \mu_i + u_{it} \tag{7}$$

where β is the slope parameter, $X_t = \frac{t}{T}$, unobservable individual effects $\mu_i \sim iid(0, \sigma_\mu^2)$ and disturbance term of AR(1) $u_{it} = \rho u_{it-1} + \varepsilon_{it}, \varepsilon_{it} \sim iid(0, \sigma_\varepsilon^2)$. They propose two different methods for testing the following null hypothesis

$$H_0: \beta_t = \beta; for\ \forall t \tag{8}$$

meaning that there is no change in the model against the following alternative hypothesis

$$H_1: \beta_t = \begin{cases} \beta_1 & t = 1, \dots, k \\ \beta_2 & t = k+1, \dots, T \end{cases} \tag{9}$$

meaning that there exists a change in the k-point.

They proposed to estimate the time point of structural change according to these two methods. The first is the method based on the fluctuation test of Ploberger et al. [25]. The second method is based on the mean statistics of Andrew and Ploberger [26] and exponential Wald statistic and the Wald statistic of Andrew [27]. In testing null hypothesis with fluctuation test, if the difference

$$\max_{i=1,\dots,k}\left|\hat{\beta}_k-\hat{\beta}_T\right| \quad (10)$$

is big enough, that is when $\hat{\beta}_k$ is too much fluctuating, the null hypothesis is rejected. In other words, there is a structural break at this point and ve $\left|\hat{\beta}_k-\hat{\beta}_T\right|$ is the estimate of the date of break. In the Equation 10, $\hat{\beta}_T$ denotes the estimate of the slope parameter estimated by the OLS method over all panel data and $\hat{\beta}_k$, which is estimated with recursive OLS, is

$$\hat{\beta}_k=\frac{\sum_{i=1}^{N}\left[\sum_{t=1}^{k}(X_t-\bar{X}_k)y_{it}\right]}{\sum_{i=1}^{N}\sum_{t=1}^{k}(X_t-\bar{X}_k)^2} \quad (11)$$

where

$$\bar{X}_k=\frac{1}{k}\sum_{t=1}^{k}X_t$$

In the Wald statistic test, the breaking point is estimated to be

$$\hat{k}=\underset{[Tr^*]\le k\le T-[Tr^*]}{\operatorname{argmax}} W_1(k). \quad (12)$$

Here,

$$\tilde{\sigma}_u^2=\frac{1}{NT}\sum_{i=1}^{N}\sum_{t=1}^{T}(v_{it}-\bar{v}_i)^2 \quad (13)$$

and the estimation of σ_0^2 is

$$\sigma_0^2=\frac{\sigma_\varepsilon^2}{(1-\rho)^2} \quad (14)$$

and thus,

$$W_1(k)=\frac{\tilde{\sigma}_u^2}{3\sigma_0^2}W(k). \quad (15)$$

In addition,

$$\hat{\beta}_{1k}=\frac{\sum_{i=1}^{N}\left[\sum_{t=1}^{k}(X_t-\bar{X}_{1k})y_{it}\right]}{\sum_{i=1}^{N}\sum_{t=1}^{k}(X_t-X_{1k})^2} \quad (16)$$

$$\hat{\beta}_{2k}=\frac{\sum_{i=1}^{N}\left[\sum_{t=k+1}^{T}(X_t-\bar{X}_{2k})y_{it}\right]}{\sum_{i=1}^{N}\sum_{t=k+1}^{T}(X_t-\bar{X}_{2k})^2} \quad (17)$$

$$\bar{X}_{1k}=\frac{1}{k}\sum_{t=1}^{k}X_t$$

and

$$\bar{X}_{2k}=\frac{1}{T-k}\sum_{t=k+1}^{T}X_t$$

and then $W(k)$ is calculated as follows:

$$W(k)=\frac{1}{\hat{\sigma}_u^2}A(k). \quad (18)$$

$A(k)$ is defined as

$$A(k)=\frac{\left(\hat{\beta}_{1k}-\hat{\beta}_{2k}\right)^2}{\left(\sum_{i=1}^{N}SS(X_{1k})\right)^{-1}+\left(\sum_{i=1}^{N}SS(X_{2k})\right)^{-1}}$$

where $SS(X_{1k})=\sum_{t=1}^{k}(X_t-\bar{X}_{1k})^2$ and $SS(X_{2k})=\sum_{t=k+1}^{T}(X_t-\bar{X}_{2k})^2$.

Kim [17] considers following model with the deterministic trend and the disturbance component

$$y_{it}=d_{it}+u_{it} \quad (19)$$

where $i=1,\dots,N$ and $t=1,\dots,T$. The deterministic component d_{ti} can be considered in three different ways to be

$$d_{ti}=\begin{cases}\mu_i+\beta_i t+\gamma_i B_t & \text{Model I}\\ \mu_i+\beta_i t+\theta_i C_t+\gamma_i B_t & \text{Model II}\\ \mu_i+\beta_i t+\theta_i C_t & \text{Model III}\end{cases} \quad (20)$$

where

$$C_t=\begin{cases}0 & t\le k_0\\ 1 & t>k_0\end{cases} \quad (21)$$

and

$$B_t=(t-k_0)C_t. \quad (22)$$

Here, Equation 21 can be rewritten for all of three models, if $t\le k_0$, to be

$$d_{ti}=\mu_i+\beta_i t$$

and if $t>k_0$, then

$$d_{ti}=\begin{cases}\mu_i-k_0\gamma_i+(\beta_i+\gamma_i)t & \text{Model I}\\ \mu_i-k_0\gamma_i+\theta_i+(\beta_i+\gamma_i)t & \text{Model II}\\ \mu_i+\beta_i t+\theta_i & \text{Model III}\end{cases}$$

where Model I is called Joint Broken Trend Model, Model II is called Locally Broken Trend Model and Model III is called Mean Shift Model. Models I and II are extended to the panel data models of the models reviewed by Perron and Zhu [28] for the univariate case. Model III, on the other hand, is an extended form so as to include a deterministic trend of the mean shift model examined in Bai [5].

The regression coefficients in the model are not constrained to be common for each section. For this reason, instead of estimating the regression coefficients jointly by combining the cross-section data, the regression coefficients can be estimated separately for each equation using the OLS method. Thus, in the Kim Test, the individual OLS estimators of the regression coefficients for each equation are used for each cross section [17].

In the Kim test, it is assumed that the actual breaking date is unknown and the broken fraction defined as $\lambda_1 = k_0/T; \lambda_1 \in [\pi, 1-\pi], \pi \in (0, 1/2)$ is constant for every T. It is also assumed that the break date k_0 is common to all equations and that the broken fraction λ_1 remains constant as the sample size grows.

