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ON A STRONGER FORM OF HEREDITARY COMPACTNESS IN PRODUCT SPACES

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Abstract

The aim of this paper is to continue the study of sg-compact spaces. The class of sg-compact spaces is a proper subclass of the class of hereditarily compact spaces. In our paper we shall consider sg-compactness in product spaces. Our main result says that if a product space is sg-compact, then either all factor spaces are finite, or exactly one factor space is infinite and sg-compact and the remaining ones are finite and locally indiscrete.

Keywords: sg-compact, Hereditarily compact, C_2-space, Semi-open, sg-open, sg-closed, hsg-closed.

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1. Introduction

If a topological space (X, τ) is hereditarily compact, then under some additional assumptions either X or τ might become finite (or countable). For example, if (X, τ) is a second countable hereditarily compact space, then τ is finite. Hence, if (X, τ) is a second countable hereditarily compact T_0-space, then X must be countable. Moreover, it is well-known that every maximally hereditarily compact space and every hereditarily compact Hausdorff (even kc-) space is finite. For more information about hereditarily compact spaces we refer the reader to A.H. Stone's paper [15].

In 1995 and in 1996, a stronger form of hereditary compactness was introduced independently in three different papers. Caldas [3], Devi, Balachandran and Maki [6] and Tapi, Thakur and Sonwalkar [17] considered topological spaces in which every cover by sg-open sets has a finite subcover. These spaces have been called *sg-compact* and were further studied by the present authors in [7].

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As the property sg-compactness is much stronger than hereditary compactness (for even spaces with finite topologies need not be sg-compact), the general behavior of sg-compactness becomes more 'unusual' than the one of hereditarily compact spaces. This will be especially the case in product spaces.

It is well-known that the finite product of hereditarily compact spaces is hereditarily compact, and that if a product space is hereditarily compact, then every factor space is hereditarily compact. What we want to show here is the following: If the product space of an arbitrary family of spaces is sg-compact, then all but one factor spaces must be finite and the remaining one must be (at most) sg-compact. Maki, Balachandran and Devi [14, Theorem 3,7] showed (under the additional assumption that the product space satisfies the weak separation axiom T_{gs}) that if the product of two spaces is sg-compact, then every factor space is sg-compact. Tapi, Thakur and Sonwalkar [17, Theorem 2.7] stated the result for two spaces but their proof is wrong as they claimed that the projection mapping is sg-irresolute. They [17] used a wrong lemma from [16] saying that the product of sg-closed sets is sg-closed (we will show that this is not true even for two sets).

We recall some definitions. A set A is called *semi-open* if $A \subseteq \mathrm{cl}(\mathrm{int}(A))$ and *semi-closed* if $\mathrm{int}(\mathrm{cl}(A)) \subseteq A$. The *semi-interior* (resp. *semi-kernel*) of A, denoted by $\mathrm{sint}(A)$ (resp. $\mathrm{sker}(A)$, is the union (resp. intersection) of all semi-open subsets (resp. supersets) of A. The *semi-closure* of A, denoted by $\mathrm{scl}(A)$, is the intersection of all semi-closed supersets of A. It is well known that $\mathrm{sint}(A) = A \cap \mathrm{cl}(\mathrm{int}(A))$ and that $\mathrm{scl}(A) = A \cup \mathrm{int}(\mathrm{cl}(A))$. Observe that $\mathrm{sint}(A)$ is semi-open and that $\mathrm{scl}(A)$ is semi-closed. A subset A of a topological space (X, τ) is called *sg-open* [2] (resp. *g-open* [12]) if every semi-closed (resp. closed) subset of A is included in the semi-interior (resp. interior) of A. A topological space (X, τ) is called *sg-compact* [3, 6, 17] (resp. *go-compact* [1]) if every cover of X by sg-open (resp. g-open) sets has a finite subcover.

