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Approximate solutions of coupled Ramani equation by using RDTM with compared DTM and exact solutions

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Abstract: In this paper, we present a new approximate solutions of famous coupled Ramani Equation. In order to obtain the solution, we use the semi-analytical methods differential transform method (DTM) and reduced form of DTM called reduced differential transform method (RDTM). We compare the RDTM solutions with exact solution and DTM. Numerical results show clearly that DTM and RDTM are very effective and also provide very accurate solutions. Also, one can conclude that RDTM is used easier than DTM and converges faster than the DTM for these kind of problems.

Keywords: Reduced differential transform method (RDTM), differential transform method (DTM), coupled ramani equation, approximate solution.

1 Introduction

Partial differential equations are the fundamental of applied mathematics and they are frequently used in physic, engineering, chemistry and etc. In real life, many events can be modeled by a nonlinear partial differential equation such as evolution equations. Particularly in nonlinear sciences, one of the important and outstanding evolution equation is the famous coupled Ramani Equation that is presented as follow [1], [2], [3], [4].

$$\begin{aligned} u(x,t)_{xxxxxx} + 15u(x,t)_{xx}u(x,t)_{xxx} + 15u(x,t)_{x}u(x,t)_{xxxx} + 45\left(u(x,t)_{x}\right)^{2}u(x,t)_{xx} \\ -5u(x,t)_{tt} + 18v(x,t)_{x} - 5u(x,t)_{xxxt} - 15u(x,t)_{xx}u(x,t)_{t} - 15u(x,t)_{x}u(x,t)_{xt} = 0 \\ v(x,t)_{t} - v(x,t)_{xxx} - 3v(x,t)_{x}u(x,t)_{x} - 3v(x,t)u(x,t)_{xx} = 0 \end{aligned} \tag{1}$$

In literature, a great number of researchers have studied the system (1) to obtain exact and approximate solutions. Ablowitz and Clarkson [5], Ito [6], Zhang [7], Feng [8], Malfiet and Hereman [9] have investigated the solitons and inverse scattering, extensions, exact traveling wave solutions and traveling solitary wave solutions of nonlinear evolution equations respectively. Li has presented exact traveling wave solutions of six order Ramani and a coupled Ramani equation in [10]. In [11], Nadjafikhah and Shirvani-Sh have found Lie symmetries and conservation laws of Hirota-Ramani equation. Further, Yusufoglu and Bekir have obtained the two exact traveling wave solutions of coupled Ramani equation by applying tanh method as following [12].

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$$u(x,t) = a_0 + 2\alpha \tanh(\alpha(x-\beta t)) \tag{2}$$
$$v(x,t) = -\frac{4}{9}\beta\alpha^4 - \frac{16}{27}\alpha^6 + \frac{5}{9}\alpha^2\beta^2 - \frac{5}{54}\beta^3 + \left[\frac{20}{9}\beta\alpha^4 + \frac{16}{9}\alpha^6 - \frac{5}{9}\alpha^2\beta^2\right]\tanh^2(\alpha(x-\beta t))$$

and

$$u(x,t) = a_0 - 2\alpha \tan(\alpha(x-\beta t)) \qquad \left(|\alpha(x-\beta t)| < \frac{\pi}{2}\right) \tag{3}$$
$$v(x,t) = -\frac{4}{9}\beta\alpha^4 + \frac{16}{27}\alpha^6 - \frac{5}{9}\alpha^2\beta^2 - \frac{5}{54}\beta^3 + \left[-\frac{20}{9}\beta\alpha^4 + \frac{16}{9}\alpha^6 - \frac{5}{9}\alpha^2\beta^2\right]\tan^2(\alpha(x-\beta t))$$

where a_0, α and β are arbitrary constants.

Recently, Wazwaz and Triki [13], Wazwaz [14], Jafarian et all [15] and Wazwaz [16] have presented the multiple soliton solutions and approximate solution of eq. (1) respectively.
The main goal of this study is to obtain accurate, convergent and efficient approximate solution of coupled Ramani equation (1) by using differential transform (DTM) and reduced differential transform (RDTM) methods. For the purpose of efficiency and accuracy, our results are compared with exact solutions (2) and (3). Numerical considerations are revealed that RDTM is very effective and more convergent than DTM. In addition, RDTM can be applied easier than DTM and ensures very accurate solutions as shown in **Table** (3)-(8) and **Fig.** (1)-(4). Moreover, RDTM is also faster than DTM in point of CPU times of computational process as seen in **Table** (9).

2 Basic properties of two dimensional reduced differential transform method (RDTM) and differential transform method (DTM)

2.1 Two dimensional DTM

Differential transform method (DTM) is a numerical method based on Taylor expansion. This method is related to find coefficients of series expansion of unknown function term by term. The concept of DTM was first proposed by Zhou [17]. By, considering the literature [17]-[26], we give the following definition of two dimensional DTM;

Definition 1. *Let $u(x,t)$ be an analytic differentiable function, then two dimensional transform is follow*

$$U(k,h) = \frac{1}{k!h!}\left[\frac{\partial^{k+h}}{\partial x^k \partial t^h}u(x,t)\right]_{x=x_0,t=t_0} \tag{4}$$

where $U(k,h)$ is the transformed function of $u(x,t)$. The transformation is called T-function. Hence, the differential inverse transform of $U(k,h)$ is defined as

$$u(x,t) = \sum_{k=0}^{\infty}\sum_{h=0}^{\infty} U(k,h)(x-x_0)^k(t-t_0)^h \tag{5}$$

From the eqs. (4) and (5), it can be written

$$u(x,t) = \sum_{k=0}^{\infty}\sum_{h=0}^{\infty} \frac{1}{k!h!}\left[\frac{\partial^{k+h}}{\partial x^k \partial t^h}u(x,t)\right]_{x=x_0,t=t_0}(x-x_0)^k(t-t_0)^h \tag{6}$$

In terms of applicability, we rearrange the eq. (6) as follow

$$u(x,t)=\sum_{k=0}^{n}\sum_{h=0}^{m}\frac{1}{k!h!}\left[\frac{\partial^{k+h}}{\partial x^k\partial t^h}u(x,t)\right]_{x=0,t=0}x^kt^h+R_{nm}(x,t) \tag{7}$$

where (x_0,t_0) *are taken as* $(0,0)$ *and* $R_{nm}(x,t)=\sum_{k=n+1}^{\infty}\sum_{h=m+1}^{\infty}U(k,h)x^kt^h$. *Here,* $R_{nm}(x,t)$ *is negligibly small terms. Some of the transform form of functions are given as **Table** 1 and their proofs can be found in [17]-[20].*

Table 1: Some two dimensional DTM operations with transformed forms.

