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## A numerical method for solving continuous population models for single and interacting species

Elçin Gökmen<sup>1\*</sup>, Elçin Çelik<sup>2</sup>

### Abstract

The main purpose of this article is to suggest an approximation method for solving continuous population models. Using the collocation method and matrix operations, the problems are reduced into a system of nonlinear algebraic equations. The desired approximate solutions are obtained by solving this system via Maple 15. The error analysis for the proposed method is also introduced using the residual function. Numerical experiments are given to demonstrate the efficiency of the method. The results obtained from the proposed method are compared with the known results.

**Keywords:** continuous population models, nonlinear differential equations and their systems, Taylor polynomials and series, collocation points.

### 1. INTRODUCTION

Differential equations and their systems have an importance in science and engineering since they have been used for modelling many problem in these areas [1]. Population models have been examined in various fields such as ecology, biology, medicine. The first studies on population models were made by Thomas Malthus (1798). Pierre-Francois Verhulst (1838) has made a linear approach to the population problem by assuming that the population is a continuous function of time. This equation is called logistic differential equation which is also known Verhulst-Pearl equations [2]. The logistic model is commonly written in the form:

$$\begin{cases} y'(t) = ry(t)\left(1 - \frac{y(t)}{k}\right), & 0 \leq t \leq b, \\ y(0) = \lambda, \end{cases} \quad (1)$$

where  $r > 0$  is the intrinsic growth rate and  $k > 0$  is the carrying capacity of the population [3]. Eq.(1) only assumes that the growth rate of a population at any time  $t$  depends on the relative number of individuals at that time.

If there are more than one species, these species interact. There are three main types of relation between these species. These predator-prey situations are competition and mutualism or symbiosis [4].

In this work we deal with predator-prey situation which means when the growth rate of one population is decreased, the other is increased. The first predator-prey model with aftereffect was proposed by Volterra [5]. This model is a system as follow:

$$\begin{cases} \frac{dy_1}{dt} = y_1(t)(a - by_2(t)), & y_1(0) = \lambda_1, \\ \frac{dy_2}{dt} = -y_2(t)(c - dy_1(t)), & y_2(0) = \lambda_2, \end{cases} \quad (2)$$

where  $y_1(t)$  and  $y_2(t)$  indicate respectively the populations of prey and predator at time  $t$ , and  $y_1(t)y_2(t)$  denotes the amount of the two organisms encounter each other,  $a > 0$  growth rate of prey,  $b > 0$  the efficiency of the predator's ability to capture the prey,  $c > 0$  the death rate of the predator,  $d > 0$  the growth rate of the predator [14].

Up to now, the decomposition method [6], He's homotopy perturbation method [7], Bessel collocation method [8] have been used for solving logistic equation numerically. Also, Biazar et al. [9-11] used the Adomian decomposition method (ADM) and power series method, Rafei et al. and Pamuk [6,12] used the homotopy perturbation method (HPM) and the decomposition method, Rafei et al. [13] and Yusufoglu et al.[14] used variational iteration method (VIM) and Yuzbasi used Bessel collocation method [8] to solve the system (2) numerically.

In recent years, various matrix and collocation methods and the polynomial approximations [18-21] have been used. In this paper, Taylor matrix and collocation method is modified and developed to obtain the solution of model (1) and (2). Our aim is to approach solutions of these models which are expressed in the truncated Taylor series forms respectively;

$$y(t) = \sum_{n=0}^N y_n t^n, \quad y_n = \frac{y^{(n)}(0)}{n!}, \quad 0 \leq t \leq b \quad (3)$$

$$y_i(t) = \sum_{n=0}^N y_{i,n} t^n, \quad y_{i,n} = \frac{y_i^{(n)}(0)}{n!}, \quad i=1,2, \quad 0 \leq t \leq b \quad (4)$$

where  $y_n, y_{i,n}, n=0,1,\dots,N, i=1,2$  are unknown Taylor coefficients to be determined.

## 2. APPLICATION OF THE METHOD FOR MODEL (1)

During this part of the work we want to convert (1) or equivalently

$$y'(t) = ry(t) - \frac{r}{k} y^2(t) \quad (5)$$

to a matrix equation. For this purpose we first consider the approximate solution  $y(t)$  and its

derivative defined by truncated Taylor series (3). Then we write (3) and its derivative in the matrix form we obtain relations (6), (7)

$$y(t) = \mathbf{T}(t) \mathbf{Y}, \quad (6)$$

$$y'(t) = \mathbf{T}(t) \mathbf{B} \mathbf{Y}, \quad (7)$$

where

$$\mathbf{T}(t) = \begin{bmatrix} 1 & t & t^2 & \dots & t^N \end{bmatrix}_{1 \times (N+1)},$$

$$\mathbf{B} = \begin{bmatrix} 0 & 1 & 0 & 0 & \dots & 0 \\ 0 & 0 & 2 & 0 & \dots & 0 \\ 0 & 0 & 0 & 3 & \dots & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ 0 & 0 & 0 & 0 & \dots & N \\ 0 & 0 & 0 & 0 & \dots & 0 \end{bmatrix}_{(N+1) \times (N+1)}, \quad \mathbf{Y} = \begin{bmatrix} y_0 \\ y_1 \\ y_2 \\ \vdots \\ y_N \end{bmatrix}_{(N+1) \times 1}.$$

