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E-Bochner curvature tensor on $N(k)$-contact metric manifolds

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Abstract

The object of the present paper is to study E-Bochner curvature tensor B^e satisfying $R.B^e = 0$, $B^e.R = 0$, $B^e.B^e = 0$ and $B^e.S = 0$ in an n-dimensional $N(k)$-contact metric manifold.

Keywords: $N(k)$-contact metric manifold, Sasakian manifold, extended Bochner curvature tensor.

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1. Introduction

In modern mathematics the study of contact geometry has become a matter of growing interest due to its role in explaining physical phenomena in the context of mathematical physics. An important class of contact manifold is Sasakian manifold introduced by S. Sasaki [17]. Among the geometric properties of manifolds symmetry is an important one. A Riemannian manifold M is called locally symmetric if its curvature tensor R is parallel, that is, $\nabla R = 0$, where ∇ denotes the Levi-Civita connection. As a generalization of locally symmetric spaces, many geometers have considered semisymmetric spaces and in turn their generalizations. A Riemannian manifold M is said to be semisymmetric if its curvature tensor R satisfies

$$R(X, Y).R = 0, \qquad X, Y \in T(M),$$

where $R(X, Y)$ acts on R as a derivation. In contact geometry, S. Tanno [18] showed that a semisymmetric K-contact manifold M is locally isometric to the unit sphere $S^n(1)$.

On the other hand, S. Bochner [8] introduced a Kähler analogue of the Weyl conformal curvatur tensor by purely formal considerations, which is now well known as the Bochner curvature tensor. A geometric meaning of the Bochner curvature

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tensor is given by D. E. Blair [6]. By using the Boothby-Wang's fibration [10], M. Matsumoto and G. Chuman [14] constructed C-Bochner curvature tensor from the Bochner curvature tensor. The C-Bochner curvature tensor is given by

$$
\begin{aligned}
B(X,Y)Z &= R(X,Y)Z + \frac{1}{n+3}[S(X,Z)Y - S(Y,Z)X \\
&\quad + g(X,Z)QY - g(Y,Z)QX + S(\phi X,Z)\phi Y \\
&\quad - S(\phi Y,Z)\phi X + g(\phi X,Z)Q\phi Y \\
&\quad - g(\phi Y,Z)Q\phi X + 2S(\phi X,Y)\phi Z \\
&\quad + 2g(\phi X,Y)Q\phi Z - S(X,Z)\eta(Y)\xi \\
&\quad + S(Y,Z)\eta(X)\xi - \eta(X)\eta(Z)QY \\
&\quad + \eta(Y)\eta(Z)QX] - \frac{p+n-1}{n+3}[g(\phi X,Z)\phi Y \\
&\quad - g(\phi Y,Z)\phi X + 2g(\phi X,Y)\phi Z] \\
&\quad - \frac{p-4}{n+3}[g(X,Z)Y - g(Y,Z)X] \\
&\quad + \frac{p}{n+3}[g(X,Z)\eta(Y)\xi - g(Y,Z)\eta(X)\xi \\
&\quad + \eta(X)\eta(Z)Y - \eta(Y)\eta(Z)X],
\end{aligned} \tag{1.1}
$$

where S is the Ricci tensor of type $(0,2)$, Q is the Ricci operator defined by $g(QX,Y) = S(X,Y)$ and $p = \frac{n+r-1}{n+1}$, r being the scalar curvature of the manifold.

In [11], H. Endo extended the concept of C-Bochner curvature tensor to E-Bochner curvature tensor as follows:

$$
\begin{aligned}
B^e(X,Y)Z &= B(X,Y)Z - \eta(X)B(\xi,Y)Z \\
&\quad - \eta(Y)B(X,\xi)Z - \eta(Z)B(X,Y)\xi.
\end{aligned} \tag{1.2}
$$

Then, he showed that a K-contact manifold with vanishing E-Bochner curvature tensor is a Sasakian manifold. In a similar way Kim, Choi, Özgür and Tripathi [13] extended the contact conformal curvature tensor in a $N(k)$-contact metric manifold. Again, Özgür and Sular studied $N(k)$-contact metric manifold with extended contact conformal curvature tensor in [15]. In [16], C-Bochner semisymmetric $N(k)$-contact metric manifolds are studied. Again, $N(k)$-contact metric manifolds satisfying $B(\xi,X).R = 0$ and $B(\xi,X).B = 0$ are studied in [12]. Beside these, Sasakian manifolds satisfying $B.S = 0$ has been studied in [1]. Motivated by these studies, we consider E-Bochner semisymmetry in a $N(k)$-contact metric manifold which is defined as follows:

1.1. Definition. An n-dimensional $N(k)$-contact metric manifold is said to be E-Bochner semisymmetric if

$$R(X,Y).B^e = 0, \tag{1.3}$$

where B^e is the E-Bochner curvature tensor.