Original functions	Transformed forms
$u(x,t)=v(x,t)\pm w(x,t)$	$U(k,h)=V(k,h)\pm W(k,h)$
$u(x,t)=\lambda v(x,t)$	$U(k,h)=\lambda V(k,h)$
$u(x,t)=\frac{\partial}{\partial x}v(x,t)$	$U(k,h)=(k+1)V(k+1,h)$
$u(x,t)=\frac{\partial}{\partial t}v(x,t)$	$U(k,h)=(h+1)V(k,h+1)$
$u(x,t)=\frac{\partial^{m+n}}{\partial x^m\partial t^n}v(x,t)$	$U(k,h)=\frac{(k+m)!}{k!}\frac{(h+n)!}{h!}V(k+m,h+n)$
$u(x,t)=v(x,t)w(x,t)$	$U(k,h)=\sum_{r=0}^{k}\sum_{s=0}^{h}V(r,h-s)W(k-r,s)$
$u(x,t)=v(x,t)w(x,t)q(x,t)$	$U(k,h)=\sum_{r=0}^{k}\sum_{p=0}^{k-r}\sum_{s=0}^{h}\sum_{z=0}^{h-s}V(r,h-s-z)W(p,s)Q(k-r-p,z)$
$u(x,t)=x^mt^n$	$U(k,h)=\delta(k-m,h-n)=\delta(k-m)\delta(h-n),\quad \delta(k-m)=\begin{cases}1 & k=m\\ 0 & otherwise\end{cases}$

2.2 Two dimensional RDTM

Reduced differential transform method (RDTM) which has an alternative approach of problems is presented to overcome the demerit complex calculation, discretization, linearization or perturbations of well-known numerical and analytical methods such as Adomian decomposition, Differential transform, Homotopy perturbation and Variational iteration. RDTM was first introduced by Keskin and Oturanc [28]-[31]. The main advantage of RDTM is providing an analytic approximation, in many cases exact solutions, in rapidly convergent sequence with elegantly computed terms [26]-[39]. And also, unlike the DTM, RDTM is based on the Poisson series coefficients expansion. By using the literature [26]-[39], we present the RDTM as follow.

Definition 2. *Let* $u(x,t)$ *be an analytic differentiable function and assumed that it can be demonstrated as a product of two functions which are single variable* $u(x,t)=h(x)g(t)$. *By making use of differential transform properties,* $u(x,t)$ *can be written as*

$$u(x,t)=\sum_{i=0}^{\infty}H(i)x^i\sum_{j=0}^{\infty}G(j)t^j=\sum_{k=0}^{\infty}U_k(x)t^k. \tag{8}$$

Here $U_k(x)$ *is called* t *dimensional spectrum function of* $u(x,t)$. *If function* $u(x,t)$ *is analytic and differentiated continuously with respect to time* t *and space* x *in the domain of interest, than*

$$U_k(x)=\frac{1}{k!}\left[\frac{\partial^k}{\partial t^k}u(x,t)\right]_{t=t_0} \tag{9}$$

where $U_k(x)$ is transformed function of $u(x,t)$. The differential inverse transform of $U_k(x)$ is defined as

$$u(x,t) = \sum_{k=0}^{\infty} U_k(x)(t-t_0)^k. \tag{10}$$

Combining (8)-(10), we can write

$$u(x,t) = \sum_{k=0}^{\infty} \frac{1}{k!}\left[\frac{\partial^k}{\partial t^k}u(x,t)\right]_{t=t_0}(t-t_0)^k. \tag{11}$$

In real applications, we use the finite series form of (11), therefore we rewrite the solution as

$$\tilde{u}_n(x,t) = \sum_{k=0}^{n} U_k(x)t^k \tag{12}$$

where n is order of approximation. Hence, the RDTM solution is given by

$$u(x,t) = \lim_{n\to\infty} \tilde{u}_n(x,t) \tag{13}$$

*here n is taken as sufficiently big to get convergent solution. In **Table** 2, transformed form of mathematical operation of some functions are given and their proofs are shown in ref. [28], [29].*

Table 2: Some two dimensional RDTM operations with transformed forms.

Original functions	Transformed forms
$u(x,t) = v(x,t) \pm w(x,t)$	$U_k(x) = V_k(x) \pm W_k(x)$
$u(x,t) = \lambda v(x,t)$	$U_k(x) = \lambda V_k(x)$
$u(x,t) = \frac{\partial}{\partial x}v(x,t)$	$U_k(x) = \frac{\partial}{\partial x}V_k(x)$
$u(x,t) = \frac{\partial^r}{\partial t^r}v(x,t)$	$U_k(x) = \frac{(k+r)!}{k!}V_{k+r}(x)$
$u(x,t) = v(x,t)w(x,t)$	$U_k(x) = \sum_{r=0}^{k} V_r(x)W_{k-r}(x) = \sum_{r=0}^{k} W_r(x)V_{k-r}(x)$
$u(x,t) = v(x,t)w(x,t)q(x,t)$	$U_k(x) = \sum_{r=0}^{k}\sum_{p=0}^{k-r} V_r(x)W_p(x)Q_{k-r-p}(x)$
$u(x,t) = x^m t^n$	$U_k(x) = x^m\delta(k-n), \quad \delta(k-n) = \begin{cases} x^m & k=n \\ 0 & otherwise \end{cases}$

3 Solution procedures of Ramani equation by DTM and RDTM

3.1 DTM methodology

Let's consider the coupled Ramani equation (1) with two different initial conditions as [4], [10],[12]-[16],

$$u(x,0) = a_0 + 2\alpha\tanh(\alpha x) \tag{14}$$
$$v(x,0) = -\frac{4}{9}\beta\alpha^4 - \frac{16}{27}\alpha^6 + \frac{5}{9}\alpha^2\beta^2 - \frac{5}{54}\beta^3 + \left[\frac{20}{9}\beta\alpha^4 + \frac{16}{9}\alpha^6 - \frac{5}{9}\alpha^2\beta^2\right]\tanh^2(\alpha x)$$

and

$$u(x,0) = a_0 - 2\alpha\tan(\alpha x) \tag{15}$$
$$v(x,0) = -\frac{4}{9}\beta\alpha^4 + \frac{16}{27}\alpha^6 - \frac{5}{9}\alpha^2\beta^2 - \frac{5}{54}\beta^3 + \left[-\frac{20}{9}\beta\alpha^4 + \frac{16}{9}\alpha^6 - \frac{5}{9}\alpha^2\beta^2\right]\tan^2(\alpha x).$$

