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Robust model selection criteria for robust S and LTS estimators

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Abstract

Outliers and multi-collinearity often have large influence in the model/variable selection process in linear regression analysis. To investigate this combined problem of multi-collinearity and outliers, we studied and compared Liu-type S (liuS-estimators) and Liu-type Least Trimmed Squares (liuLTS) estimators as robust model selection criteria. Therefore, the main goal of this study is to select subsets of independent variables which explain dependent variables in the presence of multi-collinearity, outliers and possible departures from the normality assumption of the error distribution in regression analysis using these models.

Keywords: Liu-estimator, robust-Liu estimator, M-estimator, robust cp, robust Tp, robust model selection.

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1. Introduction

Traditional variable selection methods are based on classical estimators and tests which depend on normality assumption of errors. Even though many robust alternatives to the traditional model selection methods have been offered in the past 30 years, the associated variable selection problem has been somewhat neglected. For instance, in regression analysis, Mallows's Cp (Mallows, 1973) is a powerful selection procedure. But, since the Cp statistics is based on least squares estimation, it is very sensitive to outliers and other departures from the normality assumption on the error distribution. The need for robust selection procedures is obvious, because using Mallow's Cp variable selection method cannot estimate and select parameters robustly. Ronchetti (1985) and Ronchetti et. al. (1997) proposed and investigated the properties of a robust version of Akaike's Information Criterion (AIC). Hampel (1983) suggested a modified version of it. Hurvich and Tsai(1990) compared several model selection procedures for L1 regression. Ronchetti

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and Staudte (1994) proposed a robust version of Mallows's Cp. Sommer and Huggins (1996) proposed a robust Tp criterion based on Wald Statistics.

Consider the linear regression model

$$(1) \qquad \boldsymbol{Y} = \boldsymbol{X}\boldsymbol{\beta} + \boldsymbol{\epsilon}$$

where $\boldsymbol{Y}$ is an $n \times 1$ response vector; $\boldsymbol{X}$ is an $n \times p$ full rank matrix of predictors; $\boldsymbol{\beta}$ is an p vector of unknown parameters; $\boldsymbol{\epsilon}$, is an error vector with mean 0 and variance $\sigma^2 I$. For convenience, it is assumed that the $\boldsymbol{X}$ variables are standardized so that $\boldsymbol{X}^{'}\boldsymbol{X}$ has the form of correlation matrix.

Multi-collinearity and outliers are two main problems in regression methods. To cope with multi-collinearity, some techniques are proposed. Ridge regression estimator is one of the most widely used estimators to overcome multi-collinearity. Ridge regression estimator is defined as

$$(2) \qquad \hat{\beta}_r(k) = (X^{'}X + kI)^{-1}X^{'}X\hat{\beta}_{OLS}$$

where $k > 0$ is the shrinkage parameter. Since $\hat{\beta}_R(k)$ is sensitive to outliers in the y-direction, an alternative robust ridge M-estimator has been proposed by Sivapulle (1991). Since $\hat{\beta}_R(k)$ is a complicated function of k, Liu (1993) proposes a new biased estimator for β. Liu estimator

$$(3) \qquad \hat{\beta}_L(d) = (X^{'}X + I)^{-1}(X^{'}X + dI)\hat{\beta}_{OLS}$$

is obtained by shrinking the ordinary least squares (OLS) estimator using the matrix $(X^{'}X+I)^{-1}(X^{'}X+dI)$ Where $0 < d < 1$ is a shrinking parameter. Since OLS is used in Liu estimator, the presence of outliers in y direction may affect $\hat{\beta}_L(d)$. To overcome this problem, Arslan and Billor (2000) proposed an alternative class of Liu-type M-estimators (LM) which is defined as:

$$(4) \qquad \hat{\beta}_{LM}(d) = (X^{'}X + I)^{-1}(X^{'}X + dI)\hat{\beta}_M$$

LM estimator is obtained by shrinking an M-estimator ($\hat{\beta}_M$) instead of the OLS estimator using the matrix $(X^{'}X + I)^{-1}(X^{'}X + dI)$. The main objective of this proposed estimator is to decrease the effects of the simultaneous occurrence of multicollinearity and outliers in the data set.

