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Chebyshev-type matrix polynomials and integral transforms

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Abstract

In this study we introduce a new type generalization of Chebyshev matrix polynomials of second kind by using the integral representation. We obtain their matrix recurrence relations, matrix differential equation and generating matrix functions. We investigate operational rules associated with operators corresponding to Chebyshev-type matrix polynomials of second kind. Furthermore, in order to give qualitative properties of this integral transform, we introduce the Chebyshev-type matrix polynomials of first kind.

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1. Introduction

Matrix generalization of special functions has become important in the last two decades. Extension of the matrix framework of the classical families of Hermite, Laguerre, Jacobi, Bessel, Gegenbauer and Pincherle matrix polynomials are introduced in [11, 14, 15, 19, 25, 21] and some generalized forms are studied in [1, 20, 22, 26, 28]. Moreover, some properties of the these matrix polynomials are given in [3, 4, 6, 7, 16, 24]. Chebyshev matrix polynomials of first kind are introduced by Defez and Jódar starting from the hypergeometric matrix function. Some properties such as Rodrigues formula, three-term recurrences relation and orthogonality property are studied in [10]. Second kind Chebyshev matrix polynomials are defined in [5] by using integral representation method. Furthermore generating matrix function and some families of bilinear and bilateral generating matrix functions for Chebyshev matrix polynomials of the second kind are derived in [2].

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Throughout this paper, the zero matrix and identity matrix will be denoted by $\mathbf{0}$ and I, respectively. If A is a matrix in $\mathbb{C}^{r\times r}$, its spectrum $\sigma(A)$ denotes the set of all eigenvalues of A. Its 2-norm is denoted by $\|A\|$ and defined by

$$\|A\| = \sup_{x\neq 0} \frac{\|Ax\|_2}{\|x\|_2},$$

where for a vector y in $\mathbb{C}^r$, $\|y\|_2 = \left(y^T, y\right)^{\frac{1}{2}}$ is the Euclidean norm of y. If $f(z)$ and $g(z)$ are holomorphic functions of the complex variable z, which are defined in an open set Ω of the complex plane and A is a matrix in $\mathbb{C}^{r\times r}$ such that $\sigma(A) \subset \Omega$, then from the properties of matrix functional calculus in [13, p. 558], it follows that $f(A)g(A) = g(A)f(A)$. Hence, if $B \in \mathbb{C}^{r\times r}$ is a matrix for which $\sigma(B) \subset \Omega$ and $AB = BA$, then $f(A)g(B) = g(B)f(A)$. Let A be a matrix in $\mathbb{C}^{r\times r}$ such that $\operatorname{Re}(z) > 0$ for every eigenvalues $z \in \sigma(A)$. Then we say that A is a positive stable matrix.

Let A be a positive stable matrix. Then two-variable Hermite matrix polynomials are defined in [5] by

$$H_n(x,y,A) = \sum_{k=0}^{\left[\frac{n}{2}\right]} \frac{(-1)^k n! \left(x\sqrt{2A}\right)^{n-2k} y^k}{k!\,(n-2k)!}, \tag{1.1}$$

which satisfy the recurrences

$$\frac{\partial}{\partial x} H_n(x,y,A) = \sqrt{2A}\, n H_{n-1}(x,y,A), \tag{1.2}$$

$$\frac{\partial}{\partial y} H_n(x,y,A) = -n(n-1) H_{n-2}(x,y,A), \tag{1.3}$$

$$H_{n+1}(x,y,A) = \left(x\sqrt{2A} - 2\left(\sqrt{2A}\right)^{-1} y \frac{\partial}{\partial x}\right) H_n(x,y,A). \tag{1.4}$$

Also, second order matrix differential equation

$$\left[y\frac{\partial^2}{\partial x^2} I - xA\frac{\partial}{\partial x} + nA\right] H_n(x,y,A) = \mathbf{0} \tag{1.5}$$

and the expression

$$\sum_{n=0}^{\infty} \frac{H_n(x,y,A)}{n!} t^n = \exp\left(xt\sqrt{2A} - yt^2 I\right) \tag{1.6}$$

are given in [5].

