FOUR TOPICS AND AN EXAMPLE [†]

Ivan L. Reilly[*]

Dedicated to the memory of Professor Doğan Çoker

Abstract

Four aspects of the theory of topological spaces are presented, and
an important example which illustrates all four of these aspects is de-
scribed.

1. Introduction

Throughout this paper (X, τ) will denote an arbitrary topological space. No (sepa-
ration) properties are assumed unless explicitly stated. This paper surveys some of the
work of the author and his co-authors over more than two decades. It is an expanded
version of his lecture at the Çoker memorial meeting.

2. Spaces in which the only compact subspaces are finite

Such spaces were introduced under the names *pseudo finite* spaces by Wilansky [30]
and *cf* spaces by Levine [14]. In 1976 Hutton and Reilly [11] attempted to obtain a local
characterization of such spaces, with the following definition.

2.1. Definition. A topological space (X, τ) is called *icn* (infinite complement neigh-
bourhood) if for each point p in X and for each infinite subset A of X there is an open
set G containing p and such that $A - G$ is infinite.

---

[*]Department of Mathematics, University of Auckland, Private Bag 92019, Auckland, New
Zealand. E-mail: `i.reily@auckland.ac.nz`
[†]This paper was presented to the 'Çoker memorial meeting', Hacettepe University, Ankara,
April 2003.

They showed that the *cf* property implied *icn*, and that for first countable spaces the converse was true. The general converse was left as an open question. Reilly and Vamanamurthy [24] used $\beta\,\mathbb{N}$ to show that *icn* does not imply *cf* in general. They also proved that the *icn* property is equivalent to *scf*, where X is *scf* if every sequentially compact subspace of X is finite. Furthermore, they showed that *cf* is equivalent to *icn**, where *icn** is defined by changing the condition "$A - G$ is finite" in Definition 1 to "$A - G$ is non-compact". The class of *Lc* spaces - those in which every Lindelof subspace is countable – was considered by Reilly and Vamanamurthy [24].

Bankston [1] developed a general theory which had the *cf* spaces as a particular example. Consider a topological property $\mathcal{P}$. An operation "anti" is defined on classes of spaces as follows. A space X is called *anti-$\mathcal{P}$* if the only subspaces of X having property $\mathcal{P}$ are those whose cardinalities require them to have property $\mathcal{P}$. For example, anti-compact $\equiv$ *cf*, anti-connected $\equiv$ totally disconnected, and anti-Lindelof $\equiv$ *Lc*.

Reilly and Vamanamurthy [25] continued the general study, especially for covering and separation properties. They showed that the anti$(\cdot)$ operation does not discriminate well between classes of spaces defined by different separation properties. In fact, the anti$(\cdot)$ operation distinguishes only the T_0 spaces from spaces with any higher separation property. It maps the class of T_0 spaces onto the class of indiscrete spaces, and the class of T_i spaces ($i \geq 1$) onto the class of spaces with totally ordered topologies. Here T_3 means regular and T_1, T_4 means normal and T_1 and so on.

If $\mathcal{K}$ is a topological class the *spectrum* of $\mathcal{K}$, denoted spec($\mathcal{K}$), is the class of cardinal numbers κ such that any topology on a set of power κ lies in $\mathcal{K}$. Then anti-$\mathcal{K}$ is defined to be the class of spaces X such that whenever $Y \subset X$ then $Y \in \mathcal{K}$ if and only if $|Y| \in$ spec($\mathcal{K}$).

Bankston [1, Proposition 1.2] showed the following

2.2. Proposition. *If $\mathcal{K}$ and $\mathcal{M}$ are classes of spaces, with $\mathcal{K} \subset \mathcal{M}$ and* spec($\mathcal{K}$) = spec($\mathcal{M}$), *then* anti-$\mathcal{K} \supset$ anti-$\mathcal{M}$.

Since any set containing at least two distinct points can have a non-Hausdorff topology defined on it, we have that spec($\{$ Hausdorff spaces $\}$) = $\{0, 1\}$ = 2. Thus X is anti-Hausdorff if and only if no two-point subspace of X is Hausdorff. For example, the set of real numbers $\mathbb{R}$ with the left hand topology $\mathcal{L}$, which has as a base the family of sets $\{(-\infty, a) : a \in \mathbb{R}\}$ is anti-Hausdorff. Observe that $(\mathbb{R}, \mathcal{L})$ is T_0. This is a best possible example in the sense that T_1 anti-Hausdorff spaces do not exist.