Beside this, we also study $N(k)$-contact metric manifolds satifying $B^e(\xi, X).R = 0$, $B^e(\xi, X).B^e = 0$ and $B^e(\xi, X).S = 0$.

The present paper is organized as follows:
After preliminaries in Section 3, we study E-Bochner semisymmetry in a $N(k)$-contact metric manifold and prove that the manifold is E-Bochner semisymmetric if and only if it is either a Sasakian manifold or it is locally isometric to the product $E^{n+1}(0) \times S^n(4)$. Beside this, in this section we prove that a non-Sasakian $N(k)$-contact metric manifold M^n, $(n \geq 5)$, satisfies $R(\xi, U).B^e = 0$ if and only if it is locally isometric to the product $E^{n+1}(0) \times S^n(4)$. Also, some important corollaries are given here. Section 4 deals with the $N(k)$-contact metric manifold satisfying $B^e(\xi, X).R = 0$ and we prove that a $N(k)$-contact metric manifold M^n, $(n \geq 5)$, with $k \neq 0$, satisfies $B^e(\xi, X).R = 0$ if and only if it is a Sasakian manifold. In the next Section, we prove that a $N(k)$-contact metric manifold M^n, $(n \geq 5)$, satisfies $B^e(\xi, X).B^e = 0$ if and only if it is a Sasakian manifold. Finally, in Section 6, we prove that a $N(k)$-contact metric manifold M^n, $(n \geq 5)$, satisfies $B^e(\xi, X).S = 0$ if and only if it is either a Sasakian manifold or the Ricci tensor S satisfies the relation $S(X, Y) = k(n-1)\eta(X)\eta(Y)$.

2. Preliminaries

An n-dimensional manifold M^n, $(n \geq 5)$, is said to admit an almost contact structure if it admits a tensor field ϕ of type $(1,1)$, a vector field ξ and a 1-form η satisfying ([2], [3])

(2.1) $$\phi^2 X = -X + \eta(X)\xi, \quad \eta(\xi) = 1, \quad \phi\xi = 0 \quad \text{and} \quad \eta \circ \phi = 0.$$

An almost contact metric structure is said to be normal if the induced almost complex structure J on the product manifold $M^n \times \mathbb{R}$ defined by

$$J(X, f\frac{d}{dt}) = (\phi X - f\xi, \eta(X)\frac{d}{dt})$$

is integrable, where X is tangent to M, t is the coordinate of $\mathbb{R}$ and f is a smooth function on $M^n \times \mathbb{R}$. Let g be the compatible Riemannian metric with almost contact structure (ϕ, ξ, η), that is,

(2.2) $$g(\phi X, \phi Y) = g(X, Y) - \eta(X)\eta(Y).$$

Then M^n becomes an almost contact metric manifold equipped with an almost contact metric structure (ϕ, ξ, η, g). From (2.1), it can be easily seen that

(2.3) $$g(X, \phi Y) = -g(\phi X, Y), \qquad g(X, \xi) = \eta(X),$$

for any vector fields $X, Y \in TM$. An almost contact metric structure becomes a contact metric structure if $g(X, \phi Y) = d\eta(X, Y)$, for all vector fields $X, Y \in TM$.

