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Data envelopment analysis approach for discriminating efficient candidates in voting systems by considering the priority of voters

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Abstract

There are different ways to allow the voters to express their preferences on a set of candidates. In the traditional voting systems, it is assumed that the votes of all voters have the same importance and there is no preference between them. In this paper, a new approach is proposed to express the preference of voters on a set of candidates. In the proposed approach voters are classified into several categories with different importance levels in which the vote of a higher category may have a greater importance than that of the lower category. Then two models are introduced to measure the best preference scores of the target candidate from the virtual best candidate and the virtual worst candidate point of view. After that, two obtained preference scores are aggregated together in order to obtain an overall ranking. Finally, two numerical examples are provided for illustration the applications of the proposed approach.

Keywords: Data envelopment analysis, Voting system, Preference score, Virtual candidate.

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1. Introduction

There are different ways to allow the voters to express their preferences on a set of candidates. In some voting systems, each voter selects some candidates and ranks them from most to least. Among these systems, the well-known procedures to obtain a social ranking or a winning candidate are scoring rules, where fixed scores are assigned to the different ranks. In this way, the score obtained by each candidate is the weighted sum of the scores receives in different places. The plurality rule (the winner candidate is the one who receives more votes in the first place), the Borda rule (the weight assigned to the first place equals to number of candidates and to the second place is one less than the first place and so on) are the best known instances of scoring rules. In spite of the Borda rule has interesting properties in relation to other scoring rules [5], but the utilization of a fixed scoring vector has weak point that a candidate that is not the winner with the scoring vector imposed initially could be so if another scoring is used. To avoid this problem, Cook and Kress [7] suggested evaluating each candidate with the most favorable scoring vector for him/her. With this purpose, they introduced Data Envelopment Analysis (DEA) in this context. DEA determines the most favorable weights for each candidate. Different candidates utilize different sets of weights to calculate their total scores, which are referred to as the best relative total scores and are all restricted to be less than or equal to one. The candidate with the biggest relative total score of unity is said to be efficient candidate and may be considered as a winner. The principal drawback of this method is very often leads to more than one candidate to be efficient candidate. We can judge that the set of efficient candidates is the top group of candidates, but cannot single out only one winner among them. To avoid this weakness, Cook and Kress [7], proposed to maximize the gap between consecutive weights of the scoring vector. However, Green et al. [15] noticed two important drawbacks of the previous procedure. The first one is that the choice of the intensify functions used in their model is not obvious, and that choice determines the winner. The second one is that for an important class of discrimination intensity functions the previous procedure is equivalent to imposing a common set of scores on all candidates. Therefore, when Cook and Kress's model is used with this class of discrimination intensity functions, the aim pursued by these authors (evaluating each candidate with the most favourable scoring vector for him/her) is not reached.

Due to the drawbacks mentioned above, other procedures to discriminate efficient candidates have appeared in the literature. Green et al. [15] proposed to use the cross-evaluation method, introduced by Sexton et al. [32] to discriminate efficient candidates. Hashimoto [18] used the DEA exclusion method (see Andersen and Petersen [4]) to Cook and Kress's model. Hashimoto's model is useful to discriminate efficient candidates, but it is unstable with respect to inefficient candidates too. Noguchi et al. [28] criticized the choice of discrimination intensity functions in Green et al.'s model. In their model, the weight assigned to a certain rank may be zero and, consequently, the votes granted to that rank are not considered. Furthermore, the weights corresponding to two different ranks may be equal and, therefore, the rank votes lose their meaning. To avoid the previous drawbacks, Noguchi et al. [28] gave a strong ordering constraint condition on weights. Besides the previous condition on the scoring vectors, Noguchi et al. [28] introduced two other modifications in the model of Green et al. [15]. On the one hand, in the cross-evaluation matrix each candidate utilizes the same scoring vector to evaluate each of the remaining candidates. However, Noguchi et al.'s model maintains the problems of Green et al.'s model. Obata and Ishii [29] proposed another model that does not use any information about inefficient candidates. To obtain a fair approach, they used weight vectors of the same size, by normalizing the most favorable weight vectors. But it presents other drawbacks. In their model it is necessary to determine the norm and the discrimination intensity functions to use. If these functions are zero and the L_∞-norm is used, the winning candidate coincides with the one obtained by means of a scoring rule. If L_∞-norm is replaced by L_1-norm, the outcome could be considered unfair by some candidates. Foroghi and Tamiz [13] and Foroghi et al. [12] extended and simplified their model with fewer constraints and also used it for ranking inefficient as well as efficient candidates. Llamazares and Pena [26] analyzed the principal ranking methods proposed in the literature to discriminate efficient candidates and by solving several examples showed that none of the previous proposed procedures was fully convincing. In fact, although all the previous methods do not require predetermine the weights subjectively, some of them have a serious drawback: the relative order between two candidates may be altered when the number of first, second, ..., kth ranks obtained by other candidates changes, although there is not any variation in the number of first, second, ... , kth ranks obtained by both candidates. Thus, Llamazares and Pena [26] proposed a model that allows each candidate to be evaluated with the most favorable weighting vector for him/her and avoids the mentioned drawback. Moreover, in some cases, they found a closed expression for the score assigned with their model to each candidate.

Wang and Chin [39] discriminated efficient candidates by considering their least relative total scores. But the least relative total scores and the best relative total scores are not measured within the same range. The obtained conclusion was not persuasive. They also proposed a model in which the total scores are measured within an interval. The upper bound of the interval was set to be one, but they failed to determine the value of the lower bound for the interval. After that, Wang et al. [40] proposed a method to rank multiple efficient candidates, which often happens in DEA method, by comparing the least relative total scores for each efficient candidate with the best and the least relative total scores measured in the same range.

