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The Fischer-Clifford matrices and character table of the split extension $2^6{:}S_8$

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Abstract

The sporadic simple group Fi_{22} is generated by a conjugacy class D of 3510 Fischer's 3-transpositions. In Fi_{22} there are 14 classes of maximal subgroups up to conjugacy as listed in the $\mathbb{ATLAS}$ [10] and Wilson [31]. The group $E = 2^6{:}Sp_6(2)$ is maximal subgroup of Fi_{22} of index 694980. In the present article we compute the Fischer-Clifford matrices and hence character table of a subgroup of the smallest Fischer group Fi_{22} of the form $2^6{:}S_8$ which sits maximally in E. The computations were carried out using the computer algebra systems $\mathbb{MAGMA}$ [9] and $\mathbb{GAP}$ [29].

Keywords: Fischer-Clifford matrix, extension, Fischer group Fi_{22}.

2000 AMS Classification: 20C15, 20D08.

1. Introduction

In recent years there has been considerable interest in the *Fischer-Clifford theory* for both split and non-split group extensions. Character tables for many maximal subgroups of the sporadic simple groups were computed using this technique. See for instance [1, 3, 4, 5, 7, 6], [11], [12], [16], [19], [20], [22, 23, 24] and [28]. In the present article we follow a similar approach as used in [1, 3, 4, 5, 7], [22] and [24] to compute the Fischer-Clifford matrices and character tables for many group extension.

Let $\bar{G} = N{:}G$ be the split extension of $N = 2^6$ by $G = S_8$ where N is the vector space of dimension 6 over $GF(2)$ on which G acts naturally. Let $E = 2^6{:}Sp_6(2)$ be a maximal subgroup of Fi_{22}. The group $\bar{G}$ sits maximally inside the group E. In the present article we aim to construct the character table of $\bar{G}$ by using the technique of *Fischer-Clifford matrices*. The character table of $\bar{G}$ can be constructed by using the Fischer-Clifford matrix $M(g)$ for each class representative g of G and

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the character tables of H_i's which are the inertia factor groups of the inertia groups $\bar{H}_i = 2^6{:}H_i$. We use the properties of the Fischer-Clifford matrices discussed in [1], [2], [3], [4], [5] and [22] to compute entries of these matrices.

The Fischer-Clifford matrix $M(g)$ will be partioned row-wise into blocks, where each block corresponds to an inertia group $\bar{H}_i$. Now using the columns of character table of the inertia factor H_i of $\bar{H}_i$ which correspond to the classes of H_i which fuse to the class $[g]$ in G and multiply these columns by the rows of the Fischer-Clifford matrix $M(g)$ that correspond to $\bar{H}_i$. In this way we construct the portion of the character table of $\bar{G}$ which is in the block corresponding to $\bar{H}_i$ for the classes of $\bar{G}$ that come from the coset Ng. For detailed information about this technique the reader is encouraged to consult [1], [3], [4], [5], [16] and [22].

We first use the method of coset analysis to determine the conjugacy classes of $\bar{G}$. For detailed information about the coset analysis method, the reader is referred to again [1], [4], [5] and [22]. The complete fusion of $\bar{G}$ into Fi_{22} will be fully determined.

The character table of $\bar{G}$ will be divided row-wise into blocks where each block corresponds to an inertia group $\bar{H}_i = N{:}H_i$. The computations have been carried out with the aid of computer algebra systems $\mathbb{MAGMA}$ [9] and $\mathbb{GAP}$ [29]. We follow the notation of $\mathbb{ATLAS}$ [10] for the conjugacy classes of the groups and permutation characters. For more information on character theory, see [15] and [17].

Recently, the representation theory of Hecke algebras of the generalized symmetric groups has received some special attention [8], and the computation of the Fischer-Clifford matrices in this context is also of some interest.

2. The Conjugacy Classes of $2^6{:}S_8$

The group S_8 is a maximal subgroup of $Sp_6(2)$ of index 36. From the conjugacy classes of $Sp_6(2)$, obtained using $\mathbb{MAGMA}$ [9], we generated S_8 by two elements α and β of $Sp_6(2)$ which are given by

$$\alpha = \begin{pmatrix} 0 & 1 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix} \quad \text{and} \quad \beta = \begin{pmatrix} 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ 1 & 1 & 1 & 0 & 1 & 0 \end{pmatrix}$$

where $o(\alpha) = 2$ and $o(\beta) = 7$.

Using $\mathbb{MAGMA}$, we compute the conjugacy classes of S_8 and observed that S_8 has 22 conjugacy classes of its elements. The action of S_8 on 2^6 gives rise to three orbits of lengths 1, 28 and 35 with corresponding point stabilizers S_8, $S_6 \times 2$ and $(S_4 \times S_4){:}2$ respectively. Let ϕ_1 and ϕ_2 be the permutation characters of S_8 of degrees 28 and 35. Then from $\mathbb{ATLAS}$ [10], we obtained that $\chi_{\phi_1} = 1a + 7a + 20a$ and $\chi_{\phi_2} = 1a + 14a + 20a$.

Suppose $\chi = \chi(S_8|2^6)$ is the permutation character of S_8 on 2^6. Then we obtain that

$$\chi = 1a + 1_{S_6\times 2}^{S_8} + 1_{(S_4\times S_4):2}^{S_8} = 3 \times 1a + 7a + 14a + 2 \times 20a,$$

where $1_{S_6\times 2}^{S_8}$ and $1_{(S_4\times S_4):2}^{S_8}$ are the characters of S_8 induced from identity characters of $S_6{\times}2$ and $(S_4{\times}S_4){:}2$ respectively. For each class representative $g \in S_8$, we calculate $k = \chi(S_8|2^6)(g)$, which is equal to the number of fixed points of g in 2^6. We list these values in the following table:

$[g]_{S_8}$	$1A$	$2A$	$2B$	$2C$	$2D$	$3A$	$3B$	$4A$	$4B$	$4C$	$4D$
χ_{ϕ_1}	28	16	8	4	4	10	1	6	2	0	2
χ_{ϕ_2}	35	15	7	11	3	5	2	1	5	3	1
k	64	32	16	16	8	16	4	8	8	4	4
$[g]_{S_8}$	$5A$	$6A$	$6B$	$6C$	$6D$	$6E$	$7A$	$8A$	$10A$	$12A$	$15A$
χ_{ϕ_1}	3	1	4	2	1	1	0	0	1	0	0
χ_{ϕ_2}	0	0	3	1	0	2	0	1	0	1	0
k	4	2	8	4	2	4	1	2	2	2	1

We use the method of coset analysis, developed for computing the conjugacy classes of group extensions, to obtain the conjugacy classes of $2^6{:}S_8$. For detailed information and background material relating to coset analysis and the description of the parameters f_j, we encourage the readers to consult once again [1], [4], [5] and [22].