$U(k,h)$, $V(k,h)$, which are called *T-function*, denote the transformation of the functions $u(x,t)$, $v(x,t)$ in eq. (1) respectively. Then from **Table** (1) and eqs. (4) to (7), we obtain the transformed form of eq. (1) as below

$$\begin{aligned}
5(h+1)(h+2)U(k,h+2) &= \frac{(k+6)!}{k!}U(k+6,h) + 18(k+1)V(k+1,h) \\
&+ 15\sum_{r=0}^{k}\sum_{s=0}^{h}(r+1)(k-r+1)(k-r+2)(k-r+3)(k-r+4)U(r+1,h-s)U(k-r+4,s) \\
&+ 15\sum_{r=0}^{k}\sum_{s=0}^{h}(r+1)(r+2)(k-r+1)(k-r+2)(k-r+3)U(r+2,h-s)U(k-r+3,s) \\
&- 5\frac{(k+3)!}{k!}(h+1)U(k+3,h+1) - 15\sum_{r=0}^{k}\sum_{s=0}^{h}(h-s+1)(k-r+1)(k-r+2)U(k-r+2,s)U(r,h-s+1) \\
&- 15\sum_{r=0}^{k}\sum_{s=0}^{h}(r+1)(k-r+1)(h-s+1)U(r+1,h-s)U(k-r+1,h-s+1) \\
&+ 45\sum_{r=0}^{k}\sum_{l=0}^{k-r}\sum_{s=0}^{h}\sum_{p=0}^{h-s}(r+1)(l+1)(k-r-l+1)(k-r-l+2)U(r+1,h-s-p)U(l+1,s)U(k-r-l+2,p)
\end{aligned} \tag{16}$$

$$\begin{aligned}
(h+1)V(k,h+1) &= \frac{(k+3)!}{k!}V(k+3,h) + 3\sum_{r=0}^{k}\sum_{s=0}^{h}(r+1)(k-r+1)V(r+1,h-s)U(k-r+1,s) \\
&+ 3\sum_{r=0}^{k}\sum_{s=0}^{h}(k-r+1)(k-r+2)V(r,h-s)U(k-r+2,s)
\end{aligned} \tag{17}$$

and for initial conditions (14), (15), we obtain as

$$U(k,0) = a_0\delta(k,0) + 2\alpha\left[\frac{(2\alpha)^k - k!\delta(k,0)}{(2\alpha)^k + k!\delta(k,0)}\right] \tag{18}$$
$$V(k,0) = \left(-\frac{4}{9}\beta\alpha^4 - \frac{16}{27}\alpha^6 + \frac{5}{9}\alpha^2\beta^2 - \frac{5}{54}\beta^3\right)\delta(k,0) + \left(\frac{20}{9}\beta\alpha^4 + \frac{16}{9}\alpha^6 - \frac{5}{9}\alpha^2\beta^2\right)\left(\frac{(2\alpha)^k - k!\delta(k,0)}{(2\alpha)^k + k!\delta(k,0)}\right)^2$$

and

$$U(k,0) = a_0\delta(k,0) - 2\alpha\tan(\frac{k\pi}{2}) \tag{19}$$
$$V(k,0) = \left(-\frac{4}{9}\beta\alpha^4 + \frac{16}{27}\alpha^6 - \frac{5}{9}\alpha^2\beta^2 - \frac{5}{54}\beta^3\right)\delta(k,0) + \left(-\frac{20}{9}\beta\alpha^4 + \frac{16}{9}\alpha^6 - \frac{5}{9}\alpha^2\beta^2\right)\left(\tan(\frac{k\pi}{2})\right)^2$$

We put firstly (18) into (16)-(17) and using the DTM, we get the three terms approximate traveling DTM solution of coupled Ramani equation as

$$
\begin{aligned}
U_{3,3}(x,t) &= a_0 + 2\alpha^2 x - 2/3\,\alpha^4 x^3 - 2\beta\alpha^2 t + 2\beta\alpha^4 x^2 t - 2\alpha^4\beta^2 x t^2 + 8/3\,\alpha^6\beta^2 x^3 t^2 + \left(-4\alpha^6\beta^2 + 2/3\,\alpha^4\beta^3\right) t^3 \\
&+ \left(-4\beta\alpha^6 + 8\alpha^8\beta^3\right) x t^3 + \left(28\alpha^8\beta^2 - 8/3\,\alpha^6\beta^3\right) x^2 t^3 + \left(\frac{28}{3}\beta\alpha^8 - \frac{128}{3}\alpha^{10}\beta^3\right) x^3 t^3 \\
V_{3,3}(x,t) &= -4/9\,\beta\alpha^4 - \frac{16}{27}\alpha^6 + 5/9\,\beta^2\alpha^2 - \frac{5}{54}\beta^3 + \left(\frac{20}{9}\beta\alpha^4 + \frac{16}{9}\alpha^6 - 5/9\,\beta^2\alpha^2\right)\alpha^2 x^2 \\
&+ \left(-\frac{32}{9}\beta\alpha^8 - \frac{40}{9}\alpha^6\beta^2 + \frac{10}{9}\alpha^4\beta^3\right) x t + \left(\frac{128}{27}\beta\alpha^{10} + \frac{160}{27}\alpha^8\beta^2 - \frac{40}{27}\alpha^6\beta^3\right) x^3 t \\
&+ \left(\frac{16}{9}\alpha^8\beta^2 + \frac{20}{9}\alpha^6\beta^3 - 5/9\,\beta^4\alpha^4\right) t^2 + \left(-\frac{64}{9}\alpha^{10}\beta^2 - \frac{80}{9}\alpha^8\beta^3 + \frac{20}{9}\beta^4\alpha^6\right) x^2 t^2 \\
&+ \left(\frac{128}{27}\alpha^{10}\beta^3 + \frac{160}{27}\beta^4\alpha^8 - \frac{40}{27}\alpha^6\beta^5\right) x t^3 + \left(-\frac{1360}{81}\beta^4\alpha^{10} + \frac{340}{81}\alpha^8\beta^5 - \frac{1088}{81}\alpha^{12}\beta^3\right) x^3 t^3
\end{aligned} \tag{20}
$$

and secondly put (19) into (16)-(17), we obtain the other traveling DTM solution of eq. (1) as following

$$
\begin{aligned}
U_{3,3}(x,t) &= a_0 - 2\alpha^2 x - 2/3\,\alpha^4 x^3 + 2\alpha^2\beta t + 2\beta\alpha^4 x^2 t - 2\alpha^4\beta^2 x t^2 - 8/3\,\alpha^6\beta^2 x^3 t^2 + \left(4\alpha^6\beta^2 + 2/3\,\alpha^4\beta^3\right) t^3 \\
&+ \left(4\alpha^6\beta + 8\alpha^8\beta^3\right) x t^3 + \left(28\alpha^8\beta^2 + 8/3\,\alpha^6\beta^3\right) x^2 t^3 + \left(\frac{28}{3}\alpha^8\beta + \frac{128}{3}\alpha^{10}\beta^3\right) x^3 t^3 \\
V_{3,3}(x,t) &= -4/9\,\beta\alpha^4 + \frac{16}{27}\alpha^6 - 5/9\,\beta^2\alpha^2 - \frac{5}{54}\beta^3 + \left(-\frac{20}{9}\beta\alpha^4 + \frac{16}{9}\alpha^6 - 5/9\,\beta^2\alpha^2\right)\alpha^2 x^2 \\
&+ \left(-\frac{32}{9}\alpha^8\beta + \frac{40}{9}\alpha^6\beta^2 + \frac{10}{9}\alpha^4\beta^3\right) x t + \left(-\frac{128}{27}\alpha^{10}\beta + \frac{160}{27}\alpha^8\beta^2 + \frac{40}{27}\alpha^6\beta^3\right) x^3 t \\
&+ \left(\frac{16}{9}\alpha^8\beta^2 - \frac{20}{9}\alpha^6\beta^3 - 5/9\,\beta^4\alpha^4\right) t^2 + \left(\frac{64}{9}\alpha^{10}\beta^2 - \frac{80}{9}\alpha^8\beta^3 - \frac{20}{9}\alpha^6\beta^4\right) x^2 t^2 \\
&+ \left(-\frac{128}{27}\alpha^{10}\beta^3 + \frac{160}{27}\alpha^8\beta^4 + \frac{40}{27}\alpha^6\beta^5\right) x t^3 + \left(\frac{1360}{81}\alpha^{10}\beta^4 + \frac{340}{81}\alpha^8\beta^5 - \frac{1088}{81}\alpha^{12}\beta^3\right) x^3 t^3
\end{aligned} \tag{21}
$$

Hence, it is clearly seen in **Table** (3) to (8) that solutions (20) and (21) provide the good accuracy with compared exact solutions [12].