Let $\mathcal{K}_i$ be the class of topological spaces having the separation property

$$T_i, \ i = 0, \ 1, \ 2, \ 3, \ 3\tfrac{1}{2}, \ 4, \ 5, \ \alpha, \ \beta, \ m, \ t,$$

where $T_\alpha =$ discrete, $T_\beta =$ indiscrete, $T_m =$ metrizable, and $T_t =$ totally ordered. Then

$$\mathcal{K}_0 \supset \mathcal{K}_1 \supset \mathcal{K}_3 \supset \mathcal{K}_{3\frac{1}{2}} \supset \mathcal{K}_5 \supset \mathcal{K}_m \supset \mathcal{K}_\alpha,$$

while spec($\mathcal{K}$) = $\{0, 1\}$ for all these classes. Hence Proposition 1 implies that the opposite inclusions hold for the anti ($\mathcal{K}$) classes.

2.3. Theorem.
(a) anti-$\mathcal{K}_0 = \mathcal{K}_\beta$.
(b) anti-$\mathcal{K}_i = \mathcal{K}_t$ *for* $i \in \{1, 2, 3, 3\frac{1}{2}, 4, 5, m, \alpha\}$.
(c) anti-$\mathcal{K}_\beta = \mathcal{K}_0$.
(d) anti-$\mathcal{K}_t = \mathcal{K}_1$.

2.4. Corollary.
    (a) anti-(anti-$\mathcal{K}_i$) $= \mathcal{K}_i$ *for* $i \in \{0, \beta, t\}$.
    (b) anti-(anti-$\mathcal{K}_i$) $= \mathcal{K}_1$ *for* $i \in \{1, 2, 3, 3\frac{1}{2}, 4, 5, m, \alpha\}$

If $\mathcal{P}$ is any of the properties, finiteness, compactness, countable compactness, sequential compactness or pseudocompactness then $\mathrm{spec}(\mathcal{K}) = \omega$.

If $\mathcal{P}$ is ω-compactness or Lindelofness then $\mathrm{spec}(\mathcal{K}) = \Omega$.

Reilly and Vamanamurthy [25, Theorem 3] proved the following result.

2.5. Theorem. *The following inclusions are all proper.*
    (a) *anti-pseudocompact* $\subset$ *anti-countably compact* $\subset$ *anti-compact* $\subset$ *anti-finite.*
    (b) *anti-compact* $\subset$ *anti-sequentially compact.*
    (c) *anti-countably compact* $\subset$ *anti-sequentially compact.*
    (d) *anti-Lindelof* $\subset$ *anti-σ-compact.*
    (e) *anti-compact* $\subset$ *anti-σ-compact.*

They also established [25, Theorem 8] that a space is anti-(anti-compact) if and only if it is hereditarily compact, and that a similar result holds when "compact" is replaced by "Lindelof".

The question of repeated iteration of the anti-($\cdot$) operation has been considered by Matier and McMaster [16, 18]. Indeed, McMaster and his students have studied several aspects of this topic [16, 17, 18, 19, 20, 21].

3. Spaces in which every countably infinite subset is discrete

Such spaces were considered by Potoczny [23]. They are called *cid* spaces, and are closely related to the *cf* spaces. Indeed, Potozny [23] showed that any Hausdorff cid space is anti-compact. Reilly and Vamanamurthy [26] showed that the Hausdorff condition is superfluous, that is every cid space is anti-compact. The Arens-Fort space shows that the converse is false.

Reilly and Vamanamurthy [26] showed that every infinite cid space is T_1. Recall that a space is a P space if each G_δ set is open. They proved [26, Theorem 10] that every T_1 P space is cid, so there are plenty of cid spaces.

Another analogue between T_1 spaces and cid spaces relates to their position in the lattice of topologies. Reilly and Vamanamurthy [26, Theorem 11] showed that the cocountable topology on an infinite set X is minimal cid in the lattice of topologies on X.

Ganster, Reilly and Vamanamurthy [6] were able to characterize the cid property as a separation-like property, as follows. A space X is cid if and only if for each countably infinite subset C of X and for each point $p \notin C$ there are open sets U and V such that $p \in U$, $C \subset V$, $p \notin V$, and $C \cap U = \phi$.

Similar results for more general cardinalities have been obtained by Grant and Reilly [10].

