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Analyzing the nonlinear heat transfer equation by AGM

Hadi Mirgolbabaee, Soheil Tahernejad Ledari, Davood Domiri Ganji and Esmail Karimi Valujai

Department of Mechanical Engineering, Babol University of Technology,P.O. Box 484, Babol, Iran

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Abstract: In this paper, a novel nonlinear differential equation in the field of heat transfer has been investigated and solved completely by a new method that we called it Akbari-Ganjis Method (AGM). Regarding to the previously published papers, investigating this kind of equations is a very hard project to do and the obtained solution is not accurate and reliable. This issue will be appeared after comparing the obtained solution by Numerical Method or the Exact Solution. Based on the comparison which has been made between the achieved solutions by AGM and Numerical Method (Runge-Kutte 4th), it is possible to indicate that AGM can be successfully applied to various differential equations particularly for difficult ones. Furthermore, It is necessary to mention that a summary of the excellence of this method in comparison with other approaches can be considered as follows: Boundary conditions are required in accordance with order of the differential equation, this approach can create additional new boundary conditions in regard to the own differential equation and its derivatives. Therefore, it is logical to mention which AGM is operational for miscellaneous nonlinear differential equations in comparison with the other methods.

Keywords: Akbari-Ganji'sMethod (AGM), nonlinear equation of heat transfer, numerical method.

1 Introduction

Since most of the phenomena in our world are essentially nonlinear and hence described by nonlinear equations, there has developed an ever-increasing interest of scientists and engineers in the analytical asymptotic techniques for solving nonlinear problems. All these problems and phenomena are modeled by ordinary or partial differential equations. In this case study, similarity transformation has been used to reduce the governing differential equations into an ordinary non-linear differential equation. Recently, many new numerical techniques have been widely applied to the nonlinear problems. Some of these methods are Perturbation Method (PM) [1], Homotopy Perturbation Method (HPM) [2,3,4,5,6], Variational Iteration Method (VIM) [7,8,9], Homotopy Analysis Method (HAM)[10,11], Differential Transform Method (DTM) [12] and Adomian Decomposition Method (ADM) [13,14].

The main purpose of this paper is introducing Akbari-Ganji's Method (AGM) [15,16,17,18] as new methods and by comparing it with numerical methods we can precisely conclude that the AGM has high efficiency and accuracy for solving nonlinear problems with high nonlinearity. It is necessary to mention that a summary of the excellence of this method in comparison with the other approaches can be considered as follows: Boundary conditions are needed in accordance with the order of differential equations in the solution procedure, but when the number of boundary conditions is less than the order of the differential equation, this approach can create additional new boundary conditions in regard to the own differential equation and its derivatives. Therefore, it is logical to mention that the AGM is operational for miscellaneous nonlinear differential equations in comparison with the other methods.

* Corresponding author e-mail: h.mirgolbabaee@stu.nit.ac.ir

Table 1: Nomenclature

AGM	Akbari-Ganji's Method
T	Temperature
V	Volume
A	Surface area
ρ	Density
τ	time
c	Specific heat
h	Heat transfer coefficient
β	Constant parameter
ω	Angular frequency

2 Basic Idea of Akbari-Ganji's method (AGM)

Boundary conditions and initial conditions are required for analytical methods of each linear and nonlinear differential equation according to the physics of the problem. Therefore, we can solve every differential equation with any degrees. In order to comprehend the given method in this paper, two differential equations governing on engineering processes will be solved in this new manner.

In accordance with the boundary conditions, the general manner of a differential equation is as follows. The nonlinear differential equation of p which is a function of u, the parameter u which is a function of x and their derivatives are considered as follows.

$$P_k : f\left(u, u^{'}, u^{''}, \ldots, u^{m}\right) = 0;\ u = u(x) \tag{1}$$

where P_k is the name of the nonlinear differential equation; and m is the order of the derivatives. Boundary conditions:

$$\begin{cases} u(x) = u_0, u'(x) = u_1, u^{(m-1)}(x) = u_{m-1} \ \text{at} : x = 0, \\ u(x) = u_{L_0}, u'(x) = u_{L_1}, u^{(m-1)}(x) = u_{L_{m-1}} \ \text{at} : x = L \end{cases} \tag{2}$$