For a positive stable matrix A, the second kind Chebyshev matrix polynomials with two variables are defined in [5] by

$$U_n(x,y,A) = \sum_{k=0}^{\left[\frac{n}{2}\right]} \frac{(-1)^k (n-k)! \left(x\sqrt{2A}\right)^{n-2k} y^k}{k!\,(n-2k)!}. \tag{1.7}$$

These matrix polynomials satisfy integral representation

$$U_n(x,y,A) = \frac{1}{n!} \int_0^{\infty} e^{-t} t^n H_n\left(x, \frac{y}{t}, A\right) dt. \tag{1.8}$$

It has already been shown that most of the properties of $U_n(x,y,A)$, linked to the ordinary case by

$$U_n(x,y,A) = y^{\frac{n}{2}} U_n\left(\frac{x}{\sqrt{y}}, A\right),$$

can be directly inferred from those of the $H_n(x, y, A)$ and from the integral representation given in (1.8).

The aim of this paper is to introduce a generalization for Chebyshev matrix polynomials by modifiying the integral transform. The organization of this paper is as follows. In section 2, we define Chebyshev-type matrix polynomials of second kind and give an explicit expression, recurrence relations, matrix differential equation and generating matrix functions. Besides, we focus on two index two variable second kind Chebyshev-type matrix polynomials. Section 3 deals with operational identities which yield different view for Chebyshev-type matrix polynomials of second kind. Finally in section 4, we give the definition of the Chebyshev-type matrix polynomials of the first kind.

2. Second Kind Chebyshev-type Matrix Polynomials with Two-Variable

As already remarked, integral transform relating Chebyshev and Hermite matrix polynomials are not new. Therefore we can introduce a new generalization for the second kind Chebyshev matrix polynomials with two variables by modifiying the integral transfrom as:

$$U_n(x, y, A, B) = \frac{1}{n!}\int_0^\infty e^{-Bt} t^n H_n\left(x, \frac{y}{t}, A\right) dt, \tag{2.1}$$

where A and B are positive stable matrices in $\mathbb{C}^{r\times r}$ and $AB = BA$.

We note that for the case $A = [2]_{1\times 1}$ and $B = [1]_{1\times 1}$, the expression (2.1) coincides with the formula which was proved by Dattoli ([8]) for the scalar second kind Chebyshev polynomials with two variables.

The use of the identity (1.1) allows us the explicit expression for $U_n(x, y, A, B)$ in the form

$$U_n(x, y, A, B) = \sum_{k=0}^{\left[\frac{n}{2}\right]} \frac{(-1)^k (n-k)! B^{k-n-1} \left(x\sqrt{2A}\right)^{n-2k} y^k}{k!\,(n-2k)!}. \tag{2.2}$$

It is clear from (2.2) that

$$U_{-1}(x, y, A, B) = \mathbf{0}, \quad U_0(x, y, A, B) = B^{-1}, \quad U_1(x, y, A, B) = x\sqrt{2A}B^{-2}.$$

In addition, we can write

$$U_n(x, y, A, I) = U_n(x, y, A), \qquad U_n(x, 1, A, I) = U_n(x, A),$$
$$U_n(x, 0, A, B) = B^{-(n+1)}\left(x\sqrt{2A}\right)^n, \quad U_{2n}(0, y, A, B) = (-1)^n B^{-(n+1)} y^n.$$

In order to investigate some important properties, we give the generating matrix function of second kind Chebyshev-type matrix polynomials with two variables in the following proposition.

2.1. Proposition. *Let A and B be positive stable matrices in $\mathbb{C}^{r\times r}$ and $AB = BA$. Then the second kind Chebyshev-type matrix polynomials with two variables have the generating matrix function*

$$\sum_{n=0}^{\infty} U_n(x, y, A, B) z^n = \left(B - xz\sqrt{2A} + yz^2 I\right)^{-1}, \tag{2.3}$$

where $\left\| xz\sqrt{2A} - yz^2 I \right\| < \|B\|$.