4. Bitopological spaces

If X is a set and τ_1 and τ_2 are topologies on X, then the triple (X, τ_1, τ_2) is defined to be a *bitopological space*. It seems that this term was first used by Kelly [12] in his classical paper on the topic.

Bitopological spaces arise naturally whenever one considers a non-symmetrical topological structure. This is because the original structure and its conjugate each generate

(usually different) topologies on the underlying set. Examples are *quasi-metrics, quasi-proximities, quasi-topological groups* and *quasi-uniformities.*

There is a well-developed theory of *separation properties* for bitopological spaces. Some authors distinguish between weak and strong versions of most of these properties. For example, the bitopological Hausdorff property may be defined as follows: (X, τ_1, τ_2) is *strong (weak) pairwise Hausdorff* if for each pair of distinct points x and y in X there are disjoint open sets $U \in \tau_1$ and $V \in \tau_2$ such that $x \in U$, $y \in V$ (U contains one point and V contains the other point). Weston [29] first defined this notion in its strong form and used the term "*consistent.*" Kelly [12] first used the term pairwise Hausdorff.

Fletcher Hoyle and Patty [5] defined (X, τ_1, τ_2) to be *strong pairwise T_0* if for each pair of distinct points x, y in X there is either a τ_1 open set U such that $x \in U$ and $y \notin U$ or a τ_2 open set V such that $y \in V$ and $x \notin V$. The weak version of this property is defined as follows: (X, τ_1, τ_2) is *weak pairwise T_0* if for each pair of distinct points of X there is a set which is either τ_1 open or τ_2 open containing one of the points but not the other. Similarly, (X, τ_1, τ_2) is defined to be *strong (weak) pairwise T_1* if for each pair of distinct points x and y in X there are open sets $U \in \tau_1$ and $V \in \tau_2$ such that $x \in U$, $y \notin U$ and $y \in V, x \notin V$ (either $[x \in U,\ y \notin U,\ y \in V$ and $x \notin V]$ or $[y \in U,\ x \notin U,\ x \in V$ and $y \notin V]$).

In the bitopological space (X, τ_1, τ_2), Kelly [12] defined τ_1 to be *regular* with respect to τ_2 if for each point x in X and each τ_1 closed set P such that $x \notin P$ there is a τ_1 open set U and a τ_2 open set V disjoint from U such that $x \in U$ and $P \subset V$. Then (X, τ_1, τ_2) is *strong (weak) pairwise regular* if τ_1 is regular with respect to τ_2 and (or) τ_2 is regular with respect to τ_1. In (X, τ_1, τ_2), τ_1 is defined to be *completely regular* with respect to τ_2 if for each τ_1 closed set C and each point $x \notin C$ there is a real valued function f on X into $[0, 1]$ such that $f(x) = 0$, $f(C) = 1$, and f is τ_1 upper semi-continuous and τ_2 lower semi-continuous. Furthermore, (X, τ_1, τ_2) is *strong (weak) pairwise completely regular* if τ_1 is completely regular with respect to τ_2 and (or) τ_2 is completely regular with respect to τ_1.

Bitopological normality was defined by Kelly [6] as follows: (X, τ_1, τ_2) is *pairwise normal* if for each τ_1 closed set A and τ_2 closed set B disjoint from A there is a τ_1 open set V containing B and a τ_2 open set U disjoint from V containing A. Consider the bitopological space $(\mathbb{R}, \mathcal{U}, \mathcal{L})$, where $\mathbb{R}$ is the set of real numbers and $\mathcal{U}$ and $\mathcal{L}$ are the upper and lower topologies on $\mathbb{R}$, namely $\mathcal{U} = \{\emptyset,\ \mathbb{R},\ (a, \infty) : a \in \mathbb{R}\}$ and $\mathcal{L} = \{\emptyset,\ \mathbb{R},\ (-\infty, a) : a \in \mathbb{R}\}$. Then $(\mathbb{R}, \mathcal{U}, \mathcal{L})$ is pairwise normal, and satisfies the weak version of each of the other separation properties, but does not satisfy the strong form.

By adding the appropriate form of the pairwise T_1 property to the higher separation properties, one obtains two hierarchies of bitopological separation properties - a weak one and a strong one. Section 2 of Kopperman [13] is a thorough discussion of bitopological separation properties.