To solve the first differential equation with respect to the boundary conditions at $x = L$ in Eq. (2), the series of letters in the nth order with constant coefficients which is the answer of the first differential equation is considered as follows.

$$u(x) = \sum_{i=0}^{n} a_i x^i = a_0 + a_1 x^1 + a_2 x^2 + \ldots + a_n x^n \tag{3}$$

The more precise answer of Eq. (1), the more choice of series sentences from Eq. (3). In applied problems, approximately five or six sentences in the series are enough to solve nonlinear differential equation. In the answer of differential, regarding the series from degree (n), there are $(n+1)$ unknown coefficients which need $(n+1)$ equation to satisfy. In AGM, a total answer with constant coefficients is required in order to solve differential equations. The boundary conditions of Eq. (2) are used to solve a set of equations which consist of $(n+1)$ ones. The boundary conditions are applied to the functions as follow.

(a) The application of the boundary conditions for the answer of differential Eq. (3) is in the form of when $x = 0$,

$$\begin{cases} u(0) = a_0 = u_0, \\ u'(0) = a_1 = u_1 \\ u''(0) = a_2 = u_2 \\ \vdots \quad \vdots \quad \vdots \end{cases} \tag{4}$$

When $x = L$,

$$\begin{cases} u(L) = a_0 + a_1 L + a_2 L^2 + \cdots + a_n L^n = u_{L_0}, \\ u'(L) = a_1 + 2a_2 L + 3a_3 L^2 + \cdots + n a_n L^{n-1} = u_{L_1} \\ u''(L) = 2a_2 + 6a_3 L + 12a_4 L^2 + \cdots + n(n-1) a_n L^{n-2} = u_{L_{m-1}} \\ \vdots \quad \vdots \quad \vdots \quad \vdots \quad \vdots \quad \vdots \quad \vdots \end{cases} \tag{5}$$

(b) After substituting Eq. (3) into Eq. (1), the application of the boundary conditions on differential Eq. (1) is done according to the following procedure:

$$\begin{array}{ll} p_0 : & f\left(u(0), u'(0), u''(0), \cdots, u^{(m)}(0)\right) \\ p_1 : & f\left(u(L), u'(L), u''(L), \cdots, u^{(m)}(L)\right) \\ \vdots & \vdots \quad \vdots \quad \vdots \quad \vdots \quad \vdots \end{array} \tag{6}$$

With regard to the choice of n;$(n < m)$ sentence in Eq. (3) and in order to make a set of equation which are consisted of $(n+1)$ equations and $(n+1)$ unknowns, we confront with a number of additional unknowns which are certainly the same coefficients of Eq. (3), therefore, to remove this problem, we should make m time derivation in Eq. (1) according to the additional unknowns in the aforementioned set of differential equation. This is the time to apply the boundary condition of Eq. (2).

$$\begin{array}{ll} p'_k : & f\left(u', u'', u''', \cdots, u^{(m+1)}\right) \\ p''_k : & f\left(u'', u''', u^{(IV)}, \cdots, u^{(m+2)}\right) \\ \vdots & \vdots \quad \vdots \quad \vdots \quad \vdots \quad \vdots \end{array} \tag{7}$$

(c) Application of the boundary conditions on the derivatives of the differential equation P_k in Eq. (7) is done in the form of,

$$p'_k : \begin{cases} f\left(u'(0), u''(0), u'''(0), \cdots, u^{(m+1)}(0)\right) \\ f\left(u'(L), u''(L), u'''(L), \cdots, u^{(m+1)}(L)\right) \end{cases} \tag{8}$$

$$p''_k : \begin{cases} f\left(u''(0), u'''(0), \cdots, u^{(m+2)}(0)\right) \\ f\left(u''(L), u'''(L), \cdots, u^{(m+2)}(L)\right) \end{cases} \tag{9}$$

$(n+1)$ equations can be made from Eq. (4) to (9), so that $(n+1)$ unknown coefficients of Eq. (3) will be calculated for example $a_0, a_1, a_2 a_n$. The answer of the nonlinear differential Eq. (1) will be gained by determining the coefficients of Eq. (3).