Wang et al. [42] proposed three new models to assess the weights associated with different ranking places in preference voting and aggregation. Two of them are linear programming models which determine a common set of weights for all the candidates considered and the other is a non-linear programming model that determines the most favorable weights for each candidate. Hadi-Vencheh and Mokhtarian [16] presented three counter examples to show that the three new models developed by Wang et al. [42] for preference voting and aggregation may produce a zero weight for the last ranking place and may sometimes identify two candidates as the winner in some specific situations. After that, Wang et al. [43] presented two modified linear programming models for preference voting and aggregation to avoid the zero weight for the last ranking place. In addition, Hadi-Vencheh [17] proposed two improved DEA models to determine the weights of ranking places that each of them can lead to a stable full ranking for all the candidates considered and avoid the mentioned shortcoming. Wu et al. [45] considered a preferential voting system using DEA game cross efficiency model, in which each candidate is viewed as a player that seeks to maximize its own efficiency, under the condition that the cross efficiencies of each of other DMU's does not deteriorate. Jahanshaloo et al. [22] reviewed ranked voting data and its analysis with DEA and proposed a model based on the ranking of units using common weights. Their model gives one common set of weights that is the most favorable for determining the absolute efficiency of all candidates at the same time. Bystricky [6] investigated different approaches to weighted voting systems based on preferential positions. In addition, other models have appeared in the literature in order to deal with this kind of problems [1, 3, 8, 9, 10, 11, 19, 20, 21, 25, 31, 33, 34, 36, 37, 38].

However, all previous models are based on Cook and Kress's model in which the votes of all voters have equal importance and there is no preference among them. In this paper we generalize the existing models to overcome this shortcoming. In fact, in our proposed model voters are classified into several categories with different importance levels that the vote of a higher category may have a greater importance than that of the lower category. Our main contribution in this paper will be the simplification of the model of Wu [44] (first proposed by Wang and Luo [41]) in DEA efficiency assessment for an overall ranking of candidates. We introduce two models that the first model evaluates candidates from the viewpoint of the best possible preference score and the second model evaluates them from the perspective of the worst possible preference score. The two distinctive scores are combined to form a comprehensive index such that an overall ranking for all the candidates can be obtained.

The rest of this paper is organized as follows. Section 2 gives the traditional voting model proposed by Cook and Kress [7] considering all of voters are in one category. Section 3 gives our model to determine efficient candidates by classifying voters into several groups with different importance levels. Section 4 extends the existing ranking method to discriminate the efficient candidates in terms of our proposed model assumptions. In Section 5 we illustrate our new methodology with two numerical examples. This paper is concluded in Section 6.

2. Ranked voting data

In this section we consider ranked voting data such that each voter select m candidate from n ($n \geq m$) candidates $\{A_1, A_1, \ldots,\ A_n\ \}$ and rank them from top to the place m, each place associated with a relative important weight u_i^r $(i = 1, 2, \ldots, m)$. In this way, the score obtained by the candidate A_r is $z_r = \sum_{i=1}^{m} u_i^r y_i^r$ where y_i^r is the number vote of place i that candidate A_r occupies and $(u_1^r,\ u_2^r, \ldots,\ u_m^r)$ is the scoring vector used.

2.1. Remark. In the DEA framework, many voting models are based on that of Cook and Kress [7], where the input variables u_i^r $(i = 1, 2, \ldots, m)$ are the weights, and these values are real numbers. Thus, the DEA models applied to find these weights as the relative importance of each place are not integer models.

Cook and Kress [7] suggested evaluating each candidate with the most favorable scoring vector for him/her based on DEA models. Their DEA/assurance region (DEA/AR) model is as follows:

$$
\begin{aligned}
z_p^* = \max \quad & \sum_{i=1}^{m} u_i^p y_i^p \\
s.t. \quad & \sum_{i=1}^{m} u_i^p y_i^r \leq 1, && r = 1, \cdots, n, \\
& u_i^p - \ u_{i+1}^p \geq d(i, \varepsilon), && i = 1, \cdots, m-1, \\
& u_m^p \geq d(m, \varepsilon),
\end{aligned}
\tag{2.1}
$$

In the above model, $d(., \varepsilon)$ is called the discrimination intensity function that is non-negative and monotonically increasing in a non-negative ε and satisfies $d(., 0) = 0$. The last constraints in (2.1) are called the assurance region constraints and ensure that the votes of the higher place has a greater importance that of the lower place. The model (2.1) is solved for each candidate p $(p = 1, \ldots, n)$. The resulting score z_p^* is the preference score of the candidate p. This score is used to rank of all candidates in a voting system that assumes that the votes of all voters have the same importance and there is no preference between them.

In this next section, we extend model (2.1) for situations that voters are classified into several categories with different importance levels in which the vote of a higher category may have a greater importance than that of the lower category.

3. An extended model

In this section, we introduce a new approach to allow the voters to express their preferences on a set of candidates by classifying voters into several groups with different importance levels. Suppose that in a ranked voting system, voters are classified into k distinct categories. The voters of each category, select m candidates among n ($n \geq m$) candidates $\{A_1, A_1, \ldots,\ A_n\}$ and rank them from top to the place m. Let y_{ij}^r be the votes of the candidate r being ranked in the place i from the category j. In evaluating of the candidate r, each place is associated with a relative importance weight u_i^r ($i = 1, 2, \ldots, m$) and each category is associated with a relative importance weight v_j^r($j = 1, 2, \ldots, k$). The preference score of candidate r in the place i is equal to $\sum_{j=1}^{k} v_j^r y_{ij}^r$. Thus, the total preference score of candidate r will be $z_r = \sum_{i=1}^{m} (u_i^r \sum_{j=1}^{k} v_j^r y_{ij}^r) = \sum_{i=1}^{m} \sum_{j=1}^{k} u_i^r v_j^r y_{ij}^r$.

It should be noted that if all categories have the same relative importance weights, then the preference score of candidate r in the place i will be $\sum_{j=1}^{k} y_{ij}^r$, that is exactly the number of votes in place i received by candidate r. In this case, the preference score of candidate r is equal to the one which assumes voters are in one category. Thus, this value indicates the real score of each candidate.