Let $\lambda_1 \geq \lambda_2 \geq \ldots \lambda_p$ be the eigenvalues of $\boldsymbol{X}\boldsymbol{X}^{'}$ and $q_1, q_2, \ldots, q_p$ be the corresponding eigenvectors. Let $\boldsymbol{\Lambda} = diag(\lambda_1, \lambda_2, \ldots, \lambda_p)$ and $\boldsymbol{P} = (q_1, q_2, \ldots, q_p)$ such that $\boldsymbol{X}^{'}\boldsymbol{X} = \boldsymbol{P}\boldsymbol{\Lambda}\boldsymbol{P}^{'}$. The regression model can be written in the canonical form by

$$\boldsymbol{Y} = \beta_0\boldsymbol{1} + \boldsymbol{C}\boldsymbol{\alpha} + \epsilon$$

where $\boldsymbol{C} = \boldsymbol{X}\boldsymbol{P}$ and $\boldsymbol{\alpha} = \boldsymbol{P}^{'}\boldsymbol{\beta}$.

Then, the LM-estimator of α, $\hat{\alpha}_{LM}(d)$, becomes

$$(5) \qquad \hat{\alpha}_{LM}(d) = (\Lambda + I)^{-1}(\Lambda + d\boldsymbol{I})\hat{\alpha_M}$$

This estimator is resistant to the combined problem of multicollinearity and outliers in the y direction (Arslan and Billor, 2000).

In order to obtain $\hat{\alpha}_{LM}(d)$ we used the robust choice of d given in equation (5). Robust d value is

$$(6) \qquad \hat{d}_M = 1 - \hat{A}^2\left[\sum_{i=1}^{p}\frac{1}{\lambda_i(\lambda_i+1)} / \sum_{i=1}^{p}\frac{\alpha\hat{_{Mi}}^2}{(\lambda_i+1)^2}\right]$$

where $\hat{A}^2$ is

$$\hat{A}^2 = s^2(n-p)^{-1}\sum_{i=1}^{n}\left[\Psi(r_i/s)\right]^2 / \left[\frac{1}{n}\sum_{i=1}^{n}\left[\Psi^{'}(r_i/s)\right]\right]^2 \tag{7}$$

(Arslan and Billor, 2000).

2. Model Selection Estimators

2.1. Robust Cp Criteria.

Mallow's Cp (Mallows, 1973) is a powerful technique for model selection in regression. Since the Cp is based on OLS estimation, it is sensitive to outliers and other departures from the normality assumption on the error distribution.
Ronchetti and Staudte (1994) define a robust version of Cp as follows:

$$RC_p = \frac{W_p}{\hat{\sigma}^2} - (U_p - V_p) \tag{8}$$

where $W_p = \sum_i \hat{w}_i^2 r_i^2 = \sum_i \hat{w}_i^2 (y_i - \hat{y}_i)^2$, w_i is a weight for i. th observation, and $\hat{\sigma}^2$ is a robust and consistent estimator of $\hat{\sigma}$ in the full model given by $\hat{\sigma}^2 = W_{full}/U_{full}$. W_{full}, is the weighted residual sum of squares for full model. The constants $U_p = \sum_i var(\hat{w}_i r_i)$ and $V_p = \sum_i var(\hat{w}_i x_i^T(\hat{\beta}_p - \beta))$ are computed assuming that the subsets are correct and $\sigma = 1$.
In robust Cp (RCP) criterion used by Ronchetti and Staudte (1994), Up-Vp value is constant for all models. In our study, the value of Up-Vp changes according to each subset. Moreover, Ronchetti and Staudte (1994) have used weighting least squares (WLS) while computing the estimates. However, we use Huber-type estimation and Huber weights, instead of WLS. So, Up-Vp is

$$U_p - V_p \sim nE\|\eta\|^2 - 2tr(\boldsymbol{NM^{-1}}) + tr(\boldsymbol{LM^{-1}QM^{-1}}) \tag{9}$$

(see Ronchetti and Staudte,1994).
where $E\|\eta\|^2 = \sum_{1\le i\le n} \eta^2(x_i, \epsilon_i)$, $\boldsymbol{N} = E[\eta^2\eta^{'}\boldsymbol{xx'}]$ and $\boldsymbol{L} = E[w^{'}\epsilon(w^{'}\epsilon + 4w)\boldsymbol{xx'}]$.
Mallows's Cp and RCP are useful tools for model selection in regression. However, they have several disadvantages. First they are difficult to generalize to the situations where residuals are less defined. Second, they are computer intensive and their computation, particularly in robust version, can be time consuming as they require fitting of all submodels (Sommer and Huggins, 1996). Sommer and Huggins (1996) proposed a flexible easily generalized alternative based on the Wald test (see, Wald, 1994) which requires computation of estimates only from the full model. Models, with values of RCP close to Vp or smaller than VP, will be preferred to others.