Table 3: For $u(x,t)$, numerical results of seven steps DTM and RDTM solutions of eq. (1) with compared exact solution (2) at $t = 20$ and $a_0 = 1$, $\alpha = \beta = 0.01$.

x	Exact[12] $u(x,t)$	RDTM $U_7(x)$	DTM $U_{7,7}(x,t)$	Errors of RDTM $\|u(x,t) - U_7(x)\|$	Errors of DTM $\|u(x,t) - U_{7,7}(x,t)\|$
-50	0.990726228	0.9907262252	0.9909484041	2.8×10^{-9}	2.221761×10^{-4}
-40	0.9923668212	0.9923668186	0.9925853005	2.6×10^{-9}	2.184793×10^{-4}
-30	0.9941371636	0.9941371613	0.994329091	2.3×10^{-9}	1.919273×10^{-4}
-20	0.9960140671	0.9960140653	0.9961574997	1.8×10^{-9}	1.434326×10^{-4}
-10	0.9979670454	0.9979670444	0.9980439141	1.0×10^{-9}	7.68687×10^{-5}
10	1.001953749	1.00195375	1.001876877	1.0×10^{-9}	7.6872×10^{-5}
20	1.003909050	1.003909051	1.003765614	1.0×10^{-9}	1.43436×10^{-4}
30	1.005789625	1.005789628	1.005597698	3.0×10^{-9}	1.91927×10^{-4}
40	1.007564728	1.00756473	1.007346259	2.0×10^{-9}	2.18469×10^{-4}
50	1.009210856	1.009210859	1.008988739	3.0×10^{-9}	2.22117×10^{-4}

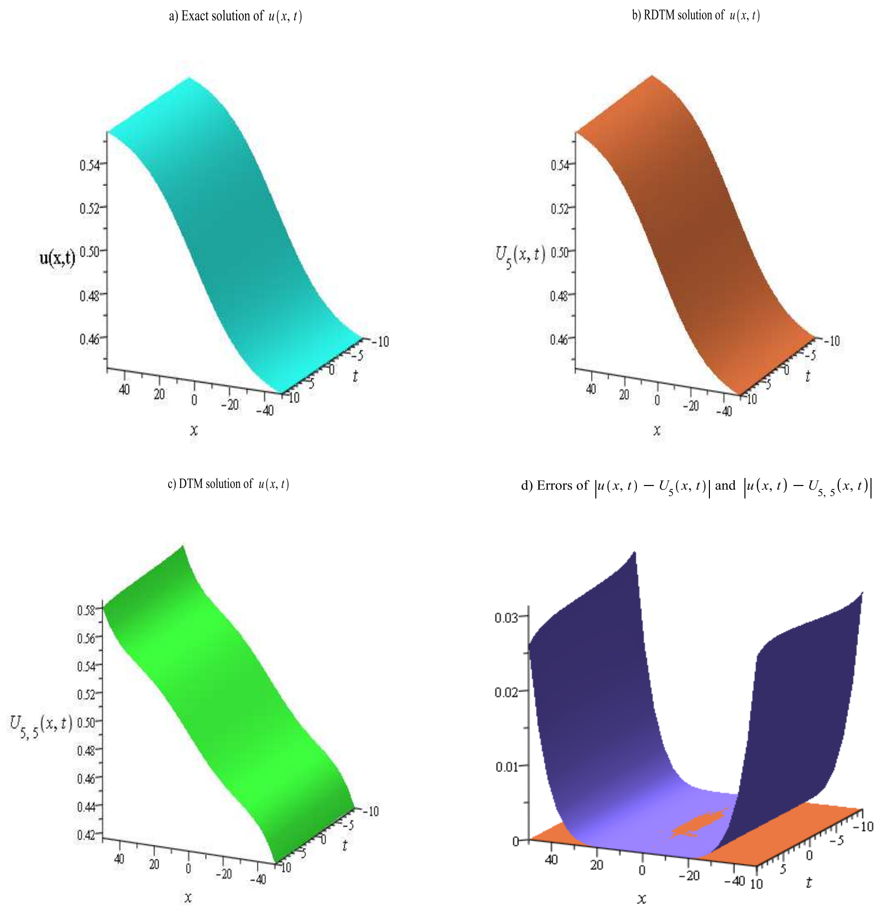

Fig. 1: Comparison between exact solution (2) and five steps RDTM, DTM solutions of $u(x,t)$ at $a_0 = \frac{1}{2}$, $\alpha = \beta = 0.03$. And comparison of errors d) **Sienna-RDTM**, **SlateBlue-DTM**.

Table 4: For $v(x,t)$, numerical results of seven steps DTM and RDTM solutions of eq. (1) with compared exact solution (2) at $t = 20$ and $a_0 = 1$, $\alpha = \beta = 0.01$.

x	Exact[12] $v(x,t)$	RDTM $V_7(x)$	DTM $V_{7,7}(x,t)$	Errors of RDTM $\lvert v(x,t) - V_7(x)\rvert$	Errors of DTM $\lvert v(x,t) - V_{7,7}(x,t)\rvert$
-50	$-8.822839648 \times 10^{-8}$	$-8.822106112 \times 10^{-8}$	$-8.823201645 \times 10^{-8}$	7.33534×10^{-12}	3.61999×10^{-12}
-40	$-8.785868735 \times 10^{-8}$	$-8.785217836 \times 10^{-8}$	$-8.785931076 \times 10^{-8}$	6.50899×10^{-12}	6.2341×10^{-13}
-30	$-8.754022597 \times 10^{-8}$	$-8.753492301 \times 10^{-8}$	$-8.754027998 \times 10^{-8}$	5.30295×10^{-12}	5.401×10^{-14}
-20	$-8.729383894 \times 10^{-8}$	$-8.729008043 \times 10^{-8}$	$-8.729383182 \times 10^{-8}$	3.75851×10^{-12}	7.12×10^{-15}
-10	$-8.713716109 \times 10^{-8}$	$-8.713520229 \times 10^{-8}$	$-8.713715553 \times 10^{-8}$	1.9588×10^{-12}	5.561×10^{-15}
10	$-8.713295226 \times 10^{-8}$	$-8.71348706 \times 10^{-8}$	$-8.713295226 \times 10^{-8}$	1.91834×10^{-12}	5.59×10^{-15}
20	$-8.72857484 \times 10^{-8}$	$-8.728947099 \times 10^{-8}$	$-8.72857608 \times 10^{-8}$	3.72259×10^{-12}	1.24×10^{-14}
30	$-8.752885527 \times 10^{-8}$	$-8.753412915 \times 10^{-8}$	$-8.752893306 \times 10^{-8}$	5.27388×10^{-12}	7.779×10^{-14}
40	$-8.784482114 \times 10^{-8}$	$-8.785130927 \times 10^{-8}$	$-8.784547184 \times 10^{-8}$	6.48813×10^{-12}	6.507×10^{-13}
50	$-8.821289529 \times 10^{-8}$	$-8.822021826 \times 10^{-8}$	$-8.821656659 \times 10^{-8}$	7.32297×10^{-12}	3.6713×10^{-12}

