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A STATISTICAL MODEL OF OCCUPATIONAL MOBILITY - A SALARY BASED MEASURE

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Abstract

Mobility models are very useful in explaining the movements of people over socio-economic and job categories. Occupational mobility deals with the movements of individuals over job categories during their employment periods. Since the time interval between successive job changes is a random variable, different occupational mobility models have been developed by scientists using modified Markov and semi-Markov processes. This phenomenon can be modelled by considering the underlying factors such as job satisfaction, salary, distance of the work place, family requirements and others. Unlike most of the previous works in this area, the present study suggests a new measure of occupational mobility based on the distribution of wages. Here a general occupational mobility model has been developed to study the pattern of mobility during the service life of employees. First the probability distribution of the number of job changes in the entire employment life of individuals has been obtained considering the inter-job offer times (within an interval) and the associated wages as random variables. Then a measure of occupational mobility based on this distribution has been developed. The results are obtained under both frequentist and Bayesian frameworks. As an application of the proposed model the results in this paper have been illustrated by using data from a recent survey among the staff members of the University of Southern Queensland, Australia.

Keywords: Occupational mobility; Distribution of wages and job offers; Measure of job changes; Distribution of order statistics; Geometric and Gamma distributions; Survey data.

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1. Introduction

In recent years there has been growing interest in the study of manpower planning. Occupational mobility plays a central role in manpower planning. In broader terms, it refers to the movement of employees between jobs. As a consequence of globalization and the expansion of the job market in the non-traditional sectors, the phenomenon of changing jobs has gained greater attention of the researchers and planners.

Unlike social mobility (cf. Prais, [18]), there is no fixed time interval between successive moves in occupational mobility. Hodge [16] studied occupational mobility as a probability process. Stewman [20] discussed occupational mobility using a Markov model. A comprehensive summary of the theoretical developments and practical applications of occupational mobility have been provided by Stewman [21]. Ginsberg [7–12] has made several attempts to describe occupational mobility patterns in terms of semi-Markov processes.

Bartholomew [2] suggested measures of occupational mobility based on the matrix of transition probabilities and the stochastic process $[m(t)]$, where $m(t)$ is the random number of time points at which individuals decide to change their existing employment during the interval $(0, t)$. Mukherjee and Chattopadhyay [16] developed a measure by considering successive changes in occupation of an individual as constituting a renewal process. Later, Mukherjee and Chattopadhyay [17] proposed a measure based on the reward structure. Chattopadhyay and Baidya [3] considered salary based occupational categories to study social mobility. Khan and Chattopadhyay [14] developed a predictive measure of occupational mobility based on the number of job offers. None of the measures available in the literature has taken into account the explicit role of the reward (remuneration) associated with job offer that directly influences the pattern of job changes.

In this study we propose a new measure of occupational mobility based on the number of job changes and the associated wages. The distribution of the total number of job changes during an individual's entire service life up to the point of study has been derived by considering times between consecutive job offers (within an interval) and the corresponding wages to be independent random variables. The proposed measure of occupational mobility based on the number of job offers and wage distribution has been suggested primarily in the general setup and then it's value has been derived for special situations where some specific assumptions regarding the number of job offers and wage distributions has been made, both under frequentist and Bayesian frameworks.

A survey has been conducted among the staff members of the University of Southern Queensland, Australia to collect data on occupational mobility. The proposed model has been fitted to the survey data to explain the occupational mobility pattern among the employees of the University. For this particular application of the occupational mobility model the number of job offers has been found to best fit the geometric distribution, and the wages best fit a gamma distribution.

The occupational mobility model has been defined and discussed in section 2. The distribution of the number of job changes is obtained in section 3. Section 4 derives a measure of occupational mobility under the general distributional setup. Some special cases for particular choices of number of job offers and wages distributions have been discussed in sections 5 and 6, under frequentist and Bayesian frameworks, respectively. Section 7 is devoted to the analysis of the survey data, and its fitting to the proposed model and measure.

2. The Model

Let the service life of an individual be comprised of k intervals of equal fixed width, t. The individual gets at least one job offer within each such interval, the worth of an offer being determined by the associated salary (reward). The individual (assumed to be in service already) decides to leave the present job or not, at the end of each interval. One moves to a new job for the first time at the end of an interval in which the maximum of the remunerations associated with different job offers (within that interval) exceeds a fixed amount. This is the minimum wage at which the individual is willing to enter the job market for the first time.

Subsequently, one changes the current job at the end of a particular interval only when the maximum of the wages associated with the offers received during that interval exceeds the wage of the current job. A change of job in this paper means that an individual may move from one occupation to another or within the same occupation.