2.2. Robust Tp criteria.

A robust version of Tp (RTp), based on generalized M-estimators of the regression parameters, is defined by

$$RT_p = \hat{\beta}_2^{'}\Sigma_{22}^{-1}\hat{\beta}_2 - k + 2p \tag{10}$$

where Σ_n is the covariance matrix,

$$\Sigma_n = \begin{bmatrix} \Sigma_{11} & \Sigma_{12} \\ \Sigma_{21} & \Sigma_{22} \end{bmatrix}$$

and

$$\hat{\beta} = (\hat{\beta}_1, \hat{\beta}_2) = (\boldsymbol{X}^{'}\boldsymbol{V}\boldsymbol{X})^{-1}\boldsymbol{X}^{'}\boldsymbol{V}\boldsymbol{Y},$$

and k and p are dimensions of full model and submodel, respectively, $\hat{\beta}_1 = (\beta_0, \beta_1, \ldots, \beta_{p-1})$ and $\hat{\beta}_2 = (\beta_p, \ldots, \beta_{k-1})$ (Hampel et all,1986). If submodel P is correct, the value of RT_p should be close to p (Sommer and Huggins, 1996).

2.3. S and LTS estimators.

Rousseeuw and Yohai (1984) proposed S-estimator which is another high breakdown point estimator having the same asymptotic properties as the M-estimator and used in model selection in linear regression analysis. It has a higher statistical efficiency than LTS estimation even though S and LTS estimates share the same breakdown value. The S-estimator minimizes the sample S-scale of the fitted residuals, while the LTS estimator minimizes the sample root mean square error. To obtain a high breakdown point estimator, which is also $\sqrt{n}$-consistent and asymptotically normal, was the motivation for the S-estimators. The ρ is considered to be a quadratic function. Let $k = E_\Phi[\rho]$ where Φ is the standard normal distribution. For any given sample $\{r_1, r_2, \ldots, r_n\}$ of residuals, an M-estimate of scale $\sigma(r_1, r_2, \ldots, r_n)$ is the solution to

$$ave\{\rho(r_i/\sigma)\}$$

where ave denotes the arithmetic mean over $i = 1, 2, \ldots, n$. For each value of β, the dispersion of the residuals $r_i = y_i - x_i^T\beta$ can be calculated using the upper equation . Then, the S-estimator $\hat{\beta}$ of β be defined as

$$arg\min_{\beta} \sigma(r_1(\beta), r_2(\beta), \ldots, r_n(\beta))$$

and the final scale estimate is $\hat{\sigma} = \sigma(r_1(\hat{\beta}), r_2(\hat{\beta}), \ldots, r_n(\hat{\beta}))$.
The least trimmed squares (LTS) estimate proposed by Rousseeuw (1984) is defined as the p-vector

$$\hat{\Theta}_{LTS} = arg\min_{\Theta} \boldsymbol{Q}_{LTS}(\Theta)$$

where

$$\boldsymbol{Q}_{LTS}(\Theta) = \sum_{i=1}^{h} r_{(i)}^2$$

$r_{(1)}^2 \leq r_{(2)}^2 \leq \cdots \leq r_{(n)}^2$ are the ordered squared residuals $r_i^2 = (y_i - x_i^T\Theta)$, $i = 1, \ldots, n$, and h is defined in range $\frac{n}{2} + 1 \leq h \leq \frac{3n+p+1}{4}$.

2.4. Suggested Model Selection Method.

In this study, in order to compute RCP and RTP criteria, we propose to use $\hat{\alpha}_S$ and $\hat{\alpha}_{LTS}$ instead of $\hat{\alpha}_M$ in equation (5), leading to

$$\hat{\alpha}_{pr}(d_M) = \hat{\alpha}_S(d_M) = (\Lambda + I)^{-1}(\Lambda + \hat{d}_M\boldsymbol{I})\hat{\alpha}_S \tag{11}$$

and

$$\hat{\alpha}_{pr}(d_M) = \hat{\alpha}_{LTS}(d_M) = (\Lambda + I)^{-1}(\Lambda + \hat{d}_M\boldsymbol{I})\hat{\alpha}_{LTS}$$

Consequently these estimators are used in (12) and (13) to estimate parameters $\hat{\beta}_{Liu.S}$ and $\hat{\beta}_{Liu.LTS}$ that are used in the calculation of selection criteria RCP and RTP.

$$\hat{\beta}_{Liu.S} = P^{'}\hat{\alpha}_S(d_M) \tag{12}$$

$$\hat{\beta}_{Liu.LTS} = P^{'}\hat{\alpha}_{LTS}(d_M) \tag{13}$$

where $\boldsymbol{P} = (q_1, q_2, \ldots, q_p)$ is the eigenvector matrix, such that $\boldsymbol{XX}' = \boldsymbol{P\Lambda P}'$.