Now having obtained the values of the k's for each class representative $g \in S_8$, we use a computer programme for $2^6{:}S_8$ (see Programme A in [1]) written for $\mathbb{MAGMA}$ [9] to find the values of f_j's corresponding to these k's. From the programme output, we calculate the number f_j of orbits Q_i's ($1 \leq i \leq k$) of the action of $N = 2^6$ on Ng, which have come together under the action of $C_{S_8}(g)$ for each class representative $g \in S_8$. We deduce that altogether we have 64 conjugacy classes of the elements of $\bar{G} = 2^6{:}S_8$, which we list in Table 1. We also list the order of $C_{\bar{G}}(x)$ for each $[x]_{\bar{G}}$ in the last column of Table 1.

Table 1: The conjugacy classes of $\bar{G} = 2^6{:}S_8$

$[g]_{S_8}$	k	f_j	$[x]_{2^6:S_8}$	$\lvert[x]_{2^6:S_8}\rvert$	$\lvert C_{2^6:S_8}(x)\rvert$
$1A$	64	$f_1 = 1$	$1A$	1	2580480
		$f_2 = 28$	$2A$	28	92160
		$f_3 = 35$	$2B$	35	73728
$2A$	32	$f_1 = 1$	$2C$	56	46080
		$f_2 = 6$	$4A$	336	7680
		$f_3 = 10$	$4B$	560	4608
		$f_4 = 15$	$2D$	840	3072
$2B$	16	$f_1 = 1$	$2E$	420	6144
		$f_2 = 1$	$2F$	420	6144
		$f_3 = 2$	$2G$	840	3072
		$f_4 = 12$	$4C$	5040	512
$2C$	16	$f_1 = 1$	$2H$	840	3072
		$f_2 = 1$	$4D$	840	3072
		$f_3 = 3$	$2I$	2520	1024
		$f_4 = 3$	$4E$	2520	1024
		$f_5 = 8$	$4F$	6720	384
$2D$	8	$f_1 = 1$	$2J$	3360	768
		$f_2 = 1$	$4G$	3360	768
		$f_3 = 3$	$4H$	10080	256
		$f_4 = 3$	$4I$	10080	256
$3A$	16	$f_1 = 1$	$3A$	448	5760
		$f_2 = 5$	$6A$	2240	1152
		$f_3 = 10$	$6B$	4480	576

Table 1: The conjugacy classes of $\bar{G}$ (continued)

$[g]_{S_8}$	k	f_j	$[x]_{2^6:S_8}$	$\lvert[x]_{2^6:S_8}\rvert$	$\lvert C_{2^6:S_8}(x)\rvert$
$3B$	4	$f_1 = 1$	$3B$	17920	144
		$f_2 = 1$	$6C$	17920	144
		$f_3 = 2$	$6D$	35840	72
$4A$	8	$f_1 = 1$	$4J$	3360	768
		$f_2 = 3$	$4K$	10080	256
		$f_3 = 4$	$8A$	13440	192
$4B$	8	$f_1 = 1$	$4L$	10080	256
		$f_2 = 1$	$4M$	10080	256
		$f_3 = 2$	$4N$	20160	128
		$f_4 = 4$	$8B$	40320	64
$4C$	4	$f_1 = 1$	$4O$	20160	128
		$f_2 = 1$	$4P$	20160	128
		$f_3 = 2$	$4Q$	40320	64
$4D$	4	$f_1 = 1$	$4R$	40320	64
		$f_2 = 1$	$8C$	40320	64
		$f_3 = 1$	$8D$	40320	64
		$f_4 = 1$	$4S$	40320	64
$5A$	4	$f_1 = 1$	$5A$	21504	120
		$f_2 = 3$	$10A$	64512	40
$6A$	2	$f_1 = 1$	$6E$	35840	72
		$f_2 = 1$	$12A$	35840	72
$6B$	8	$f_1 = 1$	$6F$	8960	288
		$f_2 = 1$	$12B$	8960	288
		$f_3 = 3$	$12C$	26880	96
		$f_4 = 3$	$6G$	26880	96
$6C$	4	$f_1 = 1$	$6H$	26880	96
		$f_2 = 1$	$12D$	26880	96
		$f_3 = 2$	$12E$	53760	48
$6D$	2	$f_1 = 1$	$6I$	107520	24
		$f_2 = 1$	$12F$	107520	24
$6E$	4	$f_1 = 1$	$6J$	53760	48
		$f_2 = 1$	$6K$	53760	48
		$f_3 = 2$	$6L$	107520	24
$7A$	1	$f_1 = 1$	$7A$	368640	7
$8A$	2	$f_1 = 1$	$8E$	161280	16
		$f_2 = 1$	$8F$	161280	16
$10A$	2	$f_1 = 1$	$10B$	129024	20
		$f_2 = 1$	$20A$	129024	20
$12A$	2	$f_1 = 1$	$12G$	107520	24
		$f_2 = 1$	$24A$	107520	24
$15A$	1	$f_1 = 1$	$15A$	172032	15

3. The Inertia Groups of $\bar{G}$

The action of G on N produces three orbits of lengths 1, 28 and 35. Hence by Brauer's theorem (see Lemma 4.5.2 of [14]) G acting on $Irr(N)$ will also produce three orbits of lengths 1, s and t such that $s + t = 63$. From $\mathbb{ATLAS}$, by checking the indices of maximal subgroups of S_8, we can see that the only possibility is that $s = 28$ and $t = 35$. We deduce that the three inertia groups are $\bar{H}_i = 2^6{:}H_i$ of indices 1, 28 and 35 in $\bar{G}$ respectively where $i \in \{1, 2, 3\}$ and $H_i \leq S_8$ are the inertia factors. We also observe that $H_1 = S_8$, $H_2 = S_6{\times}2$ and $H_3 = (S_4{\times}S_4){:}2$.

The character tables and power maps of the elements of H_1, H_2 and H_3 are given in the $\mathbb{GAP}$ [29]. Using the permutation characters of S_8 on H_2 and H_3 of degrees 28 and 35 respectively we are able to obtain partial fusions of H_2 and H_3 into $H_1 = S_8$. We completed the fusions by using direct matrix conjugation in S_8. The complete fusion of H_2 and H_3 into H_1 are given in Tables 2 and 3 respectively.

Table 2: The fusion of H_2 into H_1

$[g]_{S_6\times 2}$	$\longrightarrow$	$[h]_{S_8}$	$[g]_{S_6\times 2}$	$\longrightarrow$	$[h]_{S_8}$
$1A$		$1A$	$2A$		$2A$
$2B$		$2A$	$2C$		$2D$
$2D$		$2B$	$2E$		$2C$
$2F$		$2C$	$2G$		$2D$
$3A$		$3A$	$3B$		$3B$
$4A$		$4D$	$4B$		$4A$
$4C$		$4B$	$4D$		$4D$
$5A$		$5A$	$6A$		$6B$
$6B$		$6A$	$6C$		$6B$
$6D$		$6E$	$6E$		$6D$
$6F$		$6C$	$10A$		$10A$