It is well known that the tangent sphere bundle of a flat Riemannian manifold admits a contact metric structure satisfying $R(X, Y)\xi = 0$ [4]. Again, on a Sasakian manifold [17] we have

$$R(X, Y)\xi = \eta(Y)X - \eta(X)Y.$$

As a generalization of both $R(X,Y)\xi = 0$ and the Sasakian case : D. E. Blair, Th. Koufogiorgos and B. J. Papantoniou [5] introduced the (k,μ)-nullity distribution on a contact metric manifold and gave several reasons for studying it. The (k,μ)-nullity distribution $N(k,\mu)$ [5] of a contact metric manifold M is defined by

$$\begin{aligned} N(k,\mu) \quad : \quad & p \longrightarrow N_p(k,\mu) \\ & = \{W \in T_pM : R(X,Y)W = (kI + \mu h)(g(Y,W)X - g(X,W)Y)\}, \end{aligned}$$

for all $X, Y \in TM$, where $(k,\mu) \in \mathbb{R}^2$. A contact metric manifold M^n with $\xi \in N(k,\mu)$ is called a (k,μ)-contact metric manifold. If $\mu = 0$, the (k,μ)-nullity distribution reduces to k-nullity distribution [19]. The k-nullity distribution $N(k)$ of a Riemannian manifold is defined by [19]

$$N(k) : p \longrightarrow N_p(k) = \{Z \in T_pM : R(X,Y)Z = k[g(Y,Z)X - g(X,Z)Y]\},$$

k being a constant. If the characteristic vector field $\xi \in N(k)$, then we call a contact metric manifold as $N(k)$-contact metric manifold [7]. If $k = 1$, then the manifold is Sasakian and if $k = 0$, then the manifold is locally isometric to the product $E^{n+1}(0) \times S^n(4)$ for $n > 1$ and flat for $n = 1$ [3].

Given a non-Sasakian (k,μ)-contact manifold M, E. Boeckx [9] introduced an invariant

$$I_M = \frac{1 - \frac{\mu}{2}}{\sqrt{1-k}}$$

and showed that for two non-Sasakian (k,μ)-manifolds M_1 and M_2, we have $I_{M_1} = I_{M_2}$ if and only if up to a D-homothetic deformation, the two manifolds are locally isometric as contact metric manifolds.

Thus we see that from all non-Sasakian (k,μ)-manifolds of dimension $(2n+1)$ and for every possible value of the invariant I, one (k,μ)-manifold M can be obtained with $I_M = 1$. For $I > -1$ such examples may be found from the standard contact metric structure on the tangent sphere bundle of a manifold of constant curvature c where we have $I = \frac{1+c}{|1-c|}$. Boeckx also gives a Lie algebra construction for any odd dimension and value of $I < -1$.

Using this invariant, D. E. Blair, J-S. Kim and M. M. Tripathi [7] constructed an example of a $(2n+1)$-dimensional $N(1-\frac{1}{n})$-contact metric manifold, $n > 1$. The example is given in the following:

Since the Boeckx invariant for a $(1-\frac{1}{n}, 0)$-manifold is $\sqrt{n} > -1$, we consider the tangent sphere bundle of an $(n+1)$-dimensional manifold of constant curvature c so chosen that the resulting D-homothetic deformation will be a $(1-\frac{1}{n}, 0)$-manifold. That is, for $k = c(2-c)$ and $\mu = -2c$ we solve

$$1 - \frac{1}{n} = \frac{k + a^2 - 1}{a^2}, \qquad 0 = \frac{\mu + 2a - 2}{a}$$

for a and c. The result is

$$c = \frac{\sqrt{n} \pm 1}{n-1}, \qquad a = 1 + c$$

and taking c and a to be these values we obtain $N(1-\frac{1}{n})$-contact metric manifold.

However, for a $N(k)$-contact metric manifold M^n of dimension n, we have ([3], [5])

$$(\nabla_X\phi)Y = g(X+hX,Y)\xi - \eta(Y)(X+hX), \tag{2.4}$$

where $h=\frac{1}{2}\pounds_\xi\phi$, $\pounds$ denotes the Lie differentiation.