Complements of sg-open sets are called *sg-closed*. Alternatively, a subset A of a topological space (X, τ) is called sg-closed if $\mathrm{scl}(A) \subseteq \mathrm{sker}(A)$. If every subset of A is also sg-closed in (X, τ), then A is called *hereditarily sg-closed* (= hsg-closed) [7]. Every nowhere dense subset is hsg-closed but not conversely.

Janković and Reilly [11, Lemma 2] pointed out that in an arbitrary topological space every singleton is either nowhere dense or locally dense. Recall that a set A is said to be *locally dense* [5] (= *preopen*) if $A \subseteq \mathrm{int}(\mathrm{cl}(A))$ and that a topological space X is called *locally indiscrete* if every open subset of X is closed. We will make significant use of their result throughout this paper.

The next two results are already known in the literature. For the convenience of the reader we shall include the proofs.

1.1. Lemma. *For a topological space (X, τ) the following conditions are equivalent:*

(i) *X is locally indiscrete.*
(ii) *Every singleton is locally dense.*
(iii) *Every subset is sg-open.*

Proof. (i) ⇒ (ii): Let $x \in X$. Then $\mathrm{cl}\{x\}$ is closed and thus, by assumption, open. Hence $\{x\} \subseteq \mathrm{int}(\mathrm{cl}(\{x\}))$, i.e. $\{x\}$ is locally dense.

(ii) ⇒ (iii): Let $A \subseteq X$ and F be semi-closed such that $F \subseteq A$. If $x \in F$ then, by assumption, we have that $x \in \mathrm{int}(\mathrm{cl}(F))$ and so $F = \mathrm{int}(\mathrm{cl}(F)) \subseteq \mathrm{int}(A)$. Thus $F \subseteq \mathrm{sint}(A)$.

(iii) ⇒ (i): Let F be closed and suppose that $A = F \cap (X \setminus \mathrm{int}(F)) \neq \emptyset$. Then A is closed and nowhere dense and, by assumption, sg-open. Since $A \subseteq A$ we have $A \subseteq \mathrm{cl}(\mathrm{int}(A)) = \emptyset$, a contradiction. Thus F is open. □

1.2. Lemma.

(i) *Every open continuous surjective function preserves both semi-open sets and preopen sets.*

(ii) *Let* $(X_i)_{i \in I}$ *be a family of spaces and* $\emptyset \neq A_i \subseteq X_i$ *for each* $i \in I$*. Then,* $\prod_{i \in I} A_i$ *is preopen (resp. semi-open) in* $\prod_{i \in I} X_i$ *if and only if* A_i *is preopen (resp. semi-open) in* X_i *for each* $i \in I$ *and* A_i *is non-dense (resp.* $A_i \neq X_i$*) for only finitely many* $i \in I$*.*

(iii) *If* $f : (X, \tau) \to (Y, \sigma)$ *is open and continuous, then the preimage of every nowhere dense subset of* Y *is nowhere dense in* X*, i.e.,* f *is* δ*-open.*

Proof. (i) Suppose that $f : (X, \tau) \to (Y, \sigma)$ is open, continuous and surjective. If $S \subseteq X$ is semi-open, then there is an open set $U \subseteq X$ such that $U \subseteq S \subseteq \mathrm{cl}(U)$. Hence $f(U) \subseteq f(S) \subseteq f(\mathrm{cl}(U)) \subseteq \mathrm{cl}(f(U))$. Since $f(U)$ is open, $f(S)$ is semi-open. If $S \subseteq X$ is preopen then $f(S) \subseteq f(\mathrm{int}(\mathrm{cl}(S))) \subseteq \mathrm{int}(f(\mathrm{cl}(S)) \subseteq \mathrm{int}(\mathrm{cl}(f(S)))$, i.e. $f(S)$ is preopen.