Bitopological covering properties have proved to be much more intractable than the separation properties. Fletcher, Hoyle and Patty [5] provided an early definition of bitopological compactness. A cover $\mathcal{U}$ of the bitopological space (X, τ_1, τ_2) is defined to be *pairwise open* if $\mathcal{U} \subset \tau_1 \cup \tau_2$ and $\mathcal{U}$ contains at least one non-empty member of τ_1 and at least one non-empty member of τ_2. If each pairwise open cover of (X, τ_1, τ_2) has a finite subcover then the space (X, τ_1, τ_2) is defined to be *pairwise compact*. Note that $(\mathbb{R}, \mathcal{U}, \mathcal{L})$ is pairwise compact. Cooke and Reilly [4] considered alternative definitions and characterizations of bitopological compactness. Salbany [27] has provided the most comprehensive early discussion of this topic, based on the stronger definition that (X, τ_1, τ_2) is *pairwise compact* if the topological space $(X, \tau_1 \vee \tau_2)$ is compact. Salbany [27] has

introduced the bitopological analogue of the *Stone-Čech compactification*. A more recent development of these ideas is given in Sections 3 and 6 of Kopperman [13].

Bitopological spaces can be given a categorical treatment. The category **Bitop** has objects which are bitopological spaces and morphisms which are pairwise continuous functions. A function $f : (X, \tau_1, \tau_2) \to (Y, \sigma_1, \sigma_2)$ is defined to be *pairwise continuous* if each of the functions between topological spaces $f : (X, \tau_1) \to (Y, \sigma_1)$ and $f : (X, \tau_2) \to (Y, \sigma_2)$ is *continuous*. Brummer [2] and Salbany [27] are two of the major contributors to such a treatment. If the topological space (X, τ) is identified with the bitopological space (X, τ, τ), then it is clear that the category **Top** is a subcategory of **Bitop**.

Recently there has been some work relating bitopological spaces to topics in theoretical computer science. For example, we cite the paper of Ciesielski, Flagg and Kopperman [3].

5. Continuity properties of functions

There are many examples in the literature of properties closely related to the notion of continuity of a function. In many cases the property coincides with continuity if we change the topology on either the domain or the range or both.

5.1. Definition. A property $\mathcal{P}$ of functions between topological spaces is called a *continuity property* if to each pair (X, σ) and (Y, τ) of topological spaces there correspond new topologies σ' on X and τ^* on Y such that $f : (X, \sigma) \to (Y, \tau)$ has property $\mathcal{P}$ if and only if $f : (X, \sigma') \to (Y, \tau^*)$ is continuous. Otherwise, $\mathcal{P}$ is called a *non-continuity property*.

So a non-continuity property is something new, outside the category of topological spaces and continuous functions. However, a continuity property arises because the wrong source and/or target is taken for the morphism in the category **Top**, see Gauld, Mršević, Reilly and Vamanamurthy [9].

Studying such variations of continuity, and especially questions such as composition, restriction, preservation of appropriate classes of subsets, relationships between such properties, and equivalence to continuity under suitable conditions, from this perspective yields much insight and enhances understanding. Proofs are often made elegant, sometimes trivial.

Gauld, Greenwood and Reilly [8] have classified about 100 such properties from this point of view.

To show that a property $\mathcal{P}$ is a continuity property we need to exhibit the topology on the domain and/or range which reduces $\mathcal{P}$ to continuity. The following result of Gauld [7, Proposition 2] is the most effective tool to date for showing that $\mathcal{P}$ is a non-continuity property.

5.2. Proposition. *Let $\mathcal{P}$ be a property of functions between topological spaces, X and Y sets, F a family of functions from X to Y, and $g : X \to Y$ a function. Furthermore, suppose that*

(i) *whatever topologies are imposed on X and Y, if each member of F is continuous then g is also continuous, and*

(ii) *there are topologies on X and Y with respect to which each member of F satisfies $\mathcal{P}$ but g does not satisfy $\mathcal{P}$.*

Then $\mathcal{P}$ is not a continuity property.

One example which illustrates this situation is the case of semi-regularization, see Mršević, Reilly and Vamanamurthy [22]. If A is a subset of (X, τ), then A is called *regular open* if $A = \text{int(cl}A)$.

Let $RO(X, \tau)$ denote the collection of all regular open subsets of (X, τ).

Then $RO(X, \tau)$ forms the base of another topology on X, called the *semi-regularization* of τ, and it is denoted by τ_s.