$$R(X,Y)\xi = k[\eta(Y)X - \eta(X)Y], \tag{2.5}$$

$$R(\xi,X)Y = k[g(X,Y)\xi - \eta(Y)X], \tag{2.6}$$

$$R(X,\xi)Y = k[\eta(Y)X - g(X,Y)\xi], \tag{2.7}$$

$$S(X,Y) = (n-3)g(X,Y)+(n-3)g(hX,Y)+[(n-1)k-(n-3)]\eta(X)\eta(Y), \tag{2.8}$$

$$S(\phi X,\phi Y) = S(X,Y) - (n-1)k\eta(X)\eta(Y) - 2(n-3)g(hX,Y), \tag{2.9}$$

$$S(Y,\xi) = k(n-1)\eta(X). \tag{2.10}$$

Beside these, it can be easily verified that in a $N(k)$-contact metric manifold M^n, $n\geq 5$, the E-Bochner curvature tensor satisfies the following conditions:

$$B^e(X,Y)\xi = \frac{4(k-1)}{n+3}[\eta(X)Y - \eta(Y)X], \tag{2.11}$$

$$B^e(\xi,X)Y = \frac{4(k-1)}{n+3}[\eta(Y)X - \eta(X)\eta(Y)\xi], \tag{2.12}$$

$$B^e(X,\xi)Y = \frac{4(k-1)}{n+3}[\eta(X)\eta(Y)\xi - \eta(Y)X], \tag{2.13}$$

$$B^e(\xi,X)\xi = \frac{4(k-1)}{n+3}[X - \eta(X)\xi], \tag{2.14}$$

and

$$B^e(\xi,\xi)\xi = 0. \tag{2.15}$$

3. E-Bochner semisymmetric $N(k)$-contact metric manifolds

In this section we study E-Bochner semisymmetry in an n-dimensional $N(k)$-contact metric manifold. Therefore we have

$$(R(X,Y).B^e)(U,V)W = 0. \tag{3.1}$$

From (3.1) we have

$$\begin{aligned} R(X,Y)B^e(U,V)W - B^e(R(X,Y)U,V)W \\ - B^e(U,R(X,Y)V)W - B^e(U,V)R(X,Y)W = 0. \end{aligned} \tag{3.2}$$

Putting $X=\xi$ in (3.2) and using (2.6), we obtain

$$\begin{aligned} &k[g(Y,B^e(U,V)W)\xi - \eta(B^e(U,V)W)Y - g(Y,U)B^e(\xi,V)W \\ &+\eta(U)B^e(Y,V)W - g(Y,V)B^e(U,\xi)W + \eta(V)B^e(U,Y)W \\ &-g(Y,W)B^e(U,V)\xi + \eta(W)B^e(U,V)Y] = 0. \end{aligned} \tag{3.3}$$

From (3.3) we have either $k = 0$, or

$$
\begin{aligned}
(3.4) \qquad & [g(Y, B^e(U,V)W)\xi - \eta(B^e(U,V)W)Y - g(Y,U)B^e(\xi,V)W \\
& +\eta(U)B^e(Y,V)W - g(Y,V)B^e(U,\xi)W + \eta(V)B^e(U,Y)W \\
& -g(Y,W)B^e(U,V)\xi + \eta(W)B^e(U,V)Y] = 0.
\end{aligned}
$$

For $k = 0$, we have $R(X,Y)\xi = 0$ and hence the manifold is locally isometric to the product $E^{n+1}(0) \times S^n(4)$.

Again, replacing U and W by ξ in (3.4) and using (2.11), (2.12), (2.13) and (2.14), we obtain

$$
(3.5) \qquad \frac{4(k-1)}{n+3} g(\phi Y, \phi V)\xi = 0.
$$

Since $g(\phi Y, \phi V)\xi \neq 0$, in general, therefore we obtain from (3.5), $\frac{4(k-1)}{n+3} = 0$, that is, $k = 1$. Therefore in this case the manifold is a Sasakian manifold.

In view of the above discussions we state the following:

3.1. Proposition. Let M^n, $(n \geq 5)$, be a E-Bochner semisymmetric $N(k)$-contact metric manifold. Then the manifold is either
i) locally isometric to the product $E^{n+1}(0) \times S^n(4)$, or
ii) a Sasakian manifold.

3.1. Corollary. Any E-Bochner semisymmetric $N(k)$-contact metric manifold with $k \neq 0$, is a Sasakian manifold.

Since $\nabla B^e = 0$ implies $R.B^e = 0$, therefore from Proposition 3.1 we obtain the following:

3.2. Corollary. Any E-Bochner symmetric $N(k)$-contact metric manifold is either
i) locally isometric to the product $E^{n+1}(0) \times S^n(4)$, or
ii) a Sasakian manifold.