Using the deterministic time trend definitions given in Equation 20, the model in Equation 19 can be rewritten with matrix notation for each equation as

$$Y_i = X_{k_0}\Pi_i + U_i \tag{23}$$

where Y_i and U_i are $(T \times 1)$ dimensional vectors such as $Y_i = (y_{i1}, \dots, y_{iT})'$ and $U_i = (u_{i1}, \dots, u_{iT})'$, respectively, X_{k_0} is $(T \times 3)$or $(T \times 4)$ dimensional matrix and Π_i is (3×1)or (4×1) dimensional matrix. The variables and coefficient of Equation 23 are defined as follows:

$$X_{k_0} = \begin{cases} [\iota, \tau, B] & Model\ I \\ [\iota, \tau, C, B] & Model\ II, \\ [\iota, \tau, C] & Model\ III \end{cases}$$

$$\Pi_i = \begin{cases} (\mu_i, \beta_i, \gamma_i)' & Model\ I \\ (\mu_i, \beta_i, \theta_i, \gamma_i)' & Model\ II, \\ (\mu_i, \beta_i, \theta_i)' & Model\ III \end{cases}$$

where $\iota = (1, \dots, 1)'$, $\tau = (1, \dots, T)'$, $C = (C_1, \dots, C_T)'$, $B = (B_1, \dots, B_T)'$, X_{k_0} is the collection of all dependent variables and Π_i is the regression coefficient for the corresponding equation.

Then, the whole N equation system can be written as

$$Y = X_{k_0}\Pi + U \tag{24}$$

where $Y = [Y_1, \dots, Y_N]$, $\Pi = [\Pi_1, \dots, \Pi_N]$ and $\mathrm{U} = [\mathrm{U}_1, \dots, \mathrm{U}_N]$. Also the row vectors are defined as $\mu = (\mu_1, \dots, \mu_N)$, $\beta = (\beta_1, \dots, \beta_N)$, $\theta = (\theta_1, \dots, \theta_N)$ and $\gamma = (\gamma_1, \dots, \gamma_N)$. Then, an alternative expression for Π is $[\mu', \beta', \gamma']'$, $[\mu', \beta', \theta', \gamma']'$ and $[\mu', \beta', \theta']'$ for Model I, II and III, respectively.

A general break date and a general break fraction are denoted by k, and λ = k / T, respectively, and X_k is defined similarly to X_{k_0}. Then, the sum of residual squares for each k, can be defined as follows:

$$SSR(k) = tr[Y'(I - P_k)Y] \tag{25}$$

where $P_k = X_k(X_k'X_k)^{-1}X_k'$ and $tr[\cdot]$ is trace operator. Thus, estimated break date is the one minimizing the sum of residual squares such as

$$\hat{k} = \underset{k}{\operatorname{argmin}}\, SSR(k) \tag{26}$$

and

$$\hat{\lambda} = \frac{\hat{k}}{T}. \tag{27}$$

3. DATA GENERATION AND MONTE-CARLO SIMULATIONS

In this section, we evaluate the estimation performance of the Simple Mean Shift Model Method (hereafter referred to as Bai Test) proposed by Bai [5], the Fluctuation Test, Wald Statistic Test (hereafter referred to as Wald test) and Kim Test proposed in Kim [17], for the break date with Monte Carlo simulations.

The panel data to which the tests are to be applied are generated in accordance with the model given in Equation 1.

$$y_{it} = \mu_{i1} + u_{it} \quad t = 1, 2, \dots, k_0$$
$$y_{it} = \mu_{i2} + u_{it} \quad t = k_0 + 1, \dots, T$$

where $i = 1,2, \dots, N$, $t = 1, 2, \dots, T$, y_{it} is the observation value of the ith section unit at t, μ_{i1} is the mean of the panel data before the break date, μ_{i2} is the mean of the panel data after the break date, k_0 is the common break point and u_{it} indicates the disturbance terms. In the simulations, the disturbance terms are generated from independent and identically distributed $u_{it}\sim N(0;\ 1)$, and μ_{i1} and μ_{i2} are from $\mu_{i1}\sim N(3;\ 0{,}24)$ and $\mu_{i2}\sim N(3\times\gamma;\ 0{,}24)$ where γ denotes the break ratio.

To determine the number of repetitions, the difference between the number of repetitions and the asymptotics of the estimated breakpoints was taken into account. In the study, the number of repetitions was determined as 3000 repetitions with 0,001 difference between the average values of the break points predicted in each repetition. In total, Monte Carlo simulations are repeated as many times as the number of factor combinations depending on the level of the four factors under investigation.

Various issues have been taken into account to determine the factor levels. These issues can be summarized as follows: When examining the effects of time and cross-section length on break point estimation performance, the levels of these factors are defined as small, medium and large. The levels are chosen as 12, 32 and 120 for both time dimension T and cross-sectional dimension N.

If the break point k_0 is defined as a set of fixed values, the marginal effect of the break point can not be observed due to the coexistence of changes in the break point at different time dimension and the effects of changes in time dimension. For this reason, instead of taking the breakpoint k_0 as a member of a fixed value set in simulations, k_0 is defined as an integer between 1 and T, $k_0=[T\lambda]$, $\lambda\in(0,1)$. Thus, in the simulations, breaks are allowed to occur in the first, second and third quadrants of the panel dataset, respectively, taking into account $\lambda\in\{0{,}25;0{,}50;0{,}75\}$ to define the break fraction.

The final factor by which the effect on the break point estimation performance is investigated is the magnitude of the break $(\mu_{i2}-\mu_{i1})$. When the magnitudes of break factor levels are determined, the post-break mean is defined as

$$\mu_{i2}=\mu_{i1}*\gamma$$

where γ is the break ratio. Then, the magnitude of the break is constant and written in the following form:

$$(\mu_{i2}-\mu_{i1})=\gamma*\mu_{i1}-\mu_{i1}=(\gamma-1)\mu_{i1}$$

Thus, the magnitude of the break is defined as the ratio of the pre-break mean. Expression of the magnitude of break in this way allows it to be constant for different factor levels and to define the post-break mean to be smaller than the pre-break mean. For this reason, when examining the effect of magnitude of break on the performance of the tests, the factor break ratio, γ, is strictly defined as the pre-break mean is used. The factor levels of the break ratio are defined as $\gamma\in\{0{,}8;1{,}1;1{,}4;1{,}9\}$ so as to include the case where the post-break panel mean is smaller than the pre-break panel mean.

Simulation is carried out at a total of 108 points of the experimental design for the factors the four factors, time dimension, cross-section size, break fraction and break ratio, with 3, 3, 3 and 4 levels, respectively.

4. SIMULATION RESULTS

In this section, the results of the simulations of the predicted break point performance of the Bai, Fluctuation, Wald, and Kim tests are given. These results are based on an examination of the absolute value of the difference between the actual break point and the estimated break point obtained using the simulation design described in the third section. The use of tests that produce more efficient estimations at the predicted break point will lead to more accurate results when evaluating break point estimation performances of the tests. For this reason, the standard error values of the break point estimates have been examined in evaluating the break point estimation performance of the tests.

With the first simulation when $\gamma=0{,}8$ and $\lambda=0{,}25$, whose results are shown in Figure 1, the performance values, which is the absolute value of difference between actual and predicted break point, of the tests against various time and cross-sectional dimensions were obtained. As it is seen in Figure 1, the Bai Test gives the closest estimates to the true break point for all cross-sectional and time dimension factor levels while the worst performance is by the Fluctuation test.