We note the following properties;

(1) $\tau_s \subset \tau$

(2) If (X, τ) has the property that $\tau_s = \tau$, then (X, τ) is called a semi-regular space.

(3) If A and B are disjoint open sets in (X, τ), then $\tau \operatorname{int}(\tau \operatorname{cl} A)$ and $\tau \operatorname{int}(\tau \operatorname{cl} B)$ are disjoint open sets in (X, τ_s) containing A and B respectively.

(4) Semi-regularization topologies are preserved by products.

(5) Let (Y, σ) be regular. If $f : (X, \tau) \to (Y, \sigma)$ is continuous, then $f : (X, \tau_s) \to (Y, \sigma)$ is continuous.

(6) $(\tau_s)_s = \tau_s$.

5.3. Definition. A function $f : (X, \tau) \to (Y, \sigma)$ is

(i) *almost continuous* (AC),

(ii) *δ-continuous* (δC),

(iii) *super continuous* (SC),

if for each $x \in X$ and each σ open set V containing $f(x)$ there is a τ open set U containing x such that

(i) $f(U) \subset \sigma \operatorname{int}(\sigma \operatorname{cl} V)$,

(ii) $f(\tau \operatorname{int}(\tau \operatorname{cl} U)) \subset \sigma \operatorname{int}(\sigma \operatorname{cl} V)$,

(iii) $f(\tau \operatorname{int}(\tau \operatorname{cl} U)) \subset V$, respectively.

5.4. Proposition. *Let $f : (X, \tau) \to (Y, \sigma)$ be a function. Then:*

(i) *f is AC iff $f : (X, \tau) \to (Y, \sigma_s)$ is continuous*

(ii) *f is δC if $f : (X, \tau_s) \to (Y, \sigma_s)$ is continuous*

(iii) *f is SC iff $f : (X, \tau_s) \to (Y, \sigma)$ is continuous.*

6. An example

6.1. Definition.

(1) A point $z \in \mathbb{R}$ is a *point of density* of a measurable set $M \subset \mathbb{R}$ if

$$\lim_{h \to 0^+} \frac{1}{2h} \lambda\{M \cap (z - h, z + h)\} = 1,$$

where λ is Lebesgue measure.

(2) A measurable set $M \subset \mathbb{R}$ is called *d-open* if each point of M is a point of density of M. The collection of all d-open sets forms a topology on $\mathbb{R}$, called the *density topology* on $\mathbb{R}$.

(3) A function f defined on a neighbourhood of a point $z \in \mathbb{R}$ is *approximately continuous* at z if there is a measurable set $M \subset \mathbb{R}$ such that z is a point of density of M and

$$\lim_{\substack{x \to z \\ x \in M}} f(x) = f(z).$$

Tall [28] and Lukeš, Maly and Zajíček [15] have studied the density topology in some detail.

We wish to highlight the following properties of the density topology on $\mathbb{R}$.

(1) It is finer than the usual topology [15, Remark p. 148].

(2) It is cf (or anti-compact) - the only compact subspaces are the finite ones [15, Theorem 6.9].

(3) It is cid - every countably infinite subset is discrete [15, Theorem 6.9].

(4) It is completely regular and Hausdorff [15, Theorem 6.9].

(5) It is not normal. [15, Theorem 6.9]

(6) It satisfies the Lusin-Menchoff property, which is the pairwise normality property of an associated bitopological space, namely ($\mathbb{R}$, usual topology, density topology) [15, Theorem 6.9].

(7) A function f is approximately continuous at a point z if and only if it is density-continuous at z, or $f : (\mathbb{R}, \text{usual}) \to \mathbb{R}$ is approximately continuous iff $f : (\mathbb{R}, \text{density}) \to \mathbb{R}$ is continuous [15, Theorem 6.6].

The authors of the monograph [15] claim that the use of the Lusin-Menchoff property is "the main topic of this work".

References

[1] Bankston, P. *The total negation of a topological property,* Illionis J. Math. **23**, 241–252, 1979.

[2] Brümmer, G. C. L. *On some bitopologically induced monads in Top*, in: Structure of Topological Categories (Proc. Conf. Bremen, 1978), 13–30.

[3] Ciesielski, K., Flagg, R. C. and Kopperman, R. D. *Polish spaces, computable approximations and bitopological spaces,* Topology Appl. **119**, 241–256, 2002.