From Proposition 3.1, it follows that a non-Sasakian $N(k)$-contact metric manifold is locally isometric to the product $E^{n+1}(0) \times S^n(4)$. Conversely, suppose $k = 0$. Then from (2.6) we have $R(\xi, X)Y = 0$. Hence $R(\xi, X).B^e = 0$. Thus we can state the following:

3.1. Theorem. Let M^n, $(n \geq 5)$, be a non-Sasakian $N(k)$-contact metric manifold. Then the manifold satisfies $R(\xi, X).B^e = 0$ if and only if it is locally isometric to the product $E^{n+1}(0) \times S^n(4)$.

4. $N(k)$-contact metric manifolds with $B^e(\xi, U).R = 0$

This section deals with a $N(k)$-contact metric manifold which satisfies

(4.1) $\quad (B^e(\xi, U).R)(X, Y)Z = 0.$

From (4.1) we have

$$B^e(\xi, U)R(X, Y)Z - R(B^e(\xi, U)X, Y)Z \\ -R(X, B^e(\xi, U)Y)Z - R(X, Y)B^e(\xi, U)Z = 0. \tag{4.2}$$

Using (2.12) in (4.2), we get

$$\frac{4(k-1)}{n+3}[\eta(R(X, Y)Z)U - \eta(U)\eta(R(X, Y)Z)\xi - \eta(X)R(U, Y)Z \\ +\eta(X)\eta(U)R(\xi, Y)Z - \eta(Y)R(X, U)Z + \eta(Y)\eta(U)R(X, \xi)Z \\ -\eta(Z)R(X, Y)U + \eta(U)\eta(Z)R(X, Y)\xi] = 0. \tag{4.3}$$

From (4.3) we have either $k = 1$, or

$$\eta(R(X, Y)Z)U - \eta(U)\eta(R(X, Y)Z)\xi - \eta(X)R(U, Y)Z \\ +\eta(X)\eta(U)R(\xi, Y)Z - \eta(Y)R(X, U)Z + \eta(Y)\eta(U)R(X, \xi)Z \\ -\eta(Z)R(X, Y)U + \eta(U)\eta(Z)R(X, Y)\xi = 0. \tag{4.4}$$

For $k = 1$, the manifold is a Sasakian manifold.

Now, putting $X = Z = \xi$ in (4.4) and using (2.5) and (2.6), we obtain

(4.5) $\quad kg(\phi Y, \phi U)\xi = 0.$

The relation (4.5) yields $k = 0$, since $g(\phi Y, \phi U)\xi \neq 0$, in general. Therefore the manifold is locally isometric to the product $E^{n+1}(0) \times S^n(4)$. In view of the above discussion we state the following:

4.1. Proposition. Let M^n be a $N(k)$-contact metric manifold with $B^e(\xi, U).R = 0$. Then the manifold is either
i) locally isometric to the product $E^{n+1}(0) \times S^n(4)$, or
ii) a Sasakian manifold.

From Proposition 4.1 it follows that $N(k)$-contact metric manifolds with $k \neq 0$ satisfying the condition $B^e(\xi, U).R = 0$ is Sasakian. Conversely, in a Sasakian manifold we see from (2.12) that $B^e(\xi, X)Y = 0$. Thus we can state the following:

4.1. Theorem. Let M^n, $(n \geq 5)$, be a $N(k)$-contact metric manifold with $k \neq 0$. Then the manifold satisfies $B^e(\xi, U).R = 0$ if and only if the manifold is Sasakian.

5. $N(k)$-contact metric manifolds satisfying $B^e(\xi, U).B^e = 0$

Let M^n, $n \geq 5$, be a $N(k)$-contact metric manifold which satisfies

(5.1) $\quad (B^e(\xi, U).B^e)(X, Y)Z = 0.$

From (5.1) we have

$$B^e(\xi, U)B^e(X, Y)Z - B^e(B^e(\xi, U)X, Y)Z \\ -B^e(X, B^e(\xi, U)Y)Z - B^e(X, Y)B^e(\xi, U)Z = 0. \tag{5.2}$$