However to obtain a total ranking of candidates, we require the weight vectors $u^r = (\ u_1^r, \ldots,\ u_m^r)$ and $v^r = (\ v_1^r, \ldots,\ v_k^r)$ satisfy the following conditions:

$$\begin{aligned} & u_i^r - u_{i+1}^r \geq \overline{d}(i, \varepsilon), \quad i = 1, \cdots, m-1 \\ & u_m^r \geq \overline{d}(m, \varepsilon) \end{aligned} \tag{3.1}$$

$$\begin{aligned} & v_j^r - v_{j+1}^r \geq \overline{\overline{d}}(j, \varepsilon), \quad j = 1, \cdots, k-1 \\ & v_k^r \geq \overline{\overline{d}}(k, \varepsilon) \end{aligned} \tag{3.2}$$

It needs to point out that the constraints (3.1) are introduced in order that the vote of the higher place may have a greater importance than that of the lower place. In a similar way, the constraints (3.2) are introduced in order that the vote of voters in a higher category has a greater importance than that in a lower category. Hence, the following non-linear model evaluates candidate p with the most favorable weight vectors:

$$\begin{aligned} z_p^* = \max \quad & \sum_{i=1}^{m} \sum_{j=1}^{k} \quad u_i^p\ v_j^p\ y_{ij}^p \\ s.t. \quad & \sum_{i=1}^{m} \sum_{j=1}^{k} \quad u_i^p\ v_j^p\ y_{ij}^r \leq 1, \quad r = 1, 2, \ldots, n, \\ & u_i^p - u_{i+1}^p \geq \overline{d}(i, \varepsilon), \quad i = 1, \cdots, m-1 \\ & u_m^p \geq \overline{d}(m, \varepsilon) \\ & v_j^p - v_{j+1}^p \geq \overline{\overline{d}}(j, \varepsilon), \quad j = 1, \cdots, k-1 \\ & v_m^p \geq \overline{\overline{d}}(m, \varepsilon) \end{aligned} \tag{3.3}$$

To transform the non-linear model (3.3) into an equivalent linear model, let

$$w_{ij}^p = u_i^p v_j^p, \qquad i = 1, 2, \ldots, m \quad , \quad j = 1, 2, \ldots,\ k. \tag{3.4}$$

Now, we should change the constraint of (3.1) and (3.2) in terms of new transformations such that the priority among places and categories preserves. To this end, we multiply the constraints of (3.1) and (3.2) by $v_j^r(j = 1, 2, \ldots, k)$ and $u_i^r(i = 1, 2, \ldots, m)$, from the right and left, respectively. Thus, we have:

$$u_i^r v_j^r - u_{i+1}^r v_j^r \geq \overline{d}(\ i, \varepsilon), \quad i = 1, \cdots, m-1, \ j = 1, 2, \cdots, k$$
$$u_m^r v_j^r \geq \overline{d}(m, \varepsilon), \ j = 1, 2, \cdots, k$$

$$u_i^r v_j^r - u_i^r v_{j+1}^r \geq \overline{\overline{d}}(j, \varepsilon), \quad j = 1, \cdots, k-1, \ i = 1, \cdots, m$$
$$u_i^r v_k^r \geq \overline{\overline{d}}(k, \varepsilon), \ i = 1, \cdots, m$$

Thus, we have:

$$\begin{aligned} & w_{ij}^r - w_{i+1,j}^r \geq \overline{d}(i, \varepsilon), \quad i = 1, \cdots, m-1, \ j = 1, 2, \cdots, k \\ & w_{mj}^r \geq \overline{d}(m, \varepsilon), \ j = 1, 2, \cdots, k \end{aligned} \tag{3.5}$$

$$\begin{aligned} & w_{ij}^r - w_{i,j+1}^r \geq \overline{\overline{d}}(j, \varepsilon), \quad j = 1, \cdots, k-1, \ i = 1, \cdots, m \\ & w_{ik}^r \geq \overline{\overline{d}}(k, \varepsilon), \ i = 1, \cdots, m \end{aligned} \tag{3.6}$$

By substituting (3.4)-(3.5) into model (3.3), the following linear model is obtained:

$$\begin{aligned} z_p^* = \max \quad & \sum_{i=1}^{m} \sum_{j=1}^{k} \ w_{ij}^p \ y_{ij}^p \\ s.t. \quad & \sum_{i=1}^{m} \sum_{j=1}^{k} \ w_{ij}^p \ y_{ij}^r \leq 1, \qquad r = 1, 2, \ldots, n, \\ & w_{ij}^p - w_{i+1,j}^p \geq \overline{d}(i, \varepsilon), \quad i = 1, \cdots, m-1, \ j = 1, \cdots, k \\ & w_{mj}^p \geq \overline{d}(m, \varepsilon), \quad j = 1, \cdots, k \\ & w_{ij}^p - w_{i,j+1}^p \geq \overline{\overline{d}}(j, \varepsilon), \quad j = 1, \cdots, k-1, \ i = 1, \cdots, m \\ & w_{ik}^p \geq \overline{\overline{d}}(m, \varepsilon), \quad i = 1, \cdots, m \end{aligned} \tag{3.7}$$

In the next section, we introduce two virtual candidates called virtual best candidate (VBC) and virtual worst candidate (VWC) into voting system. The resultant voting models are referred to as the voting system with VBC and VWC candidates, respectively. The first system evaluates candidates from the viewpoint of the best possible preference score and the second system evaluates them from the perspective of the worst possible preference score. The two distinctive scores are combined to form a comprehensive index called the relative closeness (RC) to the VBC just like the well-known TOPSIS approach in multiple attribute decision making (MADM). The RC index is then used as the evidence of overall scores of each candidate, based on which an overall ranking for all the candidates can be obtained.

4. Voting systems with VBC and VWC

In this section we give some models so that a voting analysis based on TOPSIS idea can be performed. To do this, we first explore the concepts of virtual best candidate (VBC) and virtual worst candidate (VWC).

4.1. Definition. The virtual best candidate (VBC) is a virtual candidate that receives the most votes in each place among all candidates.

It needs to point out that the VBC may not exist in the voting. But he/she receives the most votes in each place among all n candidates. According to the above definition, we denote by $Y_i^{max} = (y_{i1}^{max}, \ldots, y_{ik}^{max})$ the number votes of VBC in place i, in which the votes of each category in this place are determined by $y_{ij}^{max} = \max_r\{y_{ij}^r\}$. In fact, VBC receives the most votes in each place and each category among all candidates and will be ranked in first place in any condition.