Similarly,  $y^2(t)$  which is the nonlinear term of (1) can be defined by the relation

$$[y(t)]^2 = \mathbf{T}(t) \mathbf{T}^* \bar{\mathbf{Y}}, \quad (8)$$

where

$$\mathbf{T}^*(t) = \begin{bmatrix} \mathbf{T}(t) & 0 & \dots & 0 \\ 0 & \mathbf{T}(t) & \dots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \dots & \mathbf{T}(t) \end{bmatrix}_{(N+1) \times (N+1)^2},$$

$$\bar{\mathbf{Y}} = \begin{bmatrix} y_0 \mathbf{Y} \\ y_1 \mathbf{Y} \\ \vdots \\ y_N \mathbf{Y} \end{bmatrix}_{(N+1)^2 \times 1}.$$

By substituting (6), (7), (8) into Eq. (5), we reach the primary equation as:

$$\mathbf{T}(t) \mathbf{B} \mathbf{Y} = r \mathbf{T}(t) \mathbf{Y} - \frac{r}{k} \mathbf{T}(t) \mathbf{T}^*(t) \bar{\mathbf{Y}} \quad (9)$$

or

$$[\mathbf{T}(t) \mathbf{B} - r \mathbf{T}(t)] \mathbf{Y} + \frac{r}{k} \mathbf{T}(t) \mathbf{T}^*(t) \bar{\mathbf{Y}} = 0. \quad (10)$$

If it is wanted to write Eq. (10) in the compact form then it is obtained

$$\mathbf{D}(t)\mathbf{Y} + \mathbf{A}(t)\bar{\mathbf{Y}} = \mathbf{f}(t) \tag{11}$$

where

$$\mathbf{D}(t) = \mathbf{T}(t)\mathbf{B} - r\mathbf{T}(t),$$

$$\mathbf{A}(t) = \frac{r}{k}\mathbf{T}(t)\mathbf{T}^*(t),$$

$$\mathbf{f}(t) = 0.$$

As the collocation points which are defined as

$$t_s = \frac{b}{N}s, \quad s = 0, 1, \dots, N, \tag{12}$$

substitute into (11), we have a new system

$$\mathbf{D}(t_s)\mathbf{Y} + \mathbf{A}(t_s)\bar{\mathbf{Y}} = \mathbf{f}(t_s), \quad s = 0, 1, \dots, N \tag{13}$$

or the matrix equation

$$\mathbf{D}\mathbf{Y}^* + \mathbf{A}\bar{\mathbf{Y}} = \mathbf{F} \tag{14}$$

where

$$\mathbf{D} = \begin{bmatrix} \mathbf{D}(t_0) & 0 & \dots & 0 \\ 0 & \mathbf{D}(t_1) & \dots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \dots & \mathbf{D}(t_N) \end{bmatrix}_{(N+1) \times (N+1)^2}, \quad \mathbf{Y}^* = \begin{bmatrix} \mathbf{Y} \\ \mathbf{Y} \\ \vdots \\ \mathbf{Y} \end{bmatrix}_{(N+1)^2 \times 1},$$

$$\mathbf{A} = \begin{bmatrix} \mathbf{A}(t_0) & 0 & \dots & 0 \\ 0 & \mathbf{A}(t_1) & \dots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \dots & \mathbf{A}(t_N) \end{bmatrix}_{(N+1) \times (N+1)^3}, \quad \bar{\mathbf{Y}} = \begin{bmatrix} \bar{\mathbf{Y}} \\ \bar{\mathbf{Y}} \\ \vdots \\ \bar{\mathbf{Y}} \end{bmatrix}_{(N+1)^3 \times 1},$$

$$\mathbf{F} = \begin{bmatrix} \mathbf{f}(t_0) \\ \mathbf{f}(t_1) \\ \vdots \\ \mathbf{f}(t_N) \end{bmatrix}_{(N+1) \times 1}.$$

To obtain the solution of (1) under condition we get the matrix form as:

$$\mathbf{T}(0)\mathbf{Y} = \lambda. \tag{15}$$

We replace row matrix (15) by any row of the matrix (14) and we get the new system depending on conditions. So, we obtained to a system of nonlinear algebraic equations with unknown Taylor coefficients  $y_0, y_1, \dots, y_N$ .

If these determined coefficients are substituted in (3), we reach desired approximate solution as follow:

$$y_N(t) = \sum_{n=0}^N y_n t^n.$$

Hence, when  $y_N(t)$  and its derivative are substituted into Eq. (1) the following equality is obtained

$$E(t) = \left| y'_N(t) - ry_N(t) \left( 1 - \frac{y_N(t)}{k} \right) \right| \square 0. \tag{16}$$

that it indicates  $y_N(t) \square y(t)$ .