Table 3: The fusion of H_3 into H_1

$[g]_{S_4\times S_4}$	$\longrightarrow$	$[h]_{S_8}$	$[g]_{S_4\times S_4}$	$\longrightarrow$	$[h]_{S_8}$
$1A$		$1A$	$2A$		$2C$
$2B$		$2B$	$2C$		$2A$
$2D$		$2B$	$2E$		$2C$
$2F$		$2D$	$3A$		$3A$
$3B$		$3B$	$4A$		$4A$
$4B$		$4C$	$4C$		$4B$
$4D$		$4C$	$4E$		$4D$
$4F$		$4B$	$6A$		$6C$
$6B$		$6B$	$6C$		$6E$
$8A$		$8A$	$12A$		$12A$

4. The Fischer-Clifford Matrices of $\bar{G}$

For each conjugacy class $[g]$ of G with representative $g \in G$, we construct the corresponding Fischer-Clifford matrix $M(g)$ of $\bar{G} = 2^6{:}S_8$. We use properties of the Fischer-Clifford matrices (see [1], [3], [4], [5], [22]) together with fusions of H_2 and H_3 into H_1 (Tables 2 and 3) to compute the entries of the these matrices. The Fischer-Clifford matrix $M(g)$ will be partitioned row-wise into blocks, where each block corresponds to an inertia group $\bar{H}_i$. We list the Fischer-Clifford matrices of $\bar{G}$ in Table 4.

Table 4: The Fischer-Clifford matrices of $\bar{G}$

$M(g)$	$M(g)$	$M(g)$
$M(1A) = \begin{pmatrix} 1 & 1 & 1 \\ 28 & 4 & -4 \\ 35 & -5 & 3 \end{pmatrix}$	$M(2A) = \begin{pmatrix} 1 & 1 & 1 & 1 \\ 1 & -1 & -1 & 1 \\ 15 & 5 & -3 & -1 \\ 15 & -5 & 3 & -1 \end{pmatrix}$	$M(2B) = \begin{pmatrix} 1 & 1 & 1 & 1 \\ 4 & 4 & -4 & 0 \\ 3 & 3 & 3 & -1 \\ 8 & -8 & 0 & 0 \end{pmatrix}$
$M(2C) = \begin{pmatrix} 1 & 1 & 1 & 1 & 1 \\ 2 & -2 & -2 & 2 & 0 \\ 6 & 6 & -2 & -2 & 0 \\ 1 & 1 & 1 & 1 & -1 \\ 6 & -6 & 2 & -2 & 0 \end{pmatrix}$	$M(2D) = \begin{pmatrix} 1 & 1 & 1 & 1 \\ 1 & -1 & -1 & 1 \\ 3 & -3 & 1 & -1 \\ 3 & 3 & -1 & -1 \end{pmatrix}$	$M(3A) = \begin{pmatrix} 1 & 1 & 1 \\ 10 & 2 & -2 \\ 5 & -3 & 1 \end{pmatrix}$
$M(3B) = \begin{pmatrix} 1 & 1 & 1 \\ 1 & 1 & -1 \\ 2 & -2 & 0 \end{pmatrix}$	$M(4A) = \begin{pmatrix} 1 & 1 & 1 \\ 6 & -2 & 0 \\ 1 & 1 & -1 \end{pmatrix}$	$M(4B) = \begin{pmatrix} 1 & 1 & 1 & 1 \\ 2 & 2 & -2 & 0 \\ 1 & 1 & 1 & -1 \\ 4 & -4 & 0 & 0 \end{pmatrix}$
$M(4C) = \begin{pmatrix} 1 & 1 & 1 \\ 1 & 1 & -1 \\ 2 & -2 & 0 \end{pmatrix}$	$M(4D) = \begin{pmatrix} 1 & 1 & 1 & 1 \\ 1 & 1 & -1 & -1 \\ 1 & -1 & -1 & 1 \\ 1 & -1 & 1 & -1 \end{pmatrix}$	$M(5A) = \begin{pmatrix} 1 & 1 \\ 3 & -1 \end{pmatrix}$
$M(6A) = \begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix}$	$M(6B) = \begin{pmatrix} 1 & 1 & 1 & 1 \\ 1 & -1 & -1 & 1 \\ 3 & 3 & -1 & -1 \\ 3 & -3 & 1 & -1 \end{pmatrix}$	$M(6C) = \begin{pmatrix} 1 & 1 & 1 \\ 2 & -2 & 0 \\ 1 & 1 & -1 \end{pmatrix}$
$M(6D) = \begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix}$	$M(6E) = \begin{pmatrix} 1 & 1 & 1 \\ 1 & 1 & -1 \\ 2 & -2 & 0 \end{pmatrix}$	$M(7A) = \begin{pmatrix} 1 \end{pmatrix}$
$M(8A) = \begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix}$	$M(10A) = \begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix}$	$M(12A) = \begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix}$
$M(15A) = \begin{pmatrix} 1 \end{pmatrix}$		

We use the above Fischer-Clifford matrices (Table 4) and the character tables of inertia factor groups $H_1 = S_8$, H_2 and H_3, together with the fusion of H_2 and H_3 into S_8, to obtain the character table of $\bar{G}$. The set of irreducible characters of $\bar{G} = 2^6{:}S_8$ will be partitioned into three blocks B_1, B_2 and B_3 corresponding to the inertia factors H_1, H_2 and H_3 respectively. In fact $B_1 = \{\chi_i \mid 1 \leq i \leq 22\}$, $B_2 = \{\chi_i \mid 23 \leq i \leq 44\}$ and $B_3 = \{\chi_i \mid 45 \leq i \leq 64\}$, where $Irr(2^6{:}S_8) = \bigcup_{i=1}^{3} B_i$. The character table of $\bar{G}$ is displayed in Table 5. Note that the centralizers of the elements of $\bar{G}$ are listed in the last column of Table 1.

The character table of $\bar{G} = 2^6{:}S_8$, which we computed in this paper and displayed in Table 5, has been incorporated into and available in the latest version of $\mathbb{GAP}$ [29] as well.