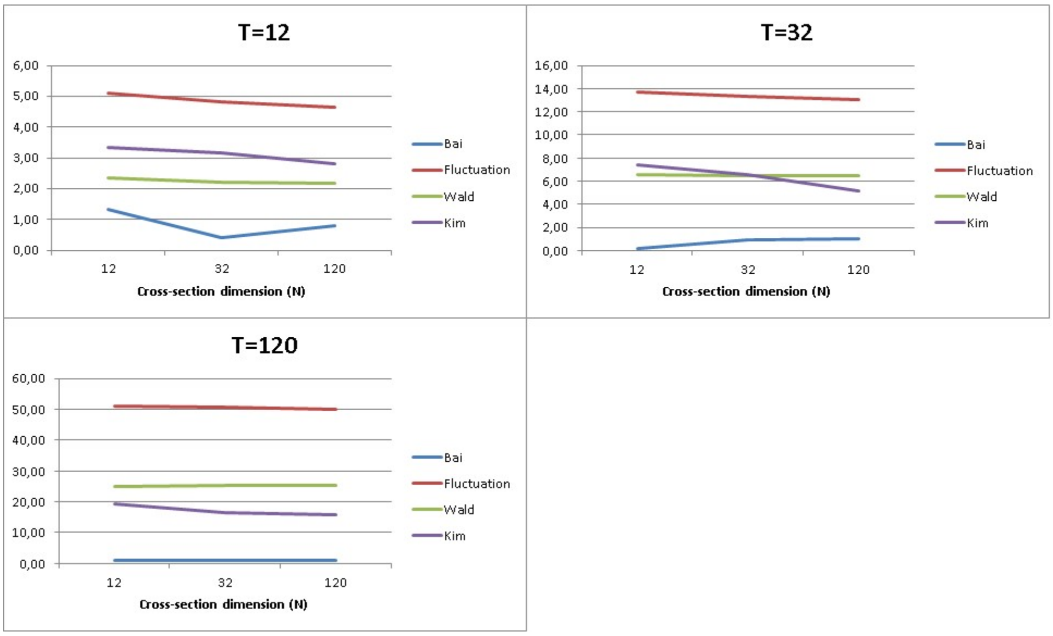

Figure 1. Simulation results for breakpoint estimates ($\boldsymbol{\gamma = 0,8}$ and $\boldsymbol{\lambda = 0,25}$)

In general, as the cross-sectional dimension increases, the estimations of Fluctuation and Kim Test slightly approach the real breakpoint values although the increase in cross-sectional size has a limited effect on difference values. Furthermore, except for the Bai test, the difference values grow as the time dimension increases, and the methods produce estimates that are farther away from the actual break point.

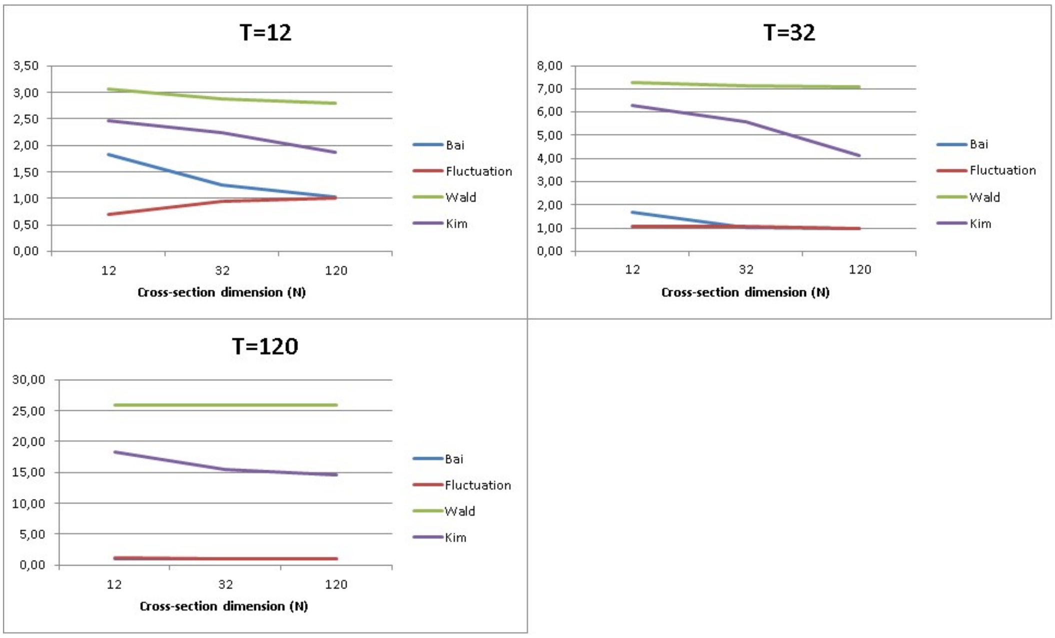

Figure 2. Simulation results for breakpoint estimates ($\boldsymbol{\gamma = 0,8}$ and $\boldsymbol{\lambda = 0,75}$)

Figure 2 shows that if the break is in the third quartile of the series, and the mean of the series is reduced by 20% after break, the closest estimates to the actual break point is obtained in the Bai and

Fluctuation tests. When the time dimension is small, as the cross-sectional dimension increases, there is a relative improvement, while as the time dimension increases, the influence of the cross-sectional size on the predictive performance of the break point disappears except for the Kim test. Where the time dimension is small at this level of the break rate and break fraction factors, the Fluctuation test produces estimates closer to the actual break point than the Bai test. As the time dimension increases, the Bai and Fluctuation tests produce estimates at the same distance to the actual break point. Also, the cross-sectional dimensions influence at most the Kim test.

When we evaluate Figure 1 and Figure 2 together, we can say that the difference values show a similar tendency towards the changes in cross-sectional dimension. However, if the break point is in the first quartile of the series, the Fluctuation test produces the farthest estimations of the true break point, and if the break occurs in the third quartile of the series, the closest predictor to the true break point produces the Fluctuation test. Thus, it can be said that the break fraction factor has a significant influence on the performance of Fluctuation test. When breaks occur in the third quarter of the series and the mean of the series increases by 40% after break, the closest estimations to the true break point are produced by the Fluctation and Bai tests (Figure 3). In addition, there is no significant effect of increase in cross section size on the series. However, the increase in cross section size for shorter time series has a positive impact on the Kim test predictive performance compared to other methods.

When the Figures 2 and 3 are evaluated together, it is seen that the difference values show a similar tendency. However, if the mean of the series decreases by 20% after break and the time series length is short, the method that produces the closest estimates to the actual breakpoint is the Fluctuation test. When the mean of the series grows by 40% after break, Bai and Fluctuation tests for all levels of the time series dimension result in estimates at the same distance to the actual break point. From this it can be said that the Bai test break point estimation performance is affected more and more positively than the Fluctuation test estimation performance at all levels of the magnitude of break.