### 3. RESIDUAL CORRECTION PROCEDURE FOR SINGLE SPECIES MODEL

In this part of the study, it will be explained an error estimation based on the residual function [15]-[17] for Taylor collocation method. Using this procedure it can be estimated the optimal  $M$  giving minimal absolute error. To modify the procedure to Eq (1) first we get the residual function for Taylor polynomial solution as

$$R = y'_N(t) - ry_N(t) + \frac{r}{K} y_N^2(t).$$

By adding  $R$  into both sides of Eq. (1), we obtain

$$e'(t) - re(t) + \frac{r}{K} e^2(t) + \frac{2r}{K} y_N(t)e(t) = -R \tag{17}$$

where

$$e(t) = y(t) - y_N(t).$$

Let  $e_M^*$  be the Taylor series solutions of (17). If

$$\|e(t) - e_M^*(t)\| < \varepsilon$$

is sufficiently small, then the absolute errors can be estimated by  $e_M^*(t)$ . Hence, the optimal  $M$  which gives the minimal absolute error can be obtained measuring the error functions  $e_M^*(t)$  in any norm.

**4. APPLICATION OF THE METHOD FOR MODEL (2)**

Now we consider the interacting species model (2) or

$$\begin{cases} y_1'(t) = ay_1(t) - by_1(t)y_2(t), & y_1(0) = \lambda_1 \\ y_2'(t) = -cy_2(t) + dy_2(t)y_1(t), & y_2(0) = \lambda_2 \end{cases} \quad (18)$$

Let's start to apply the procedure with the solution (4) and its first derivative, for  $n = 0, 1, 2, \dots, N$  then we get the matrix forms as follows:

$$y_i(t) = \mathbf{T}(t)\mathbf{Y}_i, \quad i = 1, 2 \quad (19)$$

and

$$y_i'(t) = \mathbf{T}(t)\mathbf{B}\mathbf{Y}_i, \quad i = 1, 2 \quad (20)$$

where

$$\mathbf{T}(t) = \begin{bmatrix} 1 & t & t^2 & \dots & t^N \end{bmatrix},$$

$$\mathbf{B} = \begin{bmatrix} 0 & 1 & 0 & 0 & \dots & 0 \\ 0 & 0 & 2 & 0 & \dots & 0 \\ 0 & 0 & 0 & 3 & \dots & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ 0 & 0 & 0 & 0 & \dots & N \\ 0 & 0 & 0 & 0 & \dots & 0 \end{bmatrix}, \quad \mathbf{Y}_i = \begin{bmatrix} y_{i,0} \\ y_{i,1} \\ y_{i,2} \\ \vdots \\ y_{i,N} \end{bmatrix}.$$

The expressions  $y_1(t)y_2(t)$  and  $y_2(t)y_1(t)$  can be represented by the relations respectively

$$\begin{aligned} y_1(t)y_2(t) &= \mathbf{T}(t)\mathbf{T}^*(t)\bar{\mathbf{Y}}_{1,2} \\ y_2(t)y_1(t) &= \mathbf{T}(t)\mathbf{T}^*(t)\bar{\mathbf{Y}}_{2,1} \end{aligned} \quad (21)$$

where

$$\begin{aligned} \bar{\mathbf{Y}}_{1,2} &= [y_{2,0}\mathbf{Y}_1 \quad y_{2,1}\mathbf{Y}_1 \quad \dots \quad y_{2,N}\mathbf{Y}_1]_{(N+1)^2 \times 1}^T \\ \bar{\mathbf{Y}}_{2,1} &= [y_{1,0}\mathbf{Y}_2 \quad y_{1,1}\mathbf{Y}_2 \quad \dots \quad y_{1,N}\mathbf{Y}_2]_{(N+1)^2 \times 1}^T. \end{aligned}$$

If the relations (19), (20), (21) substitute into the system (18), we get the following matrix system

$$\begin{cases} \mathbf{T}(t)\mathbf{B}\mathbf{Y}_1 - a\mathbf{T}(t)\mathbf{Y}_1 + b\mathbf{T}(t)\mathbf{T}^*(t)\bar{\mathbf{Y}}_{1,2} = 0, \\ \mathbf{T}(t)\mathbf{B}\mathbf{Y}_2 + c\mathbf{T}(t)\mathbf{Y}_2 - d\mathbf{T}(t)\mathbf{T}^*(t)\bar{\mathbf{Y}}_{2,1} = 0. \end{cases} \quad (22)$$

In relation (22) whether linear and nonlinear parts are separated then we get the relation

$$\begin{cases} \mathbf{D}_1(t)\mathbf{Y}_1 + \mathbf{A}_1(t)\bar{\mathbf{Y}}_{1,2} = 0 \\ \mathbf{D}_2(t)\mathbf{Y}_2 + \mathbf{A}_2(t)\bar{\mathbf{Y}}_{2,1} = 0 \end{cases} \quad (23)$$

where

$$\begin{aligned} \mathbf{D}_1(t) &= \mathbf{T}(t)\mathbf{B} - a\mathbf{T}(t), & \mathbf{D}_2(t) &= \mathbf{T}(t)\mathbf{B} + c\mathbf{T}(t), \\ \mathbf{A}_1(t) &= b\mathbf{T}(t)\mathbf{T}^*(t), & \mathbf{A}_2(t) &= -d\mathbf{T}(t)\mathbf{T}^*(t). \end{aligned}$$

