Hacettepe Journal of Mathematics and Statistics
Volume 43 (2) (2014), 279 – 281

Erratum and notes for *near groups on nearness approximation spaces*

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Keywords: Near set, Nearness approximation spaces, Near group.

2000 AMS Classification: 03E75, 03E99, 20A05, 20E99

Erratum and notes for: "İnan, E., Öztürk, M. A. Near groups on nearness approximation spaces, Hacet J Math Stat, 41(4), 2012, 545–558."

The authors would like to write some notes and correct errors in the original publication of the article [1]. The notes are given below:

0.1. Remark. In page 550, in Definition 3.1., (1) and (2) properties have to hold in $N_r(B)^* G$. Sometimes they may be hold in $\mathcal{O} \diagdown N_r(B)^* G$, then G is not a near group on nearness approximation space.

Example 3.3. and 3.4. are nice examples of this case. In Example 3.3., if we consider associative property $(b \cdot e) \cdot b = b \cdot (e \cdot b)$ for $b, e \in H \subset G$, we obtain $\imath = \imath$, but $\imath \in \mathcal{O} \diagdown N_r(B)^* H$. Hence, we can observe that if the associative property holds in $\mathcal{O} \diagdown N_r(B)^* H$, then H can not be a subnear group of near group G. Consequently, Example 3.3. and 3.4. are incorrect, i.e., they are not subnear groups of near group G.

0.2. Remark. Multiplying of finite number of elements in G may not always belongs to $N_r(B)^* G$. Therefore always we can not say that $x^n \in N_r(B)^* G$, for all $x \in G$ and some positive integer n. If $\left(N_r(B)^* G, \cdot\right)$ is groupoid, then we can say that $x^n \in N_r(B)^* G$, for all $x \in R$ and all positive integer n.

In Example 3.2., the properties (1) and (2) hold in $N_r(B)^* G$. Hence G is a near group on nearness approximation space.

The corrections are given below:

In page 548, in subsection 2.4.1., definition of B-lower approximation of $X \subseteq \mathcal{O}$ must be

$$B_* X = \bigcup_{[x]_B \subseteq X} [x]_B.$$

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In page 554, Theorem 3.8. must be as in Theorem 0.3:

0.3. Theorem. *Let G be a near group on nearness approximation space, H a nonempty subset of G and $N_r(B)^* H$ a groupoid. $H \subseteq G$ is a subnear group of G if and only if $x^{-1} \in H$ for all $x \in H$.*

Proof. Suppose that H is a subnear group of G. Then H is a near group and so $x^{-1} \in H$ for all $x \in H$. Conversely, suppose $x^{-1} \in H$ for all $x \in H$. By the hypothesis, since $N_r(B)^* H$ is a groupoid and $H \subseteq G$, then closure and associative properties hold in $N_r(B)^* H$. Also we have $x \cdot x^{-1} = e \in N_r(B)^* H$. Hence H is a subnear group of G. □

In page 554, Theorem 3.9. must be as in Theorem 0.4:

0.4. Theorem. *Let H_1 and H_2 be two near subgroups of the near group G and $N_r(B)^* H_1$, $N_r(B)^* H_2$ groupoids. If*

$$(N_r(B)^* H_1) \cap (N_r(B)^* H_2) = N_r(B)^* (H_1 \cap H_2),$$

then $H_1 \cap H_2$ is a near subgroup of near group G.