4.2. Definition. The virtual worst candidate (VWC) is a virtual candidate that receives the least votes in each place among all candidates.

It is also important to note that the VWC may not exist in the voting. But he/she receives the least votes in each place among all n candidates. According to the above definition, we denote by $Y_i^{min} = (y_{i1}^{min}, \ldots, y_{ik}^{min})$ the number votes of VWC in place i, in which the votes of each category in this place are determined by $y_{ij}^{min} = \min_r\{y_{ij}^r\}$. In fact, VBC receives the least votes in each place and each category among all candidates and will be ranked in last place in any condition.

It is obvious that the VBC should be able to achieve the highest/best preference score. The best preference score of VBC denoted as ϕ^* is determined by the following model:

$$
\begin{aligned}
\phi^*_{VBC} = \max \quad & \sum_{i=1}^{m}\sum_{j=1}^{k} w_{ij}^{VBC}\ y_{ij}^{max} \\
s.t. \quad & \sum_{i=1}^{m}\sum_{j=1}^{k} w_{ij}^{VBC}\ y_{ij}^{r} \le 1, \qquad r = 1, 2, \ldots, n, \\
& w_{ij}^{VBC} - w_{i+1,j}^{VBC} \ge \overline{d}(i,\varepsilon), \quad i = 1, \cdots, m-1,\ j = 1, \cdots, k \\
& w_{mj}^{VBC} \ge \overline{d}(m,\varepsilon), \quad j = 1, \cdots, k \\
& w_{ij}^{VBC} - w_{i,j+1}^{VBC} \ge \overline{\overline{d}}(j,\varepsilon), \quad j = 1, \cdots, k-1,\ i = 1, \cdots, m \\
& w_{ik}^{VBC} \ge \overline{\overline{d}}(m,\varepsilon), \quad i = 1, \cdots, m
\end{aligned} \tag{4.1}
$$

Since the above linear programming model (4.1) may have multiple optima, we utilize the following linear programming model to determine the best preference score of candidate p under the condition that the best possible preference score of the VBC remains unchanged:

$$
\begin{aligned}
z_p^* = \max \quad & \sum_{i=1}^{m}\sum_{j=1}^{k} w_{ij}^{p}\ y_{ij}^{p} \\
s.t. \quad & \sum_{i=1}^{m}\sum_{j=1}^{k} w_{ij}^{VBC}\ y_{ij}^{max} = \phi^*_{VBC} \\
& \sum_{i=1}^{m}\sum_{j=1}^{k} w_{ij}^{p}\ y_{ij}^{r} \le 1, \qquad r = 1, 2, \ldots, n, \\
& w_{ij}^{p} - w_{i+1,j}^{p} \ge \overline{d}(\ i,\varepsilon), \quad i = 1, \cdots, m-1,\ j = 1, \cdots, k \\
& w_{mj}^{p} \ge \overline{d}(\ m,\varepsilon), \quad j = 1, \cdots, k \\
& w_{ij}^{p} - w_{i,j+1}^{p} \ge \overline{\overline{d}}(\ j,\varepsilon), \quad j = 1, \cdots, k-1,\ i = 1, \cdots, m \\
& w_{mj}^{p} \ge \overline{\overline{d}}(\ m,\varepsilon), \quad i = 1, \cdots, m
\end{aligned} \tag{4.2}
$$

Similar to that in Wu [44], the following model is proposed to compute the worst possible preference score of the VWC:

$$
\begin{aligned}
\varphi^*_{VBC} = \quad & min \sum_{i=1}^{m}\sum_{j=1}^{k} w_{ij}^{VWC}\ y_{ij}^{min} \\
s.t. \quad & \sum_{i=1}^{m}\sum_{j=1}^{k} w_{ij}^{VBC}\ y_{ij}^{max} \ge \gamma, \quad \gamma \in [1, \phi^*_{VBC}] \\
& \sum_{i=1}^{m}\sum_{j=1}^{k} w_{ij}^{VWC}\ y_{ij}^{r} \le 1, \qquad r = 1, 2, \ldots, n, \\
& w_{ij}^{VWC} - w_{i+1,j}^{VWC} \ge \overline{d}(i,\varepsilon), \quad i = 1, \cdots, m-1,\ j = 1, \cdots, k \\
& w_{mj}^{VWC} \ge \overline{d}(m,\varepsilon), \quad j = 1, \cdots, k \\
& w_{ij}^{VWC} - w_{i,j+1}^{VWC} \ge \overline{\overline{d}}(j,\varepsilon), \quad j = 1, \cdots, k-1,\ i = 1, \cdots, m \\
& w_{ik}^{VWC} \ge \overline{\overline{d}}(m,\varepsilon), \quad i = 1, \cdots, m
\end{aligned} \tag{4.3}
$$

Model (4.3) aims to minimize the preference score of the VWC while at the same time keeping the preference score of the VBC no less than an appropriate parameter γ, which might be selected in a range from one and the maximal possible value. Although we note that the selection of the value of γ is flexible, we will prove by the Theorem 1 in the case of$\gamma = 1$, the model (4.3) is equivalent to the following model (see also Theorem 1 in [44]):

$$
(4.4)\quad
\begin{aligned}
\varphi^*_{VBC} = \quad & min \sum_{i=1}^{m}\sum_{j=1}^{k} \; w_{ij}^{VWC} \; y_{ij}^{min} \\
s.t. \quad & \sum_{i=1}^{m}\sum_{j=1}^{k} \; w_{ij}^{VBC} \; y_{ij}^{max} = 1 \\
& w_{ij}^{VWC} - w_{i+1,j}^{VWC} \geq \overline{d}(i,\varepsilon), \quad i = 1,\cdots,m-1, \; j = 1,\cdots,k \\
& w_{mj}^{VWC} \geq \overline{d}(m,\varepsilon), \quad j = 1,\cdots,k \\
& w_{ij}^{VWC} - w_{i,j+1}^{VWC} \geq \overline{\overline{d}}(j,\varepsilon), \quad j = 1,\cdots,k-1, \; i = 1,\cdots,m \\
& w_{ik}^{VWC} \geq \overline{\overline{d}}(m,\varepsilon), \quad i = 1,\cdots,m
\end{aligned}
$$