(ii) Let $A = \prod_{i \in I} A_i$. Suppose that A is preopen (resp. semi-open). Since the projections are open, continuous and surjective, it follows from (i) that each A_i is preopen (resp. semi-open). If A is preopen, pick any $x \in A$. Then there is a basic open set $U = \prod_{i \in I} U_i$ such that $x \in U \subseteq \mathrm{cl}(\prod_{i \in I} A_i) = (\prod_{i \in I} \mathrm{cl}(A_i))$. For only finitely many $i \in I$ we have $U_i \neq X_i$ and therefore only finitely many A_i can be non-dense. If A is semi-open, then $\mathrm{int}(A) \neq \emptyset$ since $A \neq \emptyset$. So there is a basic open set $U = \prod_{i \in I} U_i \subseteq \prod_{i \in I} A_i$. Thus $A_i \neq X_i$ for only finitely many $i \in I$. The converse follows easily from the definition of the product topology.

(iii) Let $N \subseteq Y$ be nowhere dense and let $A = f^{-1}(N)$. If $\mathrm{int}(\mathrm{cl}(A)) \neq \emptyset$, then $\emptyset \neq f(\mathrm{int}(\mathrm{cl}(A))) \subseteq \mathrm{int}(f(\mathrm{cl}(A))) \subseteq \mathrm{int}(\mathrm{cl}(f(A))) \subseteq \mathrm{int}(\mathrm{cl}(N))$, a contradiction. □

1.3. Lemma. [7, Theorem 2.6] *For a topological space* (X, τ) *the following conditions are equivalent:*

(1) X *is sg-compact.*

(2) X *is a* C_3*-space, i.e., every hsg-closed set is finite.*

1.4. Lemma. [7, Proposition 2.1] *For a subset* A *of a topological space* (X, τ) *the following conditions are equivalent:*

(1) A *is hsg-closed.*

(2) $N(X) \cap \mathrm{int}(\mathrm{cl}(A)) = \emptyset$*, where* $N(X)$ *denotes the set of nowhere dense singletons in* X*.*

2. Sg-compactness in product spaces

We will start with an example showing that Theorem 2.1 of [17] is not true. There, the authors stated (without proof) that every sg-compact space is go-compact (it is our guess that they assumed that g-open sets are sg-open).

2.1. Example. Let $\mathbb{N}$ be set of all positive integers. We consider the following topology τ on $\mathbb{N}$ given by $\tau = \{\emptyset, \mathbb{N}\} \cup \{U_n = \{n, n+1, n+2, \ldots\} \colon n \geq 3\}$.

We first show that $(\mathbb{N}, \tau)$ is sg-compact. Observe that every singleton of $(\mathbb{N}, \tau)$ is nowhere dense. Since every nonempty semi-open set has finite complement, $(\mathbb{N}, \tau)$ is semi-compact. By [7, Remark 2.7 (i)], $(\mathbb{N}, \tau)$ is sg-compact.

However, every singleton of $(\mathbb{N}, \tau)$ is g-open, and so $(\mathbb{N}, \tau)$ fails to be go-compact.

At this point, we note that from now on, all topological spaces in this paper are assumed to be non-empty.

2.2. Lemma. *Let $X = \prod_{i\in I} X_i$ be a product space. If infinitely many X_i are not indiscrete, then X contains an infinite nowhere dense subset.*

Proof. Let $J = \{i \in I : X_i \text{ is not indiscrete}\}$. Then $|J| \geq \omega$. For each $i \in J$, since X_i is indiscrete, $|X_i| > 1$. Decompose J into a disjoint union of J_1 and J_2 such that $|J_1| = |J_2| = |J|$. For each $i \in J_1$, there is a closed set $A_i \subseteq X_i$ distinct from the empty set and from X_i. Now, let $A = \prod_{i\in J_1} A_i \times \prod_{i\in I\setminus J_1} X_i$. Then A is closed and nowhere dense in X. Since $|X_i| > 1$ for all $i \in J_2$, A is also infinite. □

As a consequence of Lemma 1.3 we therefore have:

2.3. Corollary. *If a product space $X = \prod_{i\in I} X_i$ is sg-compact, then only finitely many X_i are not indiscrete.* □