Hence, system (23) is expressed as:

$$\mathbf{D}(t)\mathbf{Y} + \mathbf{A}(t)\bar{\mathbf{Y}} = \mathbf{G}(t) \quad (24)$$

where

$$\begin{aligned} \bar{\mathbf{D}}(t) &= \begin{bmatrix} \mathbf{D}_1(t) & 0 \\ 0 & \mathbf{D}_2(t) \end{bmatrix}_{2 \times 2(N+1)}, & \mathbf{Y} &= \begin{bmatrix} \mathbf{Y}_1 \\ \mathbf{Y}_2 \end{bmatrix}_{2(N+1) \times 1}, \\ \mathbf{G}(t) &= \begin{bmatrix} 0 \\ 0 \end{bmatrix}_{2 \times 1}, \\ \bar{\mathbf{A}}(t) &= \begin{bmatrix} \mathbf{A}_1(t) & 0 \\ 0 & \mathbf{A}_2(t) \end{bmatrix}_{2 \times 2(N+1)^2}, & \bar{\mathbf{Y}} &= \begin{bmatrix} \bar{\mathbf{Y}}_{1,2} \\ \bar{\mathbf{Y}}_{2,1} \end{bmatrix}_{2(N+1)^2 \times 1}. \end{aligned}$$

In Eq. (24), by using collocation points (12) we obtain the matrix equations system

$$\bar{\mathbf{D}}(t_s)\mathbf{Y} + \bar{\mathbf{A}}(t_s)\bar{\mathbf{Y}} = \mathbf{G}(t_s), \quad s = 0, 1, \dots, N \quad (25)$$

or

$$\bar{\mathbf{D}}\mathbf{Y}^* + \bar{\mathbf{A}}\bar{\mathbf{Y}} = \mathbf{G} \quad (26)$$

where

$$\begin{aligned} \bar{\mathbf{D}} &= \begin{bmatrix} \bar{\mathbf{D}}(t_0) & 0 & \dots & 0 \\ 0 & \bar{\mathbf{D}}(t_1) & \dots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \dots & \bar{\mathbf{D}}(t_N) \end{bmatrix}_{2(N+1) \times 2(N+1)^2}, & \mathbf{Y}^* &= \begin{bmatrix} \mathbf{Y} \\ \mathbf{Y} \\ \vdots \\ \mathbf{Y} \end{bmatrix}_{2(N+1)^2 \times 1}, \\ \bar{\mathbf{A}} &= \begin{bmatrix} \bar{\mathbf{A}}(t_0) & 0 & \dots & 0 \\ 0 & \bar{\mathbf{A}}(t_1) & \dots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \dots & \bar{\mathbf{A}}(t_N) \end{bmatrix}_{2(N+1) \times 2(N+1)^3}, & \bar{\mathbf{Y}} &= \begin{bmatrix} \bar{\mathbf{Y}} \\ \bar{\mathbf{Y}} \\ \vdots \\ \bar{\mathbf{Y}} \end{bmatrix}_{2(N+1)^3 \times 1}. \end{aligned}$$

In a similar way it is obtained the matrix form of initial conditions of (2) as:

$$\mathbf{T}(0)\mathbf{Y}_1 = \lambda_1 \quad (27)$$

$$\mathbf{T}(0)\mathbf{Y}_2 = \lambda_2. \quad (28)$$

To find the solutions of (2) under these conditions we should replace (27) and (28) by any two rows of (26). So we have the required augmented matrix.

By solving this system we determine the unknown coefficients  $y_{1,n}$  and  $y_{2,n}$ , ( $n = 0, 1, \dots, N$ ) and therefore we attain the numerical approaches as

$$y_{i,N}(t) = \sum_{n=0}^N y_{i,n} t^n, \quad i=1,2 \quad (29)$$

If it is wanted to check the accuracy of the approximate solutions, we put these solutions into (2) and thus equation (30) is acquired for  $t \in [0, b]$ .

$$\begin{aligned} E_{1,N}(t) &= |y'_{1,N}(t) - y_{1,N}(t)(a - b y_{2,N}(t))| \cong 0, \\ E_{2,N}(t) &= |y'_{2,N}(t) - y_{2,N}(t)(c y_{1,N}(t) - d)| \cong 0 \end{aligned} \quad (30)$$

It is expected that  $E_{i,N}(t) = 0$ ,  $i = 1, 2$  on the collocation points. If  $y_{i,N}(t)$  converges to  $y_i(t)$  then  $E_{i,N}(t)$  converges to zero for  $t \in [0, b]$ .