4.3. Theorem. *For $\gamma = 1$ model (4.3) and model (4.4) are equivalent.*

Proof. Consider the following model in the case of $\gamma = 1$:

$$
(4.5)\quad
\begin{aligned}
\varphi^*_{VBC} = \quad & min \sum_{i=1}^{m}\sum_{j=1}^{k} \; w_{ij}^{VWC} \; y_{ij}^{min} \\
s.t. \quad & \sum_{i=1}^{m}\sum_{j=1}^{k} \; w_{ij}^{VBC} \; y_{ij}^{max} \geq 1 \\
& \sum_{i=1}^{m}\sum_{j=1}^{k} \; w_{ij}^{VWC} \; y_{ij}^{r} \leq 1, \qquad r = 1,2,\ldots,n, \\
& w_{ij}^{VWC} - w_{i+1,j}^{VWC} \geq \overline{d}(i,\varepsilon), \quad i = 1,\cdots,m-1, \; j = 1,\cdots,k \\
& w_{mj}^{VWC} \geq \overline{d}(m,\varepsilon), \quad j = 1,\cdots,k \\
& w_{ij}^{VWC} - w_{i,j+1}^{VWC} \geq \overline{\overline{d}}(j,\varepsilon), \quad j = 1,\cdots,k-1, \; i = 1,\cdots,m \\
& w_{ik}^{VWC} \geq \overline{\overline{d}}(m,\varepsilon), \quad i = 1,\cdots,m
\end{aligned}
$$

Assume that $w_{ij}^{*,VWC}$to be optimal weight of model (4.5). We prove that $\sum_{i=1}^{m}\sum_{j=1}^{k} \; w_{ij}^{*,VWC} y_{ij}^{max} = 1$. Assume not, i.e. $\sum_{i=1}^{m}\sum_{j=1}^{k} \; w_{ij}^{*,VWC} y_{ij}^{max} = q > 1$ and $\sum_{i=1}^{m}\sum_{j=1}^{k} \; w_{ij}^{*,VWC} y_{ij}^{r} \leq 1 (r = 1,2,\ldots,n)$. Now set $w_{ij}^{**,VWC} = \frac{w_{ij}^{*,VWC}}{q}$.

Thus we have $\sum_{i=1}^{m}\sum_{j=1}^{k} \; w_{ij}^{**,VWC} y_{ij}^{r} < \sum_{i=1}^{m}\sum_{j=1}^{k} \; w_{ij}^{*,VWC} y_{ij}^{r} \leq 1 (r = 1,2,\ldots,n)$ and $\sum_{i=1}^{m}\sum_{j=1}^{k} \; w_{ij}^{**,VWC} y_{ij}^{max} = 1$. So, we have another feasible solution $w_{ij}^{**,VWC}$ with which the obtained value of objective function $\sum_{i=1}^{m}\sum_{j=1}^{k} \; w_{ij}^{**,VWC} y_{ij}^{min}$ is less than the assumed optimal value$\sum_{i=1}^{m}\sum_{j=1}^{k} \; w_{ij}^{*,VWC} y_{ij}^{min}$. This is a contradiction and hence the first constraint is constantly binding in any optimal solution.

Since we have $\sum_{i=1}^{m}\sum_{j=1}^{k} \; w_{ij}^{*,VWC} y_{ij}^{max} = 1$ it follows that $\sum_{i=1}^{m}\sum_{j=1}^{k} \; w_{ij}^{VWC} y_{ij}^{r} \leq \sum_{i=1}^{m}\sum_{j=1}^{k} \; w_{ij}^{*,VWC} y_{ij}^{max} = 1$. Hence the constraints $\sum_{i=1}^{m}\sum_{j=1}^{k} \; w_{ij}^{VWC} y_{ij}^{r} \leq$

1 ($r = 1, 2, \ldots, n$) are redundant and can be removed from model (4.5). This completes the proof. □

Following the same logic as before, given the worst efficiency of the VWC, the following linear programming model can be used to determine the worst possible preference score of candidate p under the condition that the worst possible preference score of the VWC stays unchanged:

$$
(4.6)\qquad
\begin{aligned}
\varphi_p^* = \min \quad & \sum_{i=1}^{m}\sum_{j=1}^{k} w_{ij}^p\, y_{ij}^p \\
s.t. \quad & \sum_{i=1}^{m}\sum_{j=1}^{k} w_{ij}^{VBC}\, y_{ij}^{min} = \varphi_{VWC}^* \\
& \sum_{i=1}^{m}\sum_{j=1}^{k} w_{ij}^p\, y_{ij}^r \le 1, \qquad r = 1, 2, \ldots, n, \\
& w_{ij}^p - w_{i+1,j}^p \ge \overline{d}(i,\varepsilon), \quad i = 1, \cdots, m-1,\ j = 1, \cdots, k \\
& w_{mj}^p \ge \overline{d}(m,\varepsilon), \quad j = 1, \cdots, k \\
& w_{ij}^p - w_{i,j+1}^p \ge \overline{\overline{d}}(j,\varepsilon), \quad j = 1, \cdots, k-1,\ i = 1, \cdots, m \\
& w_{ik}^p \ge \overline{\overline{d}}(m,\varepsilon), \quad i = 1, \cdots, m
\end{aligned}
$$

From the above discussion it is known that voting models (4.1) and (4.2) measure the best possible preference scores of VBC and the n real candidates based on VBC, while voting models (4.4)and (4.6) measure the worst possible preference scores of VWC and the n real candidates based on VWC. These two distinctive efficiency assessments may lead to quite different conclusions. Therefore, there is a need to consider them together to give an overall assessment of each candidate. In order to do so, we use the following relative closeness (RC) (Wang and Luo [41]), which is widely used in the TOPSIS approach, a well-known MADM methodology.

$$
(4.7)\qquad RC_p = \frac{(\varphi_p^* - \varphi_{VWC}^*)}{(\varphi_p^* - \varphi_{VWC}^*) + (\phi_{VBC}^* - \phi_p^*)}
$$

It is obvious that the bigger difference between $\varphi_p^*$and φ_{VWC}^* and the smaller difference between ϕ_{WBC}^* and ϕ_p^* mean the better performance of candidate p. So, the bigger RC_p value, the better the performance of candidate p. Since the RC index integrates both the best and the worst possible preference scores of each candidate, it thus provides an overall assessment for each candidate, based on which an overall ranking for the n real candidates can be easily obtained.