2.4. Theorem. *Let $(X_i, \tau_i)_{i\in I}$ be a family of topological spaces. If the product space $X = \prod_{i\in I} X_i$ is sg-compact, then either all factor spaces are finite or exactly one of them is infinite and sg-compact and the rest are finite and locally indiscrete.*

Proof. Suppose that two factor spaces, say X_i and X_j, are infinite. Let p_i denote the projection from X onto X_i for any $i \in I$. Let $k \in I$. If $x_k \in X_k$, then $p_k^{-1}(\{x_k\})$ is infinite, hence cannot be nowhere dense since X is sg-compact. Thus $\{x_k\}$ is not nowhere dense in X_k. Consequently, each factor space X_k must be locally indiscrete. By Corollary 2.3 and Lemma 1.2, each singleton in X is locally dense and so every subset of X is sg-open. Since X is sg-compact, X must be finite, a contradiction. Hence, at most one factor space can be infinite.

Now suppose that X_j is infinite and that X_i is finite for $i \neq j$. For each $x_i \in X_i$, where $i \neq j$, $p_i^{-1}(\{x_i\})$ is infinite, therefore $\{x_i\}$ cannot be nowhere dense in X_i. So X_i is locally indiscrete for $i \neq j$. By Corollary 2.3 and Lemma 1.2 it follows that for each $x \in X$, $\{x\}$ is nowhere dense in X if and only if $\{x_j\}$ is nowhere dense in X_j.

Assume now that X_j is not sg-compact. Then X_j contains an infinite hsg-closed subset, say A_j. Let $A = p_j^{-1}(A_j)$. We want to show that $N(X) \cap \text{int}(\text{cl}(A)) = \emptyset$, where $N(X)$ denotes the set of nowhere dense singletons in X. If there exists a point $x \in N(X) \cap \text{int}(\text{cl}(A))$, then x has an open neighbourhood W contained in $\text{cl}(A)$. Also, $\{x_j\}$ is nowhere dense in X_j and $x_j \in p_j(W) \subseteq p_j(\text{cl}(A)) \subseteq \text{cl}(A_j)$. So $x_j \in \text{int}(\text{cl}(A_j))$, a contradiction to the hsg-closedness of A_j. Hence, by Lemma 1.4, A is hsg-closed and infinite, a contradiction. Therefore, X_j is sg-compact. □

Tapi, Thakur and Sonwalkar [17, Theorem 2.7] stated our result for two topological spaces but their proof is wrong as they claimed the projection mapping to be sg-irresolute. They used the erroneous lemma from [16] that the product of sg-closed sets is sg-closed. The following example will correct their claims.

2.5. Example. Let $X = \{a, b, c\}$ and let $\tau = \{\emptyset, \{a, b\}, X\}$. Set $A = \{b, c\}$.

(i) First observe that A is sg-closed in (X, τ) but $A \times A$ is not sg-closed in $X \times X$, since $A \times A \subseteq X \times X \setminus \{(a, c)\}$ and $\text{scl}(A \times A) = X \times X$.

(ii) If p is the projection mapping from $X \times X$ onto X, then $p^{-1}(A)$ is not sg-closed in $X \times X$, i.e., the projection map need not be always sg-irresolute.

(iii) We already noted that if $f : (X, \tau) \to (Y, \sigma)$ is open and continuous, then the preimage of every nowhere dense subset of Y is nowhere dense in X. There is no similar result for hsg-closed sets. If σ denotes the indiscrete topology on X, then $S = \{a, b\}$ is hsg-closed in (X, σ) but $q^{-1}(S)$ is not hsg-closed in $(X, \sigma) \times (X, \tau)$, where q denotes the projection mapping from $(X, \sigma) \times (X, \tau)$ onto (X, σ).