Table 5: The character table of $\bar{G}$

$[g]_{S_8}$	$1A$			$2A$				$2B$				$2C$				
$[x]_{2^6:S_8}$	$1A$	$2A$	$2B$	$2C$	$4A$	$4B$	$2D$	$2E$	$2F$	$2G$	$4C$	$2H$	$4D$	$2I$	$4E$	$4F$
χ_1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
χ_2	1	1	1	-1	-1	-1	-1	1	1	1	1	1	1	1	1	1
χ_3	7	7	7	5	5	5	5	-1	-1	-1	-1	3	3	3	3	3
χ_4	7	7	7	-5	-5	-5	-5	-1	-1	-1	-1	3	3	3	3	3
χ_5	14	14	14	4	4	4	4	4	6	6	6	6	2	2	2	2
χ_6	14	14	14	-4	-4	-4	-4	4	6	6	6	6	2	2	2	2
χ_7	20	20	20	10	10	10	10	4	4	4	4	4	4	4	4	4
χ_8	20	20	20	-10	-10	-10	-10	4	4	4	4	4	4	4	4	4
χ_9	21	21	21	9	9	9	9	-3	-3	-3	-3	1	1	1	1	1
χ_{10}	21	21	21	-9	-9	-9	-9	-3	-3	-3	-3	1	1	1	1	1
χ_{11}	42	42	42	0	0	0	0	-6	-6	-6	-6	2	2	2	2	2
χ_{12}	28	28	28	10	10	10	10	-4	-4	-4	-4	4	4	4	4	4
χ_{13}	28	28	28	-10	-10	-10	-10	-4	-4	-4	-4	4	4	4	4	4
χ_{14}	35	35	35	5	5	5	5	3	3	3	3	-5	-5	-5	-5	-5
χ_{15}	35	35	35	-5	-5	-5	-5	3	3	3	3	-5	-5	-5	-5	-5
χ_{16}	90	90	90	0	0	0	0	-6	-6	-6	-6	-6	-6	-6	-6	-6
χ_{17}	56	56	56	4	4	4	4	8	8	8	8	0	0	0	0	0
χ_{18}	56	56	56	-4	-4	-4	-4	8	8	8	8	0	0	0	0	0
χ_{19}	64	64	64	16	16	16	16	0	0	0	0	0	0	0	0	0
χ_{20}	64	64	64	-16	-16	-16	-16	0	0	0	0	0	0	0	0	0
χ_{21}	70	70	70	10	10	10	10	-2	-2	-2	-2	2	2	2	2	2
χ_{22}	70	70	70	-10	-10	-10	-10	-2	-2	-2	-2	2	2	2	2	2
χ_{23}	28	4	-4	16	4	-4	0	4	4	-4	0	8	4	0	-4	0
χ_{24}	28	4	-4	14	6	-2	-2	-4	-4	4	0	4	8	-4	0	0
χ_{25}	28	4	-4	-16	-4	4	0	4	4	-4	0	8	4	0	-4	0
χ_{26}	28	4	-4	-14	-6	2	2	-4	-4	4	0	4	8	-4	0	0
χ_{27}	140	20	-20	-40	-20	4	8	4	4	-4	0	0	12	-8	4	0
χ_{28}	140	20	-20	40	20	-4	-8	4	4	-4	0	0	12	-8	4	0
χ_{29}	140	20	-20	50	10	-14	2	-4	-4	4	0	12	0	4	-8	0
χ_{30}	140	20	-20	-50	-10	14	-2	-4	-4	4	0	12	0	4	-8	0
χ_{31}	140	20	-20	20	0	-8	4	-12	-12	12	0	8	4	0	-4	0
χ_{32}	140	20	-20	-20	0	8	-4	-12	-12	12	0	8	4	0	-4	0

Table 5: The character table of $\bar{G}$ (continued)

$[g]_{S_8}$	$2D$				$3A$			$3B$			$4A$			$4B$			
$[x]_{2^6:S_8}$	$2J$	$4G$	$4H$	$4I$	$3A$	$6A$	$6B$	$3B$	$6C$	$6D$	$4J$	$4K$	$8A$	$4L$	$4M$	$4N$	$8B$
χ_1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
χ_2	-1	-1	-1	-1	1	1	1	1	1	1	-4	-4	-4	-4	-4	-4	-4
χ_3	1	1	1	1	4	4	4	1	1	1	3	3	3	-1	-1	-1	-1
χ_4	-1	-1	-1	-1	4	4	4	1	1	1	-3	-3	-3	1	1	1	1
χ_5	0	0	0	0	-1	-1	-1	2	2	2	-2	-2	-2	2	2	2	2
χ_6	0	0	0	0	-1	-1	-1	2	2	2	2	2	2	-2	-2	-2	-2
χ_7	2	2	2	2	5	5	5	-1	-1	-1	2	2	2	2	2	2	2
χ_8	-2	-2	-2	-2	5	5	5	-1	-1	-1	-2	-2	-2	-2	-2	-2	-2
χ_9	-3	-3	-3	-3	6	6	6	0	0	0	3	3	3	-1	-1	-1	-1
χ_{10}	3	3	3	3	6	6	6	0	0	0	-3	-3	-3	1	1	1	1
χ_{11}	0	0	0	0	-6	-6	-6	0	0	0	0	0	0	0	0	0	0
χ_{12}	2	2	2	2	1	1	1	1	1	1	-2	-2	-2	-2	-2	-2	-2
χ_{13}	-2	-2	-2	-2	1	1	1	1	1	1	2	2	2	2	2	2	2
χ_{14}	-3	-3	-3	-3	5	5	5	2	2	2	1	1	1	1	1	1	1
χ_{15}	3	3	3	3	5	5	5	2	2	2	-1	-1	-1	-1	-1	-1	-1
χ_{16}	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
χ_{17}	4	4	4	4	-4	-4	-4	-1	-1	-1	0	0	0	0	0	0	0
χ_{18}	-4	-4	-4	-4	-4	-4	-4	-1	-1	-1	0	0	0	0	0	0	0
χ_{19}	0	0	0	0	4	4	4	-2	-2	-2	0	0	0	0	0	0	0
χ_{20}	0	0	0	0	4	4	4	-2	-2	-2	0	0	0	0	0	0	0
χ_{21}	-2	-2	-2	-2	-5	-5	-5	1	1	1	-4	-4	-4	0	0	0	0
χ_{22}	2	2	2	2	-5	-5	-5	1	1	1	4	4	4	0	0	0	0
χ_{23}	4	-4	0	0	10	2	-2	1	1	-1	6	-2	0	2	2	-2	0
χ_{24}	-2	2	-2	2	10	2	-2	1	1	-1	6	-2	0	-2	-2	2	0
χ_{25}	-4	4	0	0	10	2	-2	1	1	-1	-6	2	0	-2	-2	2	0
χ_{26}	2	-2	2	-2	10	2	-2	1	1	-1	-6	2	0	2	2	-2	0
χ_{27}	4	-4	0	0	20	4	-4	-1	-1	1	-6	2	0	-2	-2	2	0
χ_{28}	-4	4	0	0	20	4	-4	-1	-1	1	6	-2	0	2	2	-2	0
χ_{29}	2	-2	2	-2	20	4	-4	-1	-1	1	6	-2	0	-2	-2	2	0
χ_{30}	-2	2	-2	2	20	4	-4	-1	-1	1	-6	2	0	2	2	-2	0
χ_{31}	0	0	4	-4	-10	-2	2	2	2	-2	-6	2	0	-2	-2	2	0
χ_{32}	0	0	-4	4	-10	-2	2	2	2	-2	6	-2	0	2	2	-2	0

Table 5: The character table of $\bar{G}$ (continued)