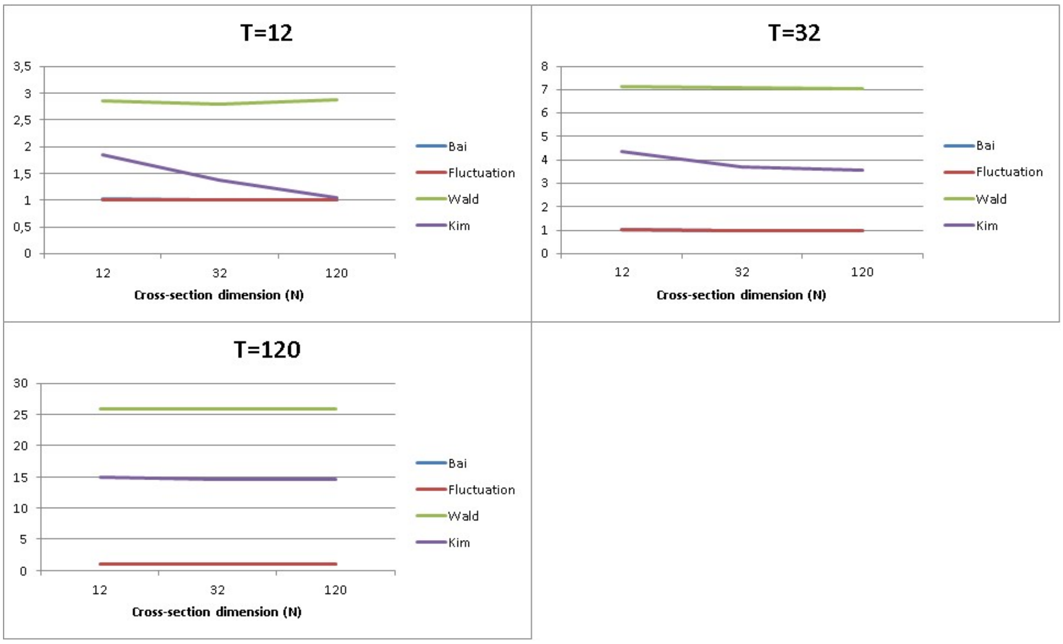

Figure 3. Simulation results for breakpoint estimates ($\gamma = 1{,}4$ and $\lambda = 0{,}75$)

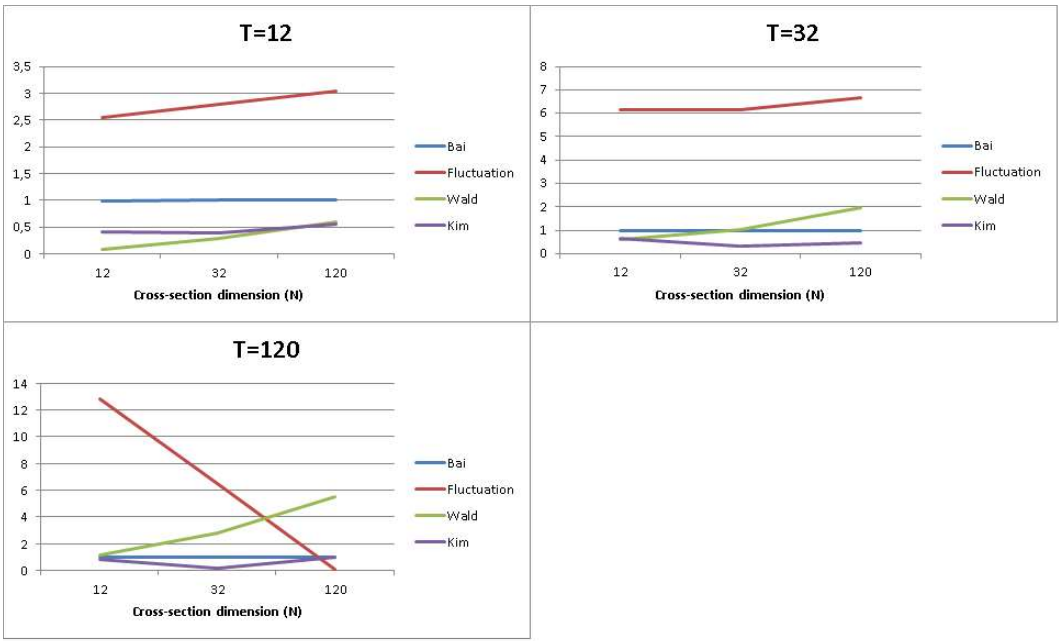

Figure 4. Simulation results for breakpoint estimates ($\gamma = 1{,}4$ and $\lambda = 0{,}5$)

While Bai, Kim, and Wald tests produce estimates that are close to the actual break point when breaks occur at mid-point and the mean of the series increases by 40% after break, the Fluctuation test produces more accurate estimates of the true break point. However, the Wald test has a good predictive performance in panels where time series is at small and medium length, while the other methods on panels with large series lengths exhibit higher estimation performance. Also, as the cross-section size increases, the estimation performance of the Wald test decreases, while there is no significant change in the estimation performance of other methods for the same length of time series.

However, the increase in the time series length of the panels has a positive effect on the Fluctuation test performance. As the section size increases for the long time series, the Fluctuation test produces estimates that are closer to the actual break point. Finally, the Kim test has a better predictive performance than the Bai test when the breaks are at the midpoint of the series and the mean of the series increases by 40% after break.

Comparing Figure 3 and 4, the Fluctuation test shows good predictive performance when the breaks are in the third quarter of the series, while the Wald and Kim tests have a good predictive performance if the breaks are at the midpoint of the series. Nevertheless, while there are significant changes on the break point estimation performances of tests based on factor levels, Bai test shows steady and high accurate break point estimation performance.