Let the individual get N_i new job offers in the i^{th} interval, and let X_{ij} be the salary corresponding to the j^{th} job offer in the i^{th} interval, for $j = 1, 2, \ldots, n_i$, and $i = 1, 2, \ldots, k$. Note that to reflect the real life situation it is necessary to assume that n_i is strictly greater than zero since no one can enter into the job market without a job offer. Both X_{ij} and N_i are assumed to be independently and identically distributed with pdf $g(x)$, $0 < x < \infty$, and pmf $h(y)$, $y = 1, 2, \ldots, \infty$ respectively. Define

$$Z_i = \max(X_{i1}, X_{i2}, \ldots, X_{in_i}). \tag{2.1}$$

Here Z_i is the maximum wage of all job offers during the i^{th} interval. Since Z_i is the largest order statistic, for a given n_i, the pdf of the conditional distribution of Z_i is

$$f(z_i|n_i) = n_i[G(x_{ij})]^{n_i-1} g(z_i),$$

where $G(\cdot)$ is the cdf of the distribution of X_{ij}. The pdf of the joint distribution of Z_i and N_i becomes

$$f(z_i, n_i) = n_i[G(x_{ij}]^{n_i-1} g(z_i) h(n_i).$$

Hence the marginal distribution of Z_i is given by

$$f(z_i) = \sum_{n_i=1}^{\infty} n_i[G(z_i)]^{n_i-1} g(z_i) h(n_i), \tag{2.2}$$

where $g(\cdot)$ and $h(\cdot)$ have the same specifications as before.

Let $F_{Z_i}(z)$ denote the the corresponding cdf. Let z_0 be the minimum wage for which the individual accepts the first job offer at the i^{ih} interval. Then we can define

$$F_{Z_i}(z_0) = P[Z_i < z_0] \tag{2.3}$$

and its complement

$$\overline{F}_{Z_i}(z_0) = 1 - F_{Z_i}(z_0) = P[Z_i > z_0]. \tag{2.4}$$

3. Distribution of the number of job changes

In this section we derive the distribution of the number of job changes during the service life of an individual. Define $N(k)$ = total number of job changes within the service life of the individual and $p_r^{(k)}$ = the probability of r job changes in the entire service life of the individual. Then

$$p_r^{(k)} = P[N(k) = r]. \tag{3.1}$$

3.1. Theorem. *Under the above definition of $F_{Z_i}(z_0)$ and $p_r^{(k)}$, we have*

$$p_r^{(k)} = \begin{cases} F^k & \text{if } r = 0, \\ F^{k-1}\bar{F}[\sum_{m=0}^{k-r}\binom{r+m-1}{m}(2F)^{-(r+m-1)}] & \text{if } 1 \le r \le k \end{cases} \tag{3.2}$$

where, for notational convenience, we write $F = F_{Z_i}(z_0)$ and $\bar{F} = 1 - F_{Z_i}(z_0)$.

Proof. (Outline) Note that $P[Z_i > Z_j] = P[Z_i < Z_j] = 0.5$ for $i, j = 1, 2, \dots, k,$, $i \neq j$; and

$$P[Z_i > \max(Z_1, Z_2, \dots, Z_{i-1})] = P[Z_i < \max(Z_1, Z_2, \dots, Z_{i-1}] = 0.5 \tag{3.3}$$

Now define S_i as the event that there is a job change in the i^{th} time interval which depends only on the maximum wages of the i^{th} and $(i-1)^{th}$ time intervals, for $i = 1, 2, \dots, k$ and T_i as the event that there is a job change in the i^{th} interval which depends on the maximum wages of all intervals up to the i^{th} including z_0, the initial minimum acceptable wage, for $i = 2, 3, \dots, k$, that is

$$S_i = [z_i > z_{i-1}], \text{ for } i = 1, 2, \dots, k \text{ and}$$
$$T_i = [z_i > \max(z_0, z_1, \dots, z_{i-1})], \text{ for } i = 2, 3, \dots, k.$$

Then $p_r^{(k)}$ can be obtained by adding together the probabilities of the $(k - r + 1)$ events E_0, $E_1 \dots, E_{k-r}$, where

E_0 = there is no change in the first $(k - r)$ intervals and r changes in the last r intervals.

E_1 = there is no change in the first $(k - r - 1)$ intervals, one change at the $(k - r)^{th}$ interval and $(r - 1)$ changes among the last r intervals.

E_2 = there is no change in the first $(k-r-2)$ intervals, one change at the $(k-r-1)^{th}$ interval and $(r - 1)$ changes among the last $(r + 1)$ intervals.

....

E_{k-r} = there is a change at the first interval and $(r - 1)$ changes among the last $(k - 1)$ intervals.

Let S^c denote the complement of the event S. Then from the fundamental rule of probability we have

$$\begin{aligned} P(E_0) &= P[S_1^c S_2^c \cdots S_{k-r}^c T_{k-r+1} S_{k-r+1} \cdots S_k] \\ &= F^{k-r}\bar{F}(0.5)^{r-1} \end{aligned} \tag{3.4}$$

Similarly,

$$P(E_1) = F^{k-r-1}\bar{F}\binom{r}{1}(0.5)^r$$

$$P(E_2) = F^{k-r-2}\bar{F}\binom{r+1}{2}(0.5)^{r+1}$$

....

$$P(E_{k-r}) = \overline{F}\binom{r + (k - r - 1)}{k - r}(0.5)^{r+(k-r-1)} \tag{3.5}$$

