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New difference-cum-ratio and exponential type estimators in median ranked set sampling

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Abstract

This paper suggests difference-cum-ratio and exponential type estimators of population mean using first or third quartiles and mean of auxiliary variable under median ranked set sampling scheme and we have extended our study in double sampling scheme when the population parameters are unknown. The bias and mean square error of estimators are derived by theoretically both of sampling designs. Empirical studies have been done to demonstrate the efficiency of proposed estimators over the existing estimators. We have found that difference-cum-ratio estimator is always more efficient than regression estimator and both of the estimators are considerable efficient than existing estimators.

Keywords: Median ranked set sampling, Exponential estimator, Ratio estimator, Efficiency.

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1. Introduction

The ranked set sampling (RSS) is conducted by selecting n random samples from the population of size n units each, and ranking each unit within each set with respect to variable of interest. Then an actual measurement is taken of the unit with the smallest rank from the first sample. From the second sample an actual measurement is taken from the second smallest rank, and the procedure is continued until the unit with the largest rank is chosen for actual measurement from the n-th sample. Median ranked set sampling (MRSS) as proposed by Muttlak [10] can be formed by selecting n random samples of size n units from the population and rank the units within each sample with respect to variable of interest. Many authors developed and modified this sampling scheme such as Al-Saleh and Al-Omari[2], Jemain and Al-Omari[4] and Jemain et al.[5], Ozturk and Jafari Jozani[11] etc. Recently Al-Omari[1] has introduced modified ratio estimators

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in median ranked set sampling. In sampling theory some authors such as Singh and Solanki[12, 13], Singh et al.[14, 15] and Solanki et al.[16] etc. proposed estimators for population parameters using auxiliary information. Bahl and Tuteja [3], Yadav et al. [18], Koyuncu and Kadilar[7], Koyuncu[8], Koyuncu et al.[9] studied the exponential estimators to get more efficient estimators than ratio and regression estimators. In this paper following Koyuncu [8], we have suggested two new estimators of population mean under Al-Omari[1] median ranked set sampling scheme, extended our results to double sampling and we have found that the suggested estimators are considerable efficient than classical ratio estimator and Al-Omari[1] estimator.

Simple Random Sampling Design

Let $(X_1, Y_1), (X_2, Y_2), \ldots, (X_n, Y_n)$ be a bivariate random sample with pdf $f(x, y)$, means μ_x, μ_y, variances σ_x^2, σ_y^2 and correlation coefficient ρ_{xy}. Assume that the ranking is performed on the auxiliary variable X to estimate the mean of the variable of interest Y. Let $(X_{11}, Y_{11}), (X_{12}, Y_{12}), \ldots, (X_{nn}, Y_{nn})$ be n independent bivariate random samples each of size n. In this sampling design Al-Omari [1] defined following estimators

$$\hat{\mu}_{YSRS1} = \bar{y}_{SRS}(\frac{\mu_x + q_1}{\bar{x}_{SRS} + q_1}), \qquad \hat{\mu}_{YSRS3} = \bar{y}_{SRS}(\frac{\mu_x + q_3}{\bar{x}_{SRS} + q_3})$$

where q_1 and q_3 are first and third quartiles of X , respectively.$\bar{x}_{SRS}$, $\bar{y}_{SRS}$ are sample means of X and Y . Al-Omari [1] rewrite estimators as:

$$\hat{\mu}_{YSRSk} = \bar{y}_{SRS}(\frac{\mu_x + q_k}{\bar{x}_{SRS} + q_k}) \tag{1.1}$$

where $\hat{\mu}_{YSRSk}$ represent $\hat{\mu}_{YSRS1}$ and $\hat{\mu}_{YSRS3}$ for values of $(k = 1, 3)$. The expression for MSE of $\hat{\mu}_{YSRSk}$ is as follows:

$$MSE(\hat{\mu}_{YSRSk}) = \lambda\sigma_y^2 + \lambda\sigma_x^2(\frac{\mu_y^2}{(\mu_x + q_k)^2} - 2\beta\frac{\mu_y}{(\mu_x + q_k)}) \tag{1.2}$$

where $\lambda = 1/n$, $\beta = \rho_{xy}\sigma_y/\sigma_x$.

Median Ranked Set Sampling Design

For the sake of brevity we follow Al-Omari [1]' sampling design and notations. Median ranked set sampling design can be described as in the following steps:

(1) Select n random samples each of size n bivariate units from the population of interest.
(2) The units within each sample are ranked by visual inspection or any other cost free method with respect to a variable of interest.
(3) If n is odd, select the $((n + 1)/2)$ th-smallest ranked unit X together with the associated Y from each set, i.e., the median of each set. If n is even, from the first $n/2$ sets select the $(n/2)$th ranked unit X together with the associated Y and from the other sets select the $((n + 2)/2)$ th ranked unit X together with the associated Y .
(4) The whole process can be repeated m times if needed to obtain a sample of size nm units.

Let $(X_{i(1)}, Y_{i[1]}), (X_{i(2)}, Y_{i[2]}), \ldots, (X_{i(n)}, Y_{i[n]})$ be the order statistics of $X_{i1}, X_{i2}, \ldots, X_{in}$ and the judgement order of $Y_{i1}, Y_{i2}, \ldots, Y_{in}$ $(i = 1, 2, \ldots, n)$, where () and [] indicate that the ranking of X is perfect and ranking of Y has errors. For odd and even sample sizes the units measured using MRSS are denoted by MRSSO and MRSSE, respectively. For odd sample size let $(X_{1(\frac{n+1}{2})}, Y_{1[\frac{n+1}{2}]}), (X_{2(\frac{n+1}{2})}, Y_{2[\frac{n+1}{2}]}), \ldots, (X_{n(\frac{n+1}{2})}, Y_{n[\frac{n+1}{2}]})$ denote the observed units by MRSSO. $\bar{x}_{MRSSO} = \frac{1}{n}\sum_{i=1}^{n} X_{i(\frac{n+1}{2})}$ and $\bar{y}_{MRSSO} = \frac{1}{n}\sum_{i=1}^{n} X_{i[\frac{n+1}{2}]}$ be the sample mean of X and Y respectively.

For even sample size let $(X_{1(\frac{n}{2})}, Y_{1[\frac{n}{2}]}), (X_{2(\frac{n}{2})}, Y_{2[\frac{n}{2}]}), \ldots, (X_{\frac{n}{2}(\frac{n}{2})}, Y_{\frac{n}{2}[\frac{n}{2}]})$, $(X_{\frac{n+2}{2}(\frac{n+2}{2})}, Y_{\frac{n+2}{2}[\frac{n+2}{2}]}), (X_{\frac{n+4}{2}(\frac{n+2}{2})}, Y_{\frac{n+4}{2}[\frac{n+2}{2}]}), \ldots, (X_{n(\frac{n}{2})}, Y_{n[\frac{n}{2}]})$ denote the observed units by MRSSE. $\bar{x}_{MRSSE} = \frac{1}{n}(\sum_{i=1}^{\frac{n}{2}} X_{i(\frac{n}{2})} + \sum_{i=\frac{n+2}{2}}^{n} X_{i(\frac{n+2}{2})})$ and $\bar{y}_{MRSSE} = \frac{1}{n}(\sum_{i=1}^{\frac{n}{2}} Y_{i[\frac{n}{2}]} + \sum_{i=\frac{n+2}{2}}^{n} Y_{i[\frac{n+2}{2}]})$ be the sample mean of X and Y respectively.

To obtain the bias and the mean square error (MSE), let us define

$$\varepsilon_{0(j)} = \frac{\bar{y}_{MRSS(j)} - \mu_y}{\mu_y},\ \varepsilon_{1(j)} = \frac{\bar{x}_{MRSS(j)} - \mu_x}{\mu_x},\ \varepsilon_{2(j)} = \frac{s_{yx(j)} - \sigma_{yx(j)}}{\sigma_{yx(j)}},$$

$$\varepsilon_{3(j)} = \frac{s^2_{x(j)} - \sigma^2_{x(j)}}{\sigma^2_{x(j)}}$$

where $j = (E, O)$ denote the sample size even or odd. If sample size n is odd we can write

$$E(\varepsilon^2_{0(O)}) = \frac{1}{n\mu_y^2}\sigma^2_{y[\frac{n+1}{2}]},\ E(\varepsilon^2_{1(O)}) = \frac{1}{n\mu_x^2}\sigma^2_{x(\frac{n+1}{2})},$$

$$E(\varepsilon_{0(O)}\varepsilon_{1(O)}) = \frac{1}{n\mu_x\mu_y}\sigma_{xy[\frac{n+1}{2}]}$$

If sample size n is even we can write

$$E(\varepsilon^2_{0(E)}) = \frac{1}{2n\mu_y^2}(\sigma^2_{y[\frac{n}{2}]} + \sigma^2_{y[\frac{n+2}{2}]}),\ E(\varepsilon^2_{1(E)}) = \frac{1}{2n\mu_x^2}(\sigma^2_{x(\frac{n}{2})} + \sigma^2_{x(\frac{n+2}{2})}),$$

$$E(\varepsilon_{0(E)}\varepsilon_{1(E)}) = \frac{1}{2n\mu_x\mu_y}(\sigma_{yx[\frac{n}{2}]} + \sigma_{yx[\frac{n+2}{2}]})$$