[4] Cooke, I. E. and Reilly, I. L. *On bitopological compactness*, J. London Math. Soc. **9** (2), 518–522, 1975.

[5] Fletcher, P., Hoyle, H. B. and Patty, C. W. *The comparison of topologies*, Duke Math. J. **36**, 325–331, 1969

[6] Ganster, M., Reilly, I. L. and Vamanamurthy, M. K. *On spaces whose denumerable subspaces are discrete*, Mat. Vesnik **39**, 283–292, 1987.

[7] Gauld, D. B. *Topologies related to notions of near continuity*, Kyungpook Math. J. **21**, 195–204, 1981.

[8] Gauld, D., Greenwood, S. and Reilly, I. L. *On variations of continuity*, Topology Atlas, Invited Contributions **4**, http://at.yorku.ca/t/a/i/c/32.htm, 1–54, 1999.

[9] Gauld, D., Mršević, M., Reilly, I. L. and Vamanamurthy, M. *Continuity properties of functions*, Colloq. Math. Soc. Janos Bolyai **41**, 311–322, 1983.

[10] Grant, D. L. and Reilly, I. L. *Spaces in which a bound on cardinality implies discreteness*, Mat. Vesnik **42**, 1–7, 1990.

[11] Hutton, B. and Reilly, I. L. *On compactness and finiteness in topological spaces*, Mat. Vesnik **13**, 59–61, 1976.

[12] Kelly, J. C. *Bitopological spaces.*, Proc. London Math. Soc. **13**, 71–89, 1963.

[13] Kopperman, R. D. *Asymmetry and duality in topology*, Topology Appl. **66**, 1–39, 1995.

[14] Levine, N. *On the equivalence of compactness and finiteness in topology*, Amer. Math. Monthly **75**, 178–180, 1968.

[15] Lukes, J., Maly, J. and Zajicek, L. *Fine topology methods in real analysis and potential theory*, (Lecture Notes in Mathematics 1189, Springer-Verlag, Berlin, 1986).

[16] Matier, J. and McMaster, T. B. M. *Iteration of Bankston's anti operation* J. Inst. Math. Comp. Sci. **3**, 31–35, 1990.

[17] Matier, J. and McMaster, T. B. M. *Total negations of hereditary properties,* Rend. Circ. Mat. Palermo **29**, 563–573, 1992.

[18] Matier, J. and McMaster, T. B. M. *Iteration of the anti operation in ordered topological spaces and in other categorical contexts,* Bull. U.M.I. **7B**, 369–378, 1993.

[19] Matier, J. and McMaster, T. B. M. *Total negation in general topology*, Irish Math. Soc. Bull. **25**, 26–37, 1990.

[20] Mathews, P. T. and McMaster, T. B. M. *Minimal spaces, maximal pre-antis*, Rend. Inst. Mat. Univ. Trieste **25**, 353–361, 1993.

[21] McMaster, T. B. M. and Turner, C. R. *Total negation under constraint: pre-anti properties*, Boll. U.M.I (**8**) 3-B, 267–374, 2000.

[22] Mršević, M., Reilly, I. L. and Vamanamurthy, M. *On semi-regularization topologies*, J. Australian Math. Soc. **38**, 40–54, 1985.

[23] Potoczny, H. B. *A class of spaces in which compactness and finiteness are equivalent*, Mat. Vesnik **36**, 159–160, 1984.

[24] Reilly, I. L. and Vamanamurthy, M. K. *Compactness and finiteness*, Colloq.Math.Soc. Janos Bolyai **23**, 1005–1014, 1978.

[25] Reilly, I. L. and Vamanamurthy, M. K. *Some topological anti-properties*, Illinois J. Math. **24**, 384–389, 1980.

[26] Reilly, I. L. and Vamanamurthy, M. K. *On spaces in which every denumerable subspace is discrete*, Mat. Vesnik **38**, 97–102, 1986.

[27] Salbany, S. *A bitopological view of topology and order*, in: Categorical Topology, (Proc. Toledo Conf. Heldermann, Berlin, 481–594, 1984).

[28] Tall, F. D. *The density topology*, Pacific J. Math. **62**, 275–284, 1976.

[29] Weston, J. D. *On the comparison of topologies*, J. London Math. Soc. **32**, 342–354, 1957.

[30] Wilansky, A. *Between T_1 and T_2*, Amer. Math. Monthly **74**, 261–266, 1967.