3 Cooling of a lumped system with variable specific heat

Consider the cooling of a lumped system [19]. Let the system have volume V, surface area A, density , specific heat c and initial temperature T_i. At time $t = 0$, the system is exposed to a convective environment at temperature T_a with convective heat transfer coefficient h. Assume that the specific heat c is a linear function temperature of the form,

$$C = C_a[1 + \beta(T - T_a)] \tag{10}$$

where C_a is the specific heat, at temperature T_a and β is a constant. The cooling equation and the initial condition are

$$\rho VC\frac{dT}{dt}+hA(T-T_a)=0,\ \ T(0)=T_i. \tag{11}$$

Introducing Eq. (10) and using the dimensionless parameters,

$$\theta=\frac{T-T_a}{T_i-T_a},\ \ \tau=\frac{t}{\rho VC_a/(hA)},\ \ \varepsilon=\beta(T-T_a). \tag{12}$$

Transforms Eq. (11) to

$$(1+\varepsilon\theta)\frac{d\theta}{d\tau}+\theta=0,\ \ \theta(0)=1. \tag{13}$$

3.1 Solving the nonlinear equation by AGM

First of all we rewrite the problem Eq. (13) in the following order

$$(1+\varepsilon\theta)\frac{d\theta}{d\tau}+\theta=0. \tag{14}$$

In AGM, the answer of the differential equation is considered as a finite series of polynomials with constant coefficients, as follows

$$u(\tau)=\sum_{i=0}^{6}a_i\tau^i=a_6\tau^6+a_5\tau^5+a_4\tau^4+a_3\tau^3+a_2\tau^2+a_1\tau^1+a_0. \tag{15}$$

It is notable that in the aforementioned equation, the constant coefficients a_0 to a_8 are obtained by applying the introduced boundary conditions. For this part to write the equations which will be obtained through the solving procedure because the equation become prolongation we write the equations with the mentioned physical ailments as the previous part which is

$$\varepsilon=0.2. \tag{16}$$

3.2 Applying boundary conditions

In AGM, the boundary conditions are applied in order to compute constant coefficients of Eq. (15) in two ways as follows

(i) Applying the boundary conditions on Eq. (16) is expressed as follows

$$\theta=\theta(B.C). \tag{17}$$

It is notable that BC is the abbreviation of boundary conditions. According to the above explanations, the boundary conditions are applied on Eq. (15) in the following form

$$\theta(0)=1\rightarrow a_0=1. \tag{18}$$

(ii) Applying the boundary condition on the main differential equation, which is this case study is Eq. (14), and also on its derivatives is done after substituting Eq. (15) into the main differential equation as follows

$$g(\theta(\tau)):\rightarrow g(\theta(B.C)),g'(\theta(B.C)). \tag{19}$$

So after substituting Eq. (15) which has been considered as the answer of the main differential equation into Eq. (14), the initial conditions are applied on the obtained equation and also on its derivatives on the basis of Eq. (19)

as follows.

$$g(\theta'(0)) :\rightarrow a_1 + 0.2a_0a_1 + a_0 = 0 \quad (20)$$

$$g'(\theta'(0)) :\rightarrow 2a_2 + 0.2a_1^2 + 0.4a_0a_2 + a_1 = 0 \quad (21)$$

$$g''(\theta'(0)) :\rightarrow 6a_3 + 1.2a_1a_2 + 1.2a_0a_3 + 2a_2 = 0 \quad (22)$$

$$g'''(\theta'(0)) :\rightarrow 24a_4 + 2.4a_2^2 + 4.8a_1a_3 + 4.8a_0a_4 + 6a_3 = 0 \quad (23)$$

$$g^{(4)}(\theta'(0)) :\rightarrow 120a_5 + 24a_2a_3 + 24a_1a_4 + 24a_0a_5 + 24a_4 = 0 \quad (24)$$

$$g^{(5)}(\theta'(0)) :\rightarrow 720a_6 + 72a_3^2 + 144a_2a_4 + 144a_1a_5 + 144a_0a_6 + 120a_5 = 0. \quad (25)$$