(i) Al-Omari(2012) estimator The estimator of population mean proposed by Al-Omari[1] as

$$\hat{\mu}_{YMRRS1} = \bar{y}_{MRSS}(\frac{\mu_x + q_1}{\bar{x}_{MRSS} + q_1}), \hat{\mu}_{YMRRS3} = \bar{y}_{MRSS}(\frac{\mu_x + q_3}{\bar{x}_{MRSS} + q_3})$$

For odd and even sample sizes the estimator can be rewritten as

$$\hat{\mu}_{YMRSSk} = \bar{y}_{MRSS(j)}(\frac{\mu_x + q_k}{\bar{x}_{MRSS(j)} + q_k}) \tag{1.3}$$

To the first degree of approximation the Bias and MSE of $\hat{\mu}_{YMRRSk}$ are respectively given by

$$Bias(\hat{\mu}_{YMRSS(j)}) \cong \begin{cases} \frac{\mu_y\psi}{n\mu_x}(\psi\frac{1}{\mu_x}\sigma_{x(\frac{n+1}{2})^2} - \frac{1}{\mu_y}\sigma_{xy[\frac{n+1}{2}]}) & \text{if } n \text{ is odd} \\ \frac{\mu_y\psi}{2n\mu_x}(\psi\frac{1}{\mu_x}(\sigma^2_{x(\frac{n}{2})} + \sigma^2_{x(\frac{n+2}{2})}) \\ -\frac{1}{\mu_y}(\sigma_{xy[\frac{n}{2}]} + \sigma_{xy[\frac{n+2}{2}]})) & \text{if } n \text{ is even} \end{cases} \tag{1.4}$$

(1.5)

$$MSE(\hat{\mu}_{YMRSS(j)}) \cong \begin{cases} \frac{1}{n}(\frac{\mu_y^2}{(\mu_x+q_k)^2}\sigma^2_{x(\frac{n+1}{2})} + \sigma^2_{y[\frac{n+1}{2}]} \\ -2\frac{\mu_y}{(\mu_x+q_k)}\sigma_{xy[\frac{n+1}{2}]}) & \text{if } n \text{ is odd} \\ \\ \frac{1}{2n}(\frac{\mu_y^2}{(\mu_x+q_k)^2}(\sigma^2_{x(\frac{n}{2})} + \sigma^2_{x(\frac{n+2}{2})})+ \\ (\sigma^2_{y[\frac{n}{2}]} + \sigma^2_{y[\frac{n+2}{2}]}) - 2\frac{\mu_y}{(\mu_x+q_k)}(\sigma_{xy[\frac{n}{2}]} + \sigma_{xy[\frac{n+2}{2}]})) & \text{if } n \text{ is even} \end{cases}$$

where $\psi = \frac{\mu_x}{\mu_x+q_k}$.

(ii) Adapted Regression estimator We can define regression type estimator in median ranked set sampling given by

$$\overline{y}_{reg(j)} = \overline{y}_{MRSS(j)} + b_{yx(j)}(\mu_x - \overline{x}_{MRSS(j)}) \tag{1.6}$$

where

$$b_{xy(j)} \cong \begin{cases} \frac{\rho_{xy[\frac{n+1}{2}]}s_{y[\frac{n+1}{2}]}}{s_{x(\frac{n+1}{2})}} & \text{if } n \text{ is odd} \\ \frac{(\rho_{xy[\frac{n}{2}]}+\rho_{xy[\frac{n+2}{2}]})(s_{y[\frac{n}{2}]}+s_{y[\frac{n+2}{2}]})}{(s_{x(\frac{n}{2})}+s_{x(\frac{n+2}{2})})} & \text{if } n \text{ is even} \end{cases}$$

$$s_{xy[\frac{n+1}{2}]} = \rho_{xy[\frac{n+1}{2}]}s_{x(\frac{n+1}{2})}s_{y[\frac{n+1}{2}]}$$

$$(s_{xy[\frac{n}{2}]} + s_{xy[\frac{n+2}{2}]}) = (\rho_{xy[\frac{n}{2}]} + \rho_{xy[\frac{n+2}{2}]})(s_{x(\frac{n}{2})} + s_{x(\frac{n+2}{2})})(s_{y[\frac{n}{2}]} + s_{y[\frac{n+2}{2}]})$$

$$MSE(\overline{y}_{reg(j)}) \cong \begin{cases} \frac{1}{n}\sigma^2_{y[\frac{n+1}{2}]}(1-\rho^2_{xy[\frac{n+1}{2}]}) & \text{if } n \text{ is odd} \\ \frac{1}{2n}(\sigma^2_{y[\frac{n}{2}]} + \sigma^2_{y[\frac{n+2}{2}]})(1-(\rho^2_{xy[\frac{n+1}{2}]} + \rho^2_{xy[\frac{n+2}{2}]})) & \text{if } n \text{ is even} \end{cases} \tag{1.7}$$

2. Suggested estimators in median ranked set sampling

Following Koyuncu [8], we propose difference-cum-ratio estimator estimating the population mean of the study variable in median ranked set sampling as follows:

$$\overline{y}_{Nk(M)} = [k_{1(j)}\overline{y}_{MRSS(j)} + k_{2(j)}(\mu_x - \overline{x}_{MRSS(j)})](\frac{\mu_x + q_k}{\overline{x}_{MRSS(j)} + q_k}) \tag{2.1}$$

where $k_{1(j)}$ and $k_{2(j)}$ are determined so as to minimize the MSE of $\overline{y}_{Nk(M)}$. Expressing $\overline{y}_{Nk(M)}$ in terms of $\varepsilon_{(j)}$'s up to the second degree and extracting μ_y both sides we have

$$\begin{aligned} \overline{y}_{Nk(M)} - \mu_y =& (k_{1(j)} - 1)\mu_y + k_{1(j)}\mu_y\varepsilon_{0(j)} - k_{2(j)}\mu_x\varepsilon_{1(j)} - k_{1(j)}\mu_y\psi\varepsilon_{1(j)} \\ &+ k_{1(j)}\mu_y\psi\varepsilon_{0(j)}\varepsilon_{1(j)} + k_{2(j)}\mu_x\psi\varepsilon^2_{1(j)} + k_{1(j)}\mu_y\psi^2\varepsilon^2_{1(j)} \end{aligned} \tag{2.2}$$

Taking expectation in equation in (2.2), we obtain

(2.3)

$$Bias(\overline{y}_{Nk(M)}) \cong \begin{cases} (k_{1(O)}-1)\mu_y - k_{1(O)}\mu_y\psi\frac{1}{n\mu_y\mu_x}\sigma_{xy[\frac{n+1}{2}]} + (k_{2(O)}\mu_x\psi \\ +k_{1(O)}\mu_y\psi^2)\frac{1}{n\mu_x^2}\sigma^2_{x(\frac{n+1}{2})} & \text{if } n \text{ is odd} \\ \\ (k_{1(E)}-1)\mu_y - k_{1(E)}\mu_y\psi\frac{1}{2n\mu_y\mu_x}(\sigma_{xy[\frac{n}{2}]} + \sigma_{xy[\frac{n+2}{2}]}) \\ +(k_{2(E)}\mu_x\psi + k_{1(E)}\mu_y\psi^2)\frac{1}{2n\mu_x^2}(\sigma^2_{x(\frac{n}{2})} + \sigma^2_{x(\frac{n+2}{2})}) & \text{if } n \text{ is even} \end{cases}$$

Squaring both sides in (2.2), then taking expectation, we obtain the MSE of the estimator $\overline{y}_{Nk(M)}$, as given by

$$(2.4)\quad MSE_{min}(\overline{y}_{Nk(M)}) \cong \begin{cases} (1-\frac{\psi^2}{n\mu_x^2}\sigma^2_{x(\frac{n+1}{2})}) \\ \times\frac{MSE(\overline{y}_{reg(O)})}{1-\frac{\psi^2}{n\mu_x^2}\sigma^2_{x(\frac{n+1}{2})}+\frac{1}{\mu_y^2}MSE(\overline{y}_{reg(O)})} & \text{if } n \text{ is odd} \\ (1-\frac{\psi^2}{n\mu_x^2}(\sigma^2_{x(\frac{n}{2})}+\sigma^2_{x(\frac{n+2}{2})}) \\ \times\frac{MSE(\overline{y}_{reg(E)})}{1-\frac{\psi^2}{2n\mu_x^2}(\sigma^2_{x(\frac{n}{2})}+\sigma^2_{x(\frac{n+2}{2})})+\frac{1}{\mu_y^2}MSE(\overline{y}_{reg(E)})} & \text{if } n \text{ is even} \end{cases}$$