We are in a position to give the following algorithm for overall ranking of candidates:

Step 1. Solve the problems (4.1) for the VBC to obtain the optimal weights W_{VBC}^* and preference score ϕ_{VBC}^*, solve the problem (4.2) to compute the ϕ_p^*, $p = 1, 2, \ldots,\ n$.

Step 2. Solve the problems (4.4) for the VWC to obtain the optimal weights $W_{VWC}^*$and preference score φ_{VWC}^*, solve the problem (4.6) to compute the ϕ_p^*, $p = 1, 2, \ldots,\ n$.

Step 3. Calculate the relative closeness RC_p of candidate p using (4.7).

Step 4. Select the winner candidate q according to $RC_q^* = \max_{1 \le p \le n} RC_p$.

In this paper, it is assumed $\overline{d}_j\ (i,\ \epsilon) = \epsilon \overline{d}_i$ and $\overline{\overline{d}}_i\ (j,\ \epsilon) = \epsilon \overline{\overline{d}}_j$, in which ϵ is a sufficiently small positive value and, $\overline{d}_i$ and $\overline{\overline{d}}_j$ are the preferred values corresponding to gap i of places and gap j of categories, respectively. Without loss of the generality, throughout this paper, we assume $\overline{d}_i = \overline{\overline{d}}_j = 1$. We note that the choice of discriminating function

Table 1. Votes received by six candidates

Candidates	First Place	Second Place	Third Place	Fourth Place
A_1	3	3	4	3
A_2	4	5	5	2
A_3	6	2	3	2
A_4	6	2	2	6
A_5	0	4	3	4
A_6	1	4	3	3
VBC	6	5	5	6
VWC	0	2	2	2

Table 2. The preference scores for the six candidates

	The best score		The least score		RC		Rank	
Candidates	$\epsilon = 0.001$	$\epsilon = 0.00001$	$\epsilon = 0.001$	$\epsilon = 0.00001$	$\epsilon = 0.001$	$\epsilon = 0.00001$	$\epsilon = 0.001$	$\epsilon = 0.00001$
A_1	0.8091875	0.81246687	0.032	0.00032	0.03433845	0.00035543	4	4
A_2	1	1	0.043	0.00043	0.07699472	0.00082606	1	1
A_3	0.8196875	0.81257188	0.038	0.00038	0.04498756	0.0004621	3	3
A_4	0.1	1	0.04	0.0004	0.07006569	0.00074618	2	2
A_5	0.6758125	0.68738313	0.022	0.00022	0.01416807	0.00014542	6	6
A_6	0.6803125	0.68742813	0.025	0.000025	0.01845772	0.00018904	5	5

and also values of ϵ may be influences the results of models and it is the decision maker concern.

5. Numerical examples

In this section, we consider two numerical examples using the proposed method to illustrate its applications and show its capability in expressing preferences of voters on a set of candidates.

5.1. Example. We will examine the example taken from Cook and Kress [7], in which 20 voters are asked to rank 4 out of 6 candidates $A_1 \sim A_6$ on a ballot. The votes each candidate receives are shown in Table 1. Also the virtual candidates VBC and VWC are defined in the last two rows of Table 1. Using model (4.1), we obtain the best preference score of VBC as $\phi^*_{VBC} = 1.371625$, and $\phi^*_{VBC} = 1.37496625$ for the values of $\epsilon:\ \epsilon = 0.001$ and $\epsilon = 0.00001$, respectively. In a similar way, using model (4.4) we obtain the worst preference score of VWC as $\varphi^*_{VWC} = 0.012$ and $\varphi^*_{VWC} = 0.00012$ for the values of $\epsilon:\ \epsilon = 0.001$ and $\epsilon = 0.00001$, respectively. Based upon the optimal weights of models (4.2) and (4.6) we can calculate the best preference score and the worst preference score of each candidate as documented in Table 2. In this case, the final overall ranking order can be achieved using the systematic RC index, whose values for the six candidates are presented in Table 2 for the values of $\epsilon:\ \epsilon = 0.001$ and $\epsilon = 0.00001$. From Table 2, the full rank of candidates is as $A_2 \succ A_4 \succ A_3 \succ A_1 \succ A_6 \succ A_5$.

Now, suppose the voters classify into two distinct categories ($C1$ and $C2$) that the vote of the first category has a greater importance than that of the second category. In this case, the voters of each category are asked to rank 4 out 6 previous candidates on a ballot. The votes each candidate receives from each category are presented in Table 3. In addition the virtual candidates VBC and VWC are defined in this table.