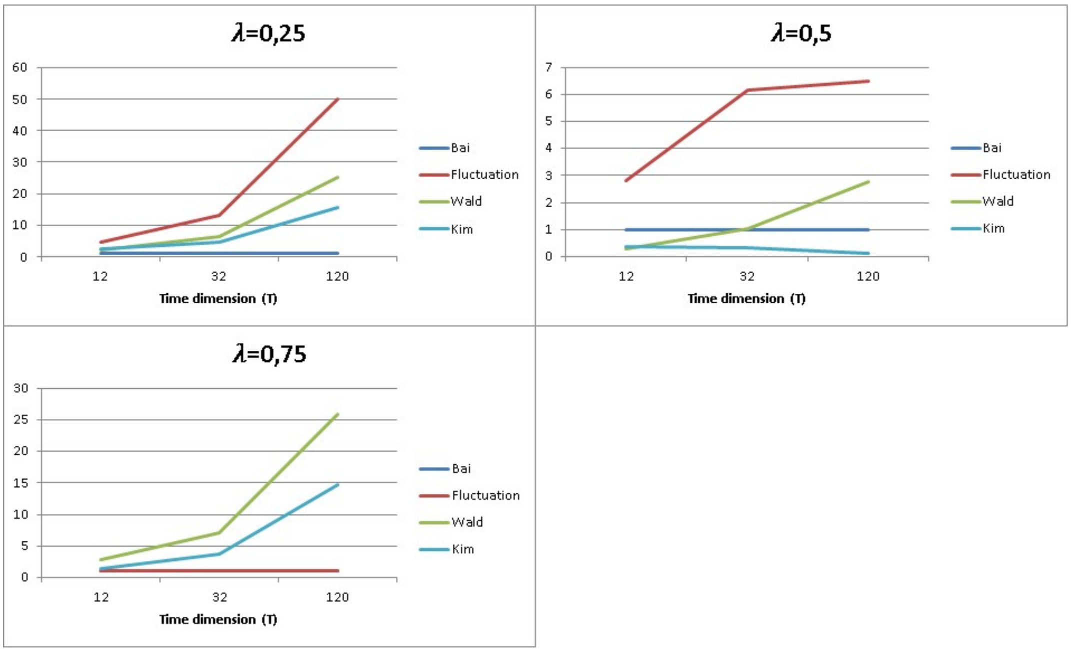

Figure 5. Simulation results for breakpoint estimates ($\gamma = 1{,}4$ and $N = 32$)

Figure 5 shows the effects of the changes at the time series size and the break fraction levels on the performance of break point estimation of the tests where the number of cross section is 32 and the mean of the series increase by 40% after structural change. As can be seen from Fig. 5, while the breakpoint estimation performance of the tests differs according to the region where breaks occur, the overall performance of the tests except the Bai Test appears to decrease as the time dimension increases. If the break is in the first quarter of the series, the methods generally produce estimates farther than the actual breakpoint. On the other hand, if the breaks are at the midpoint of the series, then all tests show better breakpoint estimation performance than if the breaks occurred in other regions of the series. The factors cross-sectional dimension and break ratio are fixed, an increase in the time dimension has a negative effect on the estimation performance of the Wald test. As a result, it can be said that both Wald and Kim tests have the best breakpoint estimation performance in the case that the breaks occur on the midpoints of the series. However, except for the case where the breaks are at the middle of the series, the predictive performance of the Kim test is adversely affected by the increase in time dimension.

According to Fig. 6, when the mean of the series with 32 cross-sections increase by 10% after structural break, the performances of the tests other than Bai decrease as the cross-sectional dimension and the break ratio increase steadily. If the break occurs in the first quarter of the series, the tests generally produce estimations farther to the true breakpoint. In case that the break occurs at the mid-point of the series, then all of the methods show a better breakpoint estimation performance than the other break fraction factor levels. Both the Wald and Kim tests show the best breakpoint estimation performance if breaks are in the middle of the series.

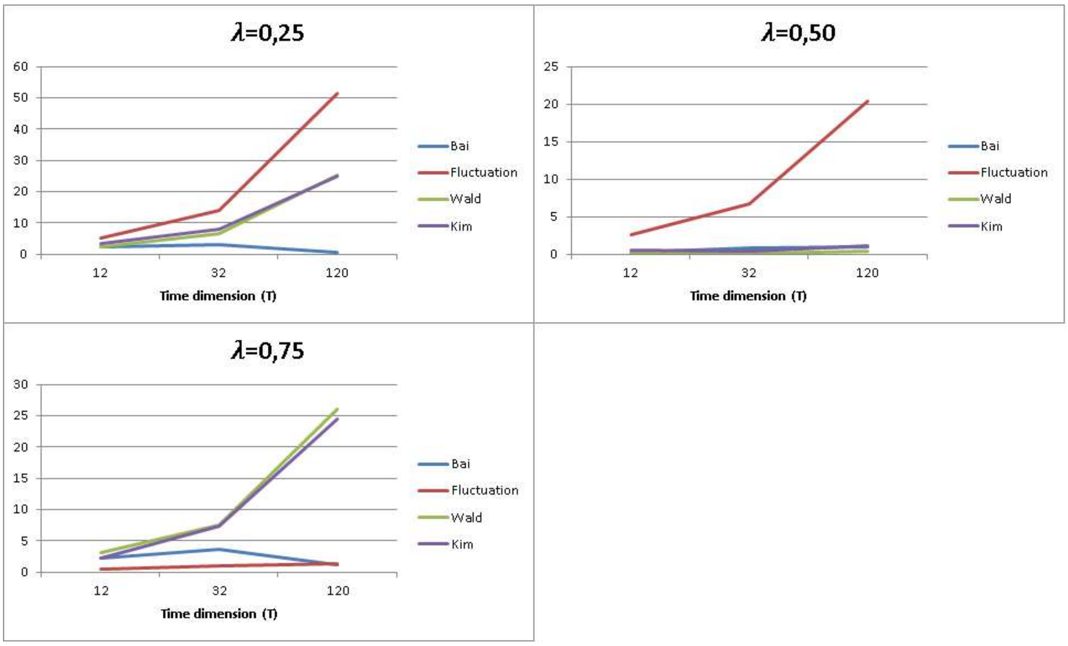

Figure 6. Simulation results for breakpoint estimates ($\gamma = 1{,}1$ and $N = 32$)

Comparison of the results obtained in Fig. 6 and Fig. 5 reveals that if the break occurs in the third quarter of the series and the means of the series increase by 40%, the Bai test breakpoint estimation performance is the same as the Fluctuation test performance. If the breaks are in the third quarter of the series and the means of the series increase by 10%, the Fluctuation test shows a better breakpoint estimation performance than the Bai test. From this, it can be said that the Bai test has higher estimation performance when the magnitude of the break of the series increases. In addition, when the break ratio is 1.4, Kim test shows higher performance than Wald test, whereas when the break ratio is 1.1, Wald test shows a higher estimation performance than Kim test. Thus, it can be said that the Kim test is more positively affected by the increase in magnitude of break than the Wald test.

Figure 7 shows the impact of the break ratio factor for the different break fraction levels on break point estimation performance of test under the assumptions that both the time and the cross-sectional dimensions are 32 and the break occurs in the first quartile or middle of the series. Under this settings except $\lambda = 0.75$ the Fluctuation test sets out estimates that are farthest to the actual breakpoint. If the break occurs in the third quarter of the series, the Fluctuation test, along with the Bai test, produces the closest estimates to the actual breakpoint. Generally, estimates close to the actual breakpoint are obtained with the Bai test. When the break size is increased, the Bai and Kim tests give closer estimates to the actual breakpoint than the other methods. However, if the break occurs at the mid-point of the series, as the size of break increases, the Wald test produces farther estimations of the actual breakpoint. Additionally, in the case where the break is in the third quartile of the series, the Wald test shows the lowest estimation performance.