Hence the proof is completed by adding the above probabilities. □

3.2. Illustration. Consider the situation when $k = 3$ and $r = 2$.

$$\begin{aligned}
p_2^{(3)} &= P(S_1S_2S_3^c) + P(S_1S_2^cT_3) + P(S_1^cT_2S_3)\\
&= P(Z_1 > z_0)P(Z_2 > Z_1)P(Z_3 < Z_2)\\
&\quad + P(Z_1 > z_0)P(Z_2 < Z_1)P(Z_3 > Z_1) + P(Z_1 < z_0)P(Z_2 > z_0)P(Z_3 > Z_2)\\
&= \overline{F}(0.5)(0.5) + \overline{F}(0.5)(0.5) + F\overline{F}(0.5)\\
&= (0.5)\overline{F}(1 + F) \qquad (3.6)
\end{aligned}$$

When $r = 0$,

$$\begin{aligned}
p_0^{(3)} &= P(S_1^cS_2^cS_3^c)\\
&= P(Z_1 < z_0)P(Z_2 < z_0)P(Z_3 < z_0) = F^3 \qquad (3.7)
\end{aligned}$$

3.3. Theorem. *Under the above setup, $p_r^{(k)}$ is a probability distribution, i.e.*

$$\sum_{r=0}^{k} p_r^{(k)} = 1 \qquad (3.8)$$

Proof. Write $s = r + m - 1$. Then

$$\begin{aligned}
\sum_{r=0}^{k} p_r^{(k)} &= F^k + F^{k-1}\overline{F}\Bigg[\sum_{r=1}^{k}\sum_{m=0}^{k-r}\binom{r+m-1}{m}(0.5F)^{(r+m-1)}\Bigg]\\
&= F^k + F^{k-1}\overline{F}\Bigg[\sum_{s=0}^{k-1}(0.5F)^s\sum_{m=0}^{s}\binom{s}{m}\Bigg] = F^k + F^{k-1}\overline{F}\Bigg[\sum_{s=0}^{k-1}(0.5F)^s 2^s\Bigg]\\
&= F^k + F^{k-1}\overline{F}\Bigg[\sum_{s=0}^{k-1}(1/F)^s\Bigg] = F^k + 1 - F^k = 1. \qquad (3.9)
\end{aligned}$$

□

4. A Measure of Occupational Mobility

In this section we obtain a measure of occupational mobility using $p_r^{(k)}$ as defined in section 3. From the previous specifications the moments of the number of job changes are reasonable choices as measures of occupational mobility. For practical reasons, the first raw moment has better intuitive appeal in interpreting the phenomenon of job changes than any other moment. Therefore, we suggest, the expectation of the number of job changes can be considered as a measure of occupational mobility. This should of course be normalized with respect to k. Then we have

$$\begin{aligned}
\mathrm{E}[N(k)] &= \sum_{r=0}^{k} r p_r^{(k)}\\
&= F^{k-1}\overline{F}\sum_{r=1}^{k}\sum_{m=0}^{k-r} rC_m^{(r+m-1)}(0.5F)^{r+m-1}\\
&= [(k+1)\overline{F} - \overline{F}F^k - F(1 - F^k)]/2\overline{F}. \qquad (4.1)
\end{aligned}$$

In the computation of $\mathrm{E}[N(k)]$ various binomial and geometric series are involved. After normalization with respect to k, the measure becomes $\mathrm{E}[N(k)/k]$.

In a similar way it can be shown that,

$$\mathrm{E}[\{N(K)\}^2] = (0.25)F^{k-1}\overline{F}\sum_{s=0}^{k-1}(1/F)^s(s^2 + 5s + 4). \qquad (4.2)$$

Hence the variance of $N(k)$, $\mathrm{Var}[N(k)]$, is readily available from (4.1) and (4.2). As a measure of spread of the above measure of occupational mobility one uses the estimated value of $\mathrm{Var}[N(k)]/k^2$. Computing procedures for $\mathrm{E}[N(k)]$, and $\mathrm{Var}[N(k)]$ are given in section 7.

5. Some special cases

Case 1: To compute the measure of mobility, in this section, we consider specific distributions for the number of job offers and for the wages. Consider the situation where the distribution of wages is exponential and the distribution of the number of job offers is truncated Poisson with the following pdf and pmf respectively,

$$g(x) = \theta \mathrm{e}^{-\theta x}, \quad 0 < x < \infty \tag{5.1}$$

and

$$h(y) = [1/(1-\mathrm{e}^{-\lambda t})]\mathrm{e}^{-\lambda t}(\lambda t)^y/y!, \quad y = 1, 2, \ldots, \infty. \tag{5.2}$$

Note that $y = 0$ is not a valid value of the number of job offers since by assumption the study includes only those individuals who received at least one job offer. Then from (2.2) the pdf of the distribution of Z_i is

$$f(z_i) = [1/(1-\mathrm{e}^{-\lambda t})]\lambda t \theta \mathrm{e}^{-(\theta z_i + \lambda t)} \mathrm{e}^{\lambda t}(1-\mathrm{e}^{-\theta z_i}), \quad 0 < z_i < \infty, \tag{5.3}$$

and the corresponding cdf is

$$F_{Z_i}(z) = [1/(1-\mathrm{e}^{-\lambda t})](\mathrm{e}^{-\lambda t \mathrm{e}^{-\theta z}} - \mathrm{e}^{-\lambda t}). \tag{5.4}$$