Proof. Suppose H_1 and H_2 be two near subgroups of the near group G. It is obvious that $H_1 \cap H_2 \subset G$. Since $N_r(B)^* H_1$, $N_r(B)^* H_2$ are groupoids and $\left(N_r(B)^* H_1\right) \cap \left(N_r(B)^* H_2\right) = N_r(B)^* (H_1 \cap H_2)$, $N_r(B)^* (H_1 \cap H_2)$ is a groupoid. Consider $x \in H_1 \cap H_2$. Since H_1 and H_2 are near subgroups, we have $x^{-1} \in H_1$ and $x^{-1} \in H_2$, i.e., $x^{-1} \in H_1 \cap H_2$. Thus from Theorem 0.3 $H_1 \cap H_2$ is a near subgroup of G. □

In page 555, proof of Theorem 5.3. has some typos. It must be as in Theorem 0.5:

0.5. Theorem. *Let G be a near group on nearness approximation space and N a subnear group of G. N is a subnear normal group of G if and only if $a \cdot n \cdot a^{-1} \in N$ for all $a \in G$ and $n \in N$.*

Proof. Suppose N is a near normal subgroup of near group G. We have $a \cdot N \cdot a^{-1} = N$ for all $a \in G$. For any $n \in N$, therefore we have $a \cdot n \cdot a^{-1} \in N$. Suppose N is a near subgroup of near group G. Suppose $a \cdot n \cdot a^{-1} \in N$ for all $a \in G$ and $n \in N$. We have $a \cdot N \cdot a^{-1} \subset N$. Since $a^{-1} \in G$, we get $a \cdot \left(a^{-1} \cdot N \cdot a\right) \cdot a^{-1} \subset a \cdot N \cdot a^{-1}$, i.e., $N \subset a \cdot N \cdot a^{-1}$. Since $a \cdot N \cdot a^{-1} \subset N$ and $N \subset a \cdot N \cdot a^{-1}$, we obtain $a \cdot N \cdot a^{-1} = N$. Therefore N is a subnear normal group of G. □

In page 556, Theorem 6.6. must be as in Theorem 0.6:

0.6. Theorem. *Let $G_1 \subset \mathcal{O}_1$,$G_2 \subset \mathcal{O}_2$ be near groups that are near homomorphic, N near homomorphism kernel and $N_r(B)^* N$ a groupoid. Then N is a near normal subgroup of G_1.*

In page 557, Theorem 6.7. must be as in Theorem 0.7:

0.7. Theorem. *Let $G_1 \subset \mathcal{O}_1$, $G_2 \subset \mathcal{O}_2$ be near homomorphic groups, H_1 and N_1 a near subgroup and a near normal subgroup of G_1, respectively and $N_{r_1}(B)^* H_1$ groupoid. Then we have the following.*

(1) *If* $\varphi\left(N_{r_1}(B)^* H_1\right) = N_{r_2}(B)^* \varphi(H_1)$*, then* $\varphi(H_1)$ *is a near subgroup of* G_2*.*

(2) *if* $\varphi(G_1) = G_2$ *and* $\varphi\left(N_{r_1}(B)^* N_1\right) = N_{r_2}(B)^* \varphi(N_1)$*, then* $\varphi(N_1)$ *is a near normal subgroup of* G_2*.*

In page 557, Theorem 6.8. must be as in Theorem 0.8:

0.8. Theorem. *Let* $G_1 \subset \mathcal{O}_1$*,* $G_2 \subset \mathcal{O}_2$ *be near homomorphic groups,* H_2 *and* N_2 *a near subgroup and a near normal subgroup of* G_2*, respectively and* $N_{r_1}(B)^* H_1$ *groupoid. Then we have the following.*

(1) *if* $\varphi\left(N_{r_1}(B)^* H_1\right) = N_{r_2}(B)^* H_2$ *, then* H_1 *is a near subgroup of* G_1 *where* H_1 *is the inverse image of* H_2*.*

(2) *if* $\varphi(G_1) = G_2$ *and* $\varphi\left(N_{r_1}(B)^* N_1\right) = N_{r_2}(B)^* N_2$*, then* N_1 *is a near normal subgroup of* G_1 *where* N_1 *is the inverse image of* N_2*.*

We apologize to the readers for any inconvenience of these errors might have caused.

References

[1] İnan, E. and Öztürk, M. A. *Near groups on nearness approximation spaces*, Hacet J Math Stat **41** (4), 545–558, 2012.