## 5. RESIDUAL CORRECTION PROCEDURE FOR INTERACTING SPECIES MODEL

In this section, using similar procedure in Section 3 it can be estimated the optimal  $M$  giving minimal absolute error. For modifying the procedure to system (2), first we get the residual function for Taylor polynomial solution (29) as

$$\begin{aligned} R_1 &= y'_{1,N}(t) - a y_{1,N}(t) + b y_{1,N}(t) y_{2,N}(t), \\ R_2 &= y'_{2,N}(t) + c y_{2,N}(t) - d y_{1,N}(t) y_{2,N}(t) \end{aligned} \quad (31)$$

where  $y_{1,N}(x), y_{2,N}(x)$  denotes the approximate solutions (30). By adding (31) into the both side of equations (2), we have

$$\begin{aligned} e'_{1,N}(t) - a e_{1,N}(t) + b e_{1,N}(t) e_{2,N}(t) \\ + b y_{1,N}(t) e_{2,N}(t) + b y_{2,N}(t) e_{1,N}(t) = -R_1, \end{aligned} \quad (32)$$

$$\begin{aligned} e'_{2,N}(t) + c e_{2,N}(t) - d e_{1,N}(t) e_{2,N}(t) \\ - d y_{1,N}(t) e_{2,N}(t) - d y_{2,N}(t) e_{1,N}(t) = -R_2 \end{aligned}$$

where

$$e_{1,N}(x) = y_1(x) - y_{1,N}(x), \quad e_{2,N}(x) = y_2(x) - y_{2,N}(x).$$

If

$$\begin{aligned} \|e_{1,N} - e_{1,M}^*\| &< \varepsilon_1 \\ \|e_{2,N} - e_{2,M}^*\| &< \varepsilon_2 \end{aligned}$$

are sufficiently small where  $e_{1,M}^*$  and  $e_{2,M}^*$  are the Taylor series solutions of system (33), then the absolute errors can be estimated via  $e_{1,M}^*$  and  $e_{2,M}^*$ . Thus the optimal  $M$  can be defined measuring the error functions  $e_{1,M}^*$  and  $e_{2,M}^*$ .

**Corollary.** If  $y_{1,N}(x)$  and  $y_{2,N}(x)$  are the Taylor series solutions of (2), then  $y_{1,N} + e_{1,M}^*$  and  $y_{2,N} + e_{2,M}^*$  are also approximate solutions of (2) and they are defined as corrected Taylor polynomial solution. Error function for this corrected solutions are respectively  $E_{1,M} = e_{1,N} - e_{1,M}^*$  and  $E_{2,M} = e_{2,N} - e_{2,M}^*$ .

## 6. NUMERICAL APPLICATIONS

To show the forcefulness of the process two numerical examples are examined. All the computations and graphs are performed by a code written in Maple 15.

**Example 1.** [6],[7] Let us first consider the following problem

$$\begin{cases} y'(t) = y(t)(1 - y(t)), & 0 \leq t \leq 1 \\ y(0) = 2 \end{cases}$$

where  $r = 1$ ,  $k = 1$ ,  $\lambda = 2$ . The exact solutions of this equation is  $y(t) = \frac{2}{2 - e^{-t}}$ . We obtain the

approximate solutions by the method based on Taylor polynomials for  $N = 3, 4, \dots, 14$ . Table 1 shows that comparison of the exact solution and the numerical solutions acquired by present method and the methods used in ref [6,7]. In Table 2, it is demonstrated that the absolute error

functions for different values of  $N$  and the error estimate function  $e_{10}^*(t)$ . In addition to in Figure 1, the absolute error function  $e_{11}(t)$  and error estimate function  $e_{10}^*(t)$  are plotted.

Table 1. Exact and numerical solutions for Example 1.

Exact solution		Present method		
$t_i$	$y(t_i) = \frac{2}{2-e^{-t_i}}$	$y_5(t_i)$	$y_8(t_i)$	$y_{11}(t_i)$
0	2	2.00000000	2.00000000	2.00000000
0.2	1.69309410	1.69247244	1.69308560	1.69309391
0.4	1.50412134	1.50394442	1.50411547	1.50412121
0.6	1.37818084	1.37785840	1.37817656	1.37818075
0.8	1.28976421	1.28997910	1.28976100	1.289764126
1.0	1.225399673	1.218195545	1.225623964	1.225392154

HHPM [7]			DM[6]	
$t_i$	$y_6(t_i)$	$y_8(t_i)$	$y_5(t_i)$	$y_8(t_i)$
0	2.000000000	2.000000000	2.000000000	2.000000000
0.2	1.693280533	1.693109627	1.692448000	1.693109627
0.4	1.523618133	1.510614146	1.470336000	1.510614146
0.6	1.659780800	1.589181662	1.052864000	1.589181662
0.8	3.116808533	3.723526257	-0.29324800	3.723526257
1.0	8.908333333	17.21641865	-4.10000000	17.21641865

Table 2. The  $\infty$  norms of the absolute errors, estimations of the absolute errors, the  $\infty$  norms of the corrected absolute errors and upper bonds of the absolute errors for Example1.