By solving a set of algebraic equations which is consisted of eight equations with eight unknowns from Eq. (20) to Eq. (25), the constant coefficients of Eq. (15) can easily be gained as follows

$$\begin{array}{l} a_0 = 1, a_1 = -0.83333333, a_2 = 0.2893518519, a_3 = -0.04018775720 \\ a_4 = -0.004186224709, a_5 = 0.002054351014, a_6 = 0.00006729399287. \end{array} \quad (26)$$

Eq. (15) which is the solution of the proposed problem is rewritten form of

$$\begin{array}{l} \theta(\tau) = 0.00006729399287\tau^6 + 0.002054351014\tau^5 - 0.004186224709\tau^4 \\ -0.04018775720\tau^3 + 0.2893518519\tau^2 - 0.83333333\tau + 1 \end{array} \quad (27)$$

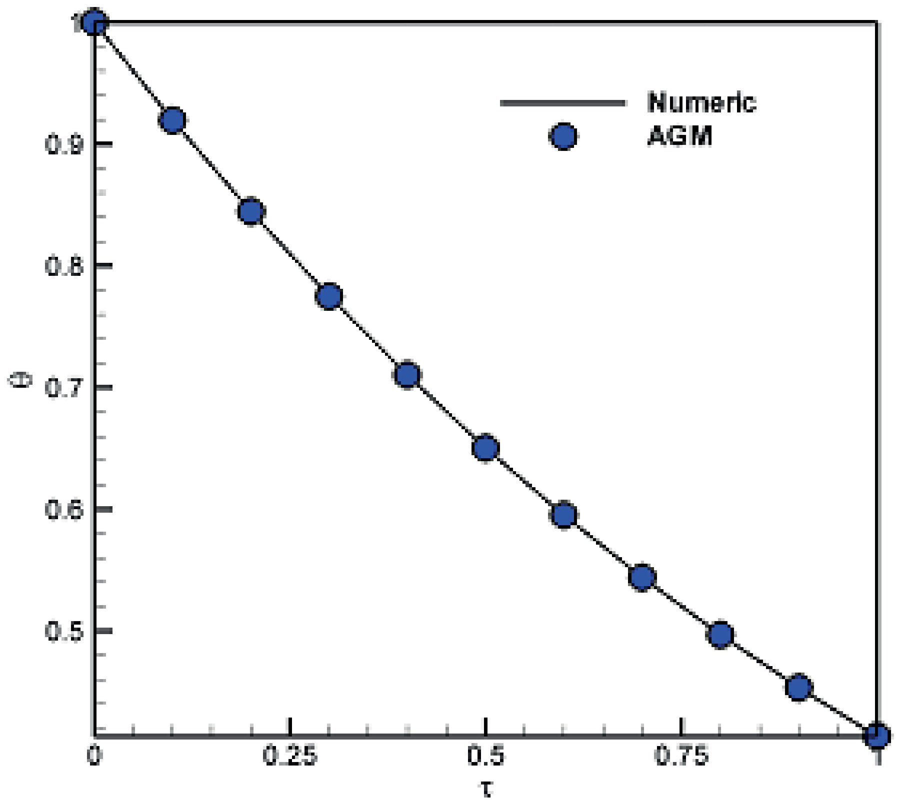

Fig. 1: Comparison of the solution via AGM and exact solution for $\theta(tau)$ at $\varepsilon = 0.2$.