In addition, we propose to use S estimator and LTS (least trimmed square) estimator in (12) for the calculation of selection criteria RCP and RTP, given in (8) and (10), respectively. In this way, $\hat{\boldsymbol{\beta}}$ for Vp in (8) and $\hat{\boldsymbol{\beta}}$ in (10) are obtained by using S estimator ($\hat{\beta}_{Liu.S}$), LTS estimator ($\hat{\beta}_{Liu.LTS}$). Equation (12) and Equation (13) are referred as robust Liu-S and robust Liu-LTS estimator, respectively.

2.5. Simulation Study.

In this section, a simulation study was performed to investigate and compare the robust variables selection criteria using S and LTS estimators. First, five independent variables were generated from Uniform distribution $(-1, 1)$. The data were obtained according to the $M1 = \beta = (5, 3, \sqrt{6}, 0, 0)$ and $M2 = \beta = (2\sqrt{5}, 4, \sqrt{3}, 1, 0)$ models. These parameters were obtained by considering $\beta^{'}\beta/\sigma^2$ (non-central signal-to-noise parameter) and $\phi = \sum_i V_j^{'}\beta/\sqrt{\sum_j \beta_i^2 \sum_{ij} V_{ij}^2}$ criteria and also used by Gunst and Mason(1977) and Erar(1982). In order to search of the effects of multicolinearity and outlier together over the robust selection criteria, a powerful linear dependency structure and a divergent-value were formed in the data sets between the x1 and x4; x2 and x3 variables in these models. Moreover, robust-d value is used in the robust Liu.M, robust Liu.S and robust Liu.LTS estimators, which are given equation 12-13. These estimators are used for computation of robust Tp (RTP) selection criteria based on Wald tests and robust Cp (RCP).

Furthermore, another goal of this simulation study is to see the results of RCP and RTP selection criteria with liu-S and liu-LTS estimators and to compare the results of RCP and RTP selection criteria with Liu and robust Liu.M estimators used in the previous studies (Çetin, 2009).

In order to obtain the percentages of subsets of criteria, a program was coded by using S-Plus function in this study. The results of the two models are given in the tables below. The numbers in these tables are shown how many times each subset is selected.

Table 1 shows that, the RTP, which is calculated by using both Liu.S and Liu. LTS estimators selects the real model which includes x1, x2, x3, with a proportion of %82 in a hundred repetitions, and it picked optional models x1, x2, x3, x4 and x1, x2 ,x3, x5 respectively in the proportions of %100 and %98. However, RCP criteria which is calculated by Liu.S and Liu.LTS estimators do not determine any subsets. On the contrary to RCP criteria calculated by Liu estimator gives better results than RTP criteria (Çetin, 2009).

As it can be seen from Table 2, RCP and RTP, both Liu.S and Liu.LTS estimators give the same results in case of multicollinearity. However, RTP criteria tend to choose multivariable models more often.

Table 3 gives the results under the assumption of multicollinearity and outliers together. RCP criteria do not work well when both the multicollinearity and outliers are present in the data. RTP criteria results are similar to those given in Table 2.

If we investigate Table 4,5 and 6, we can say that the results are similar to the result of model 1. Thus, we can say that Liu.s estimators with RCP and RTP criteria do not show

Table 1. Proportions of subsets order selected by criteria without outlier and multicollinearity for M1 model