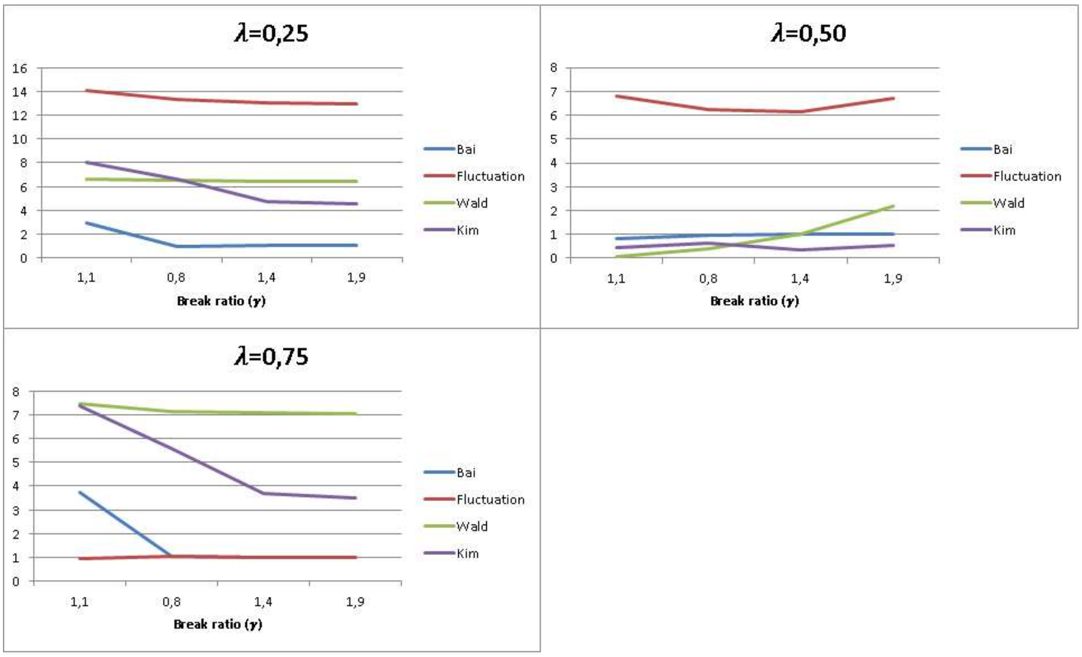

Figure 7. Simulation results for breakpoint estimates ($N = 32$ and $T = 32$)

Figure 8 shows the impact of the break fraction factor for the different break ratio levels on break point estimation performance of test under the assumptions that both time and cross-section dimensions are 32. According to Fig. 8, the Bai test estimations are limitedly affected by break fraction variations. The Fluctuation test estimations approach the actual breakpoint as the break fraction increases. Kim and Wald tests show the best estimation performance when the breaks are in the middle of the series. In general, the Bai test estimates are closer to the actual break point. However, if the break is in the third quartile of the series, the closest estimations are achieved by the Fluctuation test, while for the breaks at the middle of the series, Kim Test produces the closest estimations to the actual break point.

Depending on the changes in the cross-section or time dimension of the panel, the methods may tend to predict the same values, and also they may predict the same breakpoint depending on where the actual breaks are located in the series or magnitude of the break. Therefore, when examining the estimation performance of the breakpoints of the tests, as well as examining the differences between the breakpoint estimates and the actual breakpoints, examining the standard errors of the estimates, may be useful in evaluating the performance of the tests. Accordingly, the simulation outputs for the standard errors of estimations of the methods are given in this section.

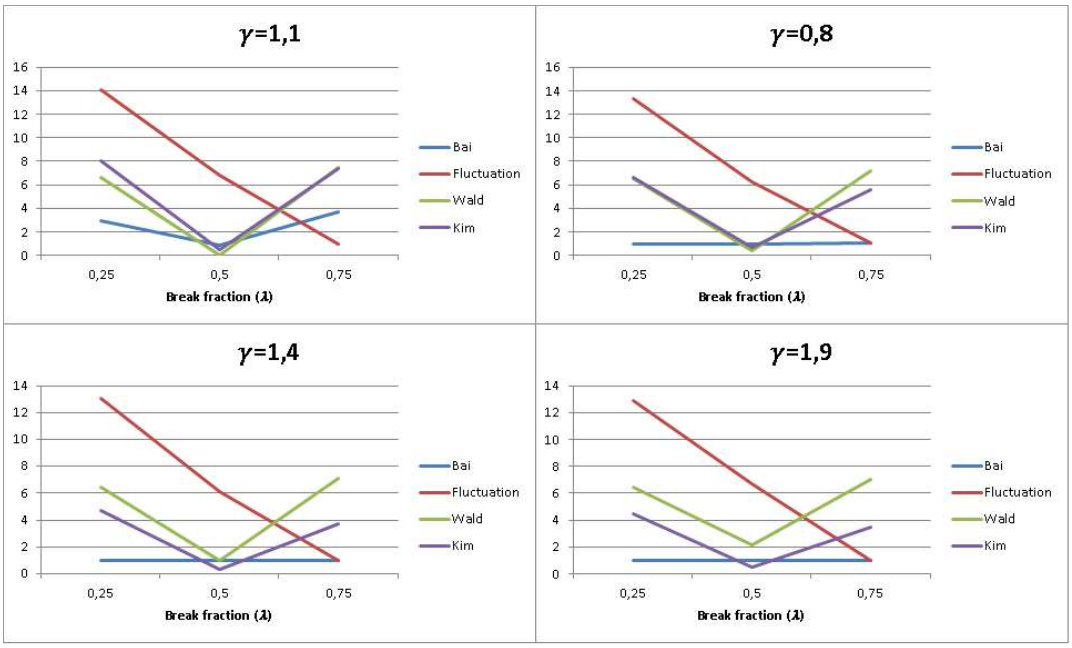

Figure 8. Simulation results for breakpoint estimates ($\boldsymbol{N} = \mathbf{32}$ and $\boldsymbol{T} = \mathbf{32}$)

Figure 9 shows the effect of the break ratios and break fraction factors on the standard errors of the breakpoint estimate of the tests when T = 32 and $N = 32$. When the break occurs in the middle of the series, the standard errors of the breakpoint estimations of the tests except the Bai test seem to increase. The Wald test standard errors are usually smaller than the standard errors of the predicted breakpoint by other methods than the Bai test. In the case where the break is in the third quarter of the series, estimates with the smallest standard deviation are obtained by the Fluctuation test. In the Bai Test, standard errors are smaller when the breaks occur in the middle of the series. In other words, as magnitude of the break increases, the methods provide more consistent breakpoint estimates.