The following result shows when the inverse image of a hsg-closed set is also hsg-closed. Recall that a function $f : (X,\tau) \to (Y,\sigma)$ is called *almost open* if the image of every regular open set is open. We say that $f : (X,\tau) \to (Y,\sigma)$ is *anti-δ-open* if the image of every nowhere dense singleton is nowhere dense. Observe that if Y is dense-in-itself and T_D (= singletons are locally closed, i.e. the intersection of an open and a closed set), then any function $f : (X,\tau) \to (Y,\sigma)$ is always anti-δ-open; in particular every real-valued function is anti-δ-open.

2.6. Proposition. *If $f : (X,\tau) \to (Y,\sigma)$ is an almost open, continuous, anti-δ-open surjection, then the inverse image of every hsg-closed set is hsg-closed.*

Proof. Let B be hsg-closed in Y and set $A = f^{-1}(B)$. If for some nowhere dense singleton $\{x\}$ of X we have $x \in \text{int}(\text{cl}(A))$, then $f(x) \in f(\text{int}(\text{cl}(A))) \subseteq \text{int}(f(\text{cl}(A))) \subseteq \text{int}(\text{cl}(f(A))) = \text{int}(\text{cl}(B))$. Since $\{f(x)\}$ is nowhere dense in Y, B is not hsg-closed. By contradiction, A is hsg-closed. □

2.7. Remark. (i) Let A be an infinite set with $p \notin A$. Let $X = A \cup \{p\}$ and $\tau = \{\emptyset, A, X\}$. We observed in [7] that $X \times X$ contains an infinite nowhere dense subset, so even the finite product of sg-compact spaces need not be sg-compact.

(ii) It is rather unexpected that the projection map fails to be sg-irresolute in general, since it is always irresolute and gs-irresolute.

The two examples of infinite sg-compact spaces in [7] and the infinite sg-compact space from Example 2.1 are not even weakly Hausdorff (however one of them is T_1). As every hereditarily compact kc-space must be finite, it is natural to ask whether there are any infinite sg-compact semi-Hausdorff spaces (there do exist infinite hereditarily compact semi-Hausdorff spaces). Recall here that a topological space (X,τ) is called *semi-Hausdorff* [13] if every two distinct points of X can be separated by disjoint semi-open sets.

Recall additionally that a space (X,τ) is called *hyperconnected* if every open subset of X is dense, or equivalently, every pair of nonempty open sets has nonempty intersection. In the opposite case X is called *hyperdisconnected*. If every infinite open subspace of X is hyperdisconnected, then we will say that X is *quasi-hyperdisconnected*. Note that not only Hausdorff spaces but also semi-Hausdorff spaces are quasi-hyperdisconnected (but not vice versa).

2.8. Proposition. *Every quasi-hyperdisconnected sg-compact space (X,τ) is finite.*

Proof. Assume that X is infinite. Let U and V be disjoint non-empty open subsets of X. Note that either $X \setminus U$ or $X \setminus V$ is infinite. Assume that $X \setminus U$ is infinite. Since $\text{cl}(U) \setminus U$ is hsg-closed (in fact even nowhere dense), by Lemma 1.3, $\text{cl}(U) \setminus U$ is finite and hence $X \setminus \text{cl}(U)$ is infinite and open. Set $A_1 = U$. Since X is quasi-hyperdisconnected, proceeding as above, we can construct an open subset of $X \setminus \text{cl}(U)$ and hence of X, say U_2, such that the complement of the closure of U_2 in $X \setminus \text{cl}(A_1)$ is infinite. Using the method above, we can construct an infinite pairwise disjoint family $A_1, A_2, \ldots$ of non-empty open subsets of (X,τ). Since sg-compact spaces are semi-compact and thus satisfy the finite chain condition, X must be finite. □

2.9. Corollary. *Every sg-compact, semi-Hausdorff space is finite.*

We have just seen that under some very low separation axioms, sg-compact spaces very easily become finite. If we replace the weak separation axiom with a weaker form of strong irresolvability, we again have finiteness. By definition, a nonempty topological space (X,τ) is called *resolvable* [10] if X is the disjoint union of two dense (or equivalently codense) subsets. In the opposite case X is called *irresolvable*. A topological space (X, τ) is *strongly irresolvable* [8] if no nonempty open set is resolvable.