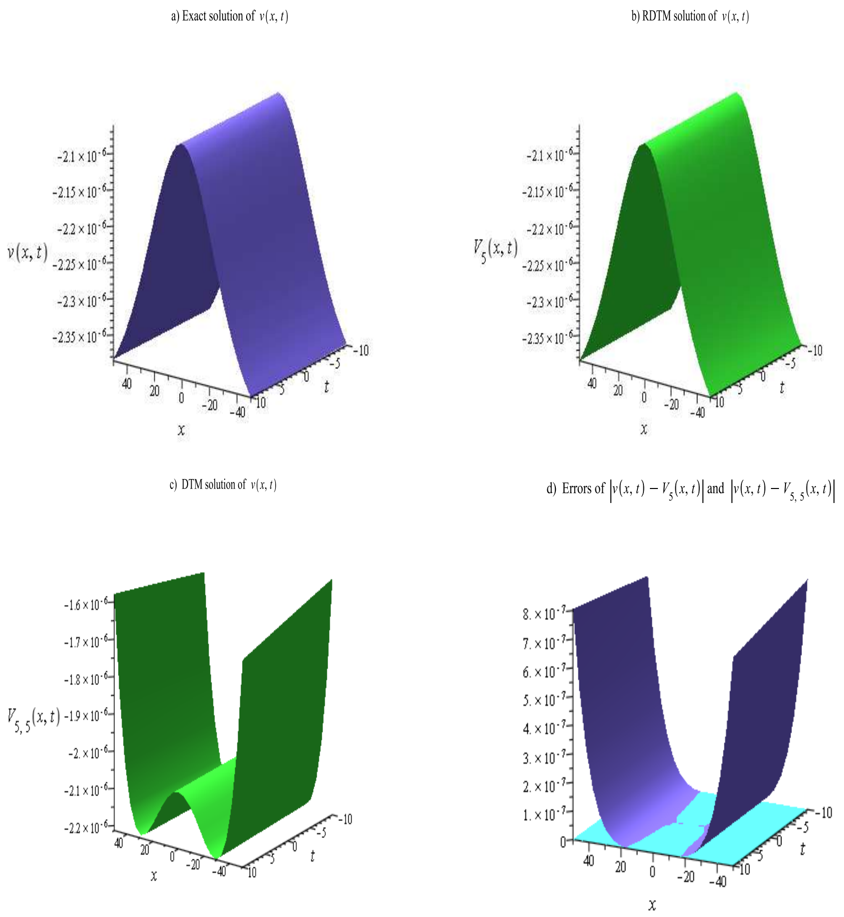

Fig. 2: Comparison between exact solution (2) and five steps RDTM, DTM solutions of $v(x,t)$ at $a_0 = \frac{1}{2}$, $\alpha = \beta = 0.03$. And comparison of errors d) **MediumTurquoise-RDTM**, **SlateBlue-DTM**.

3.2 RDTM methodology

As the same manner, again we consider the eq. (1) with initial conditions (14)-(15) to obtain the RDTM solutions. $U_k(x)$, $V_k(x)$, which are called t dimensional spectrum functions, denote the transformation of the functions $u(x,t)$, $v(x,t$ in eq. (1) respectively. Then from **Table** 2 and eqs. (8) to (13), we obtain the transformed form of eq. (1) as below

$$\begin{aligned} 5(k+1)(k+2)U_{k+2}(x) = {} & \frac{d^6}{dx^6}U_k(x) + 18\frac{d}{dx}V_k(x) + 15\sum_{r=0}^{k}\frac{d^2}{dx^2}U_{k-r}(x)\frac{d^3}{dx^3}U_r(x) + 15\sum_{r=0}^{k}\frac{d}{dx}U_{k-r}(x)\frac{d^4}{dx^4}U_r(x) \\ & - 15\sum_{r=0}^{k}\frac{d^2}{dx^2}U_r(x)(k-r+1)U_{k-r+1}(x) - 15\sum_{r=0}^{k}\frac{d}{dx}U_r(x)(k-r+1)\frac{d}{dx}U_{k-r+1}(x) \\ & + 45\sum_{r=0}^{k}\sum_{s=0}^{k-r}\frac{d}{dx}U_r(x)\frac{d}{dx}U_s(x)\frac{d^2}{dx^2}U_{k-r-s}(x) - 5(k+1)\frac{d^3}{dx^3}U_{k+1}(x) \end{aligned} \tag{22}$$

$$(k+1)V_{k+1}(x) = \frac{d^3}{dx^3}V_k(x) + 3\sum_{r=0}^{k}\frac{d}{dx}V_r(x)\frac{d}{dx}U_{k-r}(x) + 3\sum_{r=0}^{k}V_r(x)\frac{d^2}{dx^2}U_{k-r}(x) \tag{23}$$

and for initial conditions (14)-(15), we obtain reduced transform form as respectively

$$
\begin{aligned}
U_0(x) &= a_0 + 2\alpha\tanh(\alpha x) \\
V_0(x) &= \left(-\frac{4}{9}\beta\alpha^4 - \frac{16}{27}\alpha^6 + \frac{5}{9}\alpha^2\beta^2 - \frac{5}{54}\beta^3\right) + \left(\frac{20}{9}\beta\alpha^4 + \frac{16}{9}\alpha^6 - \frac{5}{9}\alpha^2\beta^2\right)\tanh^2(\alpha x)
\end{aligned} \tag{24}
$$

and

$$
\begin{aligned}
U_0(x) &= a_0 - 2\alpha\tan(\alpha x) \\
V_0(x) &= \left(-\frac{4}{9}\beta\alpha^4 + \frac{16}{27}\alpha^6 - \frac{5}{9}\alpha^2\beta^2 - \frac{5}{54}\beta^3\right) + \left(-\frac{20}{9}\beta\alpha^4 + \frac{16}{9}\alpha^6 - \frac{5}{9}\alpha^2\beta^2\right)\tan^2(\alpha x)
\end{aligned} \tag{25}
$$