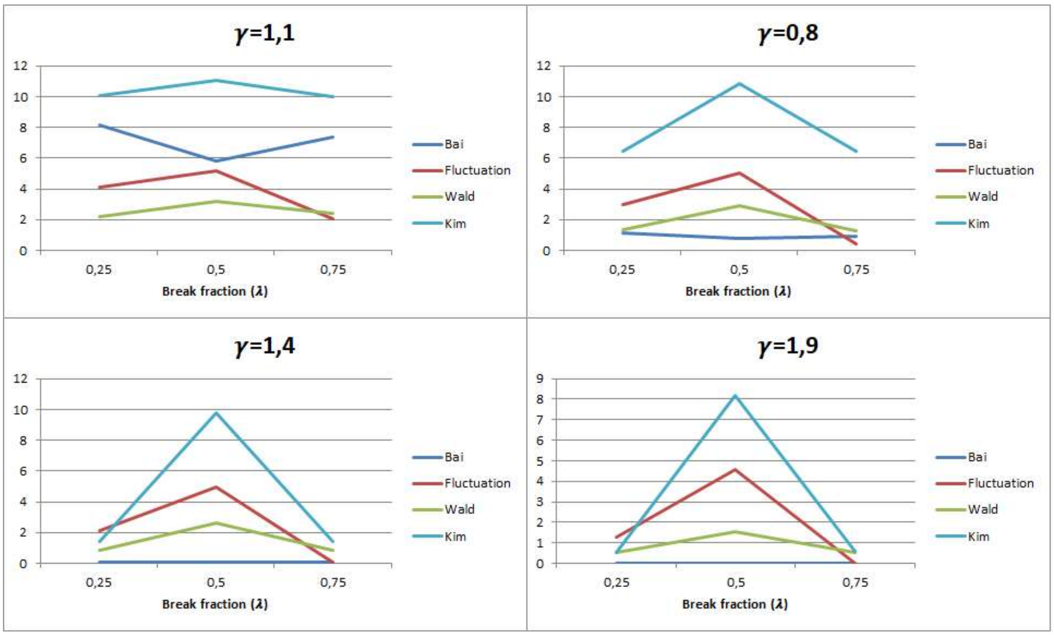

Figure 9. Simulation results for errors of breakpoint estimates ($\boldsymbol{N} = \mathbf{32}$ and $\boldsymbol{T} = \mathbf{32}$)

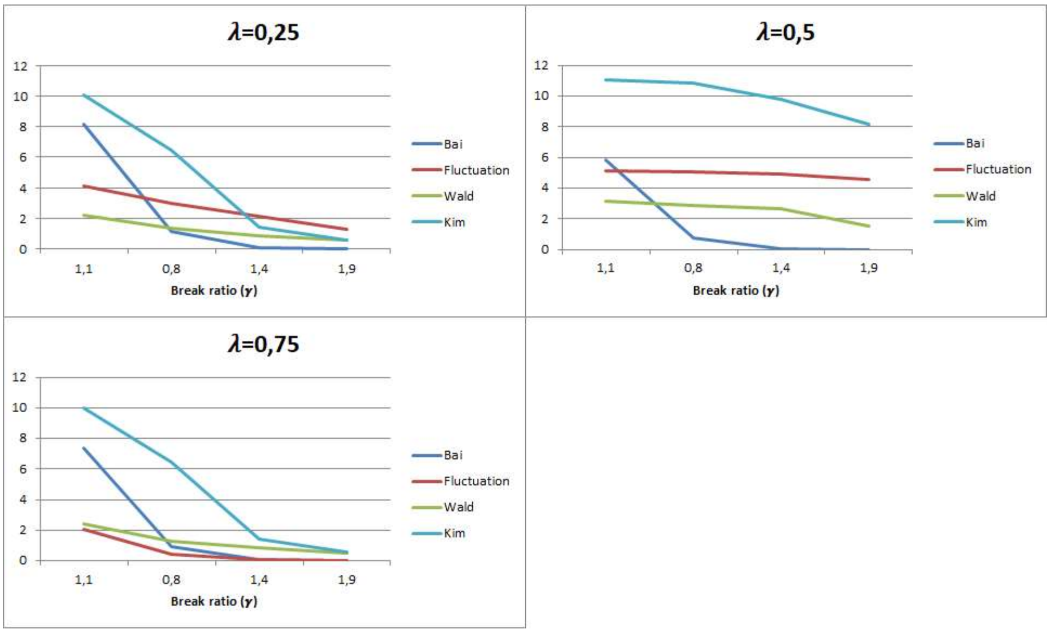

Figure 10. Simulation results for errors of breakpoint estimates ($N = 32$ and $T = 32$)

According to Fig. 10, when both the cross-section and the time dimension are 32, as the break ratio increases, the standard error of estimations decreases. Moreover, as the magnitude of the break increases, the tests always tend to predict the same value. The Bai test is the most affected method by the increases in magnitude of the break, and when the break ratio is greater than 1.4, the estimated standard error by Bai test becomes zero. If the estimated breakpoint by Bai test at these and higher break ratios is close but not equal to the actual break point, the actual break point can be predicted with the Bai test at this factor level, but only with the addition of a certain constant to the obtained estimate. Another important point seen in Figure 10 is that the standard error of the Bai Test estimates is smaller than the Wald and Kim Test standard errors of estimates when the breaks occur at the midpoint of the series. While the Wald and Kim tests show the highest breakpoint estimation performance when the breaks are in the middle of the series, the Bai test also has an estimation performance close to these tests. Because the Bai test has a small standard error, it can be preferred to predict the break point.

Where the number of time series forming the panel is 32 and the mean of the post-break decreases by 20% after the break, as the cross-sectional dimension increases, the standard errors of estimation of the breakpoint of the tests also decrease (Fig 11). As the cross-sectional dimension increases, the standard errors of the estimations of the breakpoint of the series are expected As a matter of fact, one of the main purposes of using panel data in estimating breakpoint is to acquire more consistent estimates. The results obtained in this study confirm this expectation. In addition, the standard errors of the breakpoint estimates obtained by the Bai test in general are smaller than the standard errors of the estimates by other methods. In case of breaks occurring in the third quarter of the series, the smallest standard errors belong to the Fluctuation test breakpoint estimates.

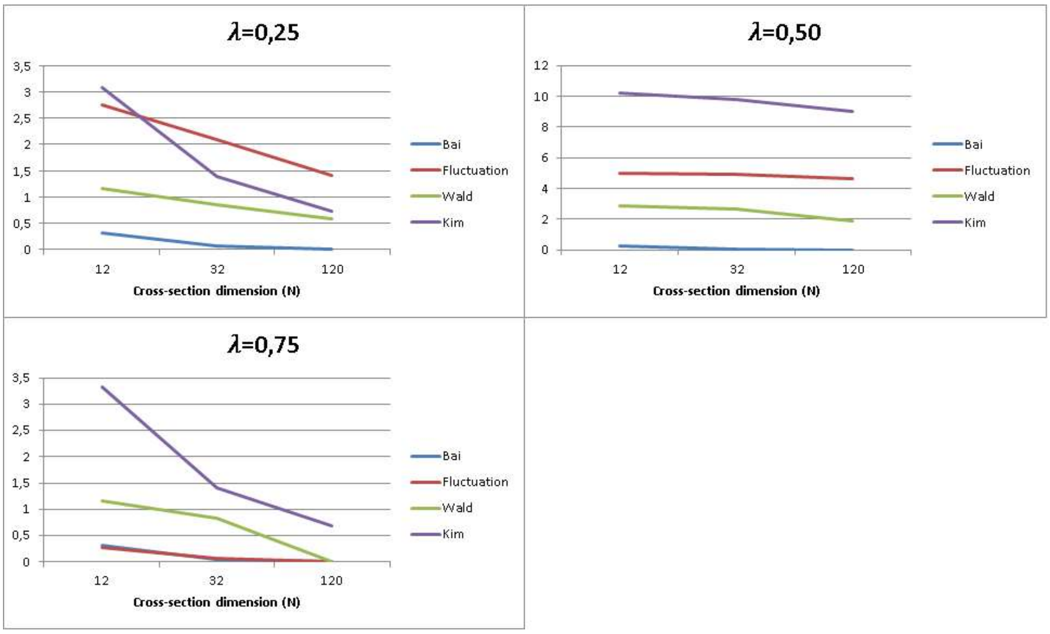

Figure 11. Simulation results for errors of breakpoint estimates ($T = 32$ and $\gamma = 0{,}8$)