Proof. Multiplying both sides of (2.1) by z^n, summing up over n, using (1.6) and then integrating over t, we have (2.3). $\square$

2.2. Theorem. *Let A and B be positive stable matrices in $\mathbb{C}^{r\times r}$ and $AB = BA$. Then the second kind Chebyshev-type matrix polynomials with two variables have the generating matrix function*

$$\sum_{n=0}^{\infty} \frac{(n+m)!}{n!m!} U_{n+m}(x,y,A,B)\, z^n$$
$$= \left(B - x\sqrt{2A}z + yz^2 I\right)^{-(m+1)} U_m\left(xI - \left(\sqrt{\frac{A}{2}}\right)^{-1} yz, \left(B - x\sqrt{2A}z + yz^2 I\right)y, A\right),$$

where $\left\|xz\sqrt{2A} - yz^2 I\right\| < \|B\|$.

Proof. From (2.1) we have,

$$\sum_{n=0}^{\infty} \frac{(n+m)!}{n!m!} U_{n+m}(x,y,A,B)\, z^n = \frac{1}{m!}\int_0^{\infty} e^{-Bt} t^m \sum_{n=0}^{\infty} H_{n+m}\left(x, \frac{y}{t}, A\right) \frac{(zt)^n}{n!} dt.$$

By using generalized form of the identity [18]:

$$\sum_{n=0}^{\infty} H_{n+m}(x,y,A) \frac{t^n}{n!} = \exp\left(x\sqrt{2A}t - yt^2 I\right) H_m\left(xI - \left(\sqrt{\frac{A}{2}}\right)^{-1} yt, y, A\right),$$

we have

$$\sum_{n=0}^{\infty} \frac{(n+m)!}{n!m!} U_{n+m}(x,y,A,B)\, z^n$$
$$= \frac{1}{m!}\int_0^{\infty} e^{-t\left(B - x\sqrt{2A}z + yz^2 I\right)} t^m H_m\left(xI - \left(\sqrt{\frac{A}{2}}\right)^{-1} yz, \frac{y}{t}, A\right) dt.$$

This completes the proof. $\square$

2.3. Corollary. *Let A be a positive stable matrix in $\mathbb{C}^{r\times r}$. Then the second kind Chebyshev matrix polynomials with two variables have the generating matrix function*

$$\sum_{n=0}^{\infty} \frac{(n+m)!}{n!m!} U_{n+m}(x,y,A)\, z^n$$
$$= \left(I - xz\sqrt{2A} + yz^2 I\right)^{-(m+1)} U_m\left(xI - \left(\sqrt{\frac{A}{2}}\right)^{-1} yz, \left(I - xz\sqrt{2A} + yz^2 I\right)y, A\right),$$

where $\left\|xz\sqrt{2A} - yz^2 I\right\| < 1$.

Now, let us get matrix recurrence relations for Chebyshev-type matrix polynomials with two-variable by using the integral representation.

2.4. Proposition. *Let A and B be positive stable matrices in $\mathbb{C}^{r\times r}$ and $AB = BA$. Then the second kind Chebyshev-type matrix polynomials with two variables satisfy*

$$y\frac{\partial}{\partial x} U_{n-1}(x,y,A,B) = \sqrt{\frac{A}{2}}\left(x\frac{\partial}{\partial x} - n\right) U_n(x,y,A,B) \tag{2.4}$$

and

$$x\frac{\partial}{\partial x} U_n(x,y,A,B) = \left(n - 2y\frac{\partial}{\partial y}\right) U_n(x,y,A,B). \tag{2.5}$$

Proof. From (2.1) and (1.2), we have

$$\begin{aligned} y\frac{\partial}{\partial x}U_{n-1}(x,y,A) &= \frac{1}{(n-1)!}\int_0^\infty e^{-Bt}t^{n-1}y\frac{\partial}{\partial x}H_{n-1}\left(x,\frac{y}{t},A\right)dt \\ &= \frac{\left(\sqrt{2A}\right)^{-1}}{n!}\int_0^\infty e^{-Bt}t^n\frac{y}{t}\frac{\partial^2}{\partial x^2}H_n\left(x,\frac{y}{t},A\right)dt. \end{aligned}$$