$N$	3	4	5	6
$\ f - y_N\ _\infty$	0.078	0.02255	0.720E-2	0.220E-2
$\ e_{10}^*\ _\infty$	0.080	0.02253	0.722E-2	0.218E-2
$\ f - y_N - e_{10}^*\ _\infty$	0.231E-4	0.231E-4	0.231E-4	0.231E-4
$N$	7	8	9	10
$\ f - y_N\ _\infty$	0.716E-3	0.224E-3	0.730E-4	0.231E-4
$\ e_{10}^*\ _\infty$	0.739E-3	0.201E-3	0.961E-4	0.153E-4
$\ f - y_N - e_{10}^*\ _\infty$	0.231E-4	0.231E-4	0.231E-4	0.231E-4
$N$	11	12	13	14
$\ f - y_N\ _\infty$	0.752E-5	0.238E-5	0.776E-6	0.247E-6
$\ e_{10}^*\ _\infty$	0.379E-5	0.517E-6	0.209E-6	0.209E-6
$\ f - y_N - e_{10}^*\ _\infty$	0.372E-5	0.290E-5	0.986E-6	0.372E-7

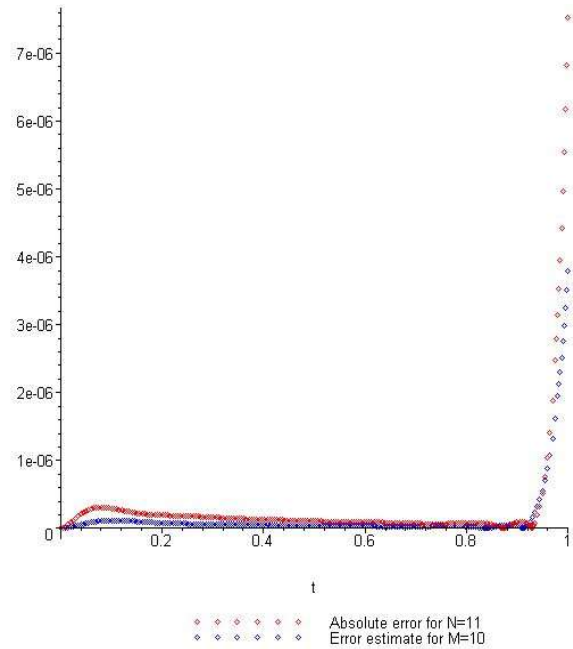


Figure 1. Comparison of the absolute error function for  $N = 11$  and error estimate function for  $M = 10$  for Example1.

**Example 2.**[6],[7] As a second problem, we consider the following problem

$$\begin{cases} \frac{dy_1}{dt} = y_1(t)(1 - y_2(t)), & y_1(0) = 1.3, \\ \frac{dy_2}{dt} = y_2(t)(-1 + y_1(t)), & y_2(0) = 0.6. \end{cases}$$

After we apply the mentioned method for different  $N$  values as in Section 4, we compare the outputs from different methods as you can see in Table 3.

In Figure 2 and Figure 3, it is plotted that the numerical approaches gained by our method and the decomposition method for  $N = 4$ . Also, we compare the accuracies of the approximate solutions in Table 4 and we show this results in Figure 4, Figure 5, Figure 6, Figure 7. We can see

from Table 4 and Figure 4-7 that larger values of  $N$  yields better accuracies.

0.6	0.041294	0.039170	0.68925 E-2	0.14187 E-2
0.8	0.094218	0.090957	0.012695	0.67535 E-2
1.0	0.17418	0.17098	0.014979	0.022811

Table 3. The approximate solutions of Example 2 obtained from different methods.

Present method				
$t_i$	$y_{1,4}(t_i)$	$y_{1,8}(t_i)$	$y_{2,4}(t_i)$	$y_{2,8}(t_i)$
0	1.300000000	1.300000000000	0.60000000	0.6000000000
0.	1.402560367	1.40250074928	0.64364900	0.6437090909
2	47		004	7
0.	1.497481204	1.49742150279	0.70442621	0.7044582738
4	92		561	1
0.	1.576531213	1.57645686870	0.78455663	0.7845888922
6	24		580	7
0.	1.630024557	1.62988609772	0.88557147	0.8856351392
8	70		092	3
1.	1.646820867	1.64800015244	1.00830815	1.0070221569
0	99		245	8

DM[6]- HHPM [7]				
$t_i$	$y_{1,3}(t_i)$	$y_{1,4}(t_i)$	$y_{2,3}(t_i)$	$y_{2,4}(t_i)$
0	1.300000000	1.300000000000	0.60000000	0.6000000000
0.	1.402582400	1.40257444800	0.64369520	0.6437110400
2	00		0000	00
0.	1.498739200	1.49861196800	0.70428160	0.7045350400
4	00		0000	00
0.	1.583084800	1.58244068800	0.78401040	0.7852934400
6	00		0000	00
0.	1.650233600	1.64819788800	0.88513280	0.8891878400
8	00		0000	00
1.	1.694800000	1.68983000000	1.00990000	1.0198000000
0	00		0000	00