Table 5: For $u(x,t)$, numerical results of seven steps DTM and RDTM solutions of eq. (1) with compared exact solution (3) at $t = 20$ and $a_0 = 1$, $\alpha = \beta = 0.01$.

x	Exact[12] $u(x,t)$	RDTM $U_7(x)$	DTM $U_{7,7}(x,t)$	Errors of RDTM $\lvert u(x,t) - U_7(x)\rvert$	Errors of DTM $\lvert u(x,t) - U_{7,7}(x,t)\rvert$
-50	1.010978045	1.010978054	1.010257505	9.0×10^{-9}	7.2054×10^{-4}
-40	1.008503055	1.00850306	1.008041336	5.0×10^{-9}	4.61719×10^{-4}
-30	1.006230580	1.006230583	1.005938409	3.0×10^{-9}	2.92171×10^{-4}
-20	1.004095861	1.004095864	1.003922985	3.0×10^{-9}	1.72876×10^{-4}
-10	1.002047104	1.002047105	1.001966559	1.0×10^{-9}	8.0545×10^{-5}
10	0.9980337012	0.9980337003	0.9981142416	9.0×10^{-10}	8.05404×10^{-5}
20	0.9959874261	0.995987424	0.9961602952	2.1×10^{-9}	1.728691×10^{-4}
30	0.9938570755	0.9938570716	0.9941492333	3.9×10^{-9}	2.921578×10^{-4}
40	0.9915912460	0.9915912401	0.9920529338	5.9×10^{-9}	4.616878×10^{-4}
50	0.9891258314	0.9891258222	0.9898462633	9.2×10^{-9}	7.204319×10^{-4}

Table 6: For $v(x,t)$, numerical results of seven steps DTM and RDTM solutions of eq. (1) with compared exact solution (3) at $t = 20$ and $a_0 = 1$, $\alpha = \beta = 0.01$.

x	Exact[12] $v(x,t)$	RDTM $V_7(x)$	DTM $V_{7,7}(x,t)$	Errors of RDTM $\lvert v(x,t) - V_7(x)\rvert$	Errors of DTM $\lvert v(x,t) - V_{7,7}(x,t)\rvert$
-50	$-9.993227215\times10^{-8}$	$-9.991393181\times10^{-8}$	$-9.992708042\times10^{-8}$	1.834034×10^{-11}	5.16173×10^{-12}
-40	$-9.923603996\times10^{-8}$	$-9.922331725\times10^{-8}$	$-9.923517293\times10^{-8}$	1.272271×10^{-11}	8.6703×10^{-13}
-30	$-9.875256259\times10^{-8}$	$-9.874398369\times10^{-8}$	$-9.875245418\times10^{-8}$	8.5789×10^{-12}	1.0841×10^{-13}
-20	$-9.843424659\times10^{-8}$	$-9.842893011\times10^{-8}$	$-9.843422759\times10^{-8}$	5.31648×10^{-12}	1.9×10^{-14}
-10	$-9.825251278\times10^{-8}$	$-9.824995748\times10^{-8}$	$-9.825250579\times10^{-8}$	2.5553×10^{-12}	6.99×10^{-15}
10	$-9.824782982\times10^{-8}$	$-9.825033777\times10^{-8}$	$-9.824783681\times10^{-8}$	2.50795×10^{-12}	6.99×10^{-15}
20	$-9.842449482\times10^{-8}$	$-9.842975832\times10^{-8}$	$-9.842450789\times10^{-8}$	5.2635×10^{-12}	1.307×10^{-14}
30	$-9.873690092\times10^{-8}$	$-9.874541644\times10^{-8}$	$-9.873685319\times10^{-8}$	8.51552×10^{-12}	4.773×10^{-14}
40	$-9.921301115\times10^{-8}$	$-9.922565351\times10^{-8}$	$-9.921226004\times10^{-8}$	1.264236×10^{-11}	7.5111×10^{-13}
50	-9.98994945×10^{-8}	-9.99177279×10^{-8}	$-9.989455455\times10^{-8}$	1.82334×10^{-11}	4.93995×10^{-12}

As in the DTM solution process, by using RDTM algorithm we put firstly (24) into (22)-(23), we get the three terms RDTM solution of coupled Ramani equation as

$$\begin{aligned} U_3(x) &= a_0 + 2\alpha\tanh(\alpha x) - 2\alpha^2\beta\left(1-\tanh^2(\alpha x)\right)t - \frac{2\sinh(\alpha x)\alpha^3 t^2\left(-48\alpha^4\cosh^2(\alpha x)\right)}{\cosh^7(\alpha x)} \\ &- \frac{2\sinh(\alpha x)\alpha^3 t^2\left(72\alpha^4 - 12\alpha^2\beta\cosh^2(\alpha x) + \beta^2\cosh^4(\alpha x)\right)}{\cosh^7(\alpha x)} \\ &- \frac{32}{15\cosh^{10}(\alpha x)}\left[1260\alpha^8\beta\cosh^4(\alpha x) + 8820\alpha^{10}\cosh^4(\alpha x) - 945\beta\alpha^8\cosh^2(\alpha x) + 11340\alpha^{10}\right]t^3 \\ &- \frac{32}{15\cosh^{10}(\alpha x)}\left[12\alpha^8\beta\cosh^8(\alpha x) - 18900\alpha^{10}\cosh^2(\alpha x)\right]t^3 \\ &- \frac{32}{15\cosh^{10}(\alpha x)}\left[-1020\alpha^{10}\cosh^6(\alpha x) + 8\alpha^{10}\cosh^8(\alpha x) - 378\alpha^8\beta\cosh^6(\alpha x)\right]t^3 \end{aligned} \tag{26}$$

$$\begin{aligned} V_3(x) &= -\frac{4}{9}\beta\alpha^4 - \frac{16}{27}\alpha^6 + \frac{5}{9}\beta^2\alpha^2 - \frac{5}{54}\beta^3 + \left(\frac{20}{9}\beta\alpha^4 + \frac{16}{9}\alpha^6 - \frac{5}{9}\beta^2\alpha^2\right)\tanh^2(\alpha x) \\ &+ \frac{8}{9}\frac{\sinh(\alpha x)\left(\cosh^2(\alpha x)-3\right)\left(20\alpha^2\beta + 16\alpha^4 - 5\beta^2\right)\alpha^5 t}{\cosh^5(\alpha x)} \\ &- \frac{9}{8}\frac{\alpha^8\left(20\alpha^2\beta + 16\alpha^4 - 5\beta^2\right)\left[4\cosh^6(\alpha x) - 126\cosh^4(\alpha x) + 420\cosh^2(\alpha x) - 315\right]t^2}{\cosh^8(\alpha x)} \\ &+ \frac{128}{27}\frac{\left(6615\cosh^4(\alpha x) - 18900\cosh^2(\alpha x)\right)\left[20\alpha^2\beta + 16\alpha^4 - 5\beta^2\right]\alpha^{11}\sinh(\alpha x)t^3}{\cosh^{11}(\alpha x)} \\ &+ \frac{128}{27}\frac{\left(2\cosh^8(\alpha x) - 510\cosh^6(\alpha x) + 14175\right)\left[20\alpha^2\beta + 16\alpha^4 - 5\beta^2\right]\alpha^{11}\sinh(\alpha x)t^3}{\cosh^{11}(\alpha x)} \end{aligned} \tag{27}$$