$[g]_{S_8}$	$4C$			$4D$				$5A$		$6A$		$6B$			
$[x]_{2^6:S_{S_8}}$	$4O$	$4P$	$4Q$	$4R$	$8C$	$8D$	$4S$	$5A$	$10A$	$6E$	$12A$	$6F$	$12B$	$12C$	$6G$
χ_1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
χ_2	1	1	1	1	1	1	1	1	1	-1	-1	-1	-1	-1	-1
χ_3	-1	-1	-1	1	1	1	1	2	2	-1	-1	2	2	2	2
χ_4	-1	-1	-1	1	1	1	1	2	2	1	1	-2	-2	-2	-2
χ_5	2	2	2	0	0	0	0	-1	-1	-2	-2	1	1	1	1
χ_6	2	2	2	0	0	0	0	-1	-1	2	2	-1	-1	-1	-1
χ_7	0	0	0	0	0	0	0	0	0	1	1	1	1	1	1
χ_8	0	0	0	0	0	0	0	0	0	-1	-1	-1	-1	-1	-1
χ_9	1	1	1	-1	-1	-1	-1	1	1	0	0	0	0	0	0
χ_{10}	1	1	1	-1	-1	-1	-1	1	1	0	0	0	0	0	0
χ_{11}	2	2	2	-2	-2	-2	-2	2	2	0	0	0	0	0	0
χ_{12}	0	0	0	0	0	0	0	-2	-2	1	1	1	1	1	1
χ_{13}	0	0	0	0	0	0	0	-2	-2	-1	-1	-1	-1	-1	-1
χ_{14}	-1	-1	-1	-1	-1	-1	-1	0	0	2	2	-1	-1	-1	-1
χ_{15}	-1	-1	-1	-1	-1	-1	-1	0	0	-2	-2	1	1	1	1
χ_{16}	2	2	2	2	2	2	2	0	0	0	0	0	0	0	0
χ_{17}	0	0	0	0	0	0	0	1	1	1	1	-2	-2	-2	-2
χ_{18}	0	0	0	0	0	0	0	1	1	-1	-1	2	2	2	2
χ_{19}	0	0	0	0	0	0	0	-1	-1	-2	-2	-2	-2	-2	-2
χ_{20}	0	0	0	0	0	0	0	-1	-1	2	2	2	2	2	2
χ_{21}	-2	-2	-2	0	0	0	0	0	0	1	1	1	1	1	1
χ_{22}	-2	-2	-2	0	0	0	0	0	0	-1	-1	-1	-1	-1	-1
χ_{23}	0	0	0	2	0	0	-2	3	-1	1	-1	4	2	-2	0
χ_{24}	0	0	0	0	-2	2	0	3	-1	-1	1	2	4	0	-2
χ_{25}	0	0	0	2	0	0	-2	3	-1	-1	1	-4	-2	2	0
χ_{26}	0	0	0	0	-2	2	0	3	-1	1	-1	-2	-4	0	2
χ_{27}	0	0	0	-2	0	0	2	0	0	-1	1	2	-2	-2	2
χ_{28}	0	0	0	-2	0	0	2	0	0	1	-1	-2	2	2	-2
χ_{29}	0	0	0	0	2	-2	0	0	0	-1	1	2	-2	-2	2
χ_{30}	0	0	0	0	2	-2	0	0	0	1	-1	-2	2	2	-2
χ_{31}	0	0	0	-2	0	0	-2	0	0	2	-2	2	4	0	-2
χ_{32}	0	0	0	-2	0	0	-2	0	0	-2	2	-2	-4	0	2

Table 5: The character table of $\bar{G}$ (continued)

$[g]_{S_8}$	$6C$			$6D$		$6E$			$7A$	$8A$		$10A$		$12A$		$15A$
$[x]_{2^6:S_{S_8}}$	$6H$	$12D$	$12E$	$6I$	$12F$	$6J$	$6K$	$6L$	$7A$	$8E$	$8F$	$10B$	$20A$	$12G$	$24A$	$15A$
χ_1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
χ_2	1	1	1	-1	-1	1	1	1	1	-1	-1	-1	-1	-1	-1	1
χ_3	0	0	0	1	1	-1	-1	-1	0	-1	-1	0	0	0	0	-1
χ_4	0	0	0	-1	-1	-1	-1	-1	0	1	1	0	0	0	0	-1
χ_5	-1	-1	-1	0	0	0	0	0	0	0	0	-1	-1	1	1	-1
χ_6	-1	-1	-1	0	0	0	0	0	0	0	0	1	1	-1	-1	-1
χ_7	1	1	1	-1	-1	1	1	1	-1	0	0	0	0	-1	-1	0
χ_8	1	1	1	1	1	1	1	1	-1	0	0	0	0	1	1	0
χ_9	-2	-2	-2	0	0	0	0	0	0	1	1	-1	-1	0	0	1
χ_{10}	-2	-2	-2	0	0	0	0	0	0	-1	-1	1	1	0	0	1
χ_{11}	2	2	2	0	0	0	0	0	0	0	0	0	0	0	0	-1
χ_{12}	1	1	1	-1	-1	-1	-1	-1	0	0	0	0	0	1	1	1
χ_{13}	1	1	1	1	1	-1	-1	-1	0	0	0	0	0	-1	-1	1
χ_{14}	1	1	1	0	0	0	0	0	0	-1	-1	0	0	1	1	0
χ_{15}	1	1	1	0	0	0	0	0	0	1	1	0	0	-1	-1	0
χ_{16}	0	0	0	0	0	0	0	0	-1	0	0	0	0	0	0	0
χ_{17}	0	0	0	1	1	-1	-1	-1	0	0	0	-1	-1	0	0	1
χ_{18}	0	0	0	-1	-1	-1	-1	-1	0	0	0	1	1	0	0	1
χ_{19}	0	0	0	0	0	0	0	0	1	0	0	1	1	0	0	-1
χ_{20}	0	0	0	0	0	0	0	0	1	0	0	-1	-1	0	0	-1
χ_{21}	-1	-1	-1	1	1	1	1	1	0	0	0	0	0	-1	-1	0
χ_{22}	-1	-1	-1	-1	-1	1	1	1	0	0	0	0	0	1	1	0
χ_{23}	2	-2	0	1	-1	1	1	-1	0	0	0	1	-1	0	0	0
χ_{24}	-2	2	0	1	-1	-1	-1	1	0	0	0	-1	1	0	0	0
χ_{25}	2	-2	0	-1	1	1	1	-1	0	0	0	-1	1	0	0	0
χ_{26}	-2	2	0	-1	1	-1	-1	1	0	0	0	1	-1	0	0	0
χ_{27}	0	0	0	1	-1	1	1	-1	0	0	0	0	0	0	0	0
χ_{28}	0	0	0	-1	1	1	1	-1	0	0	0	0	0	0	0	0
χ_{29}	0	0	0	-1	1	-1	-1	1	0	0	0	0	0	0	0	0
χ_{30}	0	0	0	1	-1	-1	-1	1	0	0	0	0	0	0	0	0
χ_{31}	2	-2	0	0	0	0	0	0	0	0	0	0	0	0	0	0
χ_{32}	2	-2	0	0	0	0	0	0	0	0	0	0	0	0	0	0

Table 5: The character table of $\bar{G}$ (continued)