Hence from (3.2) and (3.3) we have

$$p_r^{(k)} = \begin{cases} ((\mathrm{e}^{-\lambda t \mathrm{e}^{-\theta z_0}} - \mathrm{e}^{-\lambda t})/(1-\mathrm{e}^{-\lambda t}))^k & \text{if } r = 0, \\ \left[\frac{(1-\mathrm{e}^{\lambda t \mathrm{e}^{-\theta z_0}})(\mathrm{e}^{-\lambda t \mathrm{e}^{-\theta z_0}} - \mathrm{e}^{-\lambda t})^{k-1}}{(1-\mathrm{e}^{-\lambda t})^k}\right] & \\ \times \left[\sum_{m=0}^{k-r} \binom{r+m-1}{m} \left(\frac{(1-\mathrm{e}^{-\lambda t})}{2(\mathrm{e}^{-\lambda t \mathrm{e}^{-\theta z_0}} - \mathrm{e}^{-\lambda t})}\right)^{r+m-1}\right] & \text{if } 1 \le r \le k. \end{cases} \tag{5.5}$$

Now, from (5.4), (4.1) becomes

$$\mathrm{E}[N(k)] = \left[(k+1) - (k+2)\frac{\mathrm{e}^{-\lambda t \mathrm{e}^{-\theta z_0}} - \mathrm{e}^{-\lambda t}}{(1-\mathrm{e}^{-\lambda t})} + \frac{(\mathrm{e}^{-\lambda t \mathrm{e}^{-\theta z_0}} - \mathrm{e}^{-\lambda t})}{(1-\mathrm{e}^{-\lambda t})} \times \eta_1\right] \times \eta_2, \tag{5.6}$$

where

$$\eta_1 = \frac{2(\mathrm{e}^{-\lambda t \mathrm{e}^{-\theta z_0}} - \mathrm{e}^{-\lambda t})}{(1-\mathrm{e}^{-\lambda t})} - 1, \quad \eta_2 = \frac{1-\mathrm{e}^{-\lambda t}}{2(1-\mathrm{e}^{-\lambda t \mathrm{e}^{-\theta z_0}})}. \tag{5.7}$$

Given the values of the parameters λ and θ one can compute the value of the above measure of occupational mobility for different choices of t. The parameters can be estimated from the sample data. For details refer to Cohen [4].

Case 2: Here we take the distribution of wages as exponential and the number of job changes as truncated Binomial. For the above choices we have

$$g(x) = \theta \mathrm{e}^{-\theta x}, \quad 0 < x < \infty, \tag{5.8}$$

and

$$h(y) = (1/(1-q^{kt}))\binom{kt}{y} p^y q^{kt-y}, \quad y = 1, 2, \ldots, kt, \tag{5.9}$$

where $q = 1 - p$.

Then from (2.2) the pdf of the distribution of Z_i is given by

$$(5.10) \qquad f(z_i) = [(\theta q^{kt-1})/(1-q^{kt})]\mathrm{e}^{-\theta z_i} kt(p/q) \sum_{n_{1i}=0}^{kt-1} \binom{kt-1}{n_{1i}} [(p/q)(1-\mathrm{e}^{-\theta z_i})]^{n_{1i}},$$

where $n_{1i} = n_i - 1$ and the corresponding cdf is given by

$$(5.11) \qquad F(z) = [(\theta kt q^{kt})/(1-q^{kt})] \sum_{n_{1i}=0}^{kt-1} (p/q)^{n_{1i}+1} \int_0^z \mathrm{e}^{-\theta z}(1-\mathrm{e}^{-\theta z})^{n_{1i}} dz.$$

Hence following the same procedure as in case 1, $E[N(k)/k]$ can be computed for given values of the parameters θ and p.

6. Special cases under the Bayesian framework

The occupational mobility measure for the above special cases of the distributions of job offers and wages are obtained here under the Bayesian framework.

Case 1: Consider the special case 1 of section 5 with

$$(6.1) \qquad g(x) = \theta \mathrm{e}^{-\theta x}, \quad 0 < x < \infty, \quad \theta > 0.$$

Taking the conjugate prior associated with the Poisson distribution, λ has the following pdf,

$$(6.2) \qquad p(\lambda) = (1/\beta^{\alpha}\Gamma\alpha)\mathrm{e}^{-\lambda/\beta}\lambda^{\alpha-1}, \quad \alpha > 0, \ \beta > 0, \quad 0 < \lambda < \infty.$$