and secondly put (25) into (22)-(23), we obtain the other RDTM solution of eq. (1) as following

$$\begin{aligned} U_3(x) &= a_0 - 2\alpha\tan(\alpha x) + 2\alpha^2\beta\left(1+\tan^2(\alpha x)\right)t - \frac{2\sin(\alpha x)\alpha^3 t^2\left(-48\alpha^4\cos^2(\alpha x)\right)}{\cos^7(\alpha x)} \\ &- \frac{2\sin(\alpha x)\alpha^3 t^2\left(72\alpha^4 + 12\alpha^2\beta\cos^2(\alpha x) + \beta^2\cos^4(\alpha x)\right)}{\cos^7(\alpha x)} \\ &+ \frac{32}{15\cos^{10}(\alpha x)}\left[\left(8820\alpha^2\cos^4(\alpha x) + 945\beta\cos^2(\alpha x) + 378\beta\cos^6(\alpha x) + 11340\alpha^2\right)\alpha^8\right]t^3 \\ &+ \frac{32}{15\cos^{10}(\alpha x)}\left[\left(-1260\beta\cos^4(\alpha x) - 1020\alpha^2\cos^6(\alpha x) - 18900\alpha^2\cos^2(\alpha x)\right)\alpha^8\right]t^3 \\ &+ \frac{32}{15\cos^{10}(\alpha x)}\left[\left(-12\beta\cos^8(\alpha x) + 8\alpha^2\cos^8(\alpha x)\right)\alpha^8\right]t^3 \end{aligned} \tag{28}$$

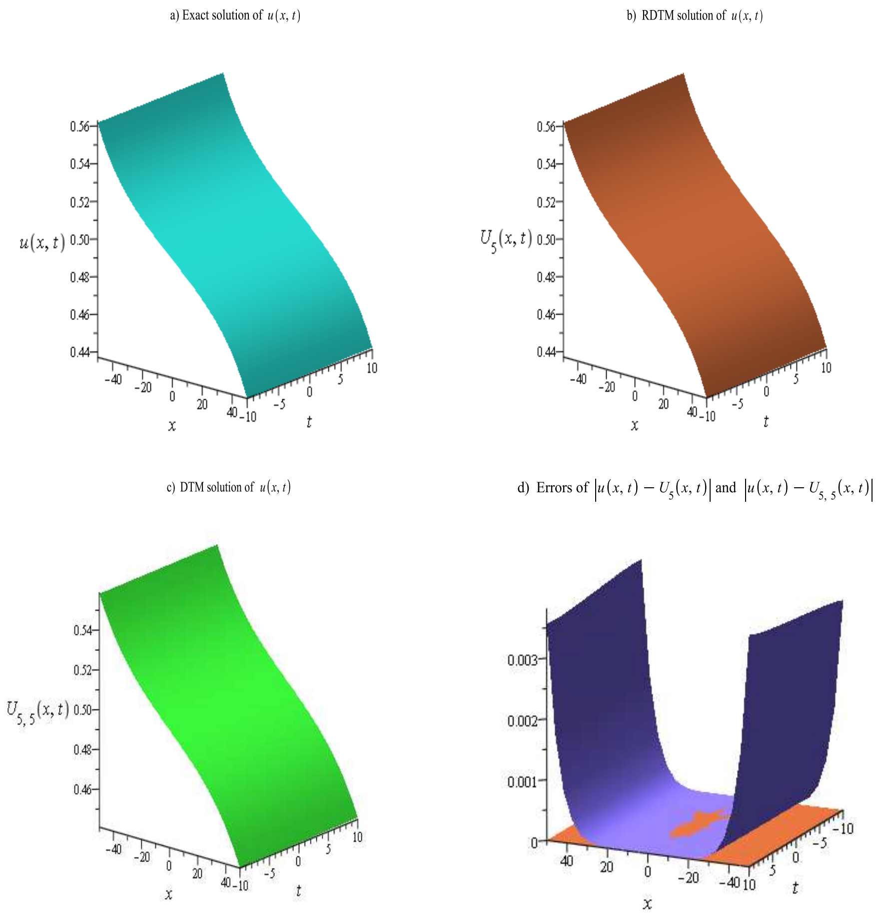

Fig. 3: Comparison between exact solution (3) and five steps RDTM, DTM solutions of $u(x,t)$ at $a_0 = \frac{1}{2}$, $\alpha = \beta = 0.02$. And comparison of errors d) **Sienna-RDTM**, **SlateBlue-DTM**.

$$
\begin{aligned}
V_3(x) = & -\frac{4}{9}\beta\alpha^4 + \frac{16}{27}\alpha^6 - \frac{5}{9}\beta^2\alpha^2 - \frac{5}{54}\beta^3 + \left(\frac{-20}{9}\beta\alpha^4 + \frac{16}{9}\alpha^6 - \frac{5}{9}\beta^2\alpha^2\right)\tan^2(\alpha x) \\
& - \frac{8}{9}\frac{\sin(\alpha x)\alpha^5 t\left(\cos^2(\alpha x) - 3\right)\left(-20\alpha^2\beta + 16\alpha^4 - 5\beta^2\right)}{\cos^5(\alpha x)} \\
& - \frac{9}{8}\frac{\alpha^8\left(-20\alpha^2\beta + 16\alpha^4 - 5\beta^2\right)\left[-126\cos^4(\alpha x) + 4\cos^6(\alpha x) + 420\cos^2(\alpha x) - 315\right]t^2}{\cos^8(\alpha x)} \\
& + \frac{128}{27}\frac{\left(6615\cos^4(\alpha x) - 18900\cos^2(\alpha x)\right)\left[-20\alpha^2\beta + 16\alpha^4 - 5\beta^2\right]\alpha^{11}\sin(\alpha x)t^3}{\cos^{11}(\alpha x)} \\
& + \frac{128}{27}\frac{\left(2\cos^8(\alpha x) - 510\cos^6(\alpha x) + 14175\right)\left[-20\alpha^2\beta + 16\alpha^4 - 5\beta^2\right]\alpha^{11}\sin(\alpha x)t^3}{\cos^{11}(\alpha x)}
\end{aligned}
\qquad (29)
$$

Thus, it is obviously noted on the **Table** 3 to 8 and **Fig.** (1)-(4) that solutions (26)-(29) provide the good accuracy with compared exact [12] and DTM solutions. Also from **Table** 9, RDTM is more faster than DTM.