Using (1.5), we get (2.4). (2.5) can be proved similarly. □

2.5. Proposition. *Let A and B be positive stable matrices in $\mathbb{C}^{r\times r}$ and $AB=BA$. Then the second kind Chebyshev-type matrix polynomials with two variables satisfy the three-term recurrence relation*

$$BU_{n+1}(x,y,A,B) = x\sqrt{2A}U_n(x,y,A,B) - yU_{n-1}(x,y,A,B)\,. \tag{2.6}$$

Proof. Equation (2.6) follows from differentiating both side of (2.3) with respect to z, making the necessary arrangements and identification of the coefficients of z^n. □

Now, let us get the matrix differential equation of second kind Chebyshev-type matrix polynomials with two variables. The recurrences given by (2.4) and (2.6) can be expressed as the definition of rising and lowering operators for $U_n(x,y,A,B)$. We can write

$$U_{n-1}(x,y,A,B) = \sqrt{\frac{A}{2}}\frac{1}{y}\widehat{D}_x^{-1}\left[x\frac{\partial}{\partial x}-n\right]U_n(x,y,A,B) \tag{2.7}$$

and

$$U_{n+1}(x,y,A,B) = \left[xB^{-1}\sqrt{2A} - B^{-1}\sqrt{\frac{A}{2}}\widehat{D}_x^{-1}\left[x\frac{\partial}{\partial x}-n\right]\right]U_n(x,y,A,B)\,, \tag{2.8}$$

where $\widehat{D}_x^{-1}$ denotes the inverse derivative operator and is defined by

$$\widehat{D}_x^{-n}[f(x)] = \frac{1}{(n-1)!}\int_0^x (x-\xi)^{n-1}f(\xi)\,d\xi.$$

(see [12] for details). So that for $f(x)=1$, we have

$$\widehat{D}_x^{-n}[1] = \frac{x^n}{n!}.$$

Equations (2.7) and (2.8) allow us to introduce of the rising and lowering operators

$$\begin{aligned} \widehat{M} &= \left[xB^{-1}\sqrt{2A} - B^{-1}\sqrt{\frac{A}{2}}\widehat{D}_x^{-1}\left[x\frac{\partial}{\partial x}-\widehat{n}\right]\right], \\ \widehat{P} &= \left[\sqrt{\frac{A}{2}}\frac{1}{y}\widehat{D}_x^{-1}\left[x\frac{\partial}{\partial x}-\widehat{n}\right]\right], \end{aligned} \tag{2.9}$$

where $\widehat{n}$ is a number operator in the sense $\widehat{n}u_s(x,y,A,B) = su_s(x,y,A,B)$. Using (2.9) $U_n(x,y,A,B)$ can be rewriten as

$$\widehat{M}\widehat{P}U_n(x,y,A,B) = U_n(x,y,A,B)\,,$$

namely,

$$U_n(x,y,A,B) = \sqrt{\frac{A}{2}}\frac{1}{y}\widehat{D}_x^{-1}\left[x\frac{\partial}{\partial x} - (n+1)\right]$$
$$\times\left\{xB^{-1}\sqrt{2A} - B^{-1}\sqrt{\frac{A}{2}}\widehat{D}_x^{-1}\left[x\frac{\partial}{\partial x} - n\right]\right\}U_n(x,y,A,B).$$

After some arrangements and use of the obvious identity

$$\partial x\widehat{D}_x^{-1} = \widehat{1},$$

we arrive at the following theorem.