The optimum values of $k_{1(j)}$ and $k_{2(j)}$ for odd and even sample sizes are given respectively

$$k^*_{1(O)} = \frac{1-\frac{\psi^2}{n\mu_x^2}\sigma^2_{x(\frac{n+1}{2})}}{1-\frac{\psi^2}{n\mu_x^2}\sigma^2_{x(\frac{n+1}{2})})+\frac{1}{\mu_y^2}MSE(\overline{y}_{reg(O)})}$$

$$k^*_{1(E)} = \frac{1-\frac{\psi^2}{2n\mu_x^2}(\sigma^2_{x(\frac{n}{2})}+\sigma^2_{x(\frac{n+2}{2})})}{1-\frac{\psi^2}{2n\mu_x^2}(\sigma^2_{x(\frac{n}{2})}+\sigma^2_{x(\frac{n+2}{2})})+\frac{1}{\mu_y^2}MSE(\overline{y}_{reg(E)})}$$

$$k^*_{2(O)} = \frac{\mu_y}{\mu_x}(\psi + \frac{(\frac{1}{\mu_y\mu_x}\sigma_{xy[\frac{n+1}{2}]} - 2\psi\frac{1}{\mu_x^2}\sigma^2_{x(\frac{n+1}{2})})(1-\frac{\psi^2}{n\mu_x^2}\sigma^2_{x(\frac{n+1}{2})})}{\frac{1}{\mu_x^2}\sigma^2_{x(\frac{n+1}{2})}((1-\frac{\psi^2}{n\mu_x^2}\sigma^2_{x(\frac{n+1}{2})})+\frac{1}{\mu_y^2}MSE(\overline{y}_{reg(O)})})$$

$$k^*_{2(E)} = \frac{\mu_y}{\mu_x}(\psi + \frac{(\frac{1}{\mu_y\mu_x}(\sigma_{xy[\frac{n}{2}]}+\sigma_{xy[\frac{n+2}{2}]}) - 2\psi\frac{1}{\mu_x^2}(\sigma^2_{x(\frac{n}{2})}+\sigma^2_{x(\frac{n+2}{2})}))(1-\frac{\psi^2}{2n\mu_x^2}(\sigma^2_{x(\frac{n}{2})}+\sigma^2_{x(\frac{n+2}{2})}))}{\frac{1}{\mu_x^2}(\sigma^2_{x(\frac{n}{2})}+\sigma^2_{x(\frac{n+2}{2})})((1-\frac{\psi^2}{2n\mu_x^2}(\sigma^2_{x(\frac{n}{2})}+\sigma^2_{x(\frac{n+2}{2})}))+\frac{1}{\mu_y^2}MSE(\overline{y}_{reg(E)})})$$

Note that the optimum choice of the constants involve unknown parameters. These quantities can be guessed quite accurately through pilot sample survey or sample data or experience gathered in due course of time as mentioned in Upadhyaya and Singh[17], and Koyuncu and Kadilar[6].

Secondly following exponential estimator is proposed:

$$(2.5)\quad \overline{y}_{Kk(M)} = [w_{1(j)}\overline{y}_{MRSS(j)} + w_{2(j)}(\frac{\overline{x}_{MRSS(j)}}{\mu_x})^{\gamma}]exp(\frac{\mu_x-\overline{x}_{MRSS(j)}}{\mu_x-\overline{x}_{MRSS(j)}+2q_k})$$

where $w_{1(j)}$ and $w_{2(j)}$ are determined so as to minimize the MSE of $\overline{y}_{Kk(M)}$.Expressing $\overline{y}_{Kk(M)}$ in terms of ε_j's up to the second degree and extracting μ_y both sides, we have

(2.6)

$$\overline{y}_{Kk(M)} - \mu_y = \{w_{1(j)}\mu_y - \mu_y + w_{1(j)}\mu_y\varepsilon_{0(j)} + w_{2(j)} + w_{2(j)}\gamma\varepsilon_{1(j)} + w_{2(j)}\frac{\gamma(\gamma-1)}{2}\varepsilon_{1(j)}^2 - \frac{1}{2}w_{1(j)}\psi\mu_y\varepsilon_{1(j)} - \frac{1}{2}w_{1(j)}\psi\mu_y\varepsilon_{0(j)}\varepsilon_{1(j)} - \frac{1}{2}w_{2(j)}\psi\varepsilon_{1(j)} - \frac{1}{2}w_{2(j)}\psi\gamma\varepsilon_{1(j)}^2 + \frac{3}{8}w_{1(j)}\psi^2\mu_y\varepsilon_{1(j)}^2 + \frac{3}{8}w_{2(j)}\psi^2\varepsilon_{1(j)}^2\}$$

Taking expectation in equation in (2.6), we obtain

(2.7)

$$Bias(\overline{y}_{Kk(M)}) \cong \begin{cases} w_{1(O)}\mu_y - \mu_y + w_{2(O)} - \frac{w_{1(O)}\psi}{2n\mu_x}\sigma_{xy[\frac{n+1}{2}]} \\ +(\frac{w_{2(O)}}{2}(-\psi\gamma + \frac{3}{4}\psi^2 + \gamma(\gamma-1)) \\ +\frac{3}{8}w_{1(O)}\psi^2\mu_y)\frac{1}{n\mu_x^2}\sigma^2_{x(\frac{n+1}{2})} & \text{if } n \text{ is odd} \\ \\ w_{1(E)}\mu_y - \mu_y + w_{2(E)} - \frac{w_{1(E)}\psi}{4n\mu_x}(\sigma_{xy[\frac{n}{2}]} + \sigma_{xy[\frac{n+2}{2}]}) \\ +(\frac{w_{2(E)}}{2}(-\psi\gamma + \frac{3}{4}\psi^2 + \gamma(\gamma-1)) \\ +\frac{3}{8}w_{1(E)}\psi^2\mu_y)\frac{1}{2n\mu_x^2}(\sigma^2_{x(\frac{n}{2})} + \sigma^2_{x(\frac{n+2}{2})}) & \text{if } n \text{ is even} \end{cases}$$

Squaring both sides in (2.6), then taking expectation, we obtain the MSE of the estimator $\overline{y}_{Kk(M)}$, as given by

(2.8)

$$MSE(\overline{y}_{Kk(M)}) = \mu_y^2 + w_{1(j)}^2\mu_y^2 + A_j + w_{2(j)}^2 B_j + w_{1(j)}\mu_y^2 + D_j + w_{2(j)}\mu_y + G_j + w_{1(j)}w_{2(j)}\mu_y F_j$$

where

$$A_{(j)} = 1 + E(\varepsilon_{0(j)}^2) + \psi^2 E(\varepsilon_{1(j)}^2) - 2\psi E(\varepsilon_{0(j)}\varepsilon_{1(j)})$$

$$B_{(j)} = 1 + (\gamma^2 + \psi^2 + \gamma(\gamma-1) - 2\gamma\psi)E(\varepsilon_{1(j)}^2)$$

$$D_{(j)} = -2 + \psi E(\varepsilon_{0(j)}\varepsilon_{1(j)}) - \frac{3}{4}\psi^2 E(\varepsilon_{1(j)}^2)$$

$$G_{(j)} = -2 + (\psi\gamma - \gamma(\gamma-1) - \frac{3}{4}\psi^2)E(\varepsilon_{1(j)}^2)$$

$$F_{(j)} = 2 + (\gamma(\gamma-1) - 2\gamma\psi + 2\psi^2)E(\varepsilon_{1(j)}^2) + 2(\gamma - \psi)E(\varepsilon_{0(j)}\varepsilon_{1(j)})$$

Minimization of (2.8) with respect to $w_{1(j)}$ and $w_{2(j)}$ yields its optimum value when

$$w_{1(j)} = \frac{F_{(j)}G_{(j)} - 2B_{(j)}D_{(j)}}{4B_{(j)}A_{(j)} - F_{(j)}^2}, \qquad w_{2(j)} = \mu_y\frac{D_{(j)}F_{(j)} - 2A_{(j)}G_{(j)}}{4B_{(j)}A_{(j)} - F_{(j)}^2} \quad (2.9)$$

Substituting optimum values of $w_{1(j)}$ and $w_{2(j)}$ in (2.9), we get minimum MSE of $\overline{y}_{Kk(M)}$ as

$$MSE_{min}(\overline{y}_{Kk(M)}) = \mu_y^2[1 - \frac{B_{(j)}D_{(j)}^2 + A_{(j)}G_{(j)}^2 - D_{(j)}F_{(j)}G_{(j)}}{4B_{(j)}A_{(j)} - F_{(j)}^2}] \quad (2.10)$$

3. Theoretical comparison

Firstly, we compare the MSE of proposed difference-cum-ratio estimator $\overline{y}_{Kk(M)}$ with the MSE of regression estimator $\overline{y}_{Reg(O)}$ when sample size is odd.

$$MSE_{min}(\overline{y}_{Nk(M)}) < MSE(\overline{y}_{Reg(O)})$$

$$(1-\frac{\psi^2}{n\mu_x^2}\sigma^2_{x(\frac{n+1}{2})})\frac{MSE(\overline{y}_{reg(O)})}{1-\frac{\psi^2}{n\mu_x^2}\sigma^2_{x(\frac{n+1}{2})}+\frac{1}{\mu_y^2}MSE(\overline{y}_{reg(O)})} < MSE(\overline{y}_{Reg(O)})$$

$$0 < \frac{1}{\mu_y^2}MSE(\overline{y}_{Reg(O)}) \tag{3.1}$$

From (3.1), we can conclude that $\overline{y}_{Nk(M)}$ is always more efficient that $\overline{y}_{Reg(O)}$. Secondly, we compare the suggested exponential estimator $\overline{y}_{Nk(M)}$ with the regression estimator $\overline{y}_{Reg(E)}$ when sample size n is even.

$$MSE_{min}(\overline{y}_{Nk(M)}) < MSE(\overline{y}_{Reg(E)})$$

$$(1-\frac{\psi^2}{n\mu_x^2}(\sigma^2_{x(\frac{n}{2})}+\sigma^2_{x(\frac{n+2}{2})})\frac{MSE(\overline{y}_{reg(E)})}{1-\frac{\psi^2}{2n\mu_x^2}(\sigma^2_{x(\frac{n}{2})}+\sigma^2_{x(\frac{n+2}{2})})+\frac{1}{\mu_y^2}MSE(\overline{y}_{reg(E)})} < MSE(\overline{y}_{Reg(E)})$$

$$0 < \frac{1}{\mu_y^2}MSE(\overline{y}_{Reg(E)}) \tag{3.2}$$

From (3.2), we can conclude that $\overline{y}_{Nk(M)}$ is always more efficient than regression estimator.