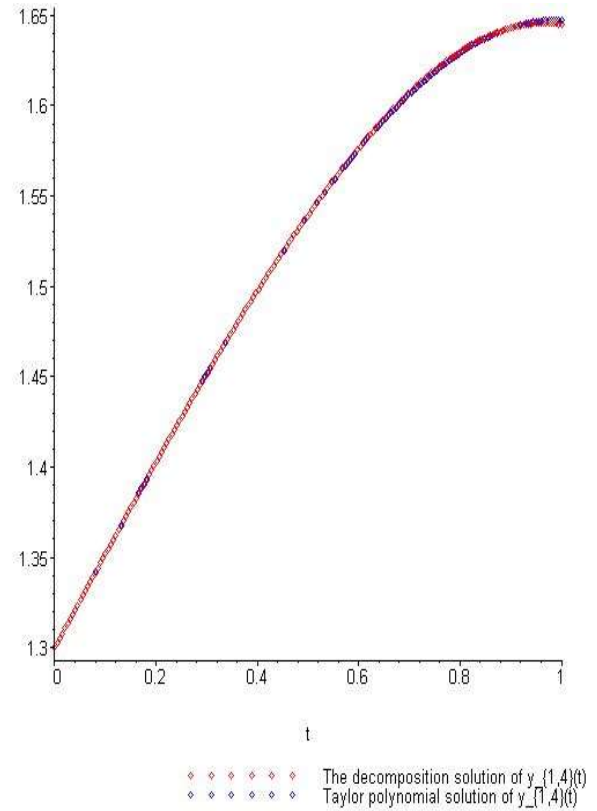


Figure 2. Comparison of approximate solution  $y_{1,4}(t)$  acquired by the decomposition method and the present method for Example 2.

Table 4. Comparison of the accuracy of solutions given in Equation (31) for  $t_i$  value of Example 2.

Present method				
$t_i$	$E_{1,4}(t_i)$	$E_{1,8}(t_i)$	$E_{2,4}(t_i)$	$E_{2,8}(t_i)$
0	0	0	0	0
0.2	0.152582 E-3	0.133054 E-6	0.185271 E-3	0.990741 E-7
0.4	0.223599 E-3	0.347114 E-7	0.265203 E-3	0.262717 E-7
0.6	0.382779 E-3	0.542112 E-7	0.445184 E-3	0.415516 E-7
0.8	0.907477 E-3	0.599012 E-6	0.103823 E-2	0.463089 E-6
1.0	0.0144644486	0.587349 E-4	0.016321765	0.455900 E-4

DM[6]- HHPM [7]				
$t_i$	$E_{1,3}(t_i)$	$E_{1,4}(t_i)$	$E_{2,3}(t_i)$	$E_{2,4}(t_i)$
0	0	0	0	0
0.2	0.15892 E-2	0.14552 E-2	0.31236 E-3	0.31834 E-5
0.4	0.012539	0.011684	0.23408 E-2	0.15680 E-3



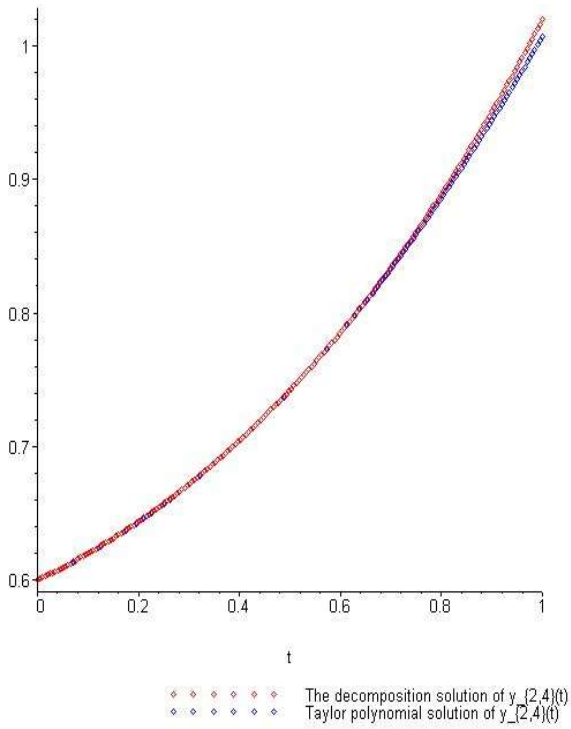


Figure 3. Comparison of approximate solution  $y_{2,4}(t)$  acquired by the decomposition method and the present method for Example 2.

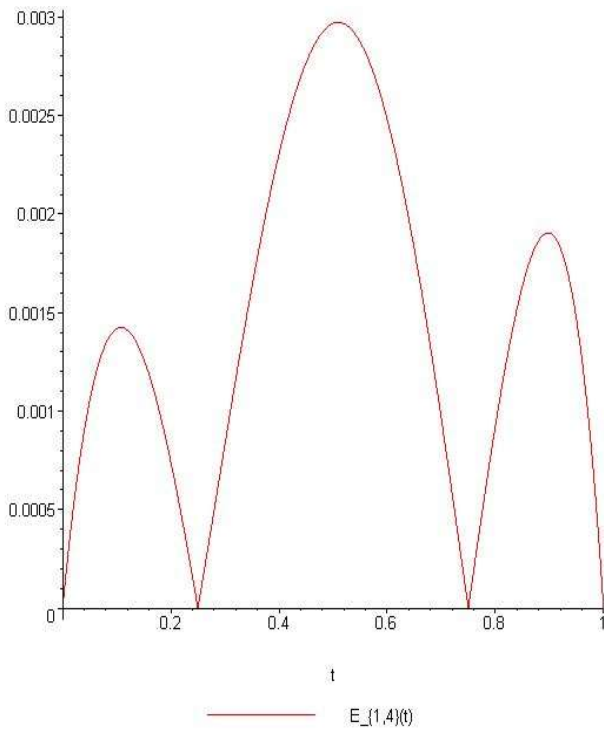


Figure 4. The accuracy of Taylor series solution  $E_{1,4}(t)$  for Example 2.