Using (2.12) the relation (5.2) yields

$$\frac{4(k-1)}{n+3}[\eta(B^e(X, Y)Z)U - \eta(U)\eta(B^e(X, Y)Z)\xi - \eta(X)B^e(U, Y)Z \\ +\eta(X)\eta(U)B^e(\xi, Y)Z - \eta(Y)B^e(X, U)Z + \eta(Y)\eta(U)B^e(X, \xi)Z \\ -\eta(Z)B^e(X, Y)U + \eta(U)\eta(Z)B^e(X, Y)\xi] = 0. \tag{5.3}$$

The equation (5.3) yields either $k = 1$, or

$$\eta(B^e(X, Y)Z)U - \eta(U)\eta(B^e(X, Y)Z)\xi - \eta(X)B^e(U, Y)Z \\ +\eta(X)\eta(U)B^e(\xi, Y)Z - \eta(Y)B^e(X, U)Z + \eta(Y)\eta(U)B^e(X, \xi)Z \\ -\eta(Z)B^e(X, Y)U + \eta(U)\eta(Z)B^e(X, Y)\xi = 0. \tag{5.4}$$

Now, putting $Z = \xi$ in (5.4) and using (2.11), (2.12) and (2.14), we get

$$B^e(X, Y)U = \frac{4(k-1)}{n+3}\eta(U)[\eta(X)Y - \eta(Y)X]. \tag{5.5}$$

For $k = 1$, the manifold is Sasakian. Conversely, in the first case if the manifold is Sasakian then from (2.12) we obtain $B^e(\xi, X)Y = 0$. Hence $B^e(\xi, U).B^e = 0$ is satisfied.

In the second case, it follows from (5.5) that $B^e(X, Y)U = 0$ if and only if the manifold is Sasakian. Hence in this case we also obtain that $B^e(\xi, U).B^e = 0$ holds if and only if the manifold is Sasakian.

Thus we can state the following:

5.1. Theorem. A $N(k)$-contact metric manifold M^n, $(n \geq 5)$, satisfies $B^e(\xi, U).B^e = 0$ if and only if the manifold is Sasakian.

Therefore from (5.5) we can conclude the following:

5.2. Theorem. Let M^n, $n \geq 5$, be a $N(k)$-contact metric manifold satisfying $B^e(\xi, U).B^e = 0$. Then either
i) the manifold is Sasakian, or
ii) the E-Bochner curvature tensor vanishes if and only if the manifold is a Sasakian manifold.

6. $N(k)$-contact metric manifolds satisfying $B^e(\xi, X).S = 0$

We devote this section to study $N(k)$-contact metric manifolds satisfying $B^e(\xi, X).S = 0$. Therefore we have

$$S(B^e(\xi, X)U, V) + S(U, B^e(\xi, X)V) = 0. \tag{6.1}$$

Using (2.12) in (6.1), we get

$$\frac{4(k-1)}{n+3}[\eta(U)S(X,V)-\eta(X)\eta(U)S(\xi,V) \\ +\eta(V)S(U,X)-\eta(X)\eta(V)S(X,\xi)]=0. \tag{6.2}$$

The relation (6.2) we have either $k=1$, or

$$-\eta(X)\eta(U)S(\xi,V)+\eta(U)S(X,V)-\eta(X)\eta(V)S(X,\xi)+\eta(V)S(U,X)=0. \tag{6.3}$$

Putting $U=\xi$ and using (2.10) and $\eta(\xi)=1$ in (6.3) yields

$$S(X,V)=(n-1)k\eta(X)\eta(V). \tag{6.4}$$

Again, if the manifold satisfies the relation (6.4), then in view of (2.12) we have

$$\begin{aligned} B^e(\xi,X).S(U,V) &= -S(B^e(\xi,X)U,V)-S(U,B^e(\xi,X)V) \\ &= -(n-1)k[\eta(B^e(\xi,X)U)\eta(V)+\eta(U)\eta(B^e(\xi,X)V)] \\ &= 0. \end{aligned} \tag{6.5}$$

Again, if the manifold is Sasakian then we easily obtain from (2.12) that $B^e(\xi,X).S=0$. In view of above discussion we state the following:

6.1. Theorem. Let M^n, $n\geq 5$, is a $N(k)-$contact metric manifold. Then the relation $B^e(\xi,X).S=0$ if and only if the manifold is either Sasakian or the Ricci tensor satisfies the relation $S(X,Y=k(n-1)\eta(X)\eta(Y)$.

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