4. Estimation of population mean when μ_x is unknown

In practice, when the population mean of auxiliary variable is unknown, double sampling method can be used to estimate μ_x. In this section we assume that mean of auxiliary variable is unavailable. Thus following the procedure outlined in Al-Omari[1], in SRS, a large sample of size $n^{'}$ is selected to estimate μ_x. Then a sub sample of size $n^{''}$ is selected from the target population in order to study the characteristic variable Y. In MRSS, simple random sampling is used at first phase and median ranked set sampling is used at second phase where $n^{'}=n^2$ and $n^{''}=n$. Let $\overline{x}^{'}_{SRS(j)}$ and $\overline{x}^{'}_{MRSS(j)}$ be the unbiased sample means of μ_x obtained using SRS and MRSS, respectively. Al-Omari [1] defined following estimator in double sampling

$$\hat{\mu}^{'}_{YSRSk} = \bar{y}_{SRS}(\frac{\overline{x}^{'}_{SRS}+q_k}{\overline{x}_{SRS}+q_k}) \tag{4.1}$$

In order to obtain the bias and mean square of the estimator in (4.1), let us define

$$e_0=\frac{\overline{y}_{SRS}-\mu_y}{\mu_y},\ e_1=\frac{\overline{x}_{SRS}-\mu_x}{\mu_x},\ e_1^{'}=\frac{\overline{x}^{'}_{SRS}-\mu_x}{\mu_x}.$$

Using these notations, the expectations are defined as $E(e_0)=E(e_1)=E(e_1)^{'}$

$$E(e_0^2)=\frac{1}{n^{''}}\frac{\sigma_y^2}{\mu_y^2},\ E(e_1^2)=\frac{1}{n^{''}}\frac{\sigma_x^2}{\mu_x^2},\ E(e_1^{'2})=\frac{1}{n^{'}}\frac{\sigma_x^2}{\mu_x^2},\ E(e_0e_1)=\frac{1}{n^{''}}\frac{\sigma_{yx}}{\mu_x\mu_y},$$

$$E(e_1 e_1^{'}) = \frac{1}{n^{'}} \frac{\sigma_x^2}{\mu_x^2},\ E(e_0 e_1^{'}) = \frac{1}{n^{'}} \frac{\sigma_{yx}}{\mu_x \mu_y}.$$

Applying the same procedure for double sampling the bias and MSE of $\hat{\mu}_{YSRSk}^{'}$ are obtained respectively as,

$$Bias(\hat{\mu}_{YSRSk}^{'}) = (\frac{1}{n^{''}} - \frac{1}{n^{'}})(\frac{\mu_y}{(\mu_x + q_k)^2}\sigma_x^2 - \frac{1}{\mu_x + q_k}\sigma_{yx}) \tag{4.2}$$

$$MSE(\hat{\mu}_{YSRSk}^{'}) = \frac{1}{n^{''}}\sigma_y^2 + (\frac{1}{n^{''}} - \frac{1}{n^{'}})\sigma_x^2(\frac{\mu_y^2}{(\mu_x + q_k)^2} - 2\beta\frac{\mu_y}{\mu_x + q_k}) \tag{4.3}$$

Secondly Al-Omari [1] defined following estimator in double sampling

$$\hat{\mu}_{YMRSSk}^{'} = \overline{y}_{MRSS(j)}(\frac{\overline{x}_{MRSS(j)}^{'} + q_k}{\overline{x}_{MRSS(j)} + q_k}) \tag{4.4}$$

To obtain the bias and the MSE, let us define

$$\delta_{0(j)} = \frac{\overline{y}_{MRSS(j)} - \mu_y}{\mu_y},\ \delta_{1(j)} = \frac{\overline{x}_{MRSS(j)} - \mu_x}{\mu_x},\ \delta_{1(j)^{'}} = \frac{\overline{x}_{MRSS(j)}^{'} - \mu_x}{\mu_x}.$$

such that $E(\delta_{0(j)}) = E(\delta_{1(j)}) = E(\delta_{1(j)}^{'})$ where $(j) = O, E$ represents the sample size is odd or even. If sample size $n^{''}$ is odd we can write

$$E(\delta_{0(O)}^2) = \frac{1}{n^{''}} \frac{\sigma_{y[\frac{n+1}{2}]}^2}{\mu_y^2},\ E(\delta_{1(O)}^2) = \frac{1}{n^{''}} \frac{\sigma_{x(\frac{n+1}{2})}^2}{\mu_x^2},\ E(\delta_{0(O)}\delta_{1(O)}) = \frac{1}{n^{''}} \frac{\sigma_{xy[\frac{n+1}{2}]}^2}{\mu_x \mu_y},$$

$$E(\delta_{1(O)}^{'2}) = \frac{1}{n^{'}} \frac{\sigma_{x(\frac{n+1}{2})}^2}{\mu_x^2},\ E(\delta_{0(O)}\delta_{1(O)}^{'}) = \frac{1}{n^{'}} \frac{\sigma_{xy(\frac{n+1}{2})}}{\mu_x \mu_y},\ E(\delta_{1(O)}\delta_{1(O)}^{'}) = \frac{1}{n^{'}} \frac{\sigma_{x(\frac{n+1}{2})}^2}{\mu_x^2}.$$

If sample size $n^{''}$ is even we can write

$$E(\delta_{0(E)}^2) = \frac{1}{2n^{''}} \frac{\sigma_{y[\frac{n}{2}]}^2 + \sigma_{y[\frac{n+2}{2}]}^2}{\mu_y^2},\ E(\delta_{1(E)}^2) = \frac{1}{2n^{''}} \frac{\sigma_{x(\frac{n}{2})}^2 + \sigma_{x(\frac{n+2}{2})}^2}{\mu_x^2},$$

$$E(\delta_{0(E)}\delta_{1(E)}) = \frac{1}{2n^{''}} \frac{\sigma_{yx[\frac{n}{2}]} + \sigma_{yx[\frac{n+2}{2}]}}{\mu_x \mu_y},\ E(\delta_{1(E)}^2) = \frac{1}{2n^{''}} \frac{\sigma_{x(\frac{n}{2})}^2 + \sigma_{x(\frac{n+2}{2})}^2}{\mu_x^2},$$

$$E(\delta_{0(E)}\delta_{1(E)}^{'}) = \frac{1}{2n^{'}} \frac{\sigma_{yx[\frac{n}{2}]} + \sigma_{yx[\frac{n+2}{2}]}}{\mu_x \mu_y},\ E(\delta_{1(E)}\delta_{1(E)}^{'}) = \frac{1}{2n^{'}} \frac{\sigma_{x(\frac{n}{2})}^2 + \sigma_{x(\frac{n+2}{2})}^2}{\mu_x^2}.$$

Using the defined expectations the bias and MSE of $\hat{\mu}_{YMRSS(j)}^{'}$ are obtained respectively as,

$$Bias(\hat{\mu}_{YMRSS(j)}^{'}) \cong \begin{cases} \frac{\mu_y \psi}{\mu_x}(\frac{1}{n^{''}} - \frac{1}{n^{'}})(\psi\frac{1}{\mu_x}\sigma_{x(\frac{n+1}{2})^2} - \frac{1}{\mu_y}\sigma_{xy[\frac{n+1}{2}]}) & \text{if } n \text{ is odd} \\ \frac{\mu_y \psi}{2\mu_x}(\frac{1}{n^{''}} - \frac{1}{n^{'}})(\psi\frac{1}{\mu_x}(\sigma_{x(\frac{n}{2})}^2 + \sigma_{x(\frac{n+2}{2})}^2) \\ -\frac{1}{\mu_y}(\sigma_{xy[\frac{n}{2}]} + \sigma_{xy[\frac{n+2}{2}]})) & \text{if } n \text{ is even} \end{cases} \tag{4.5}$$

(4.6)

$$MSE(\hat{\mu}^{'}_{YMRSS(j)}) \cong \begin{cases} \frac{1}{n''}\sigma^2_{y[\frac{n+1}{2}]} + (\frac{1}{n''} - \frac{1}{n'})\frac{\mu_y^2}{(\mu_x+q_k)^2}\sigma^2_{x(\frac{n+1}{2})} \\ -2(\frac{1}{n''} - \frac{1}{n'})\frac{\mu_y}{(\mu_x+q_k)}\sigma_{xy[\frac{n+1}{2}]} & \text{if } n \text{ is odd} \\ \\ \frac{1}{2}[\frac{1}{n''}(\sigma^2_{y[\frac{n}{2}]} + \sigma^2_{y[\frac{n+2}{2}]}) + (\frac{1}{n''} - \frac{1}{n'})\frac{\mu_y^2}{(\mu_x+q_k)^2}(\sigma^2_{x(\frac{n}{2})} + \sigma^2_{x(\frac{n+2}{2})}) \\ -2(\frac{1}{n''} - \frac{1}{n'})\frac{\mu_y}{(\mu_x+q_k)}(\sigma_{xy[\frac{n}{2}]} + \sigma_{xy[\frac{n+2}{2}]})] & \text{if } n \text{ is even} \end{cases}$$