$[g]_{S_8}$		$1A$			$2A$				$2B$					$2C$	
$[x]_{2^6:S_8}$	$1A$	$2A$	$2B$	$2C$	$4A$	$4B$	$2D$	$2E$	$2F$	$2G$	$4C$	$2H$	$4D$	$2I$	$4E$	$4F$
χ_{33}	140	20	-20	10	10	2	-6	12	12	-12	0	4	8	-4	0	0
χ_{34}	140	20	-20	-10	-10	-2	6	12	12	-12	0	4	8	-4	0	0
χ_{35}	452	36	-36	-36	-24	0	12	-12	-12	12	0	0	12	-8	4	0
χ_{36}	452	36	-36	36	24	0	-12	-12	-12	12	0	0	12	-8	4	0
χ_{37}	452	36	-36	54	6	-18	6	12	12	-12	0	12	0	4	-8	0
χ_{38}	452	36	-36	-54	-6	18	-6	12	12	-12	0	12	0	4	-8	0
χ_{39}	280	40	-40	40	0	-16	8	-8	-8	8	0	-8	-16	8	0	0
χ_{40}	280	40	-40	-40	0	16	-8	-8	-8	8	0	-8	-16	8	0	0
χ_{41}	280	40	-40	20	20	4	-12	8	8	-8	0	-16	-8	0	8	0
χ_{42}	280	40	-40	-20	-20	-4	12	8	8	-8	0	-16	-8	0	8	0
χ_{43}	448	64	-64	16	-16	-16	16	0	0	0	0	0	0	0	0	0
χ_{44}	448	64	-64	-16	16	16	-16	0	0	0	0	0	0	0	0	0
χ_{45}	35	-5	3	15	-5	3	-1	11	-5	3	-1	7	-5	-1	3	-1
χ_{46}	35	-5	3	-15	5	-3	1	-5	11	3	-1	7	-5	-1	3	-1
χ_{47}	35	-5	3	-15	5	-3	1	11	-5	3	-1	7	-5	-1	3	-1
χ_{48}	35	-5	3	15	-5	3	-1	-5	11	3	-1	7	-5	-1	3	-1
χ_{49}	70	-10	6	0	0	0	0	6	6	6	-2	-10	14	6	-2	-2
χ_{50}	140	-20	12	-30	10	-6	2	12	12	12	-4	4	4	4	4	-4
χ_{51}	140	-20	12	30	-10	6	-2	12	12	12	-4	4	4	4	4	-4
χ_{52}	140	-20	12	0	0	0	0	-4	28	12	-4	4	4	4	4	-4
χ_{53}	140	-20	12	0	0	0	0	28	-4	12	-4	4	4	4	4	-4
χ_{54}	210	-30	18	-30	10	-6	2	-6	-6	-6	2	-10	14	6	-2	-2
χ_{55}	210	-30	18	30	-10	6	-2	-6	-6	-6	2	-10	14	6	-2	-2
χ_{56}	210	-30	18	-60	20	-12	4	-6	-6	-6	2	14	-10	-2	6	-2
χ_{57}	210	-30	18	60	-20	12	-4	-6	-6	-6	2	14	-10	-2	6	-2
χ_{58}	315	-45	27	-45	15	-9	3	-21	27	3	-1	3	-9	-5	-1	3
χ_{59}	315	-45	27	-45	15	-9	3	27	-21	3	-1	3	-9	-5	-1	3
χ_{60}	315	-45	27	45	-15	9	-3	-21	27	3	-1	3	-9	-5	-1	3
χ_{61}	315	-45	27	45	-15	9	-3	27	-21	3	-1	3	-9	-5	-1	3
χ_{62}	420	-60	36	-30	10	-6	2	-12	-12	-12	4	4	4	4	4	-4
χ_{63}	420	-60	36	30	-10	6	-2	-12	-12	-12	4	4	4	4	4	-4
χ_{64}	630	-90	54	0	0	0	0	6	6	6	-2	-18	6	-2	-10	6

Table 5: The character table of $\bar{G}$ (continued)

$[g]_{S_8}$	$2D$				$3A$			$3B$			$4A$			$4B$			
$[x]_{2^6:S_8}$	$2J$	$4G$	$4H$	$4I$	$3A$	$6A$	$6B$	$3B$	$6C$	$6D$	$4J$	$4K$	$8A$	$4L$	$4M$	$4N$	$8B$
χ_{33}	-6	6	2	-2	-10	-2	2	2	2	-2	-6	2	0	2	2	-2	0
χ_{34}	6	-6	-2	2	-10	-2	2	2	2	-2	6	-2	0	-2	-2	2	0
χ_{35}	0	0	4	-4	0	0	0	0	0	0	6	-2	0	2	2	-2	0
χ_{36}	0	0	-4	4	0	0	0	0	0	0	-6	2	0	-2	-2	2	0
χ_{37}	6	-6	-2	2	0	0	0	0	0	0	-6	2	0	2	2	-2	0
χ_{38}	-6	6	2	-2	0	0	0	0	0	0	6	-2	0	-2	-2	2	0
χ_{39}	-8	8	0	0	10	2	-2	1	1	-1	0	0	0	0	0	0	0
χ_{40}	8	-8	0	0	10	2	-2	1	1	-1	0	0	0	0	0	0	0
χ_{41}	4	-4	4	-4	10	2	-2	1	1	-1	0	0	0	0	0	0	0
χ_{42}	-4	4	-4	4	10	2	-2	1	1	-1	0	0	0	0	0	0	0
χ_{43}	0	0	0	0	-20	-4	4	-2	-2	2	0	0	0	0	0	0	0
χ_{44}	0	0	0	0	-20	-4	4	-2	-2	2	0	0	0	0	0	0	0
χ_{45}	3	3	-1	-1	5	-3	1	2	-2	0	1	1	-1	5	-3	1	-1
χ_{46}	-3	-3	1	1	5	-3	1	2	-2	0	-1	-1	1	3	-5	-1	1
χ_{47}	-3	-3	1	1	5	-3	1	2	-2	0	-1	-1	1	-5	3	-1	1
χ_{48}	3	3	-1	-1	5	-3	1	2	-2	0	1	1	-1	-3	5	1	-1
χ_{49}	0	0	0	0	10	-6	2	4	-4	0	0	0	0	0	0	0	0
χ_{50}	-6	-6	2	2	5	-3	1	-4	4	0	-2	-2	2	-2	-2	-2	2
χ_{51}	6	6	-2	-2	5	-3	1	-4	4	0	2	2	-2	2	2	2	-2
χ_{52}	0	0	0	0	-10	6	-2	2	-2	0	0	0	0	0	0	0	0
χ_{53}	0	0	0	0	-10	6	-2	2	-2	0	0	0	0	0	0	0	0
χ_{54}	6	6	-2	-2	15	-9	3	0	0	0	-4	-4	4	0	0	0	0
χ_{55}	-6	-6	2	2	15	-9	3	0	0	0	4	4	-4	0	0	0	0
χ_{56}	0	0	0	0	15	-9	3	0	0	0	-2	-2	2	2	2	2	-2
χ_{57}	0	0	0	0	15	-9	3	0	0	0	2	2	-2	-2	-2	-2	2
χ_{58}	3	3	-1	-1	0	0	0	0	0	0	3	3	-3	3	-5	-1	1
χ_{59}	3	3	-1	-1	0	0	0	0	0	0	3	3	-3	-5	3	-1	1
χ_{60}	-3	-3	1	1	0	0	0	0	0	0	-3	-3	3	-3	5	1	-1
χ_{61}	-3	-3	1	1	0	0	0	0	0	0	-3	-3	3	5	-3	1	-1
χ_{62}	-6	-6	2	2	-15	9	-3	0	0	0	2	2	-2	2	2	2	-2
χ_{63}	6	6	-2	-2	-15	9	-3	0	0	0	-2	-2	2	-2	-2	-2	2
χ_{64}	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0