2.6. Theorem. *Let A and B be positive stable matrices in $\mathbb{C}^{r\times r}$ and $AB = BA$. Then the second kind Chebyshev-type matrix polynomials with two variables are a solution of the second order matrix differential equation of the form:*

$$\left[\left(2yB - x^2A\right)\frac{\partial^2}{\partial x^2} - 3Ax\frac{\partial}{\partial x} + An(n+2)\right]U_n(x,y,A,B) = \mathbf{0}.$$

2.7. Corollary. *Let A be a positive stable matrix in $\mathbb{C}^{r\times r}$. Then the second kind Chebyshev matrix polynomials are a solution of the second order matrix differential equation of the form:*

$$\left[\left(2I - x^2A\right)\frac{d^2}{dx^2} - 3Ax\frac{d}{dx} + An(n+2)\right]U_n(x,A) = \mathbf{0}. \tag{2.10}$$

It is now interesting to extend the above results to generalized forms of Chebyshev-type matrix polynomials with two-variable. We define generalized Chebyshev-type matrix polynomials with two-variable by

$$U_{n,m}(x,y,A,B) = \sum_{k=0}^{\left[\frac{n}{m}\right]}\frac{(-1)^k(n-mk+k)!B^{(m-1)k-n-1}\left(x\sqrt{mA}\right)^{n-mk}y^k}{k!(n-mk)!},$$

which can be written in terms of $H_{n,m}(x,y,A)$ as

$$U_{n,m}(x,y,A,B) = \frac{1}{n!}\int_0^\infty e^{-Bt}t^nH_{n,m}\left(x,\frac{y}{t^{m-1}},A\right)dt, \tag{2.11}$$

where $H_{n,m}(x,y,A)$ is two-index two-variable Hermite matrix polynomials, defined by [23]:

$$H_{n,m}(x,y,A) = \sum_{k=0}^{\left[\frac{n}{m}\right]}\frac{(-1)^k n!\left(x\sqrt{mA}\right)^{n-mk}y^k}{k!(n-mk)!}. \tag{2.12}$$

Since $H_{n,m}(x,y,A)$ has the generating matrix function as

$$\sum_{n=0}^{\infty}\frac{H_{n,m}(x,y,A)}{n!}t^n = \exp\left(xt\sqrt{mA} - yt^mI\right), \tag{2.13}$$

we find from (2.11) that the generating matrix function of $U_{n,m}(x,y,A,B)$ is

$$\sum_{n=0}^{\infty}U_{n,m}(x,y,A,B)z^n = \left(B - xz\sqrt{mA} + yz^mI\right)^{-1}, \tag{2.14}$$

where A, B are positive stable matrices in $\mathbb{C}^{r\times r}$, $AB = BA$ and $\left\|xz\sqrt{mA} - yz^mI\right\| < \|B\|$.

Taking $A = [m]_{1\times 1}$ and $B = [b]_{1\times 1}$in (2.14), the polynomials $U_{n,m}(x, y, m, b)$ reduce to the special case of the generalized Humbert polynomials (see [27]). The properties of this special matrix polynomials can be studied in further research.

3. Different Considerations for Chebyshev-type Matrix Polynomials of Second Kind

We will try to understand more deeply the role played by the integral transform connecting Hermite and the second kind Chebyshev matrix polynomials. It is obvious that both $H_n(x, y, A)$ and $U_n(x, y, A, B)$ reduce to ordinary form for $y = 1$.

It is easy to find that

$$U_n(x, A, B) = \frac{1}{n!} \int_0^\infty e^{-Bt} t^{\frac{n}{2}} H_n\left(x\sqrt{t}, A\right) dt. \tag{3.1}$$

After suitable change of variable, (3.1) yields

$$U_n(x, A, B) = \frac{2}{n! x^{n+2}} \int_0^\infty s^{n+1} \exp\left(-\frac{Bs^2}{x^2}\right) H_n(s, A)\, ds.$$

So, $U_n(x, A, B)$ can be viewed as a kind of Mellin transform of the function

$$f(\xi, A, B) = \exp\left(-\frac{B\xi^2}{x^2}\right) H_n(\xi, A).$$

Let us now consider the problem from an operational point of view. Let $f(x)$ be an appropriate function. Then one can easily get

$$\exp\left(\lambda x \frac{d}{dx}\right) f(x) = f(x \exp \lambda).$$

So, we obtain from (3.1) that

$$U_n(x, A, B) = \frac{1}{n!} \int_0^\infty e^{-Bt} t^{\frac{n}{2}} t^{\frac{1}{2} x \frac{d}{dx}} dt H_n(x, A). \tag{3.2}$$

Using the well-known definition of the Γ- function

$$\Gamma(s) = \int_0^\infty e^{-t} t^{s-1} dt,$$

we can rewrite (3.2) in the form

$$n! U_n(x, A, B) = B^{-\widehat{Q}} \Gamma\left(\widehat{Q}\right) H_n(x, A),$$

where $\widehat{Q} = \left[1 + \frac{1}{2}\left(n + x\frac{d}{dx}\right)\right]$.