For a fixed value of λ, the joint pmf of the distribution of N_i on the basis of a sample of size m is given by,

$$(6.3) \qquad h(y|\lambda) = [1/(1-\mathrm{e}^{-\lambda t})^m]\mathrm{e}^{-m(\lambda t)}(\lambda t)^{\sum_{i=1}^m y_i}/\Pi_{i=1}^m(y_i!)$$

and hence the Bayes estimator of λ is obtained as,

$$\lambda^B = \frac{1}{t} \times \frac{\int \lambda h(y|\lambda)p(\lambda)d\lambda}{(\int h(y|\lambda)p(\lambda)d\lambda)}$$

$$(6.4) \qquad = \frac{1}{t}\frac{[\Gamma(\sum y_i + \alpha + 1)/(m+\frac{1}{\beta})^{\sum y_i + \alpha + 1}] + \Omega_1}{[\{\Gamma(\sum y_i + \alpha)/(m+\frac{1}{\beta})^{\sum y_i+\alpha}\} + \Omega_2]},$$

where

$$\Omega_1 = \sum_{j=1}^{\infty}\left[(1/j!)m(m+1)\dots(m+j-1)\left(\frac{\Gamma(\sum y_i + \alpha + 1)}{(m+\frac{1}{\beta}+j)^{\sum y_i+\alpha+1}}\right)\right],$$

$$\Omega_2 = \sum_{j=1}^{\infty}\left[(1/j!)m(m+1)\dots(m+j-1)\left(\frac{\Gamma(\sum y_i + \alpha)}{(m+\frac{1}{\beta}+j)^{\sum y_i+\alpha}}\right)\right].$$

Note that λ^B can be estimated from the sample values of y_i for some known values of the prior parameters α and β.

Hence $E[N(k)]$ in (5.6) can be estimated by replacing λ with λ^B. If needed, starting with initial values of α and β (say α_0 and β_0) one can generate a sample from the gamma distribution with parameters α_0 and β_0 and on the basis of the simulated sample α and β can be estimated.

Case 2: Considering the special case 2 of section 5 we have

$$(6.5) \qquad g(x) = \theta \mathrm{e}^{-\theta x}, \quad 0 < x < \infty \quad \theta > 0.$$

Assuming that p is a random variable, using the conjugate prior associated with the Binomial distribution, p has the following pdf

$$p^*(p) = [1/B(a,b)]p^{a-1}(1-p)^{b-1}, \quad 0 < p < 1,\ a > 0,\ b > 0. \tag{6.6}$$

Given p, the joint pmf of the distribution of N_i for a sample of size m is given by

$$h(y_i|p) = [1/(1-(1-p)^{kt})^m] \prod_{i=1}^{m} \binom{kt}{y_i} p^{\sum y_i}(1-p)^{mkt-\sum y_i}, \tag{6.7}$$

where $y_i = 1, 2, \ldots, kt$. The Bayes estimator of p is found to be

$$p^B = \frac{B(a+\sum y_i+1, b+mkt-\sum y_i)+\Psi_1}{B(a+\sum y_i, b+mkt-\sum y_i)+\Psi_2}, \tag{6.8}$$

where

$$\Psi_1 = \sum_{j=1}^{\infty} \frac{m(m+1)\ldots(m+j-1)B(a+\sum y_i+1, b+kt(m+j)-\sum y_i)}{j!}$$

$$\Psi_2 = \sum_{j=1}^{\infty} \frac{m(m+1)\ldots(m+j-1)B(a+\sum y_i, b+kt(m+j)-\sum y_i)}{j!}.$$

Note that p^B can be estimated from the sample values of y_i for some given values of the prior parameters a and b. Hence $E[N(k)]$ can be estimated by replacing p with p^B in (5.11). If needed, one can generate a sample from the Beta distribution with initial values a_0 and b_0 as parameters and on the basis of that simulated sample a and b can be estimated.

7. Modelling Survey Data

With a view to applying the proposed measure of occupational mobility, a survey among the employees of the University of Southern Queensland (USQ), Australia was conducted. The main objective of the survey was to gather data on the number of job offers received by the individual employees during the entire employment period, including the offer(s) of the current employer. In addition, wages associated with each of the job offers for the employees were collected. The data on the number of job offers and the associated wages have been classified according to the staff category, academic and non-academic, as well as gender, male and female. Separate analysis of the data have been provided based on the above four categories and the values of the proposed measure of occupational mobility have been obtained for all those cases. An overall analysis of the data from all the respondents across the categories has been also been provided.

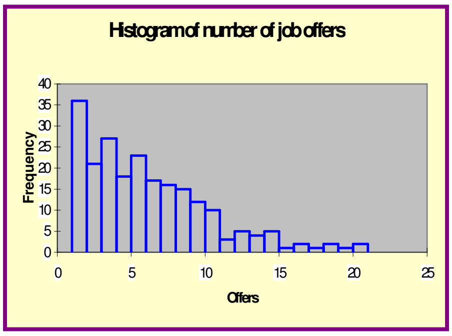
The sample consisted of 221 employees of the USQ. This comprises of 83 academic and 138 non-academic staff. Among these respondents there were 93 males and 128 females. In the survey we limited the maximum number of offers for any individual to 20 and the range of wages has been equally divided into 6 intervals. In the computation of the measure of occupational mobility and all associate functions as well as fitting of different distributions we have used the MATLAB and SPSS packages. Some Pascal programming has also been used for the fitting of the distribution.