Table 5: The character table of $\bar{G}$ (continued)

$[g]_{S_8}$	$4C$			$4D$				$5A$		$6A$		$6B$			
$[x]_{2^6:S_{S_8}}$	$4O$	$4P$	$4Q$	$4R$	$8C$	$8D$	$4S$	$5A$	$10A$	$6E$	$12A$	$6F$	$12B$	$12C$	$6G$
χ_{33}	0	0	0	0	2	-2	0	0	0	-2	2	4	2	-2	0
χ_{34}	0	0	0	0	2	-2	0	0	0	2	-2	-4	-2	2	0
χ_{35}	0	0	0	2	0	0	-2	-3	1	0	0	0	0	0	0
χ_{36}	0	0	0	2	0	0	-2	-3	1	0	0	0	0	0	0
χ_{37}	0	0	0	0	-2	2	0	-3	1	0	0	0	0	0	0
χ_{38}	0	0	0	0	-2	2	0	-3	1	0	0	0	0	0	0
χ_{39}	0	0	0	0	0	0	0	0	0	1	-1	-2	-4	0	2
χ_{40}	0	0	0	0	0	0	0	0	0	-1	1	2	4	0	-2
χ_{41}	0	0	0	0	0	0	0	0	0	-1	1	-4	-2	2	0
χ_{42}	0	0	0	0	0	0	0	0	0	1	-1	4	2	-2	0
χ_{43}	0	0	0	0	0	0	0	3	-1	-2	2	-2	2	2	-2
χ_{44}	0	0	0	0	0	0	0	3	-1	2	-2	2	-2	-2	2
χ_{45}	3	-1	-1	1	-1	-1	1	0	0	0	0	3	-3	1	-1
χ_{46}	-1	3	-1	1	-1	-1	1	0	0	0	0	-3	3	-1	1
χ_{47}	3	-1	-1	1	-1	-1	1	0	0	0	0	-3	3	-1	1
χ_{48}	-1	3	-1	1	-1	-1	1	0	0	0	0	3	-3	1	-1
χ_{49}	-2	-2	2	-2	2	2	-2	0	0	0	0	0	0	0	0
χ_{50}	0	0	0	0	0	0	0	0	0	0	0	3	-3	1	-1
χ_{51}	0	0	0	0	0	0	0	0	0	0	0	-3	3	-1	1
χ_{52}	-4	4	0	0	0	0	0	0	0	0	0	0	0	0	0
χ_{53}	4	-4	0	0	0	0	0	0	0	0	0	0	0	0	0
χ_{54}	2	2	-2	0	0	0	0	0	0	0	0	3	-3	1	-1
χ_{55}	2	2	-2	0	0	0	0	0	0	0	0	-3	3	-1	1
χ_{56}	-2	-2	2	0	0	0	0	0	0	0	0	-3	3	-1	1
χ_{57}	-2	-2	2	0	0	0	0	0	0	0	0	3	-3	1	-1
χ_{58}	3	-1	-1	-1	1	1	-1	0	0	0	0	0	0	0	0
χ_{59}	-1	3	-1	-1	1	1	-1	0	0	0	0	0	0	0	0
χ_{60}	3	-1	-1	-1	1	1	-1	0	0	0	0	0	0	0	0
χ_{61}	-1	3	-1	-1	1	1	-1	0	0	0	0	0	0	0	0
χ_{62}	0	0	0	0	0	0	0	0	0	0	0	3	-3	1	-1
χ_{63}	0	0	0	0	0	0	0	0	0	0	0	-3	3	-1	1
χ_{64}	-2	-2	2	2	-2	-2	2	0	0	0	0	0	0	0	0

Table 5: The character table of $\bar{G}$ (continued)

$[g]_{S_8}$	$6C$			$6D$		$6E$			$7A$	$8A$		$10A$		$12A$		$15A$
$[x]_{2^6:S_{S_8}}$	$6H$	$12D$	$12E$	$6I$	$12F$	$6J$	$6K$	$6L$	$7A$	$8E$	$8F$	$10B$	$20A$	$12G$	$24A$	$15A$
χ_{33}	-2	2	0	0	0	0	0	0	0	0	0	0	0	0	0	0
χ_{34}	-2	2	0	0	0	0	0	0	0	0	0	0	0	0	0	0
χ_{35}	0	0	0	0	0	0	0	0	0	0	0	-1	1	0	0	0
χ_{36}	0	0	0	0	0	0	0	0	0	0	0	1	-1	0	0	0
χ_{37}	0	0	0	0	0	0	0	0	0	0	0	-1	1	0	0	0
χ_{38}	0	0	0	0	0	0	0	0	0	0	0	1	-1	0	0	0
χ_{39}	-2	2	0	1	-1	1	1	-1	0	0	0	0	0	0	0	0
χ_{40}	-2	2	0	-1	1	1	1	-1	0	0	0	0	0	0	0	0
χ_{41}	2	-2	0	1	-1	-1	-1	1	0	0	0	0	0	0	0	0
χ_{42}	2	-2	0	-1	1	-1	-1	1	0	0	0	0	0	0	0	0
χ_{43}	0	0	0	0	0	0	0	0	0	0	0	1	-1	0	0	0
χ_{44}	0	0	0	0	0	0	0	0	0	0	0	-1	1	0	0	0
χ_{45}	1	1	-1	0	0	2	-2	0	0	1	-1	0	0	1	-1	0
χ_{46}	1	1	-1	0	0	-2	2	0	0	1	-1	0	0	-1	1	0
χ_{47}	1	1	-1	0	0	2	-2	0	0	-1	1	0	0	-1	1	0
χ_{48}	1	1	-1	0	0	-2	2	0	0	-1	1	0	0	1	-1	0
χ_{49}	2	2	-2	0	0	0	0	0	0	0	0	0	0	0	0	0
χ_{50}	1	1	-1	0	0	0	0	0	0	0	0	0	0	1	-1	0
χ_{51}	1	1	-1	0	0	2	-2	0	0	1	-1	0	0	-1	1	0
χ_{52}	-2	-2	2	0	0	2	-2	0	0	0	0	0	0	0	0	0
χ_{53}	-2	-2	2	0	0	-2	2	0	0	0	0	0	0	0	0	0
χ_{54}	-1	-1	1	0	0	0	0	0	0	0	0	0	0	-1	1	0
χ_{55}	-1	-1	1	0	0	0	0	0	0	0	0	0	0	1	-1	0
χ_{56}	-1	-1	1	0	0	0	0	0	0	0	0	0	0	1	-1	0
χ_{57}	-1	-1	1	0	0	0	0	0	0	0	0	0	0	-1	1	0
χ_{58}	0	0	0	0	0	0	0	0	0	-1	1	0	0	0	0	0
χ_{59}	0	0	0	0	0	0	0	0	0	1	-1	0	0	0	0	0
χ_{60}	0	0	0	0	0	0	0	0	0	1	-1	0	0	0	0	0
χ_{61}	0	0	0	0	0	0	0	0	0	-1	1	0	0	0	0	0
χ_{62}	1	1	-1	0	0	0	0	0	0	0	0	0	0	-1	1	0
χ_{63}	1	1	-1	0	0	0	0	0	0	0	0	0	0	1	-1	0
χ_{64}	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0

5. The Fusion of $\bar{G}$ into Fi_{22}

We use the results of the conjugacy classes of $\bar{G}$ which are given in Section 2, to compute the power maps of the elements of $\bar{G}$ which we list in Table 6.