Using model (4.1), the best preference scores of VBC are obtained as $\phi^*_{VBC} = 1.93931579$, and $\phi^*_{VBC} = 1.94728789$ for the values of $\epsilon:\ \epsilon = 0.001$ and $\epsilon = 0.00001$, respectively. In addition, using model (4.4) we obtain the worst preference score of VWC as $\varphi^*_{VWC} = 0.014$ and $\varphi^*_{VWC} = 0.00014$ for the values of $\epsilon:\ \epsilon = 0.001$ and $\epsilon = 0.00001$,

Table 3. Votes received by six candidates from two categories

	First Place		Second Place		Third Place		Fourth Place	
Candidates	C 1	C 2	C 1	C 2	C 1	C 2	C 1	C 2
A_1	2	1	1	2	1	3	2	1
A_2	1	3	1	4	3	2	2	0
A_3	3	3	1	1	1	2	1	1
A_4	1	5	1	1	1	1	1	5
A_5	0	0	3	1	1	2	1	3
A_6	1	0	1	3	1	2	1	2
VBC	3	5	3	4	3	3	2	3
VWC	0	0	1	1	1	2	1	0

Table 4. The preference scores for the six candidates based our approach

	The best score		The least score		RC		Rank	
Candidates	$\epsilon = 0.001$	$\epsilon = 0.00001$	$\epsilon = 0.001$	$\epsilon = 0.00001$	$\epsilon = 0.001$	$\epsilon = 0.00001$	$\epsilon = 0.001$	$\epsilon = 0.00001$
A_1	0.91647368	0.92100684	0.038	0.00038	0.02292609	0.0002338	4	4
A_2	1	1	0.05	0.0005	0.03691112	0.00037989	1	1
A_3	1	1	0.044	0.00044	0.03094967	0.00031659	2	2
A_4	0.994	0.99994	0.044	0.00044	0.03075927	0.00031657	3	3
A_5	0.80610526	0.81569263	0.027	0.00027	0.01134172	0.00011487	5	5
A_6	0.73057895	0.73677947	0.029	0.00029	0.01225754	0.0001239	6	6

respectively. Based upon the optimal weights of models (4.2) and (4.6) we can determine the best preference score and the worst preference score of each candidate as documented in Table 4. Thus, the final overall ranking order can be obtained using the systematic RC index, whose values for the six candidates are presented in Table 4 for the values of ϵ : $\epsilon = 0.001$ and $\epsilon = 0.00001$. From Table 4, the full rank of candidates is as $A_2 \succ A_3 \succ A_4 \succ A_1 \succ A_6 \succ A_5$. As can be seen from Table 4, our model also identifies the candidate A_2 as the first winner when $\epsilon = 0.001$ and $\epsilon = 0.00001$. Moreover, by considering the systematic RC index of candidates A_3 and A_4, we see the candidate A_3 is more efficient than the candidate A_4. That is, in our opinion the candidate A_3 is the second winner and the candidate A_4 is the third winner. Thus, there is a different in rank of second and third winner candidate comparing with models that assume all votes have a same importance. In fact, the ability to identify efficient candidates based on our approach is stronger than the previous approach.

5.2. Example. We will examine the example taken from Wang et al. [40], in which 155 voters are asked to rank 4 out of 10 candidates $A \sim J$ on a ballot. The votes each candidate receives are shown in Table 5. In addition, the virtual candidates VBC and VWC are defined in the last two rows of Table 5.

The model (4.1) gives the best preference score of VBC as $\phi^*_{VBC} = 1.28020626$, and $\phi^*_{VBC} = 1.28314864$ under two different values of $\epsilon = 0.001$ and $\epsilon = 0.00001$, respectively. In a similar way, model (4.4) gives the worst preference score of VWC as $\varphi^*_{VWC} = 0.54628571$ and $\varphi^*_{VWC} = 0.53582$ when ϵ takes 0.001 and 0.00001, respectively. Based upon the optimal weights of models (4.2) and (4.6) we can calculate the best preference score and the worst preference score of each candidate as reported in Table 6. Thus, the total ranking order can be determined using the systematic RC index, whose values for the ten candidates are given in Table 6 when ϵ takes 0.001 and 0.00001. From Table 6, the full rank of candidates is as $G \succ A \succ E \succ I \succ J \succ C \succ B \succ H \succ D \succ F$.

Table 5. Votes received by ten candidates

Candidates	First Place	Second Place	Third Place	Fourth Place
A	20	14	13	11
B	14	16	16	17
C	14	14	19	21
D	14	13	22	11
E	19	14	12	19
F	14	13	9	11
G	18	17	15	9
H	14	13	20	20
I	14	20	15	20
J	14	21	14	16
VBC	20	21	22	21
VWC	14	13	9	9

Table 6. The preference scores for ten candidates

	The best score		The least score		RC		Rank	
Candidates	$\epsilon = 0.001$	$\epsilon = 0.00001$	$\epsilon = 0.001$	$\epsilon = 0.00001$	$\epsilon = 0.001$	$\epsilon = 0.00001$	$\epsilon = 0.001$	$\epsilon = 0.00001$
A	0.99414793	0.99408349	0.69328571	0.67479	0.33944619	0.3246696	2	2
B	0.93709298	0.93911734	0.57728571	0.53613	0.0828626	0.00090027	7	7
C	0.97228487	0.97712775	0.58128571	0.53617	0.10206421	0.00114241	6	6
D	0.94033136	0.94668142	0.57428571	0.5361	0.07611283	0.00083148	9	9
E	1	1	0.67957143	0.65501	0.32234172	0.29624299	3	3
F	0.76114455	0.7599445	0.54828571	0.53584	0.00383832	0.00003822	10	10
G	1	1	0.7075102	0.68896286	0.36523167	0.35101041	1	1
H	0.9654328	0.97120124	0.57928571	0.53615	0.09488936	0.00105675	8	8
I	1	1	0.59028571	0.53626	0.13571607	0.00155154	4	4
J	0.97759172	0.97634396	0.58728571	0.53623	0.11931975	0.00133457	5	5

Table 7. Votes received by six candidates from two categories

	First Place			Second Place			Third Place			Fourth Place		
Candidates	C 1	C 2	C 3	C 1	C 2	C 3	C 1	C 2	C 3	C 1	C 2	C 3
A	5	11	4	5	8	1	4	4	5	2	2	7
B	2	9	3	4	9	3	4	8	4	4	11	2
C	2	12	0	3	8	3	11	4	4	5	10	6
D	3	7	4	4	2	7	5	10	7	7	2	2
E	1	13	5	12	1	1	3	1	8	13	2	4
F	1	13	0	3	4	6	2	4	3	2	2	7
G	5	1	12	1	1	15	13	1	1	1	7	1
H	1	4	9	3	1	9	5	11	4	5	5	10
I	1	2	11	5	2	13	3	3	9	16	3	1
J	2	2	10	7	5	9	1	2	11	4	11	1
VBC	5	13	12	12	9	15	13	11	11	16	11	10
VWC	1	1	0	1	1	1	1	1	1	1	2	1

Now we suppose the 155 voters are divided into three categories ($C1$, $C2$ and $C3$) based on their priorities and proficiencies. The votes each candidate receives from each category are shown in Table 7. Also the virtual candidates VBC and VWC are defined in this table.