We have suggested new double sampling estimators as follows:

$$\text{(4.7)} \qquad \overline{y}^{'}_{reg(j)} = \overline{y}_{MRSS(j)} + b_{yx(j)}(\overline{x}^{'}_{MRSS(j)} - \overline{x}_{MRSS(j)})$$

The MSE of $\overline{y}^{'}_{reg(j)}$ is obtained as,

(4.8)

$$MSE(\overline{y}_{reg(j)}) \cong \begin{cases} \sigma^2_{y[\frac{n+1}{2}]}(\frac{1}{n''} - (\frac{1}{n''} - \frac{1}{n'})\rho^2_{xy[\frac{n+1}{2}]}) & \text{if } n \text{ is odd} \\ \frac{1}{2}(\sigma^2_{y[\frac{n}{2}]} + \sigma^2_{y[\frac{n+2}{2}]})[\frac{1}{n''} - (\frac{1}{n''} - \frac{1}{n'})(\rho^2_{xy[\frac{n+1}{2}]} + \rho^2_{xy[\frac{n+2}{2}]})] & \text{if } n \text{ is even} \end{cases}$$

In double sampling our suggested estimator can be defined as

$$\text{(4.9)} \qquad \overline{y}^{'}_{Nk(M)} = [k_{1(j)}\overline{y}_{MRSS(j)} + k_{2(j)}(\overline{x}^{'}_{MRSS(j)} - \overline{x}_{MRSS(j)})](\frac{\overline{x}^{'}_{MRSS(j)} + q_k}{\overline{x}_{MRSS(j)} + q_k})$$

The bias and MSE of $\overline{y}^{'}_{Nk(M)}$ are obtained respectively as,

(4.10)

$$Bias(\overline{y}^{'}_{Nk(M)}) \cong \begin{cases} (k_{1(O)} - 1)\mu_y - k_{1(O)}\mu_y\psi\frac{1}{\mu_y\mu_x}(\frac{1}{n''} - \frac{1}{n'})\sigma_{xy[\frac{n+1}{2}]} \\ +(k_{2(O)}\mu_x\psi + k_{1(O)}\mu_y\psi^2)(\frac{1}{n''} - \frac{1}{n'})\frac{1}{\mu_x^2}\sigma^2_{x(\frac{n+1}{2})} & \text{if } n \text{ is odd} \\ \\ (k_{1(E)} - 1)\mu_y - k_{1(E)}\mu_y\psi\frac{1}{2\mu_y\mu_x}(\frac{1}{n''} - \frac{1}{n'})(\sigma_{xy[\frac{n}{2}]} + \sigma_{xy[\frac{n+2}{2}]}) \\ +(k_{2(E)}\mu_x\psi + k_{1(E)}\mu_y\psi^2)\frac{1}{2\mu_x^2}(\frac{1}{n''} - \frac{1}{n'})(\sigma^2_{x(\frac{n}{2})} + \sigma^2_{x(\frac{n+2}{2})}) & \text{if } n \text{ is even} \end{cases}$$

(4.11)

$$MSE_{min}(\overline{y}^{'}_{Nk(M)}) \cong \begin{cases} (1 - \frac{\psi^2}{\mu_x^2}(\frac{1}{n''} - \frac{1}{n'})\sigma^2_{x(\frac{n+1}{2})}) \\ \times\frac{MSE(\overline{y}^{'}_{reg(O)})}{1 - \frac{\psi^2}{\mu_x^2}(\frac{1}{n''} - \frac{1}{n'})\sigma^2_{x(\frac{n+1}{2})} + \frac{1}{\mu_y^2}MSE(\overline{y}^{'}_{reg(O)})} & \text{if } n \text{ is odd} \\ \\ (1 - \frac{\psi^2}{\mu_x^2}(\frac{1}{n''} - \frac{1}{n'})(\sigma^2_{x(\frac{n}{2})} + \sigma^2_{x(\frac{n+2}{2})})) \\ \times\frac{MSE(\overline{y}^{'}_{reg(E)})}{1 - \frac{\psi^2}{2\mu_x^2}(\frac{1}{n''} - \frac{1}{n'})(\sigma^2_{x(\frac{n}{2})} + \sigma^2_{x(\frac{n+2}{2})}) + \frac{1}{\mu_y^2}MSE(\overline{y}^{'}_{reg(E)})} & \text{if } n \text{ is even} \end{cases}$$

Our second estimator can be defined as in double sampling

$$\text{(4.12)} \qquad \overline{y}^{'}_{Kk(M)} = [w_{1(j)}\overline{y}_{MRSS(j)} + w_{2(j)}(\frac{\overline{x}_{MRSS(j)}}{\overline{x}^{'}_{MRSS(j)}})^{\gamma}]exp(\frac{\overline{x}^{'}_{MRSS(j)} - \overline{x}_{MRSS(j)}}{\overline{x}_{MRSS(j)} + \overline{x}_{MRSS(j)} + 2q_k})$$

The bias and MSE of $\overline{y}'_{Kk(M)}$ are obtained respectively as,

(4.13)

$$Bias(\overline{y}'_{Kk(M)}) \cong \begin{cases} w_{1(O)}\mu_y - \mu_y + w_{2(O)} - \frac{w_{1(O)}\psi}{2\mu_x}(\frac{1}{n''} - \frac{1}{n'})\sigma_{xy[\frac{n+1}{2}]} \\ +(\frac{w_{2(O)}}{2}(-\psi\gamma + \frac{3}{4}\psi^2 + \gamma(\gamma-1)) \\ +\frac{3}{8}w_{1(O)}\psi^2\mu_y)\frac{1}{\mu_x^2}(\frac{1}{n''} - \frac{1}{n'})\sigma^2_{x(\frac{n+1}{2})} & \text{if } n \text{ is odd} \\ \\ w_{1(E)}\mu_y - \mu_y + w_{2(E)} - \frac{w_{1(E)}\psi}{4\mu_x}(\frac{1}{n''} - \frac{1}{n'})(\sigma_{xy[\frac{n}{2}]} + \sigma_{xy[\frac{n+2}{2}]}) \\ +(\frac{w_{2(E)}}{2}(-\psi\gamma + \frac{3}{4}\psi^2 + \gamma(\gamma-1)) \\ +\frac{3}{8}w_{1(E)}\psi^2\mu_y)\frac{1}{2\mu_x^2}(\frac{1}{n''} - \frac{1}{n'})(\sigma^2_{x(\frac{n}{2})} + \sigma^2_{x(\frac{n+2}{2})}) & \text{if } n \text{ is even} \end{cases}$$

$$(4.14) \quad MSE_{min}(\overline{y}'_{Kk(M)}) = \mu_y^2[1 - \frac{B'_{(j)}D'^2_{(j)} + A'_{(j)}G'^2_{(j)} - D'_{(j)}F'_{(j)}G'_{(j)}}{4B'_{(j)}A'_{(j)} - F'^2_{(j)}}]$$

$$A'_{(j)} = 1 + E(\delta^2_{0(j)}) + \psi^2 E(\delta^2_{1(j)}) - \psi^2 E(\delta'^2_{1(j)}) - 2\psi E(\delta_{0(j)}\delta_{1(j)}) + 2\psi E(\delta_{0(j)}\delta'_{1(j)})$$

$$B'_{(j)} = 1 + (\gamma^2 + \psi^2 + \gamma(\gamma-1) - 2\gamma\psi)(E(\delta^2_{1(j)}) - E(\delta'^2_{1(j)}))$$

$$D'_{(j)} = -2 + \psi E(\delta_{0(j)}\delta_{1(j)}) - \psi E(\delta_{0(j)}\delta'_{1(j)}) - \frac{3}{4}\psi^2 E(\delta^2_{1(j)}) + \frac{3}{4}\psi^2 E(\delta'^2_{1(j)})$$

$$G'_{(j)} = -2 + (\psi\gamma - \gamma(\gamma-1) - \frac{3}{4}\psi^2)(E(\delta^2_{1(j)}) - E(\delta'^2_{1(j)}))$$

$$F'_{(j)} = 2 + (\gamma(\gamma-1) - 2\gamma\psi + 2\psi^2)(E(\delta^2_{1(j)}) - E(\delta'^2_{1(j)})) + 2(\gamma - \psi)(E(\delta_{0(j)}\delta_{1(j)}) - E(\delta_{0(j)}\delta'_{1(j)}))$$

5. Simulation study

In this section, we conducted a simulation study to investigate the properties of proposed estimators. . In the simulation study, we consider finite populations of size $N = 10000$ generated from a bivariate normal distribution $N(\mu_x, \mu_y, \sigma_x^2, \sigma_y^2, \rho_{xy})$. The samples were generated from a bivariate normal distribution using mvrnorm function in R programme. In the simulation, we considered $\mu_x = 2$, $\mu_y = 4$, $\sigma_x^2 = \sigma_y^2 = 1$ and different values of ρ_{xy}. We have computed mean square errors (MSEs) and percent relative efficiencies (PREs) of estimators with respect to $\hat{\mu}_{YSRSk}$ for $n = 3, 4, 5, 6$ on the basis of 60.000 replications using q_k and displayed in Table1 and Table4. When the mean of auxiliary variable is unknown, we used double sampling method to estimate μ_x and we have calculated MSEs and PREs of estimators given in (4.1)-(4.14). Findings are summarized in Table5 and Table8.