With robust Liu.S estimators			**With robust Liu.LTS estimators**	
subsets	**RCP < VP**	**RTP < P**	**RCP < VP**	**RTP < P**
X1	0	0	0	0
X2	0	0	0	0
X3	0	0	0	0
X4	0	0	0	0
X5	0	0	0	0
X1 x2	12	0	14	0
X1 x3	0	0	0	0
X1 x4	0	0	0	0
X1 x5	0	0	0	0
X2 x3	0	0	0	0
X2 x4	0	0	0	0
X2 x5	0	0	0	0
X3 x4	0	0	0	0
X3 x5	0	0	0	0
X4 x5	0	0	0	0
X1 x2 x3	8	82	6	86
X1 x2 x4	2	0	0	0
X1 x2 x5	0	0	0	0
X1 x3 x4	2	0	1	0
x1 x3 x5	0	0	0	0
x1 x4 x5	5	0	1	0
x2 x3 x4	0	0	1	0
x2 x3 x5	0	0	0	0
x2 x4 x5	0	0	0	0
x3 x4 x5	0	0	0	0
x1 x2 x3 x4	0	100	0	100
x1 x2 x3 x5	0	98	0	99
x1 x2 x4 x5	6	0	0	0
x1 x3 x4 x5	0	0	0	0
x2 x3 x4 x5	11	0	16	0

any improvements. Liu.lts estimators with RCP and RTP criteria show better results. Under multi-collinearity condition, RTP and RCP criteria selected the true model (1234) but RTP tend to select other four variable models as well. Moreover, Liu.S and Liu.lts estimators with RCP and RTP criteria did not perform well under outliers and multicollinearity.

Table 2. Proportions of subsets order selected by criteria in case of multicollinearity for M1 model

With robust Liu.S estimators			**With robust Liu.LTS estimators**	
subsets	**RCP < VP**	**RTP < P**	**RCP < VP**	**RTP < P**
x1	0	3	1	4
x2	28	0	23	0
x3	0	0	0	0
x4	0	0	1	0
x5	0	0	0	0
x1 x2	58	21	42	25
x1 x3	76	22	57	24
x1 x4	0	7	0	5
x1 x5	0	33	1	31
x2 x3	32	1	10	1
x2 x4	78	0	59	2
x2 x5	49	1	41	1
x3 x4	36	1	32	1
x3 x5	3	1	6	1
x4 x5	0	7	0	5
x1 x2 x3	52	72	46	43
x1 x2 x4	7	87	25	83
x1 x2 x5	77	73	70	45
x1 x3 x4	57	93	44	91
x1 x3 x5	83	74	72	45
x1 x4 x5	11	78	45	63
x2 x3 x4	84	91	63	90
x2 x3 x5	75	10	66	5
x2 x4 x5	80	88	75	90
x3 x4 x5	87	100	73	100
x1 x2 x3 x4	5	95	5	96
x1 x2 x3 x5	7	62	3	32
x1 x2 x4 x5	1	90	4	92
x1 x3 x4 x5	0	100	0	100
x2 x3 x4 x5	0	100	0	100

Table 3. Proportions of subsets order selected by criteria in case of multicollinearity and outliers for M1 model

With robust Liu.S estimators			With robust Liu.LTS estimators	
subsets	**RCP < VP**	**RTP < P**	**RCP < VP**	**RTP < P**
x1	4	0	3	1
x2	10	0	13	0
x3	3	0	3	0
x4	2	2	6	2
x5	1	0	6	0
x1 x2	10	1	20	2
x1 x3	11	1	13	2
x1 x4	20	4	18	9
x1 x5	13	1	11	1
x2 x3	3	0	9	0
x2 x4	13	11	16	11
x2 x5	3	0	16	0
x3 x4	10	11	13	12
x3 x5	11	0	24	0
x4 x5	10	7	19	9
x1 x2 x3	11	9	8	4
x1 x2 x4	14	75	24	64
x1 x2 x5	10	10	8	4
x1 x3 x4	18	78	31	66
x1 x3 x5	8	10	8	4
x1 x4 x5	18	62	20	40
x2 x3 x4	3	79	3	68
x2 x3 x5	1	0	3	1
x2 x4 x5	4	99	3	93
x3 x4 x5	1	100	0	100
x1 x2 x3 x4	1	76	0	66
x1 x2 x3 x5	7	14	5	5
x1 x2 x4 x5	1	97	4	91
x1 x3 x4 x5	1	100	4	100
x2 x3 x4 x5	4	100	5	100

Table 4. Proportions of subsets order selected by criteria without outlier and multicollinearity for M2 model