Table 8. The preference scores for the ten candidates based on our approach

	The best score		The least score		RC		Rank	
Candidates	$\epsilon = 0.001$	$\epsilon = 0.00001$	$\epsilon = 0.001$	$\epsilon = 0.00001$	$\epsilon = 0.001$	$\epsilon = 0.00001$	$\epsilon = 0.001$	$\epsilon = 0.00001$
A	1	1	0.327	0.20127	0.19162271	0.11214341	1	1
B	0.96464852	0.96810428	0.26055556	0.09060556	0.143802	0.02125852	8	6
C	1	1	0.28533333	0.13485333	0.1647688	0.06008753	3	3
D	0.94407853	0.95292174	0.27133333	0.13471333	0.14863913	0.05757814	5	4
E	0.9702937	0.06859667	0.25966667	0.96515685	0.14384231	0.0016434	6	8
F	0.71743712	0.06800667	0.20066667	0.71535221	0.08401168	0.00090331	10	10
G	0.99470197	1	0.26766667	0.11817667	0.15218444	0.04604364	4	5
H	0.966497831	0.98307375	0.23566667	0.06835667	0.12680534	0.00144938	9	9
I	1	1	0.25466667	0.06854667	0.14383597	0.00165028	7	7
J	1	1	0.28933333	0.13489333	0.16742395	0.06012072	2	2

Using model (4.1), the best preference scores of VBC are obtained as $\phi^*_{VBC} = 2.04761608$, and $\phi^*_{VBC} = 2.06472518$ for the values of $\epsilon :\ \epsilon = 0.001$ and $\epsilon = 0.00001$, respectively. In addition, using model (4.4) we obtain the worst preference score of VWC as $\varphi^*_{VWC} = 0.07866667$ and $\varphi^*_{VWC} = 0.06678667$ for the values of $\epsilon :\ \epsilon = 0.001$ and $\epsilon = 0.00001$, respectively. Based upon the optimal weights of models (4.2) and (4.6) we can determine the best preference score and the worst preference score of each candidate as documented in Table 8. Then, the final overall ranking order can be obtained using the systematic RC index, whose values for the ten candidates are presented in Table 8 for the values of $\epsilon :\ \epsilon = 0.001$ and $\epsilon = 0.00001$. From Table 8, when ϵ takes 0.001, the full rank of candidates is obtained as $A \succ J \succ C \succ G \succ D \succ E \succ I \succ B \succ H \succ F$.

As can be seen from Table 8, there is a different in total rank based on our approach comparing with that approach which assumes all votes have equal importance. Our method identifies the candidates A as the first winner and the candidate G as the forth winner while that approach identifies the candidate A as the second winner and the candidate G as the first winner. However, different from that approach, our approach considers the priority of voters and so the votes in a higher category have more importance than that in a lower category. Thus, the preference scores are measured in a persuasive way.

It is necessary to notice that as we discussed in the end of Section 4, the value of ϵ may be influences the order of candidates. This point has been illustrated in Example 5.2. From Table 8, when $\epsilon :\ \epsilon = 0.001$ and $\epsilon = 0.00001$, candidate B is the eighth winner and sixth winner, candidate D is the fifth winner and fourth winner, candidate E is the sixth winner and eighth winner and candidate D is the fourth winner and fifth winner, respectively. This means that there is a small difference in the rank of candidates B, D, E and G when ϵ varies. However, as can be seen from Table 8, candidates A, C, F, H, I and G should take the first place, the third place, the tenth place, the ninth place, the seventh place and the second place, respectively under the both values of ϵ. This is, based on two different values of $\epsilon :\ \epsilon = 0.001$ and $\epsilon = 0.00001$, candidates A and F should be the first winner and the last winner, respectively.

6. Conclusion

It is often necessary in decision making framework to rank a group of candidates in voting systems. In ranked voting systems, each voter selects a subset of candidates and rank them from most to least preferred and hence the score obtained by each candidate is the weighted sum of the scores receives in different places. The principal drawback of such scoring rules is that they assume the votes of all voters have equal importance and there is no preference among them. In this paper, we generalized the existing scoring rules to overcome the mentioned drawback. The ability to identify efficient candidates of our approach is stronger than the existing scoring rules. We also introduced two models that the first model evaluated candidates from the viewpoint of the best possible preference score and the second model evaluated them from the perspective of the worst possible preference score. The two distinctive scores have been combined to form a comprehensive index such that an overall ranking for all the candidates can be obtained. Finally we illustrated our method with two examples. In addition, the extension of some other ranking methods to rank of decision making units in DEA framework such as the proposed approach by Golam Abri et al. [14] can be interesting for ranking of efficient candidates in voting systems as a research work.

In our opinion, we feel that there are many other ranking methods in DEA and should be considered for voting systems later on. Some of these methods are discussed below.

(1) Ramazani-Tarkhorani et al. [30] obtained a common set of weights (CSW) to create the best efficiency score of a group composed of efficient units in DEA. Development of their method for ranking of efficient candidates in voting systems may also produce interesting results.
(2) Jahanshaloo et al. [23] defined an ideal line determined a CSW for efficient units in DEA and then a new efficiency score obtained and ranked them with it. In the second method, they introduced a special line and then compared all efficient units with it and ranked them. Extending of these two methods can be effective for ranking of effective candidates in voting systems.
(3) Jahanshaloo et al. [24] presented a new super-efficient method to rank all decision-making units using the TOPSIS method. Development of this method for ranking of all candidates in voting systems may also give interesting results.
(4) Amirteimoori and Kordrostami [3] proposed a super-efficiency DEA model to discriminate the performance of efficient decision making units. How to apply this model to develop a more general model with sound mathematical properties in ranking of efficient candidates is a direction for future research.

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