It is observed from all tables, suggested difference-cum-ratio and exponential type estimator performs better than Al-Omari[1] estimator. We can conclude that difference-cum-ratio estimator gives always more efficient results than regression estimator as shown in theoretical comparison section. When we compare difference-cum-ratio and exponential type estimator we can say that exponential type estimator performs better even with the low correlation data sets.

6. Conclusion

In this paper we have suggested difference-cum-ratio and exponential type estimator in median ranked set sampling and extended our result to double sampling. We have found that difference-cum-ratio estimator is always more efficient than regression estimator and exponential type is better than difference-cum-ratio estimator. Both of the estimators are considerable efficient than Al-Omari[1] estimator.

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References

[1] Al-Omari, A. I. *Ratio estimation of the population mean using auxiliary information in simple random sampling and median ranked set sampling*, Statistics and Probability Letters **82**, 1883-1890, 2012.

[2] Al-Saleh, M.F. and Al-Omari, A.I. *Multistage ranked set sampling*, Journal of Statistical Planning and Inference **102**, 273-286, 2002.

[3] Bahl, S. and Tuteja, R.K. *Ratio and product exponential estimators*, Journal of Information and Optimization Sciences **12** (1), 159-164, 1991.

[4] Jemain, A.A. and Al-Omari, A.I. *Multistage median ranked set samples for estimating the population mean*, Pakistan Journal of Statistics **22**, 195-207, 2006.

[5] Jemain, A.A., Al-Omari, A.I. and Ibrahim, K. *Multistage extreme ranked set sampling for estimating the population mean*, Journal of Statistical Theory and Application **6** (4), 456-471, 2007.

[6] Koyuncu, N. and Kadilar, C. *Family of estimators of population mean using two auxiliary variables in stratified random sampling*, Communications in Statistics: Theory and Methods **38** (14), 2398-2417, 2009.

[7] Koyuncu, N. and Kadilar, C. (2010). *On improvement in estimating population mean in stratified random sampling*, Journal of Applied Statistics **37** (6), 999-1013, 2010.

[8] Koyuncu, N. *Effcient estimators of population mean using auxiliary attributes.* Applied Mathematics Computation **218**, 10900-10905, 2012.

[9] Koyuncu, N., Gupta, S. and Sousa, R. *Exponential type estimators of the mean of a sensitive variable in the presence of non-sensitive auxiliary information*, Communications in Statistics: Simulation and Computation **43**, 1583-1594, 2014.

[10] Muttlak, H.A. *Median ranked set sampling*, Journal of Applied Statistical Science **6**, 245-255, 1997.

[11] Ozturk, O. and Jafari Jozani, M. *Inclusion probabilities in partially rank ordered set sampling*, Computational Statistics and Data Analysis **69**, 122-132, 2014.

[12] Singh, H.P. and Solanki, R.S. *An alternative procedure for estimating the population mean in simple random sampling*, Pak. Jour. Statist. Oper. Res. **8** (2), 213-232, 2012.

[13] Singh, H.P. and Solanki, R.S. *Improved estimation of population mean in simple random sampling using information on auxiliary attribute*, Appl. Math. Com. **218** (15), 7798-7812, 2012.

[14] Singh, H.P., Tailor, R. and Tailor, R. *On ratio and product methods with certain known population parameters of auxiliary variable in sample surveys*, SORT **34** (2), 157-180, 2010.

[15] Singh, S., Singh, H.P. and Upadhyaya, L.N. *Chain ratio and regression type estimators for median estimation in survey sampling*, Statist. Papers **48**, 23-46, 2006.

[16] Solanki R. S., Singh H.P. and Pal S. K. *Improved estimation of finite population mean sample surveys*, J. Adv. Com. **1** (2), 70-78, 2013.

[17] Upadhyaya, L.N., Singh, H.P. *Almost unbiased ratio and product-type estimators of finite population variance in sample surveys*, Statistics in Transition **7** (5), 1087-1096, 2006.

[18] Yadav, R., Upadhyaya, L.N., Singh, H.P. and Chatterjee, S. *Improved separate ratio exponential estimator for population mean using auxiliary information*, Statist. Trans.-new series **12** (2), 401-411, 2011.

Table 1. Mean Square Error (MSE) and the Percent Relative Efficiency (PRE) of estimators with respect to $\hat{\mu}_{YSRS1}$ for using $n = 3, 4, 5, 6$ and q_1 with positive correlation

Correlation	**Estimator**	**n=3**	**n=4**	**n=5**	**n=6**
$\rho = 0.99$	$\hat{\mu}_{YSRS1}$	0.02716	0.01918	0.01473	0.01200
		100.00	100.00	100.00	100.00
	$\hat{\mu}_{YRSS1(M)}$	0.01467	0.00602	0.00657	0.00331
		185.20	318.43	224.29	362.87
	$\hat{\mu}_{K1(M)}$	0.01134	0.00441	0.00611	0.00276
		239.49	435.31	241.05	435.66
	$\hat{\mu}_{N1(M)}$	**0.00628***	**0.00236***	**0.00334***	**0.00148***
		432.73*	**812.15***	**440.79***	**811.72***
$\rho = 0.80$	$\hat{\mu}_{YSRS1}$	0.20222	0.14713	0.11344	0.09326
		100.00	100.00	100.00	100.00
	$\hat{\mu}_{YRSS1(M)}$	0.12239	0.05059	0.05039	0.02702
		165.23	290.84	225.11	345.12
	$\hat{\mu}_{K1(M)}$	**0.07085***	**0.03298***	**0.02971***	**0.01706***
		285.43*	**446.14***	**381.86***	**546.66***
	$\hat{\mu}_{N1(M)}$	0.07993	0.0344	0.03402	0.01845
		253.00	427.70	333.44	505.59
$\rho = 0.70$	$\hat{\mu}_{YSRS1}$	0.29675	0.21575	0.16607	0.13646
		100.00	100.00	100.00	100.00
	$\hat{\mu}_{YRSS1(M)}$	0.16636	0.07052	0.06580	0.03650
		178.39	305.96	252.36	373.84
	$\hat{\mu}_{K1(M)}$	**0.07411***	**0.03588***	**0.02998***	**0.01799***
		400.41*	**601.30***	**553.97***	**758.65***
	$\hat{\mu}_{N1(M)}$	0.10245	0.04585	0.04203	0.0238
		289.67	470.60	395.13	573.36
$\rho = 0.50$	$\hat{\mu}_{YSRS1}$	0.47919	0.34640	0.26697	0.22066
		100.00	100.00	100.00	100.00
	$\hat{\mu}_{YRSS1(M)}$	0.23938	0.10748	0.09296	0.05362
		200.18	322.29	287.17	411.51
	$\hat{\mu}_{K1(M)}$	**0.07394***	**0.03637***	**0.02849***	**0.01778***
		648.06*	**952.42***	**936.95***	**1240.76***
	$\hat{\mu}_{N1(M)}$	0.13382	0.06211	0.05219	0.03109
		358.10	557.71	511.54	709.69
MSE of Estimators					
PRE of Estimators					
***represent most efficient estimator (having minimum MSE and maximum PRE)**					

Table 2. (MSE) and (PRE) of estimators with respect to $\hat{\mu}_{YSRS1}$ for using $n = 3, 4, 5, 6$ and q_1 with negative correlation

Correlation	**Estimator**	**n=3**	**n=4**	**n=5**	**n=6**
$\rho = -0.99$	$\hat{\mu}_{YSRS1}$	2.09657	1.45252	1.10811	0.89932
		100.00	100.00	100.00	100.00
	$\hat{\mu}_{YRSS1(M)}$	0.79973	0.37209	0.28960	0.17612
		262.16	390.36	382.63	510.64
	$\hat{\mu}_{K1(M)}$	**0.00147***	**0.00059***	**0.00077***	**0.00036***
		142202.83*	**246394.52***	**143164.56***	**249205.74***
	$\hat{\mu}_{N1(M)}$	0.00629	0.00236	0.00336	0.00148
		33324.11	61423.61	33018.28	60570.96
$\rho = -0.80$	$\hat{\mu}_{YSRS1}$	185.513	129.121	0.98902	0.80473
		100.00	100.00	100.00	100.00
	$\hat{\mu}_{YRSS1(M)}$	0.67687	0.32443	0.24210	0.15124
		274.07	398.00	408.52	532.10
	$\hat{\mu}_{K1(M)}$	**0.01921***	**0.00829***	**0.00842***	**0.00454***
		9656.02*	**15573.14***	**11751.25***	**17726.36***
	$\hat{\mu}_{N1(M)}$	0.08075	0.03444	0.03443	0.01860
		2297.51	3749.26	2872.41	4327.54
$\rho = -0.70$	$\hat{\mu}_{YSRS1}$	1.74465	1.21628	0.93268	0.75966
		100.00	100.00	100.00	100.00
	$\hat{\mu}_{YRSS1(M)}$	0.63102	0.30480	0.22639	0.14191
		276.48	399.04	411.98	535.31
	$\hat{\mu}_{K1(M)}$	**0.02617***	**0.01163***	**0.01102***	**0.00617***
		6666.34*	**10458.08***	**8460.68***	**12320.69***
	$\hat{\mu}_{N1(M)}$	0.10380	0.04589	0.04257	0.02395
		1680.71	2650.18	2191.01	3172.07
$\rho = -0.70$	$\hat{\mu}_{YSRS1}$	1.51489	1.06044	0.81509	0.66543
		100.00	100.00	100.00	100.00
	$\hat{\mu}_{YRSS1(M)}$	0.55071	0.26779	0.19972	0.12511
		275.08	396.00	408.12	531.87
	$\hat{\mu}_{K1(M)}$	**0.03780***	**0.01755***	**0.01524***	**0.00894***
		4007.48*	**6041.93***	**5349.97***	**7445.09***
	$\hat{\mu}_{N1(M)}$	0.13334	0.06210	0.05245	0.03112
		1136.13	1707.74	1554.05	2138.37
MSE of Estimators					
PRE of Estimators					
***represent most efficient estimator (having minimum MSE and maximum PRE)**					