Table 6: The power maps of the elements of $\bar{G}$

$[g]_{S_8}$	$[x]_{2^6:S_8}$	2	3	5	7	$[g]_{S_8}$	$[x]_{2^6:S_8}$	2	3	5	7
$1A$	$1A$					$2A$	$2C$	$1A$			
	$2A$	$1A$					$4A$	$2A$			
	$2B$	$1A$					$4B$	$2A$			
							$2D$	$1A$			
$2B$	$2E$	$1A$				$2C$	$2H$	$1A$			
	$2F$	$1A$					$4D$	$2B$			
	$2G$	$1A$					$2I$	$1A$			
	$4C$	$2B$					$4E$	$2B$			
							$4F$	$2A$			
$2D$	$2J$	$1A$				$3A$	$3A$		$1A$		
	$4G$	$2A$					$6A$	$3A$	$2B$		
	$4H$	$2A$					$6B$	$3A$	$2A$		
	$4I$	$2B$									
$3B$	$3B$		$1A$			$4A$	$4J$	$2H$			
	$6C$	$3B$	$2A$				$4K$	$2H$			
	$6D$	$3B$	$2B$				$8A$	$4D$			
$4B$	$4L$	$2H$				$4C$	$4O$	$2E$			
	$4M$	$2H$					$4P$	$2F$			
	$4N$	$2H$					$4Q$	$2G$			
	$8B$	$4E$									
$4D$	$4R$	$2H$				$5A$	$5A$			$1A$	
	$8C$	$4D$					$10A$	$5A$		$2A$	
	$8D$	$4E$									
	$4S$	$2I$									
$6A$	$6E$	$3B$	$2D$			$6B$	$6F$	$3A$	$2C$		
	$12A$	$6C$	$4B$				$12B$	$6B$	$4B$		
							$12C$	$6B$	$4A$		
							$6G$	$3A$	$2D$		
$6C$	$6H$	$3A$	$2H$			$6D$	$6I$	$3B$	$2J$		
	$12D$	$6A$	$4D$				$12F$	$6C$	$4G$		
	$12E$	$6B$	$4F$								
$6E$	$6J$	$3B$	$2E$			$7A$	$7A$				$1A$
	$6K$	$3B$	$2F$								
	$6L$	$3B$	$2G$								
$8A$	$8E$	$4O$				$10A$	$10B$	$5A$		$2C$	
	$8F$	$4P$					$20A$	$10A$		$4A$	
$12A$	$12G$	$6H$	$4J$			$15A$	$15A$		$5A$	$3A$	
	$24A$	$12D$	$8A$								

Our group $\bar{G} = 2^6{:}S_8$ sits maximally inside the group $E = 2^6{:}Sp_6(2)$. Moori and Mpono in [22] computed the character table of E, which is also available in $\mathbb{GAP}$ [29]. The fusion of $\bar{G}$ into E will help us to determine the fusion of $\bar{G}$ into Fi_{22}. We give the fusion map of $\bar{G}$ into E in Table 7.

The power maps of Fi_{22} are given in the $\mathbb{ATLAS}$ and $\mathbb{GAP}$. In order to complete the fusion of $\bar{G}$ into Fi_{22} we sometimes use the technique of set intersection. For detailed information regarding the technique of set intersection we refer to [1], [4], [5], [21] and [25]. We give the complete list of class fusions of $\bar{G}$ into Fi_{22} in Table 8.

Table 7: The fusion of $\bar{G}$ into E

$[g]_{\bar{G}}$	$\longrightarrow$	$[h]_E$	$[g]_{\bar{G}}$	$\longrightarrow$	$[h]_E$	$[g]_{\bar{G}}$	$\longrightarrow$	$[h]_E$	$[g]_{\bar{G}}$	$\longrightarrow$	$[h]_E$
1A		1A	2A		2A	2B		2A	2C		2B
4A		4A	4B		4A	2D		2C	2E		2D
2F		2E	2G		2E	4C		4B	2H		2F
4D		4C	2I		2G	4E		4C	4F		4D
2J		2H	4G		4E	4H		4F	4I		4G
3A		3A	6A		6A	6B		6A	3B		3C
6C		6B	6D		6B	4J		4L	4K		4M
8A		8B	4L		4J	4M		4K	4N		4K
8B		8A	4O		4N	4P		4O	4Q		4P
4R		4Q	8C		8D	8D		8C	4S		4R
5A		5A	10A		10A	6E		6H	12A		12E
6F		6D	12B		12B	12C		12B	6G		6E
6H		6G	12D		12C	12E		12D	6I		6I
12F		12F	6J		6J	6K		6K	6L		6K
7A		7A	8E		8E	8F		8F	10B		10B
20A		20A	12G		12H	24A		24B	15A		15A

Table 8: The fusion of $\bar{G}$ into Fi_{22}

$[g]_{S_8}$	$[x]_{2^6:S_8}$	$\longrightarrow$	$[h]_{Fi_{22}}$	$[g]_{S_8}$	$[x]_{2^6:S_8}$	$\longrightarrow$	$[h]_{Fi_{22}}$
1A	1A		1A	2A	2C		2A
	2A		2B		4A		4B
	2B		2B		4B		4B
					2D		2C
2B	2E		2B	2C	2H		2B
	2F		2C		4D		4A
	2G		2B		2I		2C
	4C		4A		4E		4A
					4F		4E
2D	2J		2C	3A	3A		3A
	4G		4B		6A		6D
	4H		4E		6B		6D
	4I		4C				
3B	3B		3C	4A	4J		4B
	6C		6I		4K		4E
	6D		6I		8A		8A
4B	4L		4B	4C	4O		4A
	4M		4E		4P		4D
	4N		4B		4Q		4E
	8B		8B				
4D	4R		4E	5A	5A		5A
	8C		8B		10A		10B
	8D		8A				
	4S		4D				
6A	6E		6E	6B	6F		6A
	12A		12J		12B		12D
					12C		12D
					6G		6F
6C	6H		6D	6D	6I		6J
	12D		12B		12F		12J
	12E		12I				
6E	6J		6I	7A	7A		7A
	6K		6H				
	6L		6I				
8A	8E		8B	10A	10B		10A
	8F		8D		20A		20A
12A	12G		12D	15A	15A		15A
	24A		24B				

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