We conclude this section giving another representation for the second kind Chebyshev-type matrix polynomials. The use of the identities (1.2) and (1.3) in (2.1) for $B = \alpha I$ allow to conclude that

$$\frac{\partial}{\partial y} U_n(x, y, A, \alpha I) = \frac{\partial}{\partial \alpha} U_{n-2}(x, y, A, \alpha I),$$

$$\frac{\partial}{\partial x} U_n(x, y, A, \alpha I) = -\sqrt{2A} \frac{\partial}{\partial \alpha} U_{n-1}(x, y, A, \alpha I)$$

which can be combined to give

$$2A \frac{\partial^2}{\partial \alpha \partial y} U_n(x, y, A, \alpha I) = \frac{\partial^2}{\partial x^2} U_n(x, y, A, \alpha I).$$

Last identity and the fact that

$$U_n(x, 0, A, \alpha I) = \frac{\left(x\sqrt{2A}\right)^n}{\alpha^{n+1}},$$

allow to define $U_n(x,y,A,\alpha I)$ as

$$U_n(x,y,A,\alpha I)=\exp\left[y(2A)^{-1}\widehat{D}_\alpha^{-1}\frac{\partial^2}{\partial x^2}\right]\frac{\left(x\sqrt{2A}\right)^n}{\alpha^{n+1}},$$

where A is a positive stable matrix in $\mathbb{C}^{r\times r}$ and α is a complex number such that $\operatorname{Re}(\alpha)>0$.

4. First Kind Chebyshev-type Matrix Polynomials with Two-Variable

The two-variable Hermite matrix polynomials will be used here to define Chebyshev-type matrix polynomials of first kind. The Chebyshev polynomials of the first kind are defined by [9]:

$$T_n(x)=\frac{n}{2}\sum_{k=0}^{\left[\frac{n}{2}\right]}\frac{(-1)^k(n-k-1)!(2x)^{n-2k}}{k!(n-2k)!}. \tag{4.1}$$

Let A and B be positive stable matrices in $\mathbb{C}^{r\times r}$ and $AB=BA$. Then the first kind Chebyshev-type matrix polynomials can be defined by

$$T_n(x,A,B)=n\left(\sqrt{2A}\right)^{-1}\sum_{k=0}^{\left[\frac{n}{2}\right]}\frac{(-1)^k B^{k-n}(n-k-1)!\left(x\sqrt{2A}\right)^{n-2k}}{k!(n-2k)!}, \tag{4.2}$$

or by using (1.1)

$$T_n(x,A,B)=\frac{\left(\sqrt{2A}\right)^{-1}}{(n-1)!}\int_0^\infty e^{-Bt}t^{n-1}H_n\left(x,\frac{1}{t},A\right)dt. \tag{4.3}$$

For the case $A=[2]_{1\times1}$ and $B=[1]_{1\times1}$, (4.2) coincides with (4.1).

In a similar way, we define the Chebyshev-type matrix polynomials of the first kind with two variables as

$$T_n(x,y,A,B)=n\left(\sqrt{2A}\right)^{-1}\sum_{k=0}^{\left[\frac{n}{2}\right]}\frac{(-1)^k B^{k-n}(n-k-1)!\left(x\sqrt{2A}\right)^{n-2k}y^k}{k!(n-2k)!},$$

or

$$T_n(x,y,A,B)=\frac{\left(\sqrt{2A}\right)^{-1}}{(n-1)!}\int_0^\infty e^{-Bt}t^{n-1}H_n\left(x,\frac{y}{t},A\right)dt.$$

In this article, new special polynomials are introduced using integral representation. The possibility of combining these two approaches in order to study new families of special matrix polynomials is a problem for further research.

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