Table 3. (MSE) and (PRE) of estimators with respect to $\hat{\mu}_{YSRS3}$ for using $n = 3, 4, 5, 6$ and q_3 with positive correlation

Correlation	Estimator	n=3	n=4	n=5	n=6
$\rho = 0.99$	$\hat{\mu}_{YSRS3}$	0.01302	0.00963	0.00751	0.00625
		100.00	100.00	100.00	100.00
	$\hat{\mu}_{YRSS3(M)}$	0.00833	0.00351	0.00402	0.00198
		156.24	274.21	186.70	315.51
	$\hat{\mu}_{K3(M)}$	0.01403	0.00545	0.00761	0.00342
		92.82	176.83	98.71	182.94
	$\hat{\mu}_{N3(M)}$	**0.00613***	**0.00233***	**0.00331***	**0.00147***
		212.37*	**412.55***	**227.00***	**425.46***
$\rho = 0.80$	$\hat{\mu}_{YSRS3}$	0.12348	0.09280	0.07286	0.06073
		100.00	100.00	100.00	100.00
	$\hat{\mu}_{YRSS3(M)}$	0.08088	0.03462	0.03474	0.01869
		152.68	268.03	209.72	324.98
	$\hat{\mu}_{K3(M)}$	0.08867	0.04142	0.03733	0.02146
		139.26	224.05	195.20	283.03
	$\hat{\mu}_{N3(M)}$	**0.07802***	**0.03400***	**0.03371***	**0.01834***
		158.27*	**272.91***	**216.14***	**331.24***
$\rho = 0.70$	$\hat{\mu}_{YSRS3}$	0.18311	0.13745	0.10786	0.08983
		100.00	100.00	100.00	100.00
	$\mu_{YRSS3(M)}$	0.11056	0.04855	0.04559	0.02540
		165.62	283.15	236.59	353.62
	$\hat{\mu}_{K3(M)}$	**0.09278***	**0.04508***	**0.03768***	**0.02264***
		197.35*	**304.93***	**286.25***	**396.84***
	$\hat{\mu}_{N3(M)}$	0.10004	0.04533	0.04166	0.02366
		183.04	303.22	258.89	379.64
$\rho = 0.50$	$\hat{\mu}_{YSRS3}$	0.30341	0.22668	0.17871	0.14877
		100.00	100.00	100.00	100.00
	$\mu_{YRSS3(M)}$	0.16198	0.07396	0.06427	0.03738
		187.32	306.48	278.08	398.02
	$\hat{\mu}_{K3(M)}$	**0.09301***	**0.04591***	**0.03596***	**0.02248***
		326.21*	**493.78***	**496.96***	**661.91***
	$\hat{\mu}_{N3(M)}$	0.13094	0.06145	0.05165	0.03088
		231.71	368.88	345.98	481.71
MSE of Estimators					
PRE of Estimators					
***represent most efficient estimator (having minimum MSE and maximum PRE)**					

Table 4. (MSE) and (PRE) of estimators with respect to $\hat{\mu}_{YSRS3}$ for using $n = 3, 4, 5, 6$ and q_3 with negative correlation

Correlation	Estimator	n=3	n=4	n=5	n=6
$\rho = -0.99$	$\hat{\mu}_{YSRS3}$	1.27876	0.92691	0.72344	0.59704
		100.00	100.00	100.00	100.00
	$\mu_{YRSS3(M)}$	0.53198	0.25563	0.19968	0.12260
		240.38	362.59	362.30	486.98
	$\hat{\mu}_{K3(M)}$	**0.00170***	**0.00066***	**0.00092***	**0.00041***
		75346.28*	**140653.13***	**78496.62***	**143988.13***
	$\hat{\mu}_{N3(M)}$	0.00615	0.00234	0.00332	0.00147
		20801.50	39655.80	21769.81	40491.47
$\rho = -0.80$	$\hat{\mu}_{YSRS3}$	1.14487	0.83140	0.65053	0.53757
		100.00	100.00	100.00	100.00
	$\mu_{YRSS3(M)}$	0.45347	0.22367	0.16751	0.10561
		252.47	371.70	388.35	508.99
	$\hat{\mu}_{K3(M)}$	**0.02409***	**0.01038***	**0.01056***	**0.00568***
		4751.97*	**8012.21***	**6162.50***	**9461.94***
	$\hat{\mu}_{N3(M)}$	0.07899	0.03406	0.03409	0.01847
		1449.33	2440.67	1908.53	2910.45
$\rho = -0.70$	$\hat{\mu}_{YSRS3}$	1.07804	0.78365	0.61373	0.50746
		100.00	100.00	100.00	100.00
	$\mu_{YRSS3(M)}$	0.42290	0.21001	0.15654	0.0990
		254.92	373.14	392.05	512.48
	$\hat{\mu}_{K3(M)}$	**0.03289***	**0.01460***	**0.01386***	**0.00774***
		3277.86*	**5366.87***	**4429.80***	**6557.28***
	$\hat{\mu}_{N3(M)}$	0.10158	0.0454	0.04214	0.02379
		1061.33	1726.09	1456.49	2133.32
$\rho = -0.50$	$\hat{\mu}_{YSRS3}$	0.94135	0.68590	0.53818	0.44554
		100.00	100.00	100.00	100.00
	$\mu_{YRSS3(M)}$	0.37019	0.18468	0.13816	0.08733
		254.29	371.40	389.52	510.17
	$\hat{\mu}_{K3(M)}$	**0.04753***	**0.02208***	**0.01917***	**0.01124***
		1980.46*	**3106.64***	**2807.53***	**3963.50***
	$\hat{\mu}_{N3(M)}$	0.13054	0.06144	0.05192	0.03091
		721.10	1116.30	1036.59	1441.34
MSE of Estimators					
PRE of Estimators					
***represent most efficient estimator (having minimum MSE and maximum PRE)**					

Table 5. (MSE) and (PRE) of estimators with respect to $\hat{\mu}'_{YSRS1}$ for using $n = 3, 4, 5, 6$ and q_1 with positive correlation

Correlation	Estimator	n=3	n=4	n=5	n=6
$\rho = 0.99$	$\hat{\mu}'_{YSRS1}$	0.17912	0.11756	0.08670	0.06762
		100.00	100.00	100.00	100.00
	$\hat{\mu}'_{YRSS1(M)}$	0.23023	0.11654	0.08777	0.05530
		77.80	100.87	98.78	122.27
	$\hat{\mu}'_{K1(M)}$	**0.13146***	**0.06466***	**0.05121***	**0.03149***
		136.26*	**181.81***	**169.30***	**214.73***
	$\hat{\mu}'_{N1(M)}$	0.13819	0.06916	0.05378	0.03343
		129.62	169.98	161.21	202.28
$\rho = 0.80$	$\hat{\mu}'_{YSRS1}$	0.23888	0.16633	0.12742	0.10244
		100.00	100.00	100.00	100.00
	$\hat{\mu}'_{YRSS1(M)}$	0.28069	0.13827	0.10670	0.06626
		85.10	120.30	119.42	154.60
	$\hat{\mu}'_{K1(M)}$	**0.09576***	**0.04582***	**0.03806***	**0.02298***
		249.45*	**363.05***	**334.76***	**445.74***
	$\hat{\mu}'_{N1(M)}$	0.15289	0.07387	0.05938	0.03623
		156.25	225.17	214.58	282.76
$\rho = 0.70$	$\hat{\mu}'_{YSRS1}$	0.30028	0.21632	0.16904	0.13807
		100.00	100.00	100.00	100.00
	$\hat{\mu}'_{YRSS1(M)}$	0.32584	0.15892	0.12276	0.07609
		92.16	136.12	137.70	181.46
	$\hat{\mu}'_{K1(M)}$	**0.08211***	**0.03926***	**0.03228***	**0.01962***
		365.69*	**551.02***	**523.62***	**703.64***
	$\hat{\mu}'_{N1(M)}$	0.16068	0.07666	0.06180	0.03760
		186.87	282.19	273.51	367.23
$\rho = 0.50$	$\hat{\mu}'_{YSRS1}$	0.42384	0.31654	0.25222	0.20933
		100.00	100.00	100.00	100.00
	$\hat{\mu}'_{YRSS1(M)}$	0.40172	0.19537	0.14864	0.09282
		105.51	162.02	169.69	225.52
	$\hat{\mu}'_{K1(M)}$	**0.07532***	**0.03651***	**0.02847***	**0.01773***
		562.73*	**867.00***	**885.88***	**1180.43***
	$\hat{\mu}'_{N1(M)}$	0.16669	0.07933	0.06271	0.03849
		254.27	399.03	402.20	543.81
MSE of Estimators					
PRE of Estimators					
***represent most efficient estimator (having minimum MSE and maximum PRE)**					