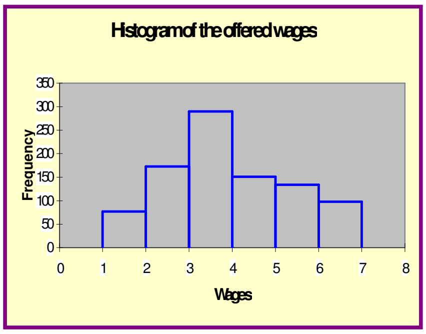
7.1. The Survey. The general distribution of the survey data with respect to the number of job offers and associated wages is provided in this subsection. Table 1 below represents the means and standard deviations of the number of job offers of the respondents by various categories.

Table 1. Summary Statistics of the Number of Job Offers by Gender and Staff Category

	Academic			Non-Academic			Total		
	Count	Mean	Std.	Count	Mean	Std.	Count	Mean	Std.
Male	58	5.40	4.26	35	6.20	4.31	93	5.70	4.27
Female	25	6.20	4.07	103	5.60	4.24	128	5.72	4.20
Total	83	5.64	4.20	138	5.75	4.25	221	5.71	4.22

The average number of job offers received by the employees in the sample is 5.71. The corresponding figure for the academics is 5.64 and that of the non-academics is 5.75. Thus the average number of job offers for non-academics is slightly higher than the academics. The female respondents have a higher average number of job offers than their male counterparts, although the difference is negligible. However, a clear dominance of the females with respect to the average number of job offers over the males is observed





The distributions of the number of job offers and the wages are given in Figure 1. The first graph in Figure 1 shows that the distribution of the number of job offers is highly skewed to the right. From the shape of the distribution it appears that the geometric distribution would be an appropriate model for the data.

The second graph gives the observed distribution of the wages of all employees.

7.2. Fitting of Distributions. First we have fitted the geometric distribution with pmf

$$h(y) = p(1-p)^{y-1}, \quad y = 1, 2, 3, \ldots, \infty \tag{7.1}$$

to the number of job offers for all respondents as well as by all the categories. Here the estimate of the parameter p is $\hat{p} = \frac{1}{\bar{y}}$ where $\bar{y}$ is the sample mean of the observed number of job offers.

From Table 2 it is evident that the data fit very well with the geometric distribution for all respondents as well over all the categories.

Table 2. Table of Expected and Observed Frequency Distributions of Job Offers by Gender and Staff Category

	Academics		Non-Acads.		Male		Female		Total	
Offers	Exp.	Obs.	Exp.	Obs.	Exp.	Obs.	Exp.	Obs.	Exp.	Obs.
1	15	12	25	24	16	15	22	21	36	39
2	12	9	20	12	13	8	18	13	21	32
3	10	11	17	16	11	14	15	13	27	26
4	8	6	14	12	9	7	13	11	18	22
5	7	11	11	12	8	9	10	14	23	18
6	6	5	9	12	6	8	9	9	17	15
7	5	7	8	9	5	6	7	10	16	12
8	4	6	6	9	4	6	6	9	15	10
9	3	3	5	9	3	5	5	7	12	8
10	3	4	4	6	3	3	4	7	10	7
11	2	2	3	1	2	1	3	2	3	6
12	2	1	3	4	2	3	3	2	5	5
13	1	0	2	4	2	2	2	2	4	4
14	1	2	2	3	1	2	2	3	5	3
15	1	0	2	1	1	0	2	1	1	3
16	1	2	1	0	1	2	1	0	2	2
17	1	0	1	1	1	0	1	1	1	2
18	1	1	1	1	1	1	1	1	2	1
19	0	0	1	1	1	0	1	1	1	1
20	0	1	1	1	0	1	1	1	2	1

Then we have fitted the gamma distribution with pdf

$$g(x) = \frac{1}{\beta^\alpha \Gamma(\alpha)} \mathrm{e}^{-\frac{x}{\beta}} x^{\alpha-1}, \quad 0 < x < \infty,\ \alpha > 0,\ \beta > 0 \tag{7.2}$$

to the wages for all respondents as well as by all the categories. Salem and Mount [19], and McDonald and Jensen [15] used gamma distribution to model the distribution of income. The estimates of the parameters α and β are $\hat{\alpha} = [\frac{\bar{x}}{s_x}]^2$ and $\hat{\beta} = [\frac{{s_x}^2}{\bar{x}}]$ respectively where $\bar{x}$ is the sample mean of the observed wages and s_x is the corresponding standard deviation. For further details see Cohen and Whitten [5]. Angle [1] used the two parameter gamma distribution to model the income distributions of blacks and the whites.