Figure 12 depicts the effect of the variation in time length on the standard errors of the breakpoint estimates of the tests when the structural change in the panels with N = 32 occurs with a 20% shift in the mean of the series. Except for the Bai test, the standard error is negatively affected by the increase in time dimension, and thus, the standard error increases. When breaks occur in the third quarter of the series, the standard errors of the Fluctuation test breakpoint estimates do not increase as the time dimension increases. In case breaks occur in the third quarter of the series compared to those occurring in the first or middle of the series, the Fluctuation test produce more consistent and closer estimates of the true break point.

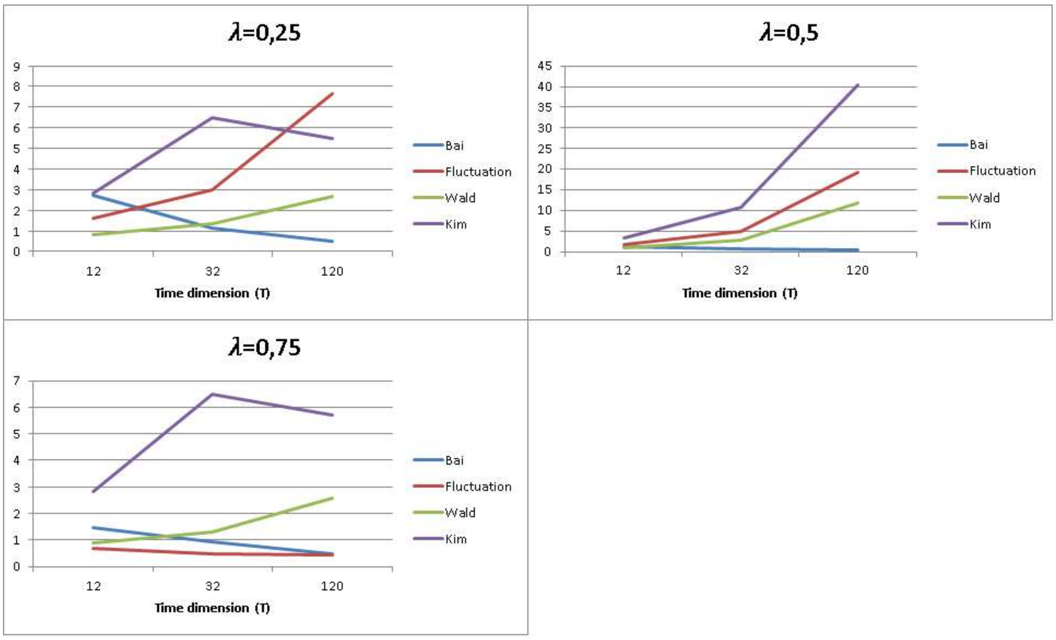

Figure 12. Simulation results for errors of breakpoint estimates ($N = 32$ and $\gamma = 0{,}8$)

5. CONCLUSION AND SUGGESTIONS

The following results are attained by examining the estimation performance of Bai, Fluctuation, Wald and Kim Tests used in determining structural break point in panel data with Monte Carlo simulations based on time dimension, cross-section size, break fraction and break ratio factors:

Estimates of the breakpoint are closest to the actual breakpoint, with the Mean Shift Model. This method proposed by Bai usually results in higher estimation performance compared to other methods.

The Fluctuation Test usually does not show a high breakpoint estimation performance when compared to other methods. However, if the breaks occur later in the series, the breakpoint estimation performance of this test improves. That is, if the breaks are in the third quartile of the series, the Fluctuation test has the highest performance, and if this break occurs before the third quarter, the Fluctuation test shows lower estimation performance than the other methods.

The Wald and Kim tests show close predictive performance and usually at moderate levels. Where breaks are in the middle of the series, the Wald and Kim tests have the highest estimation performance while the lowest performance if the breaks are located in the third quarter of the series.

The increase in cross-sectional size usually has a limited effect on the differences between the estimation and the actual breakpoints. However, as the cross-section size increases, the Fluctuation and Kim test estimates a little approach to the real breakpoint values. As the cross-sectional dimension increases, there do not exist any evidence proving that other tests have reduced breakpoint estimation performance.

In general, an increase in the time length reduces performance of the methods to estimate breakpoint. However, as the time dimension increases, only the estimation performance of the Bai test increases and gives estimates that are closer to the actual breakpoint.

When break occurs in the first quarter or middle of the series, the Fluctuation test sets out the most distant estimates of real breakpoints. On the other hand, if break occurs in the third quarter of the series, the Fluctuation and Bai tests produce the closest estimates to the actual breakpoint. In addition, the Wald test shows the worst performance in case the break is located in the third quarter of series. Therefore, it can be concluded that the estimation performance of the tests depicts significant differences depending on where the breaks occurred in the series.

The Bai test when breaks occur in the first quarter of the series, the Kim test and the Wald test when in the middle of the series, and the Bai and Fluctuation tests if in the third quarter of the series have the best estimation performance. However, the Bai test performs near the Wald and Kim tests if the breaks are in the middle of the series. While the Bai Test breakpoint estimation performance is positively influenced by the increase in time dimension, the Wald and Kim tests are negatively affected at the higher level of time dimension. Therefore, it can be concluded that if the time dimension is large and the breaks are at the mid-point of the series, the Bai test shows higher estimation performance.

No significant effect of magnitude of the break on the performance of the tests is observed. The magnitudes of the break at different levels of the break region and panel size factors have different effects on the estimation performance of the tests. In the case of breaks occurring in the first or third quartile of the series, the Kim Test results in an increase in the estimation performance, whereas breaks occur in the second quartile of the series, which has a positive impact on the Fluctuation test performance, negatively affecting the Wald test performance.

In general, as the number of cross-section increases, the standard errors of the breakpoint estimates of the test are reduced. On the other hand, as the time dimension increases, except for the Bai Test, on the contrary, the standard error of the estimated breakpoint increases. In addition, the standard errors are reduced as the break ratio increases. The test with the smallest standard error is the Bai Test. Its standard error of breakpoint estimates is equal to zero for the break ratios greater than 1.4.

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