With robust Liu.S estimators			**With robust Liu.LTS estimators**	
subsets	**RCP < VP**	**RTP < P**	**RCP < VP**	**RTP < P**
x1	33	0	9	1
x2	23	0	0	0
x3	28	0	0	0
x4	45	0	0	0
x5	7	0	2	0
x1 x2	9	0	24	0
x1 x3	8	48	7	0
x1 x4	6	0	5	0
x1 x5	8	0	1	0
x2 x3	9	0	1	0
x2 x4	10	0	3	0
x2 x5	0	0	5	0
x3 x4	9	0	1	0
x3 x5	3	0	1	0
x4 x5	6	0	2	0
x1 x2 x3	9	56	100	67
x1 x2 x4	2	0	0	2
x1 x2 x5	10	0	2	1
x1 x3 x4	8	53	4	2
x1 x3 x5	8	49	1	1
x1 x4 x5	2	0	0	1
x2 x3 x4	93	0	0	1
x2 x3 x5	6	0	0	2
x2 x4 x5	93	0	0	1
x3 x4 x5	1	0	4	3
x1 x2 x3 x4	43	60	85	100
x1 x2 x3 x5	41	35	0	22
x1 x2 x4 x5	60	0	1	12
x1 x3 x4 x5	72	71	1	1
x2 x3 x4 x5	82	0	1	1

Table 5. Proportions of subsets order selected by criteria in case of multicollinearity for M2 model

With robust Liu.S estimators			**With robust Liu.LTS estimators**	
subsets	**RCP $<$ VP**	**RTP $<$ P**	**RCP $<$ VP**	**RTP $<$ P**
x1	53	73	9	1
x2	43	0	43	1
x3	66	0	10	2
x4	45	0	11	1
x5	3	0	0	3
x1 x2	10	31	12	2
x1 x3	10	37	51	2
x1 x4	19	50	10	5
x1 x5	18	33	15	3
x2 x3	9	13	10	1
x2 x4	12	10	59	2
x2 x5	17	12	46	2
x3 x4	9	9	36	10
x3 x5	3	11	9	14
x4 x5	6	17	7	15
x1 x2 x3	100	73	100	56
x1 x2 x4	2	91	51	81
x1 x2 x5	10	73	87	25
x1 x3 x4	89	97	85	29
x1 x3 x5	83	73	98	13
x1 x4 x5	100	87	98	26
x2 x3 x4	93	96	99	39
x2 x3 x5	60	23	56	7
x2 x4 x5	93	97	78	19
x3 x4 x5	50	85	73	32
x1 x2 x3 x4	58	100	85	99
x1 x2 x3 x5	41	68	78	3
x1 x2 x4 x5	50	100	80	12
x1 x3 x4 x5	72	100	80	10
x2 x3 x4 x5	67	100	78	10

Table 6. Proportions of subsets order selected by criteria in case of multicollinearity and outliers for M2 model

With robust Liu.S estimators			**With robust Liu.LTS estimators**	
subsets	**RCP $<$ VP**	**RTP $<$ P**	**RCP $<$ VP**	**RTP $<$ P**
x1	62	53	8	3
x2	45	20	23	4
x3	68	20	10	20
x4	45	10	10	15
x5	31	10	9	3
x1 x2	10	28	7	2
x1 x3	10	33	7	5
x1 x4	31	52	11	10
x1 x5	36	31	10	3
x2 x3	19	23	14	5
x2 x4	21	10	34	9
x2 x5	17	17	46	6
x3 x4	39	19	54	11
x3 x5	1	11	7	20
x4 x5	13	17	7	15
x1 x2 x3	100	56	23	15
x1 x2 x4	100	100	78	88
x1 x2 x5	100	73	89	25
x1 x3 x4	99	54	62	29
x1 x3 x5	83	53	72	13
x1 x4 x5	100	100	85	46
x2 x3 x4	93	99	66	49
x2 x3 x5	60	20	72	37
x2 x4 x5	93	97	72	99
x3 x4 x5	50	100	72	100
x1 x2 x3 x4	75	100	83	45
x1 x2 x3 x5	79	93	72	5
x1 x2 x4 x5	64	100	74	100
x1 x3 x4 x5	79	100	86	100
x2 x3 x4 x5	89	100	77	100

3. Conclusion

RTP criteria with Liu.S and Liu.LTS estimators propose the best performance in case of the absence of any violation in the model assumptions. Despite the absence of any distortion in the assumptions, RCP criteria does not select the true model. Under the presence of outliers and multicollinearity, both RTP and RCP with Liu.S and Liu.LTS estimators do not work well. However, RCP criteria with Liu estimator showed better results (Çetin, 2009).

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