2.10. Proposition. *Every sg-compact space (X, τ) which is the topological sum of a locally indiscrete space and a strongly irresolvable space is finite.*

Proof. We will use a result in [9] which states that a space is finite if and only if every cover by β-open sets (i.e., sets which are dense in some regular closed subspace) has a finite subcover. If $\mathcal{U}$ is a cover of X by β-open sets, then by [4, Theorem 2.1] every element of $\mathcal{U}$ is sg-open. Since X is sg-compact, $\mathcal{U}$ has a finite subcover. This shows that X is finite. □

We already mentioned in Remark 2.7 that the product of two sg-compact spaces need not be sg-compact. Thus we have the natural question: When is the product of two sg-compact spaces also sg-compact? What turns out is that only in one very special case the product of an sg-compact space with another sg-compact space is also sg-compact. First we note a result whose proof is easy and hence omitted.

2.11. Proposition. *Let $(X_\alpha, \tau_\alpha)_{\alpha\in\Omega}$ be a family of pairwise disjoint topological spaces. For the topological sum $X = \sum_{\alpha\in\Omega} X_\alpha$ the following conditions are equivalent:*

(1) *X is an sg-compact space.*

(2) *Each X_α is an sg-compact space and $|\Omega| < \aleph_0$.*

2.12. Lemma. *Let (X, τ) be any space and let (Y, σ) be indiscrete. Let $A \subseteq X \times Y$ and let $p : X \times Y \to X$ denote the projection. Then* $\mathrm{int}(\mathrm{cl}(A)) = \mathrm{int}(\mathrm{cl}(p(A))) \times Y$.

Proof. If $(x, y) \in \mathrm{int}(\mathrm{cl}(A))$, there exists an open neighbourhood U_x of x such that $U_x \times Y \subseteq \mathrm{cl}(A)$. Then $x \in U_x \subseteq \mathrm{cl}(p(A))$ and so $(x, y) \in \mathrm{int}(\mathrm{cl}(p(A))) \times Y$.

Now, let $x \in \mathrm{int}(\mathrm{cl}(p(A)))$ and $y \in Y$. Choose an open set $U_x \subseteq X$ containing x such that $U_x \subseteq \mathrm{cl}(p(A))$. We claim that $U_x \times Y \subseteq \mathrm{cl}(A)$. Suppose there is a point $(x', y') \in U_x \times Y$ not in $\mathrm{cl}(A)$. Then there exists an open set $W_{x'} \subseteq U_x$ containing x' such that $(W_{x'} \times Y) \cap A = \emptyset$. Consequently, $W_{x'} \cap p(A) = \emptyset$, a contradiction. Hence, $(x, y) \in \mathrm{int}(\mathrm{cl}(A))$. □

2.13. Theorem. *If (X, τ) is sg-compact and (Y, σ) is finite and locally indiscrete, then $X \times Y$ is sg-compact.*

Proof. Since Y is a finite topological sum of indiscrete spaces, by Proposition 2.11 it suffices to assume that Y is indiscrete. Suppose that $A \subseteq X \times Y$ is hsg-closed. If A is infinite, then $p(A)$ is infinite. We claim that $p(A)$ is hsg-closed in X. Otherwise, $N(X) \cap \mathrm{int}(\mathrm{cl}(p(A))) \neq \emptyset$. If one picks any $x \in N(X) \cap \mathrm{int}(\mathrm{cl}(p(A)))$ and any $y \in Y$, then $\{(x, y)\}$ is nowhere dense in $X \times Y$. By Lemma 2.12, $(x, y) \in N(X \times Y) \cap \mathrm{int}(\mathrm{cl}(A))$, a contradiction to the hsg-closedness of A. Thus $p(A)$ is hsg-closed in X. Again, this is a contradiction, since X is sg-compact. This implies that A must be finite. Therefore, $X \times Y$ is sg-compact. □

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