Table 6. (MSE) and (PRE) of estimators with respect to $\hat{\mu}'_{YSRS1}$ for using $n = 3, 4, 5, 6$ and q_1 with negative correlation

Correlation	**Estimator**	**n=3**	**n=4**	**n=5**	**n=6**
$\rho = -0.99$	$\hat{\mu}'_{YSRS1}$	1.44310	1.12785	0.90938	0.76770
		100.00	100.00	100.00	100.00
	$\hat{\mu}'_{YRSS1(M)}$	0.97002	0.46323	0.34803	0.21602
		148.77	243.48	261.29	355.38
	$\hat{\mu}'_{K1(M)}$	**0.03720***	**0.01674***	**0.01039***	**0.00668***
		3879.01*	**6736.19***	**8751.86***	**11490.33***
	$\hat{\mu}'_{N1(M)}$	0.14743	0.07198	0.04646	0.03019
		978.84	1566.82	1957.45	2542.77
$\rho = -0.80$	$\hat{\mu}'_{YSRS1}$	1.30377	1.02123	0.82264	0.69395
		100.00	100.00	100.00	100.00
	$\hat{\mu}'_{YRSS1(M)}$	0.85105	0.41594	0.30128	0.19141
		153.20	245.52	273.05	362.54
	$\hat{\mu}'_{K1(M)}$	**0.05221***	**0.02310***	**0.01720***	**0.01023***
		2497.40*	**4421.60***	**4783.49***	**6785.34***
	$\hat{\mu}'_{N1(M)}$	0.17864	0.08184	0.06264	0.03715
		729.85	1247.78	1313.19	1868.15
$\rho = -0.70$	$\hat{\mu}'_{YSRS1}$	1.23136	0.96516	0.77731	0.65554
		100.00	100.00	100.00	100.00
	$\hat{\mu}'_{YRSS1(M)}$	0.80362	0.39541	0.28511	0.18167
		153.23	244.09	272.64	360.85
	$\hat{\mu}'_{K1(M)}$	**0.05748***	**0.02561***	**0.01925***	**0.01148***
		2142.34*	**3769.17***	**4037.59***	**5711.08***
	$\hat{\mu}'_{N1(M)}$	0.18225	0.08339	0.06404	0.03800
		675.63	1157.41	1213.80	1725.34
$\rho = -0.50$	$\hat{\mu}'_{YSRS1}$	1.09266	0.85687	0.68971	0.58123
		100.00	100.00	100.00	100.00
	$\hat{\mu}'_{YRSS1(M)}$	0.72687	0.36005	0.25968	0.16548
		150.32	237.99	265.60	351.23
	$\hat{\mu}'_{K1(M)}$	**0.06490***	**0.02952***	**0.02212***	**0.01334***
		1683.64*	**2902.88***	**3117.84***	**4357.02***
	$\hat{\mu}'_{N1(M)}$	0.18003	0.08347	0.06318	0.03790
		606.94	1026.53	1091.65	1533.57
MSE of Estimators					
PRE of Estimators					
***represent most efficient estimator (having minimum MSE and maximum PRE)**					

Table 7. (MSE) and (PRE) of estimators with respect to $\hat{\mu}'_{YSRS3}$ for using $n = 3, 4, 5, 6$ and q_3 with positive correlation

Correlation	Estimator	n=3	n=4	n=5	n=6
$\rho = 0.99$	$\hat{\mu}'_{YSRS3}$	0.15538	0.09851	0.07115	0.05438
		100.00	100.00	100.00	100.00
	$\hat{\mu}'_{YRSS3(M)}$	0.12748	0.06367	0.04988	0.03089
		121.88	154.74	142.64	176.06
	$\hat{\mu}'_{K3(M)}$	0.17019	0.08280	0.06496	0.03992
		91.30	118.97	109.54	136.23
	$\hat{\mu}'_{N3(M)}$	**0.13735***	**0.06895***	**0.05361***	**0.03335***
		113.12*	**142.89***	**132.73***	**163.03***
$\rho = 0.80$	$\hat{\mu}'_{YSRS3}$	0.19485	0.13146	0.09912	0.07844
		100.00	100.00	100.00	100.00
	$\hat{\mu}'_{YRSS3(M)}$	0.1619	0.07880	0.06320	0.03863
		120.35	166.82	156.85	203.06
	$\hat{\mu}'_{K3(M)}$	**0.12187***	**0.05821***	**0.04810***	**0.02907***
		159.88*	**225.85***	**206.12***	**269.90***
	$\hat{\mu}'_{N3(M)}$	0.15044	0.07315	0.05893	0.03602
		129.49	179.72	168.20	217.79
$\rho = 0.70$	$\hat{\mu}'_{YSRS3}$	0.23451	0.16460	0.12719	0.10263
		100.00	100.00	100.00	100.00
	$\hat{\mu}'_{YRSS3(M)}$	0.19195	0.09286	0.07424	0.04542
		122.17	177.25	171.33	225.96
	$\hat{\mu}'_{K3(M)}$	**0.10334***	**0.04963***	**0.04074***	**0.02481***
		226.93*	**331.64***	**312.17***	**413.65***
	$\hat{\mu}'_{N3(M)}$	0.15691	0.07548	0.06112	0.03727
		149.45	218.09	208.10	275.39
$\rho = 0.50$	$\hat{\mu}'_{YSRS3}$	0.31428	0.23118	0.18340	0.15113
		100.00	100.00	100.00	100.00
	$\hat{\mu}'_{YRSS3(M)}$	0.24300	0.11807	0.09222	0.05714
		129.33	195.81	198.88	264.49
	$\hat{\mu}'_{K3(M)}$	**0.09368***	**0.04586***	**0.03586***	**0.02239***
		335.49*	**504.12***	**511.49***	**675.08***
	$\hat{\mu}'_{N3(M)}$	0.16062	0.07734	0.06163	0.03795
		195.66	298.92	297.59	398.21
MSE of Estimators					
PRE of Estimators					
***represent most efficient estimator (having minimum MSE and maximum PRE)**					

Table 8. (MSE) and (PRE) of estimators with respect to $\hat{\mu}'_{YSRS3}$ for using $n = 3, 4, 5, 6$ and q_3 with negative correlation

Correlation	Estimator	n=3	n=4	n=5	n=6
$\rho = -0.99$	$\hat{\mu}'_{YSRS3}$	0.94752	0.75264	0.61419	0.52321
		100.00	100.00	100.00	100.00
	$\hat{\mu}'_{YRSS3(M)}$	0.61739	0.30211	0.22966	0.14311
		153.47	249.13	267.43	365.60
	$\hat{\mu}'_{K3(M)}$	**0.04589***	**0.02095***	**0.01306***	**0.00842***
		2064.80*	**3592.77***	**4702.58***	**6210.80***
	$\hat{\mu}'_{N3(M)}$	0.12644	0.06595	0.04363	0.02890
		749.41	1141.31	1407.83	1810.45
$\rho = -0.80$	$\hat{\mu}'_{YSRS3}$	0.86080	0.68349	0.55685	0.47373
		100.00	100.00	100.00	100.00
	$\hat{\mu}'_{YRSS3(M)}$	0.53886	0.26999	0.19753	0.12600
		159.74	253.16	281.91	375.98
	$\hat{\mu}'_{K3(M)}$	**0.06459***	**0.02887***	**0.02161***	**0.01287***
		1332.75*	**2367.33***	**2576.97***	**3679.86***
	$\hat{\mu}'_{N3(M)}$	0.15788	0.07594	0.05979	0.03588
		545.22	900.09	931.41	1320.44
$\rho = -0.70$	$\hat{\mu}'_{YSRS3}$	0.81723	0.64837	0.52778	0.44872
		100.00	100.00	100.00	100.00
	$\hat{\mu}'_{YRSS3(M)}$	0.50841	0.25640	0.18668	0.11944
		160.74	252.88	282.73	375.70
	$\hat{\mu}'_{K3(M)}$	**0.07105***	**0.03197***	**0.02416***	**0.01443***
		1150.18*	**2028.15***	**2184.89***	**3110.24***
	$\hat{\mu}'_{N3(M)}$	0.16234	0.07770	0.06130	0.03678
		503.39	834.49	860.94	1219.90
$\rho = -0.50$	$\hat{\mu}'_{YSRS3}$	0.72804	0.57617	0.46805	0.39735
		100.00	100.00	100.00	100.00
	$\hat{\mu}'_{YRSS3(M)}$	0.45513	0.23094	0.16833	0.10766
		159.96	249.48	278.05	369.07
	$\hat{\mu}'_{K3(M)}$	**0.08034***	**0.03691***	**0.02780***	**0.01679***
		906.17*	**1560.89***	**1683.76***	**2366.03***
	$\hat{\mu}'_{N3(M)}$	0.16199	0.07824	0.06071	0.03680
		449.44	736.37	770.97	1079.70
MSE of Estimators					
PRE of Estimators					
***represent most efficient estimator (having minimum MSE and maximum PRE)**					