Table 2: A view of Errors of AGM results in comparison Numeric method

	Numeric	AGM	Error
0	1	1	0
0.1	0.919519597379591	0.9195195994	0.0000000021
0.2	0.844579783987417	0.8445798691	0.0000001007
0.3	0.774927625974716	0.7749277299	0.0000001340
0.4	0.710304792769913	0.7103050914	0.0000004203
0.5	0.650450354716120	0.6504514376	0.0000016648
0.6	0.595102981629154	0.5951064624	0.0000058490
0.7	0.544002802301944	0.5440127527	0.0000182910
0.8	0.496893762198439	0.4969185197	0.0000498245
0.9	0.453524788397722	0.4535803795	0.0001225756
1	0.413651800613856	0.4137661817	0.0002765154

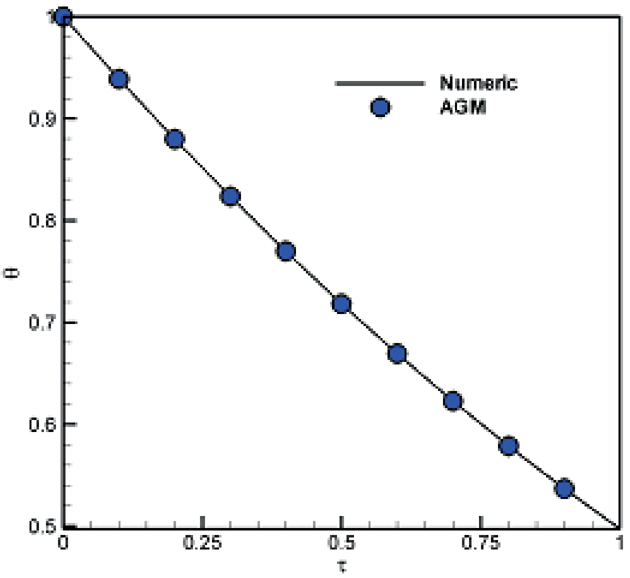

Fig. 2: Comparison of the solution via AGM and exact solution for $\theta(tau)$ at $\varepsilon = 0.6$.

4 Results and discussion

In Fig.1 the comparison of the solution between AGM and Exact results is shown and we gained a very interesting agreement between the results is observed, which with consideration of its results error in comparison with exact result from Table.1 it would confirms the excellent validity of the AGM and also in Fig.2 we compared AGM results with Exact solution with different small parameter and we found out the AGM has accurate results for various of small parameter and finally in Fig.3 we investigated a comparison amongst the obtained $\theta(tau)$ for various of ε.

5 Conclusion

In this paper, a new method (AGM) has been proposed for solving a nonlinear differential equation. The afore-mentioned procedures have been done to indicate the ability of AGM for solving differential equation which is shown in the relevant

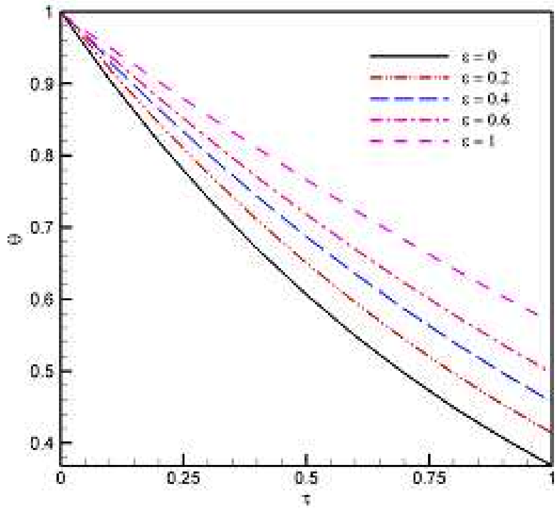

Fig. 3: Comparison of the solution via AGM and exact solution for $\theta(tau)$ for different ε.

plots and table, we could certainly claim that the obtained answer by this method is close to numerical solution. There are explicit advantages in this approach in comparison to the other semi-analytic methods such as: 1) Differential equations are directly solvable by this method; 2) with regard to the accretion of series sentences, the precision of the solution increases significantly; 3) it is not necessary to convert variables into new ones; 4) without any dimensionless procedures, we can solve the problem. In this method, the shortage of boundary condition(s) for solving differential equation(s) is terminated completely. The AGM is operational for miscellaneous nonlinear differential equations which we are hopeful in the near future and will be applied by enthusiastic young researchers.

Competing interests

The authors declare that they have no competing interests.

Authors' contributions

All authors have contributed to all parts of the article. All authors read and approved the final manuscript.

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