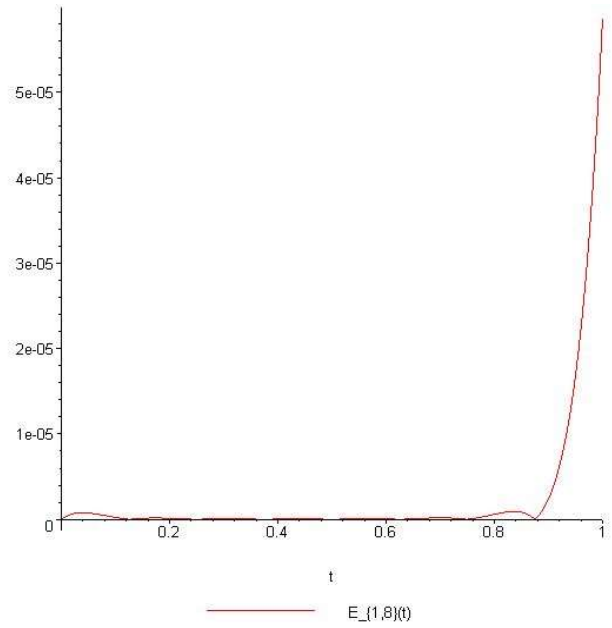


Figure 5. The accuracy of Taylor series solution  $E_{1,8}(t)$  for Example 2.

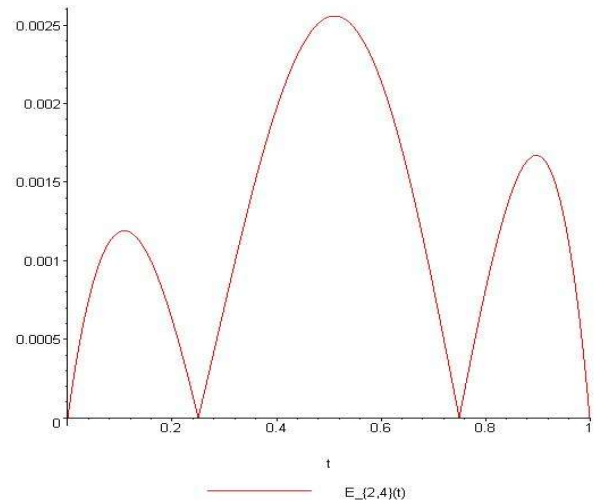


Figure 6. The accuracy of Taylor series solution  $E_{2,4}(t)$  for Example 2.

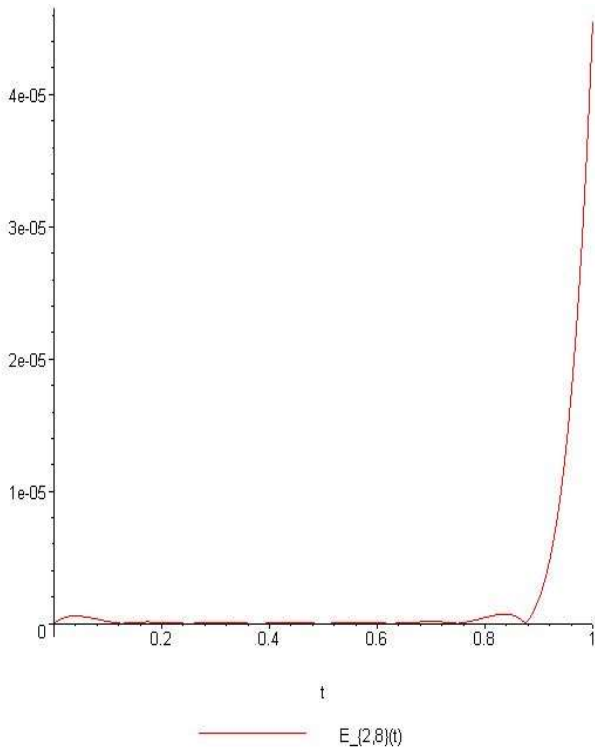


Figure 7. The accuracy of Taylor series solution  $E_{2,8}(t)$  for Example 2.

## 7. CONCLUSIONS

In this study, we perform the Taylor matrix method to obtain approximate solutions of model (1) and (2). Since the results are readily obtainable with a computer program this process is quite useful. To estimate the absolute error residual correction procedure is also improved for these models. Moreover, one can specify optimal  $N$  which gives minimum absolute errors in any norm. In Table 1, the numerical outputs obtained by different values of  $N$  are given with the exact solution. As it can be seen from the values, increasing  $N$  yields more accurate solution. In Table 2, infinite norms of the absolute errors, corrected absolute errors and upper bounds of the absolute errors are displayed.

From the results acquired, it is said that residual correction procedure estimates the absolute errors well. For Example 2, since the exact solutions set is unknown, we test the accuracies of the approximate solutions. As seen from Tables 3-4 and figures, the accuracies are better for larger values of  $N$ . For both examples, the results are shown that the convergence of our method are better.

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