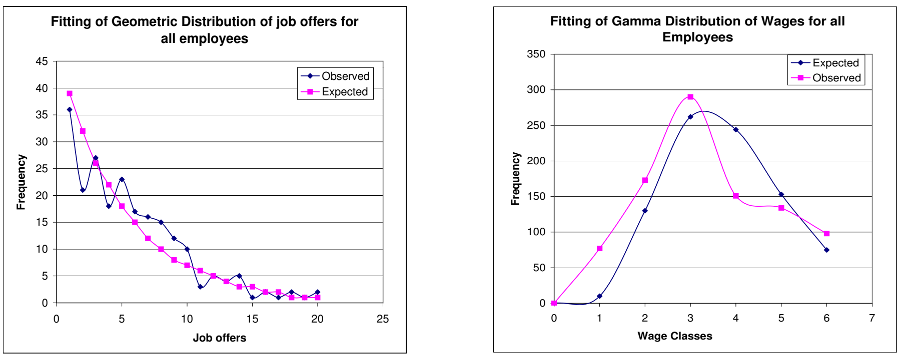
From Table 3 it is observed that the data fit more or less well with the gamma distribution for all respondents as well as over all the categories.

Table 3. Table of Expected and Observed Frequency Distributions of Wages by Gender and Staff Category

	Academics		Non-Acads		Male		Female		Total	
Wages	Exp.	Obs.	Exp.	Obs.	Exp.	Obs.	Exp.	Obs.	Exp.	Obs.
0	0	0	0	0	0	0	0	0	0	0
1	2	27	11	48	1	25	10	50	10	77
2	38	41	123	127	21	25	114	146	130	173
3	89	67	198	223	85	94	180	193	262	290
4	94	62	139	89	119	88	124	63	244	151
5	64	80	63	52	95	98	55	34	153	134
6	34	70	22	25	53	79	19	17	75	98

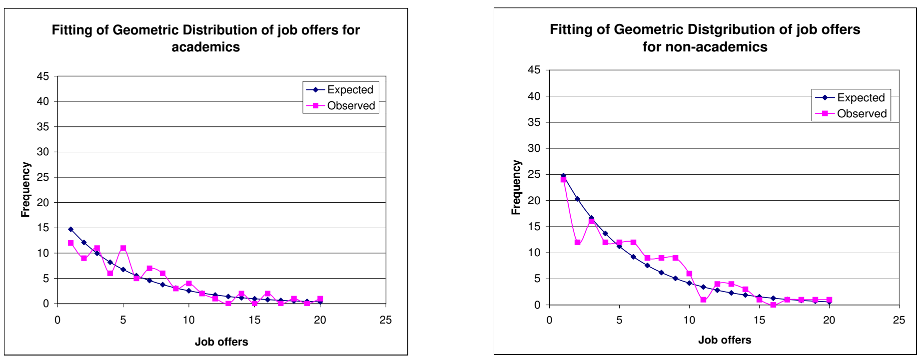
Figure 2 displays the observed and fitted distributions of the number of job offers and wages of all respondents.

Figure 2. Graphs of Observed and Fitted Distributions of Job Offers and

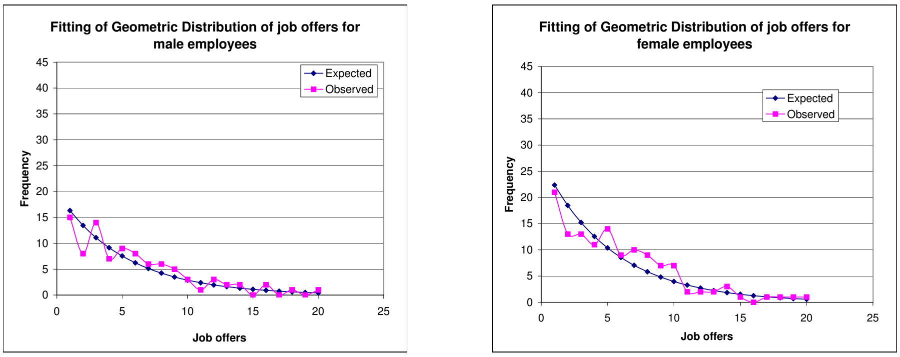


Looking at the observed and the fitted distributions of the number of job offers in the first graph it is evident that the geometric distribution fits the data very well. The same feature of the distributions of job offers for different categories of respondents is observed from Figure 3.