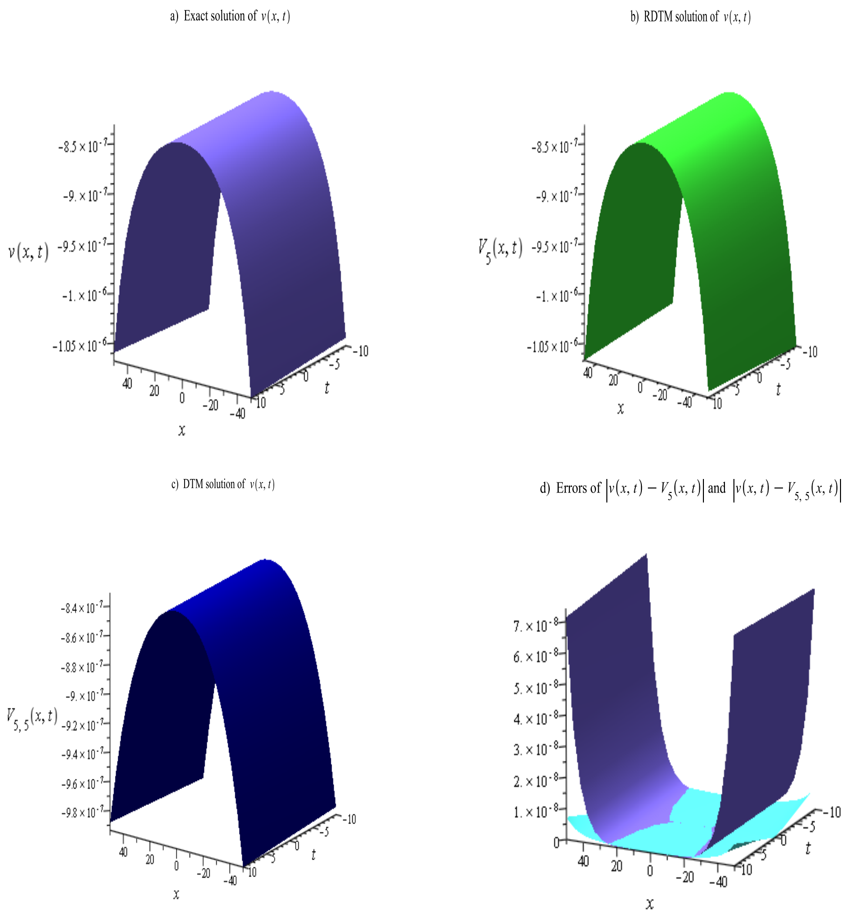

Fig. 4: Comparison between exact solution (3) and five steps RDTM, DTM solutions of $v(x,t)$ at $a_0 = \frac{1}{2}$, $\alpha = \beta = 0.02$. And comparison of errors d) **MediumTurquoise-RDTM**, **SlateBlue-DTM**.

4 Conclusion

In this paper, we consider the very famous physical problems coupled Ramani equation (1) to find two approximate traveling wave solutions by using DTM and RDTM. Moreover, we perfectly obtain approximate solutions of (1) compatible with exact solutions in [12]. In order to test efficiency, convergence and accuracy of DTM and RDTM, we perform the numerical values $a_0 = 1$, $\alpha = 0.01$, $\beta = 0.01$ and $a_0 = \frac{3}{2}$, $\alpha = 0.04$, $\beta = 0.04$ and $a_0 = \frac{3}{2}$, $\alpha = 0.02$, $\beta = 0.02$ in the seven step approximate solutions of eq. (1) which are shown in **Table** 3 to 8. Also, for $a_0 = \frac{1}{2}$, $\alpha = 0.03$, $\beta = 0.03$ and $a_0 = \frac{1}{2}$, $\alpha = 0.02$, $\beta = 0.02$, error rates for comparisons of exact [12] and seven step RDTM, DTM solutions are presented in **Fig.** (1) to (4). Furthermore, the CPU times of DTM and RDTM process are compared in **Table** 9. Results show that DTM and RDTM are efficient and powerful technique, but RDTM is more easier, fast and better than DTM.

Table 7: Comparison of errors for seven step DTM and RDTM solutions with exact solution 2 at $a_0 = \frac{3}{2}$, $\alpha = 0.04$, $\beta = 0.04$

x	Errors of RDTM		Errors of DTM	
	$\lvert u(x,t) - U_7(x)\rvert$	$\lvert v(x,t) - V_7(x)\rvert$	$\lvert u(x,t) - U_{7,7}(x,t)\rvert$	$\lvert v(x,t) - V_{7,7}(x,t)\rvert$
-50	1.03×10^{-7}	5.674312×10^{-9}	1041.757463	$1.969313611 \times 10^{-4}$
-25	5.336×10^{-6}	2.2925246×10^{-8}	4.433682835	$1.007732888 \times 10^{-6}$
-10	1.09×10^{-5}	1.9163592×10^{-8}	0.894570155	3.9147652×10^{-8}
10	9.955×10^{-6}	1.7818539×10^{-8}	0.894281382	3.9334112×10^{-8}
25	5.845×10^{-6}	2.365664×10^{-8}	4.503690346	7.2896423×10^{-7}
50	3.31×10^{-7}	5.987977×10^{-9}	1047.990895	$1.603518458 \times 10^{-4}$

Table 8: Comparison of errors for seven step DTM and RDTM solutions with exact solution 3 at $a_0 = \frac{3}{2}$, $\alpha = 0.02$, $\beta = 0.02$

x	Errors of RDTM		Errors of DTM	
	$\lvert u(x,t) - U_7(x)\rvert$	$\lvert v(x,t) - V_7(x)\rvert$	$\lvert u(x,t) - U_{7,7}(x,t)\rvert$	$\lvert v(x,t) - V_{7,7}(x,t)\rvert$
-50	1.6732×10^{-5}	1.3985587×10^{-8}	0.487909733	4.2968583×10^{-8}
-25	5.98×10^{-7}	1.35208×10^{-9}	4.6062085×10^{-2}	3.09447×10^{-10}
-10	1.33×10^{-7}	3.854221×10^{-10}	1.1065658×10^{-2}	3.51551×10^{-11}
10	1.48×10^{-7}	3.721891×10^{-10}	1.1062022×10^{-2}	3.50481×10^{-11}
25	6.2×10^{-7}	1.3290794×10^{-9}	4.6042264×10^{-2}	1.409608×10^{-10}
50	1.4049×10^{-5}	1.4859454×10^{-8}	0.487320038	3.026153×10^{-8}

Table 9: Comparison of CPU time for seven step DTM and RDTM solutions at $a_0 = 1$, $\alpha = 0.03$, $\beta = 0.03$

Iteration numbers	CPU times for RDTM	CPU times for DTM
3	0.011 second	16.297 second
5	0.015 second	132.047 second
6	0.016 second	307.406 second
7	0.026 second	647.985 second
8	0.034 second	1427.203 second

Competing interests

The authors declare that they have no competing interests.

Authors' contributions

All authors have contributed to all parts of the article. All authors read and approved the final manuscript.

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