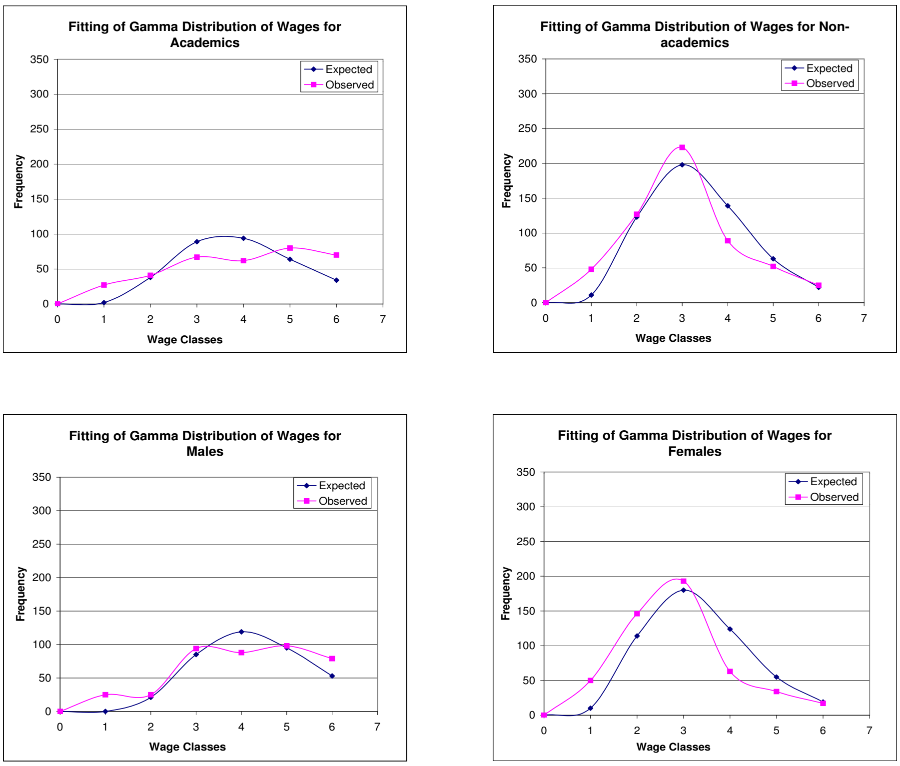
Figure 3. Graphs of Observed and Fitted Distributions of Job Offers by



The graphs of the observed and fitted distributions of job offers by gender are given in Figure 4.



In all the above mentioned graphs the geometric distribution provide a better fit than any other distribution for the number of job offers. The distributions of the observed and the expected values of the wages are given in Figure 5.



It is observed that the gamma distribution fits very well to the observed data for the non-academics and the female employees. Although for the other categories of respondents the fitting is not as good. Figure 1 displays that the observed data for the wages show some irregular pattern at some points. Hence the fitting of the gamma distribution is not so good for some parts in the right side of the distribution when all respondents are considered. Nonetheless empirically the gamma distribution provides a better fit than all other relevant distributions.

7.3. Computation of the Measure. Here we derive the expression for the measure of the occupational mobility under the above specifications of distribution of the number of job offers and wages. We also compute the values of the measure as well as the variance of the number of job changes during the employment period for all respondents. The same is also obtained for different categories of respondents to compare the mobility of employees over categories.

From (2.2), assuming α to be an integer (cf. Evan et al. [6], for instance), the pdf of the distribution of the maximum wage is

$$f(z_i) = \frac{1}{\beta^\alpha \Gamma(\alpha)} p e^{-\frac{z_i}{\beta}} z_i^{\alpha-1} \left[(1-p) \left(1 - \mathrm{e}^{-\frac{z_i}{\beta}} \sum_{r=0}^{\alpha-1} \frac{1}{r!} \left\{ \frac{z_i}{\beta} \right\}^r \right) \right]^{n_i - 1} \tag{7.1}$$

and the corresponding cdf is given by

$$F(z_0) = p \sum_{k=1}^{\infty} k(1-p)^{k-1} \tau_k(\alpha,\ \beta,\ \ z_0), \tag{7.2}$$

where

$$\tau_k(\alpha,\ \beta,\ \ z_0) = \int_0^{z_0} \frac{1}{\beta^\alpha \Gamma(\alpha)} \mathrm{e}^{-\frac{z_i}{\beta}} z_i^{\alpha-1} \left(1 - \mathrm{e}^{-\frac{z_i}{\beta}} \sum_{r=0}^{\alpha-1} \frac{1}{r!} \left\{ \frac{z_i}{\beta} \right\}^r \right)^{k-1} dz_i. \tag{7.3}$$

Table 4. Table of the Empirical Distribution Function, the Expectation, Variance of Job Changes and the Measure of Mobility by Different Categories

Variable	$\hat{\alpha}$	$\hat{\beta}$	$\hat{p}$	$F(z_0)$	$E[N(k)]$	$Var[N(k)]$	$E[\frac{N(k)}{k}]$
Academic	6.54	0.61	0.1773	0.00060	10.49969	4.73451	0.52498
Non-Acad.	6.38	0.48	0.1795	0.00200	10.49883	4.75000	0.52494
Male	8.23	0.50	0.1754	0.00020	10.49992	4.75000	0.52500
Female	6.00	0.48	0.1748	0.00400	10.49809	4.75000	0.52490
Overall	5.70	0.60	0.1751	0.00200	10.49896	4.75000	0.52495

Table 4 describes the expectation and variance of the number of job changes for all respondents as well as for the different categories, and the values of the proposed measure. From the expected values it appears that for any particular individual, on the average, the number of job offers with wage associated with a new job offer exceeding the maximum wage earned from a previous offer is a little over 10. The computed values of the measure for the survey data enable us to infer that an employee is more mobile (than expected in the job market) during his occupational life depending on whether the observed value of the measure (computed on the basis of the employee's job changes) exceeds the above computed value of the measure. This is valid under the assumption that the job changes occur only on the basis of wage consideration and for specific values of t and k. The same conclusion can be extended over all the categories considered in the study. The measure reveals the fact that the expected number of job changes is about the same for all employees regardless of gender and staff category.

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